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# Decentralized control of vehicle platoons with interconnection possessing ring topology

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***Abstract*— In this paper we design a control strategy for platoons of identical vehicles. It is assumed that each vehicle measures the distance with its immediate forward neighbor. The lead vehicle in the platoon only receives information on the position of the last vehicle in the platoon. We prove that the resulting behavior of the system is a platoon of vehicles moving at a constant velocity with constant distance between each pair of consecutive vehicles and that for a class of identical controllers this solution is asymptotically stable for sufficiently small coupling strength. An upper limit of this coupling strength is calculated, below which the solution is asymptotically stable, independent of the number of vehicles in the platoon. Moreover, simulations indicate that the platoon is string stable. To improve the behavior, integral action is added between the first and last vehicle of the platoon. The resulting behavior is determined and its stability properties are discussed.**

## I. INTRODUCTION

In this paper the systems under study are vehicular platoons. Such systems have gained importance over the years, mainly because they might offer a solution to the congestion of highways in urban areas, by increasing their capacity. The goal of these intelligent vehicle/highway systems (IHVS) is to form strings of vehicles (so-called *platoons*) moving at a desired speed with desired distances between the vehicles. Several algorithms controlling a string of vehicles have been proposed in the literature.

Ref. [1], [2] and [3] were among the first to investigate this problem and used an LQR approach controlling an *infinite* string of vehicles. Recently, this approach has been reformulated in [4], where it was shown that the problem formulation of [1], [2] was ill-posed since stabilizability and detectability decrease as a function of platoon size.

Contrary to [1] and [2], most control strategies use tuning of parameters in order to optimize some proposed controller. In most cases the control is of leader-follower type: the leading vehicle of the platoon drives at a desired speed; the other vehicles receive information from the leading vehicle (position, velocity, acceleration) either directly or indirectly through other vehicles in the platoon. Flow of information is usually directed from the head of the platoon towards its tail [5], [6]. For instance in [7] a control strategy, based on the double-graph model, is developed where each vehicle adjusts its behavior according to the leading vehicle and its neighbors.

In reference [8] one considers a platoon where each vehicle only measures the distance between itself and its immediate forward neighbor and tries to obtain and maintain a desired value for this distance. The leader vehicle drives at a desired speed. It is proved in [8] that the system cannot be string stable when identical controllers are applied. A platoon is called string stable if the transient error in the separation distance between vehicles does not grow as one proceeds down the line of vehicles. In [9] it is proved that for sufficiently weak interactions an ensemble of interconnected exponentially stable systems is string stable.

The present paper presents a novel interconnection topology using identical controllers. As in [8], only separation distances are measured. The resulting behavior and its stability properties are investigated. Stability regions in parameter space are obtained. Simulations suggest that the system is string stable.

## II. PRELIMINARY: BLOCK CIRCULANT MATRICES

In this section some notation and preliminary results on block circulant matrices [10] are introduced which will be used in the description of the dynamics of the platoon and its stability analysis.

Consider the matrix $C \in \mathbb{R}^{Nm \times Nm}$:

$$C = \begin{bmatrix} C_1 & C_2 & \cdots & C_N \\ C_N & C_1 & \cdots & C_{N-1} \\ \vdots & \vdots & \vdots & \vdots \\ C_2 & C_3 & \cdots & C_1 \end{bmatrix} \triangleq \mathrm{circ}(C_1, C_2, \ldots, C_N),$$

where $C_i \in \mathbb{R}^{m \times m}$, $\forall i \in \mathcal{N} \triangleq \{1, \ldots, N\}$.

Define the matrix $F \in \mathbb{C}^{N \times N}$:

$$F \triangleq \frac{1}{\sqrt{N}} \begin{bmatrix} 1 & 1 & \cdots & 1 \\ 1 & \omega & \cdots & \omega^{N-1} \\ \vdots & \vdots & \vdots & \vdots \\ 1 & \omega^{N-1} & \cdots & \omega^{(N-1)(N-1)} \end{bmatrix},$$

with $\omega \triangleq \exp(2\pi j/N)$, where '$j$' represents the imaginary unit. Using $F$, the matrix $C$ can be block diagonalized into a matrix $\Lambda$:

$$\Lambda = \mathrm{diag}(\Lambda_1, \ldots, \Lambda_N) = (F \otimes I_m)^* C (F \otimes I_m), \qquad (1)$$

where '$\otimes$' is the Kronecker product, '$*$' represents complex conjugate, and $I_m$ is the $m \times m$ identity matrix.

The blocks $\Lambda_i \in \mathbb{C}^{m\times m}$ on the diagonal of $\Lambda$ are given by

$$\Lambda_i = C_1+\omega^{i-1}C_2+\omega^{2(i-1)}C_3+\ldots+\omega^{(N-1)(i-1)}C_N, \quad (2)$$

for all $i \in \mathcal{N}$. The set of eigenvalues of $C$ is equal to the set of eigenvalues of the matrices $\Lambda_i, \forall i \in \mathcal{N}$.

## III. System dynamics

### A. Ring topology

The idea of coupling agents into a ring has been exploited before by [11], [12], [13] and [14]. In [11] each agent $i$ is represented by a point $z_i$ in the complex plane and the dynamics under investigation are

$$\dot{z}_i = (z_{i+1} + c_i) - z_i, \quad i \in \mathcal{N}, \quad (3)$$

where $c_i \in \mathbb{C}$, $\forall i \in \mathcal{N}$. Here, as in the remainder of the paper, the indices are evaluated modulo $N$. In the case of (3) this means that $z_{N+1} \equiv z_1$. If $c_i = 0$, $\forall i \in \mathcal{N}$, then the agents will converge to one point in the complex plane. Choosing values $c_i$ appropriately leads to desired formations of the group of agents. If the centroid of the points $c_1, \ldots, c_N$ is not at the origin, then the centroid of the agents moves off to infinity. This situation is not desirable in the setting of [11] and is thus avoided.

In [12] the topology of the interconnection networks is also a unidirectional ring, as in [11] and the present paper, but the dynamics of an individual agent is different. Each agent is represented as a kinematic unicycle with nonlinear dynamics. The control strategy is such that agent $i$ tries to reduce the distance between agent $i+1$ and itself to zero. This is done by a proportional feedback of the difference in heading or orientation of both vehicles:

$$\begin{bmatrix}\dot{x}_i\\ \dot{y}_i\\ \dot{\theta}_i\end{bmatrix} = \begin{bmatrix}\cos\theta_i & 0\\ \sin\theta_i & 0\\ 0 & 1\end{bmatrix}\begin{bmatrix} s\\ k(\theta_{i+1}-\theta_i)\end{bmatrix},$$

where $s$ and $k$ are positive real constants. The resulting equilibrium motion is all agents moving along a circle in one direction. The motion in the physical plane clearly reflects the interconnection structure.

In the present paper the vehicles are coupled in a unidirectional ring on the level of communication, similar to [11] and [12]. Contrary to [12], the ring topology is not expressed on the level of the formation: the equilibrium solution is a *string* of vehicles. The interconnection in the present paper resembles that of [11] where values for constants $c_i$ have to be chosen. In [11] this choice of $c_i$ determines the shape of the resulting formation, while in the present paper the shape of the formation will always be a string of vehicles. The choice of the parameters corresponding to the $c_i$ in [11], determines *the separation distances* inside the string. In [11] it is assumed that the centroid of the points $c_1, \ldots, c_N$ is at the origin, since otherwise the formation moves off to infinity. In the present paper we deliberately choose the centroid to be different from zero, forcing the string of vehicles to move at a certain constant velocity.

### B. System equations and equilibrium solution

Each vehicle is represented as a moving mass with second order dynamics:

$$\ddot{x}_i + p\dot{x}_i = u_i, \quad i \in \mathcal{N}, \quad (4)$$

where $x_i$ represents the position of the $i$-th vehicle, $u_i$ is the input to the $i$-th vehicle and $p \geq 0$ is a parameter representing the friction/drag coefficient per unit mass. The mass of each vehicle is taken equal to one. We propose the following control:

$$u_i = K(x_{i-1} - x_i - L_i), \quad i \in \mathcal{N}, \quad (5)$$

with $K > 0$ the coupling strength, and $L_1 \leq 0$, $L_i \geq 0$, $i = 2, \ldots, N$ real constants. Each vehicle attempts to keep the distance between itself and its immediate forward vehicle as close as possible to the set point $L_i$. The lead vehicle tries to obtain a desired distance $|L_1|$ between itself and the last vehicle of the platoon. This leads to the dynamical system

$$\begin{cases} \dot{x} = Ax + Kb,\\ x = \begin{bmatrix} x_1 & \dot{x}_1 & x_2 & \dot{x}_2 & \cdots & x_N & \dot{x}_N\end{bmatrix}^T,\\ A = \operatorname{circ}\left(\begin{bmatrix}0 & 1\\ -K & -p\end{bmatrix}, \underbrace{O_2, \ldots, O_2}_{N-2 \text{ times}}, \begin{bmatrix}0 & 0\\ K & 0\end{bmatrix}\right),\\ b = \begin{bmatrix}0 & L_1 & 0 & L_2 & \cdots & 0 & L_N\end{bmatrix}^T, \end{cases} \quad (6)$$

with $O_2$ the $2 \times 2$ null matrix.

*Theorem 1:* Each function $\varphi : \mathbb{R} \to \mathbb{R}^N; t \mapsto \varphi(t)$ defined by

$$\varphi_i(t) = \alpha t + \beta_i, \ \forall i \in \mathcal{N}, \quad (7)$$

with

$$\begin{aligned} \alpha &= \frac{-K}{Np}\sum_{j=1}^{N} L_j,\\ \beta_i - \beta_{i-1} &= \left(\frac{1}{N}\sum_{j=1}^{N} L_j\right) - L_i, \ i \in \mathcal{N}, \end{aligned} \quad (8)$$

is a solution of system (4)-(5).

*Proof:* Substitution of (7) into the system equations (6) yields

$$-K\beta_i - \alpha p + K\beta_{i-1} - KL_i = 0, \quad \forall i \in \mathcal{N}. \quad (9)$$

Equation (7) represents a solution of (4)-(5) or, equivalently, (6), if and only if $\alpha, \beta_i$ satisfy (9).

Adding all $N$ equations (9) leads to a value for $\alpha$:

$$\alpha = \frac{-K}{Np}\sum_{j=1}^{N} L_j. \quad (10)$$

Each equation of (9) can be written as

$$\beta_i - \beta_{i-1} = -L_i - \frac{\alpha p}{K}, i \in \mathcal{N}.$$

With the aid of (10) this changes into

$$\beta_i - \beta_{i-1} = \left(\frac{1}{N}\sum_{j=1}^{N} L_j\right) - L_i, i \in \mathcal{N}. \quad (11)$$

■

## C. Remarks

1) The function $t \mapsto \varphi_i(t)$ represents the evolution of the position of the $i$-th vehicle. Each solution $\varphi$ represents a string of vehicles moving at a constant velocity given by (10) with distances between consecutive vehicles defined by (11). It is the initial position of the platoon that distinguishes the solutions $\varphi$ from each other. Another way of looking at this set of solutions is by noticing that the system equations are invariant under the change of coordinates
$$x \to x + \gamma \begin{bmatrix} 1 & 0 & 1 & 0 & \cdots & 1 & 0 \end{bmatrix}^T, \quad \forall \gamma \in \mathbb{R}. \tag{12}$$
Translating the origin in the physical space does not alter the dynamics. This invariance is reflected in the spectrum of the system matrix $A$ in (6). The matrix $A$ possesses at least one zero-eigenvalue, independent of the parameter values.
2) Remark that if and only if the mean value of the set points $L_i$ is zero, the velocity of the platoon is zero, according to (10). This result was to be expected from inspection of system (3) as described in [11]. This property of the system can be exploited to control the platoon: by deliberately choosing the set points $L_i$ such that their mean value is different from zero, the string of vehicles starts to move at a constant velocity which is proportional to this mean value.
3) The solutions of (6) have two undesirable properties. At first, the separation distances between consecutive cars do not converge to the set points $L_i$. However, by (11) it is possible to compute the distances which the platoon converges to. Vice versa, if desired values $\delta_i \triangleq \beta_{i-1} - \beta_i$ for the separation distances are given, equation (11) allows us to calculate the necessary set points $L_i$.
4) A second undesirable property is the following. It is not possible to obtain a platoon driving at a certain velocity with arbitrarily small separation distances. Equations (8) can be combined into
$$\delta_i = \frac{p\alpha}{K} + L_i. \tag{13}$$
If the velocity $\alpha$ is kept constant the size of the separation distances can be reduced by increasing the coupling strength. However, as is proven in the next section, if $K$ increases too much, the solution becomes unstable. Equation (13) seems to suggest that the size of $\delta_i$ can also be reduced by decreasing the values of the set points $L_i$, while keeping the velocity constant. However, decreasing $L_i$ leads to a decreased velocity by (10).

# IV. Stability Analysis

In this section the stability of the equilibrium solution as a function of the coupling strength is investigated. In order to establish the stability properties of (7), the following change of coordinates is performed:
$$x_i = \alpha t + \beta_i + z_i,$$
where $\alpha$ and $\beta_i$ are defined by (8). This results in the system equations
$$\dot{z} = Az, \tag{14}$$
with the system matrix $A$ identical to the system matrix of the original system (4)-(5) or (6). Notice that the system (14) possesses the same translation invariance (12) as the original system.

*Theorem 2:* If and only if
$$K < p^2 \left( \frac{1 - \cos(2\pi/N)}{\sin^2(2\pi/N)} \right),$$
the solution (7)-(8) of system (6) is asymptotically stable.

*Proof:* System (14) has a line of equilibrium points. This can be concluded from the aforementioned translation invariance (12). Each equilibrium point corresponds to one of the solutions $\varphi$. The system matrix $A$ has a structural zero-eigenvalue which can be discarded from the stability analysis. If and only if the remaining $2N - 1$ eigenvalues of $A$ are located in the open left half plane, each initial condition converges to the line of equilibrium points and the solution (7)-(8) is called asymptotically stable.

Since the matrix $A$ is circulant it can be block diagonalized according to (1) and (2). The matrices appearing on the diagonal are
$$A_i = \begin{bmatrix} 0 & 1 \\ K(\omega^{(N-1)(i-1)} - 1) & -p \end{bmatrix}, \quad \forall i \in \mathcal{N}, \tag{15}$$
with $\omega = \exp(2\pi j/N)$. Since $\exp(j\phi) = \exp(j\phi + j2\pi m), \forall \phi \in \mathbb{R}, \forall m \in \mathbb{Z}$, it holds that $\omega^{(N-1)(i-1)} = \omega^{1-i}$. The eigenvalues of $A_i$ are the roots of the characteristic polynomial
$$\lambda^2 + p\lambda - K(\omega^{(1-i)} - 1),$$
which are
$$\lambda_{i(1,2)} = -\frac{p}{2} \pm \frac{1}{2} \left( p^2 + 4K(\omega^{(1-i)} - 1) \right)^{\frac{1}{2}}.$$
Now, the values of $K$ for which the eigenvalues lie in the open left half plane are determined. One immediately notices that $A_1$ yields the structural zero-eigenvalue and a strictly negative eigenvalue $-p$. Denote the eigenvalues of $A_i$ as
$$\lambda_{i(1,2)} = -\frac{p}{2} \pm \frac{1}{2}(a + jb)^{\frac{1}{2}},$$
with
$$a = p^2 + 4K\cos(2\pi(1-i)/N) - 4K,$$
$$b = 4K\sin(2\pi(1-i)/N).$$
Now set
$$-\frac{p}{2} \pm \frac{1}{2}\Re e(a + jb)^{\frac{1}{2}} < 0. \tag{16}$$
With
$$(a + jb)^{\frac{1}{2}} = \sqrt{\frac{|a + jb| + a}{2}} + j\mathrm{sgn}(b)\sqrt{\frac{|a + jb| - a}{2}},$$
(16) can be written as
$$-p \pm \sqrt{\frac{|a + jb| + a}{2}} < 0.$$

A simple calculation shows that this is equivalent to

$$K < K_{C,i} \triangleq p^2 \left( \frac{1 - \cos(2\pi(1-i)/N)}{\sin^2(2\pi(1-i)/N)} \right). \qquad (17)$$

If and only if $K < \min_{i \in \mathcal{N} \setminus \{1\}} \{K_{C,i}\}$, the system is asymptotically stable.

The function

$$f : \mathbb{R} \to \mathbb{R}; x \mapsto p^2 \left( \frac{1 - \cos x}{\sin^2 x} \right),$$

is even, convex and has a minimum at the origin. Hence

$$\min_{i \in \mathcal{N} \setminus \{1\}} \{K_{C,i}\} = p^2 \left( \frac{1 - \cos(2\pi/N)}{\sin^2(2\pi/N)} \right), \qquad (18)$$

concluding the proof. ■

If the number of vehicles tends to infinity, the upper bound on $K$ for stability determined by (18) decreases and converges to the value $p^2/2$. This yields a sufficient condition for asymptotic stability.

*Theorem 3:* If $0 < K < p^2/2$, system (6) is asymptotically stable, irrespective of the number of vehicles in the system.

The theorems shows that if no friction is present in the system (i.e. p=0) the proposed control is not stable. A possible way to overcome this problem is by having the control induce friction in each vehicle.

*Example:*

Consider system (6) with 3 vehicles and drag coefficient $p = 2$. The eigenvalues of the corresponding system matrix $A$ are plotted in Figure 1 as a function of $K$. When the vehicles are uncoupled, three eigenvalues are situated in $-p$; the remaining three eigenvalues are located at the origin. When $K$ increases two of the latter eigenvalues move into the open left half plane while two of the eigenvalues located in $-p$ start to move towards the imaginary axis. The sum of all eigenvalues is $-3p$, irrespective of the value $K$. As stated before, for all values $K > 0$ there is one eigenvalue at the origin and one in $-p$, corresponding to the matrix $A_1$ of (15). When the coupling strength exceeds the value $p^2/4$ the two rightmost eigenvalues different from zero start to move towards the imaginary axis until at $K = 2p^2$ they cross the imaginary axis simultaneously, rendering the system unstable.

## V. Simulation results

### A. String stability

In [8] a leader-follower control is applied with identical vehicles and identical controllers. It was shown that the platoon loses it string stability when the number of vehicles supersedes 20.

The simulations in the present paper lead to conclude that the equilibrium solutions of (6) are string stable. The simulations are performed for a platoon of 39 vehicles. In Figure 2, system (6) is simulated with $L_1 = -50$, $L_i = 1$, $i = 2, \ldots, N$, $p = 10$, $K = 10$. The figure presents the evolution of the distance errors $\xi_i(t) \triangleq x_i(t) - x_{i-1}(t) - L_i$ over time. For reasons of clarity, half of the distance errors, namely those with even index, are omitted from the picture. The figure shows that the maximum distance error between pairs of consecutive vehicles does not grow when proceeding towards the tail of the platoon, indicating string stability of the platoon. Figure 2 clearly shows a typical feature of the interconnection topology: each distance error rises quickly to its maximum value and then decreases to a value close to zero, but, contrary to leader follower control, after some time each distance error starts to rise *again*. It decreases again to some value near zero. This rising and decreasing is repeated periodically over time. As time evolves, the time it takes for an error to rise and fall down again increases, while the peak value decreases. One could interpret this as if there was a Mexican wave in the error value moving around in the platoon: when the wave reaches the tail of the platoon, it reappears at the leader vehicle. Notice that the Mexican wave continually decreases in amplitude while moving around in the platoon. As was to be expected from the analysis, each distance error converges to a constant different from zero.

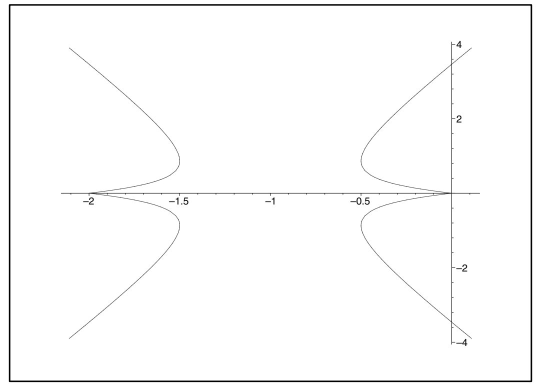


Fig. 1. The spectrum of the system consisting of three vehicles

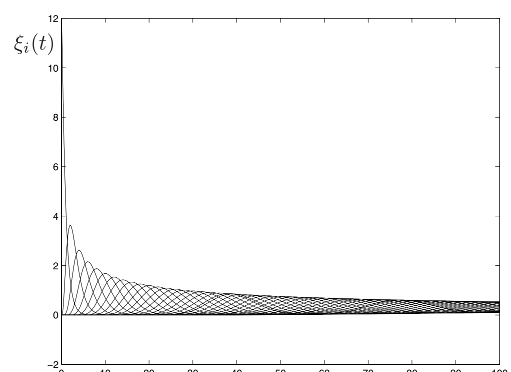


Fig. 2. Evolution of the separation distance errors for a platoon of 39 vehicles.

### B. Robustness

Assume that one of the vehicles starts to malfunction and cannot reach the velocity required by the platoon at that moment. In the case of leader-follower control this causes the leading group of vehicles to abandon the second group with the malfunctioning vehicle as leading vehicle and therefore a breaking of the platoon. The distance between both groups increases without bound.

With the interconnection topology of the present paper all vehicles adapt to the "weakest link" and the platoon starts to drive at the maximum velocity feasible by the malfunctioning vehicle. This is illustrated on the right handside plot of Figure 3: at $t = 80$ s the speed of one of the vehicles becomes bounded by $0.3$ m/s. The distance between the first and the second group remains bounded. There is a breaking of the platoon but no abandoning. The left handside plot shows the evolution of the platoon without malfunctions. For reasons of clarity, only the positions of the vehicles with an odd index are plotted.

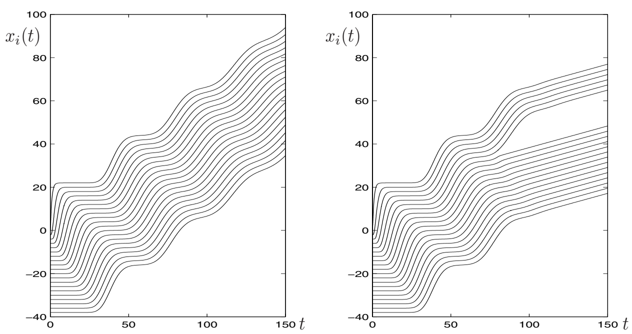


Fig. 3. Evolution of the position for a platoon of 39 vehicles. Left handside figure: no malfunctions. Right handside figure: at $t = 80$ s, the 12th vehicle starts malfunctioning and cannot drive faster than $0.3$ m/s

## VI. Adding integral control

As stated in the third remark of Section III-C the separation distances do not converge to the set points $L_i$. Although the relation between the separation distances $\delta_i$ of the equilibrium solution and the set points is known, we would like that $\delta_i = L_i$, or at least that the relation between them is as simple as possible. Since one of the key properties of the control is that the mean value of the set points is different from zero, it is impossible that $\delta_i = L_i,\ \forall i \in \mathcal{N}$: assume that $\delta_i = L_i,\ \forall i \in \mathcal{N}$. The resulting length of the platoon is then given by $-L_1$ on the one hand and by $\sum_{j=2}^{N} L_j$ on the other hand. Since $\frac{1}{N}\sum_{j=1}^{N} L_j \neq 0$ these two values are not equal, contradicting the assumption.

In this section, however, system (6) is modified by adding integral action between the leading vehicle and the last vehicle of the platoon, which drives the length of the platoon to the set point value $|L_1|$, as shown in the following analysis. Furthermore it is possible to obtain a simple relation between $\delta_i$ and the set points $L_i$.

### A. Systems dynamics and equilibrium solution

Again each vehicle is modeled by (4). The control is given by

$$\begin{aligned} u_1(t) &= K(x_N(t) - x_1(t) - L_1) + \\ & q\int_0^t (x_N(\tau) - x_1(\tau) - L_1)\,\mathrm{d}\tau, \quad \forall t \in \mathbb{R}^+, \\ u_i &= K(x_{i-1} - x_i - L_i), \quad i = 2, \dots, N, \end{aligned} \tag{19}$$

The system equations can be written as

$$\begin{cases} \dot{x}_0 = x_N - x_1 - L_1, \\ \ddot{x}_1 = -p\dot{x}_1 + K(x_N - x_1 - L_1) + q x_0, \\ \ddot{x}_i = -p\dot{x}_i + K(x_{i-1} - x_i - L_i),\ i = 2, \dots, N. \end{cases} \tag{20}$$

*Theorem 4:* Each function $\varphi : \mathbb{R} \to \mathbb{R}^{N+1}; t \mapsto \varphi(t)$ defined by

$$\begin{aligned} \varphi_0(t) &= \gamma, \\ \varphi_i(t) &= \alpha t + \beta_i,\ \forall i \in \mathcal{N}, \end{aligned} \tag{21}$$

with

$$\begin{cases} \alpha = \dfrac{-K}{(N-1)p}\displaystyle\sum_{j=1}^{N} L_j, \\ \beta_N - \beta_1 = L_1, \\ \beta_i - \beta_{i-1} = \left(\dfrac{1}{N-1}\displaystyle\sum_{j=1}^{N} L_j\right) - L_i,\ i = 2, \dots, N, \\ \gamma = -\dfrac{\alpha p}{q}, \end{cases} \tag{22}$$

is an equilibrium solution of system (20).
The proof is similar to the proof of Theorem 1.

Consider the following choice of set points:

$$\begin{aligned} L_2 = L_3 = \ldots = L_N = \mu, \\ L_1 < -(N-1)\mu, \end{aligned} \tag{23}$$

with $\mu \in \mathbb{R}_+$. The equilibrium solutions then represent a platoon of vehicles with length $L_1$ and separation distances given by

$$\begin{aligned} \delta_i &= \left(\frac{1}{N-1}\sum_{j=1}^{N} L_j\right) - L_i \\ &= \frac{L_1 + (N-1)\mu}{N-1} - \mu = \frac{L_1}{N-1}. \end{aligned} \tag{24}$$

Notice that the distances $\delta_i$ do not depend on $\mu$, but only on the set point $L_1$! This is a significant improvement with respect to the behavior of system (6) as far as simplicity of control is concerned. However, the value of $\mu$ influences the velocity of the platoon:

$$\alpha = \frac{K|L_1|}{(N-1)p} - \frac{K\mu}{p}.$$

For a platoon with fixed length $|L_1|$, the velocity can be increased by decreasing $\mu$. However the velocity cannot increase indefinitely, since $\mu \geq 0$:

$$\alpha_{\max} = \frac{K|L_1|}{(N-1)p}.$$

The velocity can also be increased by increasing $K$. Similar to the system of Section III, if $K$ gets too large, stability is lost. This is shown in the next section.

### *B. Stability properties*

The stability of (20) can be determined by investigating the eigenvalues of the system matrix. By augmenting the state space with the variable $x_0$ the system matrix loses its circulant structure. Furthermore, a parameter $q$ was added to the set of parameters. This makes it harder to establish the stability properties via mathematical analysis.

In order to obtain some qualitative results, the eigenvalues are computed by means of numerical software for some representative cases.

In Figure 4 the stability regions are presented for a 3-vehicle platoon for three different values of the drag coefficient $p$. Each time the stable region is located under the curve. When $K$ or $q$ becomes too large stability is lost. Figure 5 shows the stability region for a 7-vehicle platoon with drag coefficient $p = 2$.

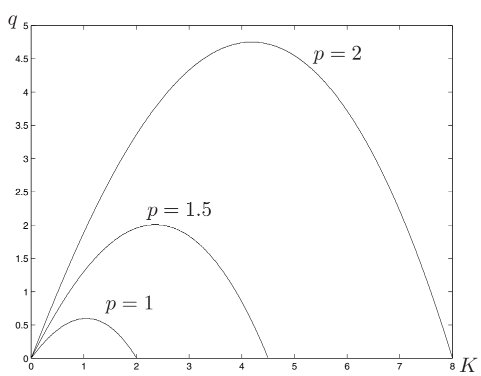


Fig. 4. Stability regions for a 3-vehicle platoon at different values of the drag coefficient.

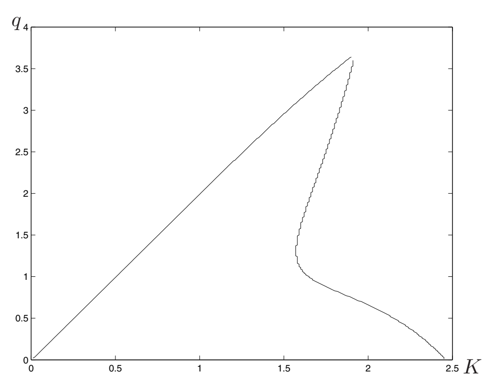


Fig. 5. Stability region for a 7-vehicle platoon with drag coefficient $p = 2$.

## VII. CONCLUSIONS

In this paper a novel control strategy for strings of vehicles is proposed. This strategy has a self-organizing property; there is no leader/master vehicle present. The coupling structure is unidirectional ring coupling. Each vehicle measures the distance with its immediate forward neighbor and the lead vehicle in the platoon receives information on the position of the last vehicle in the platoon. We proved that the resulting behavior of the system is a platoon of vehicles moving at a constant velocity with a constant distance separating consecutive vehicles. Furthermore it is proven that for a class of identical controllers the system is asymptotically stable for sufficiently small coupling strength. An upper bound of this coupling strength is calculated, below which the system is asymptotically stable, independent of the number of vehicles in the platoon. The concept of string stability of a platoon is discussed and applied to the proposed interconnection. We present some simulations supporting the claim that the system is well-behaved with respect to string stability.

To improve the behavior, adding integral action between the first and last vehicle of the platoon has been considered. The resulting behavior was determined and its stability properties were computed and discussed.


## ACKNOWLEDGMENT

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