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Stable Neural Hybrid Adaptive Control for Nonlinear Uncertain Impulsive Dynamical Systems

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Abstract

A neural network hybrid adaptive control framework for nonlinear uncertain hybrid dynamical systems is developed. The proposed hybrid adaptive control framework is Lyapunov-based and guarantees partial asymptotic stability of the closed-loop hybrid system; that is, asymptotic stability with respect to part of the closed-loop system states associated with the hybrid plant states. Finally, a numerical example is provided to demonstrate the efficacy of the proposed hybrid adaptive stabilization approach.

1. Introduction

Modern complex engineering systems involve multiple modes of operation placing stringent demands on controller design and implementation of increasing complexity. Such systems typically possess a multiechelon hierarchical *hybrid* control architecture characterized by continuous-time dynamics at the lower levels of the hierarchy and discrete-time dynamics at the higher levels of the hierarchy (see [1, 2] and the numerous references therein). The lower-level units directly interact with the dynamical system to be controlled while the higher-level units receive information from the lower-level units as inputs and provide (possibly discrete) output commands which serve to coordinate and reconcile the (sometimes competing) actions of the lower-level units. The hierarchical controller organization reduces processor cost and controller complexity by breaking up the processing task into relatively small pieces and decomposing the fast and slow control functions. Typically, the higher-level units perform logical checks that determine system mode operation, while the lower-level units execute continuous-variable commands for a given system mode of operation. The mathematical description of many of these systems can be characterized by impulsive differential equations [3–6].

In a recent paper [7], a hybrid adaptive control framework for adaptive stabilization of multivariable nonlinear uncertain impulsive dynamical systems was developed. In particular, a Lyapunov-based hybrid adaptive control framework was developed that guarantees partial asymptotic stability of the closed-loop system; that is, asymptotic stability with respect to part of the closed-loop system states associated with the hybrid plant dynamics. Furthermore, the remainder of the state associated with the adaptive controller gains was shown to be Lyapunov stable. As is the case in the adaptive control literature [8–12], the system errors in [7] are captured by a constant linearly parameterized uncertainty model of a known structure but unknown variation. This uncertainty characterization allows the system nonlinearities to be parameterized by a *finite* linear combination of basis functions within a class of function approximators such as rational functions, spline functions, radial basis functions, sigmoidal functions, and wavelets. However, this linear parametrization of basis functions in general, cannot exactly capture the unknown system nonlinearity.

Even though neural network-based adaptive control algorithms have been extensively developed in the literature, it is quite common using Lyapunov-like functions to claim that the neural network controllers can guarantee *ultimate boundedness* of the closed-loop system states. This implies that the plant states converge to a *neighborhood* of the origin (see, for example, [13–15] for continuous-time cases and [16–18] for discrete-time cases). The reason why stability in the standard sense is not guaranteed stems from the fact that uncertainties in the system dynamics cannot be perfectly captured by neural networks and the residual approximation error is characterized via *infinity norm* over a given compact set. As one can surmise, however, the ultimate boundedness claims are somewhat conservative since standard Lyapunov-like theorems that are typically used to show ultimate boundedness of the closed-loop hybrid system states provide only *sufficient conditions*, while neural network controllers may possibly achieve plant state convergence to an equilibrium point.

In this paper we develop a neural hybrid adaptive control framework for a class of nonlinear uncertain impulsive dynamical systems which ensures state convergence as well as boundedness of the neural network weighting gains. Specifically, the proposed framework is Lyapunov-based and guarantees partial asymptotic stability of the closed-loop hybrid system; that is, Lyapunov stability of the overall closed-loop states and convergence with respect to the plant state. The neuro adaptive controllers are constructed *without* requiring explicit knowledge of the hybrid system dynamics other than the fact that the plant dynamics are continuously differentiable and that the approximation error of unknown nonlinearities lies in a small gain-type *norm bounded* conic sector over a compact set. Hence, the overall neuro adaptive control framework captures the residual approximation error inherent in linear parameterizations of system uncertainty via basis functions.

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Furthermore, the proposed neuro control architecture is modular in the sense that if a nominal linear design model is available, the neuro adaptive controller can be augmented to the nominal design to account for system nonlinearities and system uncertainty.

Finally, we emphasize that we do not impose any linear growth condition on the system resetting (discrete) dynamics. Note that in the literature on classical (non-neural) adaptive control theory for discrete-time systems, it is typically assumed that the nonlinear system dynamics have the linear growth rate which is necessary in proving Lyapunov stability rather than practical stability (ultimate boundedness). Our novel characterization of system uncertainties (the small gain-type bound on the norm of the modelling error) allows us to prove asymptotic stability without requiring a linear growth condition for the system dynamics.

2. Mathematical Preliminaries

In this section we introduce notation, definitions, and some key results concerning impulsive dynamical systems [3–6, 19]. Let $\mathbb{R}$ denote the set of real numbers, $\mathbb{R}^n$ denote the set of $n \times 1$ real column vectors, $(\cdot)^{\mathrm{T}}$ denote transpose, $(\cdot)^{\dagger}$ denote the Moore-Penrose generalized inverse, $\mathcal{N}$ denote the set of nonnegative integers, $\mathbb{N}^n$ (resp., $\mathbb{P}^n$) denote the set of $n \times n$ nonnegative (resp., positive) definite matrices, and I_n denote the $n \times n$ identity matrix. Furthermore, we write $\mathrm{tr}(\cdot)$ for the trace operator, $\ln(\cdot)$ for the natural log operator, $\lambda_{\min}(M)$ (resp., $\lambda_{\max}(M)$) for the minimum (resp., maximum) eigenvalue of the Hermitian matrix M, $\sigma_{\max}(M)$ for the maximum singular value of the matrix M, $V'(x)$ for the Fréchet derivative of V at x, and $\mathrm{dist}(p, \mathcal{M})$ for the smallest distance from a point p to any point in the set $\mathcal{M}$.

In this paper, we consider controlled *state-dependent* [6] impulsive dynamical systems $\mathcal{G}$ of the form

$$\dot{x}(t) = f_{\mathrm{c}}(x(t)) + G_{\mathrm{c}}(x(t))u_{\mathrm{c}}(t), \quad x(0) = x_0, \quad x(t) \notin \mathcal{Z}_x, \quad (1)$$

$$\Delta x(t) = f_{\mathrm{d}}(x(t)) + G_{\mathrm{d}}(x(t))u_{\mathrm{d}}(t), \quad x(t) \in \mathcal{Z}_x, \quad (2)$$

where $t \geq 0$, $x(t) \in \mathcal{D} \subseteq \mathbb{R}^n$, $\mathcal{D}$ is an open set with $0 \in \mathcal{D}$, $\Delta x(t) \triangleq x(t^+) - x(t)$, $u_{\mathrm{c}}(t) \in \mathcal{U}_{\mathrm{c}} \subseteq \mathbb{R}^{m_{\mathrm{c}}}$, $u_{\mathrm{d}}(t_k) \in \mathcal{U}_{\mathrm{d}} \subseteq \mathbb{R}^{m_{\mathrm{d}}}$, t_k denotes the kth instant of time at which $x(t)$ intersects $\mathcal{Z}_x$ for a particular trajectory $x(t)$, $f_{\mathrm{c}} : \mathcal{D} \to \mathbb{R}^n$ is Lipschitz continuous and satisfies $f_{\mathrm{c}}(0) = 0$, $G_{\mathrm{c}} : \mathcal{D} \to \mathbb{R}^{n \times m_{\mathrm{c}}}$, $f_{\mathrm{d}} : \mathcal{Z}_x \to \mathbb{R}^n$ is continuous, $G_{\mathrm{d}} : \mathcal{Z}_x \to \mathbb{R}^{n \times m_{\mathrm{d}}}$ is such that $\mathrm{rank}\, G_{\mathrm{d}}(x) = m_{\mathrm{d}}$, $x \in \mathcal{Z}_x$, and $\mathcal{Z}_x \subset \mathcal{D}$ is the *resetting set*. Here, we assume that $u_{\mathrm{c}}(\cdot)$ and $u_{\mathrm{d}}(\cdot)$ are restricted to the class of *admissible* inputs consisting of measurable functions such that $(u_{\mathrm{c}}(t), u_{\mathrm{d}}(t_k)) \in \mathcal{U}_{\mathrm{c}} \times \mathcal{U}_{\mathrm{d}}$ for all $t \geq 0$ and $k \in \mathcal{N}_{[0,t)} \triangleq \{k : 0 \leq t_k < t\}$, where the constrained set $\mathcal{U}_{\mathrm{c}} \times \mathcal{U}_{\mathrm{d}}$ is given with $(0, 0) \in \mathcal{U}_{\mathrm{c}} \times \mathcal{U}_{\mathrm{d}}$. We refer to the differential equation (1) as the *continuous-time dynamics*, and we refer to the difference equation (2) as the *resetting law*. In this paper we assume that Assumptions A1 and A2 established in [6] hold for all $u_{\mathrm{d}}(\cdot) \in \mathcal{U}_{\mathrm{d}}$; that is, the resetting set is such that resetting removes $x(t_k)$ from the resetting set and no trajectory can intersect the interior of $\mathcal{Z}_x$. Hence, as shown in [6], the resetting times are well defined and distinct. Since the resetting times are well defined and distinct and since the solution to (1) exists and is unique it follows that the solution of the impulsive dynamical system (1), (2) also exists and is unique over a forward time interval.

Next, we provide a key result from [6, 19] involving an invariant set stability theorem for hybrid dynamical systems. Specifically, consider the impulsive dynamical system (1), (2) with hybrid adaptive feedback controllers $u_{\mathrm{c}}(\cdot)$ and $u_{\mathrm{d}}(\cdot)$ so that the closed-loop hybrid system $\tilde{\mathcal{G}}$ has the form

$$\dot{\tilde{x}}(t) = \tilde{f}_{\mathrm{c}}(\tilde{x}(t)), \quad \tilde{x}(0) = \tilde{x}_0, \quad \tilde{x}(t) \notin \mathcal{Z}_{\tilde{x}}, \quad (3)$$

$$\Delta\tilde{x}(t) = \tilde{f}_{\mathrm{d}}(\tilde{x}(t)), \quad \tilde{x}(t) \in \mathcal{Z}_{\tilde{x}}, \quad (4)$$

where $t \geq 0$, $\tilde{x}(t) \in \tilde{\mathcal{D}} \subseteq \mathbb{R}^{\tilde{n}}$, $\tilde{x}(t)$ denotes the closed-loop state involving the system state and the adaptive gains, $\tilde{f}_{\mathrm{c}} : \tilde{\mathcal{D}} \to \mathbb{R}^{\tilde{n}}$ and $\tilde{f}_{\mathrm{d}} : \tilde{\mathcal{D}} \to \mathbb{R}^{\tilde{n}}$ denote the closed-loop continuous-time and resetting dynamics, respectively, with $\tilde{f}_{\mathrm{c}}(\tilde{x}_{\mathrm{e}}) = 0$, where $\tilde{x}_{\mathrm{e}} \in \tilde{\mathcal{D}} \backslash \mathcal{Z}_{\tilde{x}}$ denotes the closed-loop equilibrium point, and $\tilde{n}$ denotes the dimension of the closed-loop system state. For the statement of the next result the following key assumption is needed.

Assumption 2.1 [6,19]. Let $s(t, \tilde{x}_0)$, $t \geq 0$, denote the solution of (3), (4) with initial condition $\tilde{x}_0 \in \tilde{\mathcal{D}}$. Then for every $\tilde{x}_0 \in \tilde{\mathcal{D}}$, there exists a dense subset $\mathcal{T}_{\tilde{x}_0} \subseteq [0, \infty)$ such that $[0, \infty) \backslash \mathcal{T}_{\tilde{x}_0}$ is (finitely or infinitely) countable and for every $\epsilon > 0$ and $t \in \mathcal{T}_{\tilde{x}_0}$, there exists $\delta(\epsilon, \tilde{x}_0, t) > 0$ such that if $\|\tilde{x}_0 - y\| < \delta(\epsilon, \tilde{x}_0, t)$, $y \in \tilde{\mathcal{D}}$, then $\|s(t, \tilde{x}_0) - s(t, y)\| < \epsilon$.

Assumption 2.1 is a generalization of the standard continuous dependence property for dynamical systems with continuous flows to dynamical systems with left-continuous flows. Specifically, by letting $\mathcal{T}_{\tilde{x}_0} = \overline{\mathcal{T}}_{\tilde{x}_0} = [0, \infty)$, where $\overline{\mathcal{T}}_{\tilde{x}_0}$ denotes the closure of the set $\mathcal{T}_{\tilde{x}_0}$, Assumption 2.1 specializes to the classical continuous dependence of solutions of a given dynamical system with respect to the system's initial conditions $\tilde{x}_0 \in \tilde{\mathcal{D}}$ [20]. Since solutions of impulsive dynamical systems are *not* continuous in time and solutions are *not* continuous functions of the system initial conditions, Assumption 2.1 is needed to apply the hybrid invariance principle developed in [6, 19] to hybrid adaptive systems. Henceforth, we assume that the hybrid adaptive feedback controllers $u_{\mathrm{c}}(\cdot)$ and $u_{\mathrm{d}}(\cdot)$ are such that closed-loop hybrid system (3), (4) satisfies Assumption 2.1. Necessary and sufficient conditions that guarantee that the nonlinear impulsive dynamical system $\tilde{\mathcal{G}}$ satisfies Assumption 2.1 are given in [19]. A sufficient condition that guarantees that the trajectories of the closed-loop nonlinear impulsive dynamical system (3), (4) satisfy Assumption 2.1 are Lipschitz continuity of $\tilde{f}_{\mathrm{c}}(\cdot)$ and the existence of a continuously differentiable function $\mathcal{X} : \tilde{\mathcal{D}} \to \mathbb{R}$ such that the resetting set is given by $\mathcal{Z}_{\tilde{x}} = \{\tilde{x} \in \tilde{\mathcal{D}} : \mathcal{X}(\tilde{x}) = 0\}$, where $\mathcal{X}'(\tilde{x}) \neq 0$, $\tilde{x} \in \mathcal{Z}_{\tilde{x}}$, and $\mathcal{X}'(\tilde{x})\tilde{f}_c(\tilde{x}) \neq 0$, $\tilde{x} \in \mathcal{Z}_{\tilde{x}}$. The last condition above insures that the solution of the closed-loop hybrid system is not tangent to the resetting set $\mathcal{Z}_{\tilde{x}}$ for all initial conditions $\tilde{x}_0 \in \tilde{\mathcal{D}}$. For further discussion on Assumption 2.1 see [6, 19].

The following theorem proven in [6, 19] is needed to develop the main results of this paper.

Theorem 2.1 [6, 19]. Consider the nonlinear impulsive dynamical system $\tilde{\mathcal{G}}$ given by (3), (4), assume $\tilde{\mathcal{D}}_c \subset \tilde{\mathcal{D}}$ is a compact positively invariant set with respect to (3), (4), and assume that there exists a continuously differentiable function $V : \tilde{\mathcal{D}}_c \to \mathbb{R}$ such that

$$V'(\tilde{x})\tilde{f}_c(\tilde{x}) \le 0, \quad \tilde{x} \in \tilde{\mathcal{D}}_c, \quad \tilde{x} \notin \mathcal{Z}_{\tilde{x}}, \tag{5}$$

$$V(\tilde{x} + \tilde{f}_d(\tilde{x})) \le V(\tilde{x}), \quad \tilde{x} \in \tilde{\mathcal{D}}_c, \quad \tilde{x} \in \mathcal{Z}_{\tilde{x}}. \tag{6}$$

Let $\mathcal{R} \triangleq \{\tilde{x} \in \tilde{\mathcal{D}}_c : \tilde{x} \notin \mathcal{Z}_{\tilde{x}}, V'(\tilde{x})\tilde{f}_c(\tilde{x}) = 0\} \cup \{\tilde{x} \in \tilde{\mathcal{D}}_c : \tilde{x} \in \mathcal{Z}_{\tilde{x}}, V(\tilde{x} + \tilde{f}_d(\tilde{x})) = V(\tilde{x})\}$ and let $\mathcal{M}$ denote the largest invariant set contained in $\mathcal{R}$. If $\tilde{x}_0 \in \tilde{\mathcal{D}}_c$, then $\tilde{x}(t) \to \mathcal{M}$ as $t \to \infty$. Finally, if $\tilde{\mathcal{D}} = \mathbb{R}^{\tilde{n}}$ and $V(\tilde{x}) \to \infty$ as $\|\tilde{x}\| \to \infty$, then $\tilde{x}(t) \to \mathcal{M}$ as $t \to \infty$ for all $\tilde{x}_0 \in \mathbb{R}^{\tilde{n}}$.

3. Hybrid Adaptive Stabilization for Nonlinear Hybrid Dynamical Systems using Neural Networks

In this section we consider the problem of neural hybrid adaptive stabilization for nonlinear uncertain hybrid systems. Specifically, we consider the controlled state-dependent impulsive dynamical system (1), (2) with $\mathcal{D} = \mathbb{R}^n$, $\mathcal{U}_c = \mathbb{R}^{m_c}$, and $\mathcal{U}_d = \mathbb{R}^{m_d}$, where $f_c : \mathbb{R}^n \to \mathbb{R}^n$ and $f_d : \mathbb{R}^n \to \mathbb{R}^n$ are continuously differentiable and satisfies $f_c(0) = 0$ and $f_d(0) = 0$, $G_c : \mathbb{R}^n \to \mathbb{R}^{n\times m_c}$, and $G_d : \mathbb{R}^n \to \mathbb{R}^{n\times m_d}$.

In this paper, we assume that $f_c(\cdot)$ and $f_d(\cdot)$ are unknown functions, and $f_c(\cdot)$, $G_c(\cdot)$, $f_d(\cdot)$, and $G_d(\cdot)$ are given by

$$f_c(x) = A_c x + \Delta f_c(x), \quad G_c(x) = B_c G_{cn}(x), \tag{7}$$

$$f_d(x) = (A_d - I_n)x + \Delta f_d(x), \quad G_d(x) = B_d G_{dn}(x), \tag{8}$$

where $A_c \in \mathbb{R}^{n\times n}$, $A_d \in \mathbb{R}^{n\times n}$, $B_c \in \mathbb{R}^{n\times m_c}$, and $B_d \in \mathbb{R}^{n\times m_d}$ are known matrices, $G_{cn} : \mathbb{R}^n \to \mathbb{R}^{m_c\times m_c}$ and $G_{dn} : \mathbb{R}^n \to \mathbb{R}^{m_d\times m_d}$ are known matrix functions such that $\det G_{cn}(x) \neq 0$, $x \in \mathbb{R}^n$, and $\det G_{dn}(x) \neq 0$, $x \in \mathbb{R}^n$, and $\Delta f_c : \mathbb{R}^n \to \mathbb{R}^n$ and $\Delta f_d : \mathbb{R}^n \to \mathbb{R}^n$ are unknown functions belonging to the uncertainty sets $\mathcal{F}_c$ and $\mathcal{F}_d$, respectively, given by

$$\mathcal{F}_c = \{\Delta f_c : \mathbb{R}^n \to \mathbb{R}^n : \Delta f_c(0) = 0, \Delta f_c(x) = B_c\delta_c(x), x \in \mathbb{R}^n\}, \tag{9}$$

$$\mathcal{F}_d = \{\Delta f_d : \mathbb{R}^n \to \mathbb{R}^n : \Delta f_d(0) = 0, \Delta f_d(x) = B_d\delta_d(x), x \in \mathbb{R}^n\}, \tag{10}$$

where $\delta_c : \mathbb{R}^n \to \mathbb{R}^{m_c}$ and $\delta_d : \mathbb{R}^n \to \mathbb{R}^{m_d}$ are uncertain continuously differentiable functions such that $\delta_c(0) = 0$ and $\delta_d(0) = 0$. It is important to note that since $\delta_c(x)$ and $\delta_d(x)$ are continuously differentiable and $\delta_c(0) = 0$ and $\delta_d(0) = 0$, it follows that there exist continuous matrix functions $\Delta_c : \mathbb{R}^n \to \mathbb{R}^{m_c\times n}$ and $\Delta_d : \mathbb{R}^n \to \mathbb{R}^{m_d\times n}$ such that $\delta_c(x) = \Delta_c(x)x$, $x \in \mathbb{R}^n$, and $\delta_d(x) = \Delta_d(x)x$, $x \in \mathbb{R}^n$. Furthermore, we assume that the continuous matrix functions $\Delta_c(\cdot)$ and $\Delta_d(\cdot)$ can be approximated over a compact set $\mathcal{D}_c \subset \mathbb{R}^n$ by a linear in the parameters neural network up to a desired accuracy so that

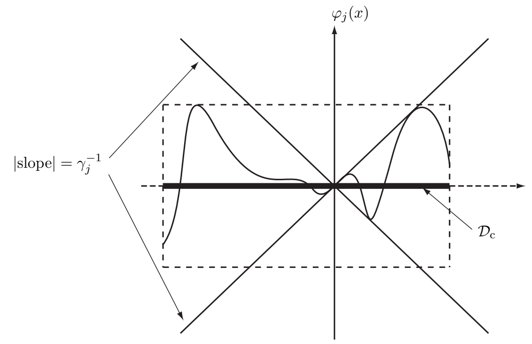

Figure 3.1: Visualization of function $\varphi_j(\cdot)$, $j = \mathrm{c}, \mathrm{d}$

$$\mathrm{col}_i(\Delta_c(x)) = W_{ci}^{\mathrm{T}}\sigma_c(x) + \varepsilon_{ci}(x), \quad x \in \mathcal{D}_c, \quad i = 1, \cdots, n, \tag{11}$$

$$\mathrm{col}_i(\Delta_d(x)) = W_{di}^{\mathrm{T}}\sigma_d(x) + \varepsilon_{di}(x), \quad x \in \mathcal{D}_c, \quad i = 1, \cdots, n, \tag{12}$$

where $\mathrm{col}_i(\Delta(\cdot))$ denotes the ith column of the matrix $\Delta(\cdot)$, $W_{ci}^{\mathrm{T}} \in \mathbb{R}^{m_c\times s_c}$ and $W_{di}^{\mathrm{T}} \in \mathbb{R}^{m_d\times s_d}$, $i = 1, \cdots, n$, are optimal *unknown* (constant) weights that minimize the approximation error over $\mathcal{D}_c$, $\varepsilon_{ci} : \mathbb{R}^n \to \mathbb{R}^{m_c}$ and $\varepsilon_{di} : \mathbb{R}^n \to \mathbb{R}^{m_d}$, $i = 1, \cdots, n$, are modeling errors such that $\sigma_{\max}(\Upsilon_c(x)) \le \gamma_c^{-1}$ and $\sigma_{\max}(\Upsilon_d(x)) \le \gamma_d^{-1}$, $x \in \mathbb{R}^n$, where $\Upsilon_c(x) \triangleq [\varepsilon_{c1}(x), \cdots, \varepsilon_{cn}(x)]$, $\Upsilon_d(x) \triangleq [\varepsilon_{d1}(x), \cdots, \varepsilon_{dn}(x)]$ and $\gamma_c, \gamma_d > 0$, and $\sigma_c : \mathbb{R}^n \to \mathbb{R}^{s_c}$ and $\sigma_d : \mathbb{R}^n \to \mathbb{R}^{s_d}$ are given basis functions.

Next, defining

$$\varphi_c(x) \triangleq \delta_c(x) - W_c^{\mathrm{T}}[x \otimes \sigma_c(x)], \tag{13}$$

$$\varphi_d(x) \triangleq \delta_d(x) - W_d^{\mathrm{T}}[x \otimes \sigma_d(x)], \tag{14}$$

where $W_c^{\mathrm{T}} \triangleq [W_{c1}^{\mathrm{T}}, \cdots, W_{cn}^{\mathrm{T}}] \in \mathbb{R}^{m_c\times ns_c}$, $W_d^{\mathrm{T}} \triangleq [W_{d1}^{\mathrm{T}}, \cdots, W_{dn}^{\mathrm{T}}] \in \mathbb{R}^{m_d\times ns_d}$, and $\otimes$ denotes Kronecker product, it follows from (11), (12), and Cauchy-Schwartz inequality that

$$\begin{aligned}\varphi_j^{\mathrm{T}}(x)\varphi_j(x) &= \|\Delta_j(x)x - W_j^{\mathrm{T}}(x \otimes \sigma_j(x))\|^2 \\ &= \|\Delta_j(x)x - \Sigma_j(x)x\|^2 \\ &= \|\Upsilon_j(x)x\|^2 \\ &\le \gamma_j^{-2}x^{\mathrm{T}}x, \quad x \in \mathcal{D}_c, \quad j = \mathrm{c}, \mathrm{d},\end{aligned} \tag{15}$$

where $\Sigma_j(x) \triangleq [W_{j1}^{\mathrm{T}}\sigma_j(x), \cdots, W_{jn}^{\mathrm{T}}\sigma_j(x)]$, $j = \mathrm{c}, \mathrm{d}$. This corresponds to a nonlinear small gain-type norm bounded uncertainty characterization for $\varphi_j(\cdot)$, $j = \mathrm{c}, \mathrm{d}$ (see Figure 3.1).

Theorem 3.1. Consider the nonlinear uncertain hybrid dynamical system $\mathcal{G}$ given by (1), (2) where $f_c(\cdot)$,

$G_{\rm c}(\cdot)$, $f_{\rm d}(\cdot)$, and $G_{\rm d}(\cdot)$ are given by (7), (8), and $\Delta f_{\rm c} : \mathbb{R}^n \to \mathbb{R}^n$ and $\Delta f_{\rm d} : \mathbb{R}^n \to \mathbb{R}^n$ belong to the uncertainty sets $\mathcal{F}_{\rm c}$ and $\mathcal{F}_{\rm d}$, respectively. For given $\gamma_{\rm c}, \gamma_{\rm d} > 0$, assume there exists a positive-definite matrix $P \in \mathbb{R}^{n\times n}$ such that

$$0 = A_{\rm cs}^{\rm T}P + PA_{\rm cs} + \gamma_{\rm c}^{-2}PB_{\rm c}B_{\rm c}^{\rm T}P + I_n + R_{\rm c}, \quad (16)$$

$$P = A_{\rm d}^{\rm T}PA_{\rm d} - A_{\rm d}^{\rm T}PB_{\rm d}(B_{\rm d}^{\rm T}PB_{\rm d})^{-1}B_{\rm d}^{\rm T}PA_{\rm d} + (\alpha+\beta)I_n + R_{\rm d}, \quad (17)$$

where $A_{\rm cs} \triangleq A_{\rm c} + B_{\rm c}K_{\rm c}$, $K_{\rm c} \in \mathbb{R}^{m_{\rm c}\times n}$, $R_{\rm c} \in \mathbb{R}^{n\times n}$ and $R_{\rm d} \in \mathbb{R}^{n\times n}$ are positive definite, $\alpha > 0$, β satisfies

$$\beta \geq \gamma_{\rm d}^{-2}\Big(\lambda_{\max}(B_{\rm d}^{\rm T}PB_{\rm d}) + a\frac{1 + x^{\rm T}Px}{c + [x\otimes\sigma_{\rm d}(x)]^{\rm T}[x\otimes\sigma_{\rm d}(x)]}\Big), \quad x \in \mathcal{Z}_x, \quad (18)$$

$$a = \max\{c, n/\lambda_{\min}(P)\}B_{\rm d}^{\rm T}PB_{\rm d}\Big(I_m + \tfrac{1}{\alpha\gamma_{\rm d}^2}B_{\rm d}^{\rm T}PB_{\rm d}\Big), \quad (19)$$

and $c > 0$. Finally, let $A_{\rm ds} \triangleq A_{\rm d} + B_{\rm d}K_{\rm d}$, where $K_{\rm d} \triangleq -(B_{\rm d}^{\rm T}PB_{\rm d})^{-1}B_{\rm d}^{\rm T}PA_{\rm d}$, and let $Q_{\rm c} \in \mathbb{R}^{m_{\rm c}}$ and $Y \in \mathbb{R}^{s_{\rm c}}$ be positive definite. Then the neural hybrid adaptive feedback control law

$$u_{\rm c}(t) = G_{\rm cn}^{-1}(x(t))\Big[K_{\rm c}x(t) - \hat{W}_{\rm c}^{\rm T}(t)[x(t)\otimes\sigma_{\rm c}(x(t))]\Big], \quad x(t) \notin \mathcal{Z}_x, \quad (20)$$

$$u_{\rm d}(t) = G_{\rm dn}^{-1}(x(t))\Big[K_{\rm d}x(t) - \hat{W}_{\rm d}^{\rm T}(t)[x(t)\otimes\sigma_{\rm d}(x(t))]\Big], \quad x(t) \in \mathcal{Z}_x, \quad (21)$$

where $\hat{W}_{\rm c}^{\rm T}(t) \in \mathbb{R}^{m_{\rm c}\times ns_{\rm c}}$, $t \geq 0$, $\hat{W}_{\rm d}^{\rm T}(t) \in \mathbb{R}^{m_{\rm d}\times ns_{\rm d}}$, $t \geq 0$, and $\sigma_{\rm c} : \mathbb{R}^n \to \mathbb{R}^{s_{\rm c}}$ and $\sigma_{\rm d} : \mathbb{R}^n \to \mathbb{R}^{s_{\rm d}}$ are given basis functions, with update laws

$$\dot{\hat{W}}_{\rm c}^{\rm T}(t) = \tfrac{1}{1+x(t)^{\rm T}Px(t)}Q_{\rm c}B_{\rm c}^{\rm T}Px(t)[x(t)\otimes\sigma_{\rm c}(x(t))]^{\rm T}Y, \quad \hat{W}_{\rm c}^{\rm T}(0) = \hat{W}_{\rm c0}^{\rm T}, \quad x(t) \notin \mathcal{Z}_x, \quad (22)$$

$$\Delta\hat{W}_{\rm c}^{\rm T}(t) = 0, \quad x(t) \in \mathcal{Z}_x, \quad (23)$$

$$\dot{\hat{W}}_{\rm d}^{\rm T}(t) = 0, \quad \hat{W}_{\rm d}^{\rm T}(0) = \hat{W}_{\rm d0}^{\rm T}, \quad x(t) \notin \mathcal{Z}_x, \quad (24)$$

$$\Delta\hat{W}_{\rm d}^{\rm T}(t) = \tfrac{1}{c+[x(t)\otimes\sigma_{\rm d}(x(t))]^{\rm T}[x(t)\otimes\sigma_{\rm d}(x(t))]}B_{\rm d}^{\dagger}[x(t^+) - A_{\rm ds}x(t)][x(t)\otimes\sigma_{\rm d}(x(t))]^{\rm T}, \quad x(t) \in \mathcal{Z}_x, \quad (25)$$

where $\Delta\hat{W}_{\rm c}^{\rm T}(t) \triangleq \hat{W}_{\rm c}^{\rm T}(t^+) - \hat{W}_{\rm c}^{\rm T}(t)$ and $\Delta\hat{W}_{\rm d}^{\rm T}(t) \triangleq \hat{W}_{\rm d}^{\rm T}(t^+) - \hat{W}_{\rm d}^{\rm T}(t)$, guarantees that there exists a positively invariant set $\mathcal{D}_\alpha \subset \mathbb{R}^n \times \mathbb{R}^{m_{\rm c}\times ns_{\rm c}} \times \mathbb{R}^{m_{\rm d}\times ns_{\rm d}}$ such that $(0, W_{\rm c}^{\rm T}, W_{\rm d}^{\rm T}) \in \mathcal{D}_\alpha$, where $W_{\rm c}^{\rm T} \in \mathbb{R}^{m_{\rm c}\times ns_{\rm c}}$ and $W_{\rm d}^{\rm T} \in \mathbb{R}^{m_{\rm d}\times ns_{\rm d}}$, and the solution $(x(t), \hat{W}_{\rm c}^{\rm T}(t), \hat{W}_{\rm d}^{\rm T}(t)) \equiv (0, W_{\rm c}^{\rm T}, W_{\rm d}^{\rm T})$ of the closed-loop system given by (1), (2), (20)–(25) is Lyapunov stable and $x(t) \to 0$ as $t \to \infty$ for all $\Delta f_{\rm c}(\cdot) \in \mathcal{F}_{\rm c}$, $\Delta f_{\rm d}(\cdot) \in \mathcal{F}_{\rm d}$, and $(x_0, \hat{W}_{\rm c0}^{\rm T}, \hat{W}_{\rm d0}^{\rm T}) \in \mathcal{D}_\alpha$.

Proof. The proof is omitted due to space limitations. □

Remark 3.1. Note that the conditions in Theorem 3.1 imply partial asymptotic stability, that is, the solution $(x(t), \hat{W}_{\rm c}^{\rm T}(t), \hat{W}_{\rm d}^{\rm T}(t)) \equiv (0, W_{\rm c}^{\rm T}, W_{\rm d}^{\rm T})$ of the overall closed-loop system is Lyapunov stable and $x(t) \to 0$ as $t \to \infty$. Hence, it follows from (22), (23) that $\dot{\hat{W}}_{\rm c}^{\rm T}(t) \to 0$ as $t \to \infty$. Furthermore, if $x(t)$, $t \geq 0$, intersects $\mathcal{Z}_x$ infinitely many times, then it follows from (24), (25) that $\hat{W}_{\rm d}(t_k^+) - \hat{W}_{\rm d}(t_k) \to 0$ as $k \to \infty$.

Remark 3.2. Since the Lyapunov function used in the proof of Theorem 3.1 is a class $\mathcal{K}_\infty$ function, in the case where the neural network approximation holds in $\mathbb{R}^n$, the control law (20), (21) ensures global asymptotic stability with respect to x. However, the existence of a global neural network approximator for an uncertain nonlinear map cannot in general be established. Hence, as is common in the neural network literature, for a given arbitrarily large compact set $\mathcal{D}_{\rm c} \subset \mathbb{R}^n$, we assume that there exists an approximator for the unknown nonlinear map up to a desired accuracy (in the sense of (11) and (12)). In the case where $\Delta_{\rm c}(\cdot)$ and $\Delta_{\rm d}(\cdot)$ are continuous on $\mathbb{R}^n$, it follows from the Stone-Weierstrass theorem that $\Delta_{\rm c}(\cdot)$ and $\Delta_{\rm d}(\cdot)$ can be approximated over an arbitrarily large compact set $\mathcal{D}_{\rm c}$. In this case, our neuro adaptive controller guarantees semiglobal partial asymptotic stability.

Remark 3.3. Note that the neuro adaptive controller (20), (21) can be constructed to guarantee partial asymptotic stability using standard *linear* H_∞ theory. Specifically, it follows from standard H_∞ theory [21] that $\|G_{\rm c}(s)\|_\infty < \gamma_{\rm c}$, where $G(s) = E_{\rm c}(sI_n - A_{\rm cs})^{-1}B_{\rm c}$ and $E_{\rm c}$ is such that $E_{\rm c}^{\rm T}E_{\rm c} = I_n + R$, if and only if there exists a positive-definite matrix P satisfying the bounded real Riccati equation (16). It is well known that (16) has a positive-definite solution if and only if the Hamiltonian matrix

$$\mathcal{H} = \begin{bmatrix} A_{\rm s} & \gamma_{\rm c}^{-2}BB^{\rm T} \\ -E^{\rm T}E & -A_{\rm s}^{\rm T} \end{bmatrix}, \quad (26)$$

has no purely imaginary eigenvalues.

It is important to note that the hybrid adaptive control law (20)–(25) does *not* require explicit knowledge of the optimal weighting matrices $W_{\rm c}$, $W_{\rm d}$, and the positive constants α and β. Theorem 3.1 simply requires the existence of $W_{\rm c}$, $W_{\rm d}$, α, and β such that (16) and (17) hold. Furthermore, no specific structure on the nonlinear dynamics $f_{\rm c}(x)$ and $f_{\rm d}(x)$ is required to apply Theorem 3.1 other than the assumption that $f_{\rm c}(x)$ and $f_{\rm d}(x)$ are continuously differentiable and that the approximation error of uncertain system nonlinearities lie in a small gain-type norm bounded conic sector. Finally, in the case where the pair $(A_{\rm d}, B_{\rm d})$ is in controllable canonical form and $R_{\rm d}$ in (17) is diagonal, it follows that $A_{\rm ds} = \begin{bmatrix} A_0 \\ 0_{m_{\rm d}\times n} \end{bmatrix}$, where $A_0 \in \mathbb{R}^{(n-m_{\rm d})\times n}$ is a known matrix of zeros and ones capturing the multivariable controllable canonical form representation [22], and hence the update law (25) is simplified as

$$\Delta\hat{W}_{\rm d}^{\rm T}(t) = \frac{1}{c+[x(t)\otimes\sigma_{\rm d}(x(t))]^{\rm T}[x(t)\otimes\sigma_{\rm d}(x(t))]}B_{\rm d}^{\dagger}\Delta x(t) \cdot [x(t)\otimes\sigma_{\rm d}(x(t))]^{\rm T}, \quad x(t)\in\mathcal{Z}_x, \quad (27)$$

since $B_{\rm d}^{\dagger}A_{\rm ds}=0$.

4. Illustrative Numerical Example

In this section we present a numerical example to demonstrate the utility of the proposed neural hybrid adaptive control framework for hybrid adaptive stabilization. Specifically, consider the nonlinear uncertain controlled hybrid system given by (1), (2) with $n=2$, $x=[x_1,x_2]^{\rm T}$,

$$f_{\rm c}(x) = \begin{bmatrix} x_2 \\ \hat{f}_{\rm c}(x)\end{bmatrix}, \quad G_{\rm c}(x)=\begin{bmatrix}0\\ b_{\rm c}\end{bmatrix}, \quad (28)$$

$$f_{\rm d}(x) = \begin{bmatrix} -x_1+x_2 \\ \hat{f}_{\rm d}(x)\end{bmatrix}, \quad G_{\rm d}(x)=\begin{bmatrix}0\\ b_{\rm d}\end{bmatrix}, \quad (29)$$

where $\hat{f}_{\rm c}:\mathbb{R}^2\to\mathbb{R}$ and $\hat{f}_{\rm d}:\mathbb{R}^2\to\mathbb{R}$ are unknown, continuously functions. Furthermore, assume that the resetting set $\mathcal{Z}_x$ is given by

$$\mathcal{Z}_x=\{x\in\mathbb{R}^2:\mathcal{X}(x)=0,\ x_2>0\}. \quad (30)$$

Here, we assume that $f_{\rm c}(x)$ and $f_{\rm d}(x)$ are unknown and can be written in the form of (7) and (8) with $A_{\rm c}=A_{\rm d}=\begin{bmatrix}0&1\\0&0\end{bmatrix}$, $\Delta f_{\rm c}(x)=[0,\hat{f}_{\rm c}(x)]^{\rm T}$, $\Delta f_{\rm d}(x)=[0,\hat{f}_{\rm d}(x)]^{\rm T}$, $B_{\rm c}=[0,b_{\rm c}]^{\rm T}$, $B_{\rm d}=[0,b_{\rm d}]^{\rm T}$, $G_{\rm cn}(x)=G_{\rm dn}(x)=1$. Here we assume that $\Delta f_{\rm c}(x)$ and $\Delta f_{\rm d}(x)$ are unknown and can be written as $\Delta f_{\rm c}(x)=B_{\rm c}\delta_{\rm c}(x)$ and $\Delta f_{\rm d}(x)=B_{\rm d}\delta_{\rm d}(x)$, where $\delta_{\rm c}(x)=\frac{1}{b_{\rm c}}\hat{f}_{\rm c}(x)$ and $\delta_{\rm d}(x)=\frac{1}{b_{\rm d}}\hat{f}_{\rm d}(x)$. Next, let $K_{\rm c}=\frac{1}{b_{\rm c}}[k_{\rm c1},k_{\rm c2}]$ and $K_{\rm d}=\frac{1}{b_{\rm d}}[k_{\rm d1},k_{\rm d2}]$, where $k_{\rm c1}$, $k_{\rm c2}$, $k_{\rm d1}$, and $k_{\rm d2}$ are arbitrary scalars, such that $A_{\rm cs}=A_{\rm c}+B_{\rm c}K_{\rm c}=\begin{bmatrix}0&1\\k_{\rm c1}&k_{\rm c2}\end{bmatrix}$ and $A_{\rm ds}=A_{\rm d}+B_{\rm d}K_{\rm d}=\begin{bmatrix}0&1\\k_{\rm d1}&k_{\rm d2}\end{bmatrix}$. Now, the proper choice of $k_{\rm c1}$, $k_{\rm c2}$, $k_{\rm d1}$, and $k_{\rm d2}$, it follows from Theorem 3.1 that if there exists $P>0$ satisfying (16) and (17), then the neural hybrid adaptive feedback controller (20) and (21) guarantees $x(t)\to 0$ as $t\to\infty$. Specifically, here we choose $k_{\rm c1}=-1$, $k_{\rm c2}=-1$, $k_{\rm d1}=-0.2$, $k_{\rm d2}=-0.5$, $\gamma_{\rm c}=10$, $\gamma_{\rm d}=20$, $b_{\rm c}=3$, $b_{\rm d}=1.4$, $c=1$, $\alpha=1$, $\sigma_{\rm d}(x)=[\tanh(0.1x_2),\ldots,\tanh(0.6x_2)]^{\rm T}$, and

$$R_{\rm c} = \begin{bmatrix}2.6947 & 2.4323\\ 2.4323 & 5.8019\end{bmatrix},$$

$$R_{\rm d} = \begin{bmatrix}8.0196 & 2.0334\\ 2.0334 & 1.0569\end{bmatrix},$$

so that P satisfying (16) and (17) is given by

$$P=\begin{bmatrix}10.0196 & 2.0334\\ 2.0334 & 12.7523\end{bmatrix}.$$

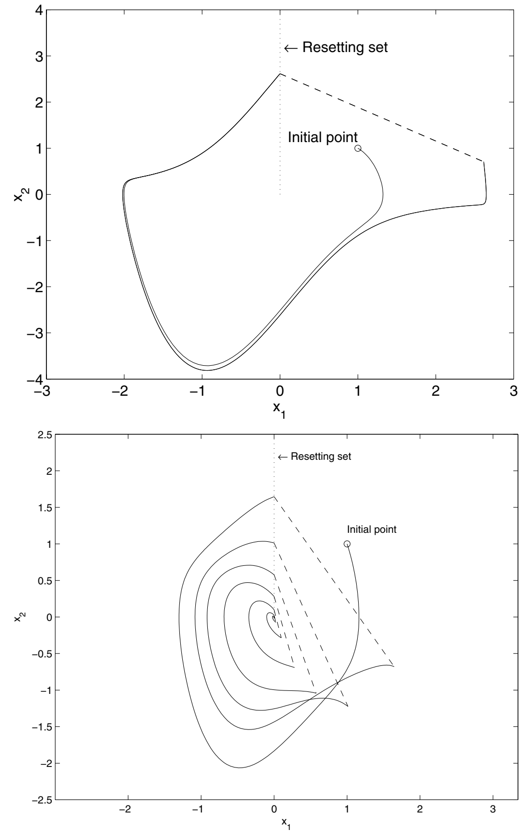

Figure 4.2: Phase portraits of uncontrolled and controlled hybrid system

With $\hat{f}_{\rm c}(x)=-a_1x_1-a_2(x_1^2-a_3)x_2$, $\hat{f}_{\rm d}(x)=-x_2-a_4x_1^2-a_5\frac{x_2^3}{1+x_2^2}-a_6x_2^3$, $a_1=1$, $a_2=2$, $a_3=1$, $a_4=-5$, $a_5=-2$, $a_6=8$, $Y=0.02I_3$, $\sigma_{\rm c}(x)=\left[\frac{1}{1+e^{-\lambda_1x_1}},\ldots,\frac{1}{1+e^{-3\lambda_1x_1}},\frac{1}{1+e^{-\lambda_2x_2}},\ldots,\frac{1}{1+e^{-3\lambda_2x_2}}\right]$, and initial conditions $x(0)=[1,1]^{\rm T}$, $\hat{W}_{\rm c}^{\rm T}(0)=0_{1\times6}$, and $\hat{W}_{\rm d}^{\rm T}(0)=0_{1\times6}$, Figure 4.2 shows the phase portraits of the uncontrolled and controlled hybrid system. Figures 4.3 and 4.4 show the state trajectories versus time and the control signals versus time, respectively. Finally, Figure 4.5 shows the adaptive gain history versus time.

5. Conclusion

A direct hybrid neuro adaptive nonlinear control framework for hybrid nonlinear uncertain dynamical systems was developed. Using Lyapunov methods the proposed framework was shown to guarantee partial asymptotic stability of the closed-loop hybrid system; that is, asymptotic stability with respect to part of the closed-loop system states associated with the hybrid plant dynamics. In the case where the nonlinear hybrid system is represented in normal form, the nonlinear hybrid adaptive controller was constructed without knowledge of the system dynamics. Finally, a numerical example was presented to show the utility of the proposed hybrid adaptive stabilization scheme.

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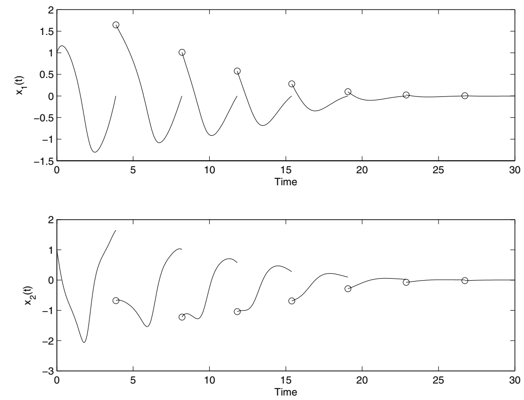

Figure 4.3: State trajectories versus time

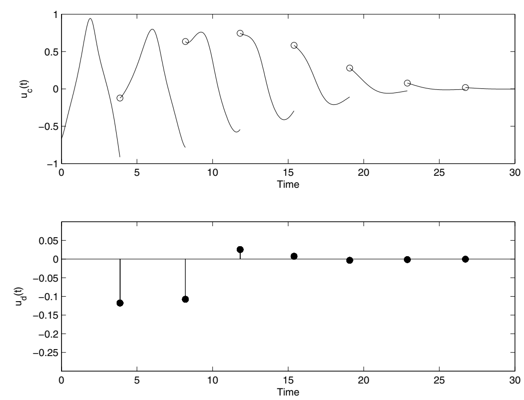

Figure 4.4: Control signals versus time

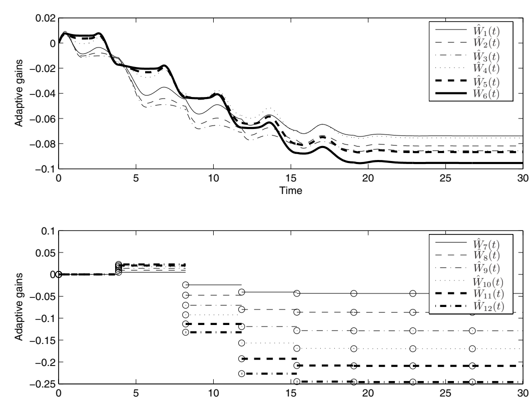

Figure 4.5: Adaptive gain history versus time