Proceedings of the
44th IEEE Conference on Decision and Control, and
the European Control Conference 2005
Seville, Spain, December 12-15, 2005

Inference-based Ambiguity Management in Decentralized Decision-Making: Decentralized Control of Discrete Event Systems

Ratnesh Kumar and Shigemasa Takai

***Abstract*— Decentralized decision-making requires the interaction of various local decision-makers in order to arrive at a global decision. Limited sensing capabilities at each local site can create ambiguities in a decision-making process at each local site. We argue that such ambiguities are of differing gradations. We propose a framework for decentralized decision-making (applied to decentralized control in particular) which allows computation of such ambiguity-gradations, and utilizes their knowledge in arriving at a global decision. Each local decision is tagged with a certain grade or level of ambiguity; zero being the minimum ambiguity level. A global decision is taken to be the same as a "winning" local decision, i.e., one having the minimum level of ambiguity. The computation of an ambiguity level for a local decision requires an assessment of the self-ambiguities as well as the ambiguities of the others, and an inference based up on such knowledge. In order to characterize the class of closed-loop behaviors achievable under the control of such an inference-based decentralized control, we introduce the notion of N-inference-observability, where N is an index representing the maximum ambiguity level of any winning local decision. We show that the C&P$\vee$D&A-coobservability is the same as the zero-inference-observability, whereas the conditional C&P$\vee$D&A-coobservability is the same as the unity-inference-observability. We also present examples of higher order inference-observable languages. Interestingly our framework does not require the existence of any a-priori partition of the controllable events into permissive/anti-permissive sets, nor does it require a global control computation based on conjunction/disjunction of local decisions; exhibiting that perhaps a more natural way to approach the problem of decentralized decision-making is based on a computation/comparison of the grades of ambiguities associated with the individual local decisions.**

I. INTRODUCTION

In any decentralized decision-making paradigm, such as decentralized control or diagnosis, multiple decision-makers, each with its limited sensing and/or control capabilities, interact to come up with the global decisions. Presence of limited sensing capabilities can lead to *ambiguity* in knowing the system state and thereby ambiguity in decision-making. Consider for example the problem of decentralized control of discrete event systems (DESs) [2], [4], [12], [10], [3], [6], [1], [8], [5], [13], [9], [11], [14]. Suppose there exist two traces that are executable in the plant and are indistinguishable to a local supervisor, and a locally controllable event that is feasible and legal following the first trace, whereas it is feasible and illegal following the second trace. Since these two traces are indistinguishable, upon receiving their observation, the local supervisor will be ambiguous about whether to enable or disable the locally controllable event.

In the past, different techniques have been suggested for the management of such ambiguity in the context of decentralized control. In the so called "conjunctive-and-permissive" (C&P) architecture of decentralized control, when a local supervisor is ambiguous about the control decision of a locally controllable event, it simply enables it. Also in this architecture, an event is globally enabled only if it is locally enabled by all local supervisors having control over that event. (The C&P architecture was used in most initial works on decentralized control of DESs; the term itself was formulated in [13].) As a result only those languages are achievable as closed-loop behaviors in which, for any controllable event that needs to be disabled, there exists at least one local supervisor which is able to do so unambiguously. The class of such languages is known as C&P-coobservable languages. Similarly, in the disjunctive-and-antipermissive (D&A) architecture [13], when a local supervisor is ambiguous about the control decision of a locally controllable event, it simply disables it. Also in this architecture, an event is globally disabled only if it is locally disabled by all local supervisors having control over that event. Consequently the class of languages achievable under the D&A architecture has the property that for any controllable event that needs to be enabled, there exists at least one local supervisor which is able to do so unambiguously. The class of such languages is known as D&A-coobservable languages and this class is incomparable to the C&P-coobservable class.

The C&P$\vee$D&A architecture [13] combines the features of both the C&P and D&A architectures. The set of controllable events is partitioned into two disjoint subsets: (i) the permissive set for which the control decision in case of ambiguity is enablement, and (ii) the anti-permissive set for which the control decision in case of ambiguity is disablement. Plus an event in the permissive (resp., anti-permissive) set is globally enabled (resp., disabled) if it is enabled (resp., disabled) by all local supervisors having control over that event. A language is achievable as a closed-loop behavior in this architecture if there exists a partition of the controllable events such that the events in the permissive (resp., anti-permissive) set can be unambiguously

The research was supported in part by the National Science Foundation under the grants NSF-ECS-0218207, NSF-ECS-0244732, NSF-EPNES-0323379, and NSF-ECS-0424048, a DoD-EPSCoR grant through the Office of Naval Research under the grant N000140110621, and MEXT under Grant-in-Aid for Scientific Research 15760321.

R. Kumar is with the Department of Electrical and Computer Engineering, Iowa State University, Ames, Iowa 50011-3060, USA, e-mail: rkumar@iastate.edu

S. Takai is with the Department of Electronics and Information Science, Kyoto Institute of Technology, Sakyo-ku, Kyoto 606-8585, Japan, e-mail: takai@dj.kit.ac.jp

0-7803-9568-9/05/$20.00 ©2005 IEEE

disabled (resp., enabled) by some local supervisor. The PSC (resp., PCX) architecture presented in [11] considered a more general 4-way (resp., 8-way) partition of the controllable event set to account for the priorities and the exclusivities of the event control. The classes of languages achievable under C&P$\vee$D&A or PSC or PCX architectures happen to be the same, and subsume the classes of languages achievable under C&P as well as D&A architectures [11].

In all the above architectures a local decision is taken purely on the basis of assessing the self-ambiguities—the ambiguities of other local decision-makers are not assessed or used. A knowledge-based mechanism for assessing the self-ambiguities was presented in [8], and later the same architecture was used for assessing the self-ambiguities as well as the ambiguities of the others in [9]. The process of utilizing the knowledge of the self-ambiguities together with the ambiguities of the others for the sake of decision-making was referred to as "inferencing" in [9] and "conditioning" in [14]. As is the case with prior non-inferencing based approaches, these inferencing based approaches require the existence of an a-priori partition of the controllable events into certain permissive/anti-permissive sets, and also these prior approaches are limited by a "single-level" of inferencing. (The notion of single- vs. multi-level of inferencing will become clear in the following.)

In this paper we introduce a framework for decentralized decision-making (decentralized control in particular) that (i) supports inferencing utilizing the knowledge of the self-ambiguities as well as the ambiguities of the other decision makers, (ii) demonstrates the partition of events into permissive/anti-permissive sets is *redundant*, and (iii) supports inferencing over an *arbitrary* number of levels of ambiguity. Each local supervisor uses its observations of the system behavior to come up its control decision together with a grade or level of ambiguity for that control decision. The computation of an ambiguity-grade of a local decision requires the assessment of the self-ambiguities together with the ambiguities of the others.

A control decision with level-zero ambiguity is taken when the local supervisor is unambiguous about its enablement/disablement decision. This happens for a locally controllable event if all the traces, producing the same observation as the one received, when extended by the locally controllable event yield traces such that the ones feasible are either all legal or all illegal. Otherwise, a higher ambiguity level control decision is issued. For example a disablement decision of level-one ambiguity is issued for a certain controllable event following a certain observation if there exist certain traces, producing the same observation as the one received, such that the extension by the controllable event is feasible and legal in some cases whereas feasible and illegal in some others. Existence of such traces is clearly the source of ambiguity for the local supervisor in question. Yet, suppose the local supervisor is able to determine that for each trace for which the controllable event extension is feasible and legal, there exists another local supervisor which can issue an enablement decision with level-zero ambiguity, then the local supervisor issues a disablement decision with level-one ambiguity.

In general a local supervisor will issue a disablement (resp., enablement) decision with an ambiguity level N for a locally controllable event following a certain observation if for each "ambiguous" trace, producing the same observation as the one received, and possessing a feasible and legal (resp., illegal) controllable event extension, there exists another local supervisor that can issue an enablement (resp., disablement) decision with an ambiguity level at most $N-1$. Clearly, a level-zero ambiguity control decision is based on assessment of only the self-ambiguities, whereas a level-N ambiguity control decision is based on assessment of the self-ambiguities together with the ambiguities of other local supervisors such that for each trace, that creates the ambiguity, there exists another local supervisor which can issue a control decision with an ambiguity level at most $N-1$.

In this manner our framework allows inferencing involving multiple-levels of ambiguities. Following the execution of each event all local supervisors receiving a new observation issue a control decision for each of their locally controllable events, tagged with a certain level of ambiguity. The global control decision for a controllable event is taken to be the same as a local control decision whose ambiguity level is the minimum. (Such a local decision can be considered to be a "winning" local decision.) We formulate the notion of *inference-observability* to characterize the class of languages achievable in the proposed framework of decentralized control.

A specification language is said to be N-inference-observable if the maximum ambiguity level of a "winning" control decision is N. We show that a language is C&P$\vee$D&A-coobservable (so that it can be achieved as a closed-loop behavior in the C&P$\vee$D&A architecture) if and only if it is zero-inference-observable. Similarly, a language is conditionally C&P$\vee$D&A-coobservable if and only if it is unity-inference-observable. Thus the framework presented here nicely subsumes the ones reported in [13], [14], [9]. Further, our framework allows identifying other higher order inference-observable languages. We provide an effective test to verify whether a given language is N-inference-observable.

An interesting feature of our framework is that it does not require the existence of any a-priori partition of the controllable events into the permissive/anti-permissive sets, nor does it require conjunction/disjunction over local decisions to arrive at a global decision. Thus our framework seems to provide a more natural formulation of inference-based decentralized decision-making than the ones conceptualized in some of the earlier works—It seems more natural to associate an ambiguity level with each local control decision and then elect the minimum ambiguity level local control decision as the candidate for the global control decision than to compute a global control decision as conjunction/disjunction of the local control decisions, which in case of ambiguity, need to be assigned certain "default" values. Moreover for the approaches which require having default control decisions, it is not clear how to generalize them for a higher-order of inferencing.

II. Notation and Preliminaries

We consider a DES modeled by an automaton $G = (Q, \Sigma, \delta, q_0, Q_m)$, where Q is the set of states, Σ is the finite set of events, a partial function $\delta : \Sigma \times Q \to Q$ is the transition function, $q_0 \in Q$ is the initial state, and $Q_m \subseteq Q$ is the set of marked states. Let Σ^* be the set of all finite traces of elements of Σ, including the empty trace ε. The function δ can be generalized to $\delta : \Sigma^* \times Q \to Q$ in the natural way. The generated and marked languages of G, denoted by $L(G)$ and $L_m(G)$, respectively, are defined as $L(G) = \{s \in \Sigma^* |\ \delta(s, q_0) \text{ is defined}\}$ and $L_m(G) = \{s \in \Sigma^* |\ \delta(s, q_0) \in Q_m\}$. Let $K \subseteq \Sigma^*$ be a language. We denote the set of all prefixes of traces in K by $\overline{K}$.

For supervisory control purposes [7], the event set Σ is partitioned into two disjoint subsets Σ_c and Σ_{uc} of controllable and uncontrollable events, respectively. A language $K \subseteq L(G)$ is said to be

- $L_m(G)$-closed if $\overline{K} \cap L_m(G) = K$.
- Controllable if $\overline{K}\Sigma_{uc} \cap L(G) \subseteq \overline{K}$.

In this paper, we consider decentralized supervisory control where n local supervisors control the system so that the controlled behavior satisfies a (global) specification. Let Σ_{ic} be the set of locally controllable events for the ith local supervisor S_i ($i \in I := \{1, 2, \cdots, n\}$), in which case,

$$\Sigma_c = \bigcup_{i \in I} \Sigma_{ic}.$$

For each controllable event $\sigma \in \Sigma_c$, we define the index set of local supervisors for which σ is controllable by $In(\sigma) = \{i \in I |\ \sigma \in \Sigma_{ic}\}$. We assume that the limited sensing capabilities of the ith local supervisor S_i ($i \in I$) can be represented as the local observation mask, $M_i : \Sigma \cup \{\varepsilon\} \to \Delta_i \cup \{\varepsilon\}$, where Δ_i is the set of locally observed symbols, and $M_i(\varepsilon) = \varepsilon$.

III. Inference-based Decentralized Control Framework

Let the set $C = \{0, 1, \phi\}$ be the set of control decisions, where "0" represents a disablement decision, "1" represents an enablement decision, and "ϕ" represents a null (or pass or don't care) decision. Each inference-based local supervisor S_i is defined as a map $S_i : M_i(L(G)) \times \Sigma_{ic} \to C \times \mathcal{N}$, where $\mathcal{N}$ denotes the set of nonnegative integers, and for each $s \in L(G)$ and $\sigma \in \Sigma_{ic}$,

$$S_i(M_i(s), \sigma) = (c_i(M_i(s), \sigma), n_i(M_i(s), \sigma)).$$

Here $c_i(M_i(s), \sigma) \in C$ denotes the control decision of S_i for a locally controllable event $\sigma \in \Sigma_{ic}$ following an observation $M_i(s) \in M_i(L(G))$, and $n_i(M_i(s), \sigma) \in \mathcal{N}$ denotes the ambiguity level of the control decision of S_i.

Let $n(s, \sigma)$ be the minimal ambiguity level of local decisions, i.e.,

$$n(s, \sigma) := \min_{i \in In(\sigma)} n_i(M_i(s), \sigma).$$

The decentralized supervisor $\{S_i\}_{i \in I}$ that consists of local supervisors S_i ($i \in I$) issues global decisions on controllable events. Formally, $\{S_i\}_{i \in I}$ is defined as a map $\{S_i\}_{i \in I} : L(G) \times \Sigma \to C$. For each $s \in L(G)$ and $\sigma \in \Sigma$, the control decision $\{S_i\}_{i \in I}(s, \sigma)$ is given as follows:

- If $\sigma \in \Sigma_c$,

$$\{S_i\}_{i \in I}(s, \sigma) = \begin{cases} 1, & \text{if } \forall i \in In(\sigma) \text{ s.t. } n_i(M_i(s), \sigma) = n(s, \sigma); \\ & c_i(M_i(s), \sigma) = 1 \\ 0, & \text{if } \forall i \in In(\sigma) \text{ s.t. } n_i(M_i(s), \sigma) = n(s, \sigma); \\ & c_i(M_i(s), \sigma) = 0 \\ \phi, & \text{otherwise.} \end{cases}$$

- If $\sigma \in \Sigma_{uc}$, $\{S_i\}_{i \in I}(s, \sigma) = 1$.

In other words, for a controllable event a global control decision is taken to be the same as the minimum ambiguity level local control decision.

In order to ensure that none of the global control decisions are "null", we next introduce the notion of "admissibility" of a decentralized supervisor. Also another useful notion of a decentralized supervisor is the largest ambiguity level $N \in \mathcal{N}$ of any "winning" enablement or disablement decision, and we refer to such a supervisor to be "N-inferring".

Definition 1: For a language $K \subseteq L(G)$, a decentralized supervisor $\{S_i\}_{i \in I} : L(G) \times \Sigma \to C$ is said to be

- *admissible* if for any $s \in \overline{K}$ and any $\sigma \in \Sigma_c$ such that $s\sigma \in L(G)$, $\{S_i\}_{i \in I}(s, \sigma) \in \{0, 1\}$.
- *N-inferring* if for any $\sigma \in \Sigma_c$, either $n^d(\sigma) \le N$ or $n^e(\sigma) \le N$, where

$$\begin{aligned} n^d(\sigma) &:= \max_{\{s \in \overline{K} |\ s\sigma \in L(G) \wedge \{S_i\}_{i \in I}(s,\sigma)=0\}} n(s, \sigma), \\ n^e(\sigma) &:= \max_{\{s \in \overline{K} |\ s\sigma \in L(G) \wedge \{S_i\}_{i \in I}(s,\sigma)=1\}} n(s, \sigma). \end{aligned}$$

Note in above, $n^d(\sigma)$ is the maximum ambiguity level of any winning disablement decision, whereas $n^e(\sigma)$ is the maximum ambiguity level of any winning enablement decision.

The generated language $L(\{S_i\}_{i \in I}/G)$ under the control action of an admissible decentralized supervisor $\{S_i\}_{i \in I}$ is defined inductively as follows:

- $\varepsilon \in L(\{S_i\}_{i \in I}/G)$,
- $(\forall s \in L(\{S_i\}_{i \in I}/G), \forall \sigma \in \Sigma)\ s\sigma \in L(\{S_i\}_{i \in I}/G) \Leftrightarrow [s\sigma \in L(G) \wedge \{S_i\}_{i \in I}(s, \sigma) = 1]$.

The marked language $L_m(\{S_i\}_{i \in I}/G)$ is defined as $L_m(\{S_i\}_{i \in I}/G) := L(\{S_i\}_{i \in I}/G) \cap L_m(G)$. If $L(\{S_i\}_{i \in I}/G) = \overline{L_m(\{S_i\}_{i \in I}/G)}$, then $\{S_i\}_{i \in I}$ is said to be *nonblocking*.

IV. Existence of Inference-based Decentralized Control

In this section we introduce the notion of N-inference-observability in order to characterize the class of languages achievable under the control of an admissible and N-inferring decentralized supervisor. For this, given a specification $K \subseteq L(G)$ of the plant language, we divide $\overline{K}$ into a set of language pairs, one pair for each controllable event

$\sigma \in \Sigma_c$. The set $D_0(\sigma) \subseteq \overline{K}$ is the set of traces in $\overline{K}$ where σ must be disabled, whereas the set $E_0(\sigma) \subseteq \overline{K}$ is the set of traces where σ must be enabled. Using these as the base step, we inductively define a monotonically decreasing sequence of language pairs $(D_k(\sigma), E_k(\sigma))$ as follows:

- Base step:

$$\begin{aligned} D_0(\sigma) &:= \{s \in \overline{K} \mid s\sigma \in L(G) - \overline{K}\}, \\ E_0(\sigma) &:= \{s \in \overline{K} \mid s\sigma \in \overline{K}\}. \end{aligned}$$

- Induction step:

$$\begin{aligned} & D_{k+1}(\sigma) \\ &:= D_k(\sigma) \cap \left(\bigcap_{i \in In(\sigma)} M_i^{-1} M_i(E_k(\sigma)) \right), \\ & E_{k+1}(\sigma) \\ &:= E_k(\sigma) \cap \left(\bigcap_{i \in In(\sigma)} M_i^{-1} M_i(D_k(\sigma)) \right). \end{aligned}$$

Note that $D_{k+1}(\sigma)$ is a sublanguage of $D_k(\sigma)$ consisting of those traces for which for each $i \in In(\sigma)$ there exists an M_i-indistinguishable trace in $E_k(\sigma)$. As a result all the local supervisors that have control over σ will be ambiguous about their control decision for σ following the execution of a trace in $D_{k+1}(\sigma)$ (and as we will see below their ambiguity level will be at least as high as $k+1$). The sublanguage $E_{k+1}(\sigma)$ of $E_k(\sigma)$ can be understood in a similar fashion.

Then we have the following definition of N-inference-observability.

Definition 2: A language $K \subseteq L(G)$ is said to be *N-inference-observable* if for any $\sigma \in \Sigma_c$, $D_{N+1}(\sigma) = \emptyset$ or $E_{N+1}(\sigma) = \emptyset$.

The following theorem shows the necessity and sufficiency of N-inference-observability for the existence of an admissible and N-inferring decentralized supervisor enforcing the given specification.

Theorem 1: For a nonempty language $K \subseteq L_m(G)$, there exists a nonblocking, admissible, and N-inferring decentralized supervisor $\{S_i\}_{i \in I} : L(G) \times \Sigma \to C$ such that $L_m(\{S_i\}_{i \in I}/G) = K$ if and only if K is $L_m(G)$-closed, controllable, and N-inference-observable.

Assume that a language $K \subseteq L(G)$ is N-inference-observable so that for each $\sigma \in \Sigma_c$ either $D_{N+1}(\sigma) = \emptyset$ or $E_{N+1}(\sigma) = \emptyset$. Note that the former implies $E_{N+2}(\sigma) = \emptyset$, whereas the latter implies $D_{N+2}(\sigma) = \emptyset$. Knowing that a specification language is N-inference-observable, a local supervisor can compute its control decision and associate a level of ambiguity with it as follows. For each $s \in L(G)$ and $\sigma \in \Sigma_{ic}$, the ith local supervisor S_i computes

$$\begin{aligned} & n_i^d(M_i(s), \sigma) \\ &\quad := \min\{k \in \mathcal{N} \mid M_i(s) \notin M_i(E_k(\sigma))\}, \quad (1) \\ & n_i^e(M_i(s), \sigma) \\ &\quad := \min\{k \in \mathcal{N} \mid M_i(s) \notin M_i(D_k(\sigma))\}. \quad (2) \end{aligned}$$

Note that since $D_{N+1}(\sigma) = E_{N+2}(\sigma) = \emptyset$ or $E_{N+1}(\sigma) = D_{N+2}(\sigma) = \emptyset$, both $n_i^d(M_i(s), \sigma)$ and $n_i^e(M_i(s), \sigma)$ are bounded above by $N+2$. Here $n_i^d(M_i(s), \sigma)$ denotes the minimum index k such that the observation $M_i(s)$ does not match with the observations of any of the traces in $E_k(\sigma)$. (Note that by virtue of N-inference-observability, such an index cannot exceed $N+2$.) Then $n_i^d(M_i(s), \sigma)$ represents the ambiguity level of a disablement decision "contemplated" by the ith supervisor for the event σ following the observation $M_i(s)$. Similarly, the notation $n_i^e(M_i(s), \sigma)$ represents the ambiguity level of an enablement decision "contemplated" by the ith supervisor for the event σ following the observation $M_i(s)$. Which of the two contemplated decisions is ultimately issued is decided by comparing the two ambiguity levels, $n_i^d(M_i(s), \sigma)$ vs. $n_i^e(M_i(s), \sigma)$, and favoring the smaller one. This is formalized next.

For a local supervisor $S_i : M_i(L(G)) \times \Sigma_{ic} \to C \times \mathcal{N}$, its control decision and ambiguity level for a controllable event $\sigma \in \Sigma_{ic}$ following an observation $M_i(s) \in M_i(L(G))$, i.e., $S_i(M_i(s), \sigma) = (c_i(M_i(s), \sigma), n_i(M_i(s), \sigma))$, is determined as follows:

$$\begin{aligned} & c_i(M_i(s), \sigma) \\ &\quad := \begin{cases} 1, & \text{if } n_i^e(M_i(s), \sigma) < n_i^d(M_i(s), \sigma) \\ 0, & \text{if } n_i^d(M_i(s), \sigma) < n_i^e(M_i(s), \sigma) \\ \phi, & \text{if } n_i^e(M_i(s), \sigma) = n_i^d(M_i(s), \sigma) \end{cases} \quad (3) \end{aligned}$$

and

$$n_i(M_i(s), \sigma) := \min\{n_i^d(M_i(s), \sigma), n_i^e(M_i(s), \sigma)\}. \quad (4)$$

The decentralized control for which the local supervisors are given by (1)–(4) enforces the given specification K whenever K satisfies the conditions of Theorem 1.

The following example illustrates a 2-inference-observable language that is not 1-inference-observable.

Example 1: We consider a DES modeled by the automaton G shown in Fig. 1(a), which is a modified version of the DES considered in [14]. A double circle is used to identify a marked state. Let $n = 2$, $\Sigma_c = \Sigma_{1c} = \Sigma_{2c} = \{c\}$,

$$\begin{aligned} M_1(\sigma) &= \begin{cases} \sigma, & \text{if } \sigma \in \{a, a', d\} \\ \varepsilon, & \text{otherwise,} \end{cases} \\ M_2(\sigma) &= \begin{cases} \sigma, & \text{if } \sigma \in \{b, b', d\} \\ \varepsilon, & \text{otherwise.} \end{cases} \end{aligned}$$

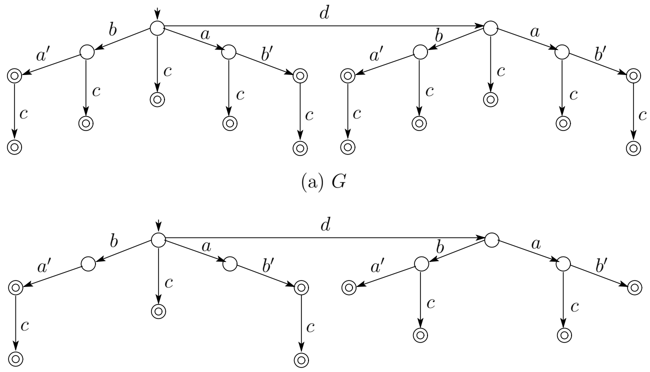
Also, let $K \subseteq L(G)$ be a language marked by the automaton G_K shown in Fig. 1(b). Clearly, K is $L_m(G)$-closed and controllable.

We show that K is 2-inference-observable. Initially, we have

$$\begin{aligned} D_0(c) &= \{a, b, d, dab', dba'\}, \\ E_0(c) &= \{\varepsilon, ab', ba', da, db\}. \end{aligned}$$

Since

$$\begin{aligned} D_1(c) &= \{a, b, d\} \neq \emptyset, \\ E_1(c) &= \{\varepsilon, da, db\} \neq \emptyset, \\ D_2(c) &= \{d\} \neq \emptyset, \\ E_2(c) &= \{\varepsilon\} \neq \emptyset, \end{aligned}$$



(a) G

(b) G_K

Fig. 1. Automata G and G_K of Example 1.

K is neither 0-inference-observable nor 1-inference-observable. Also, we have

$$D_3(c) = \emptyset,\ E_3(c) = \emptyset.$$

Thus, K is 2-inference-observable.

The local decisions of S_1 and S_2 computed using (1)–(4) are shown in Table I. For example, $S_1(\varepsilon, c)$ is computed as follows. By (1) and (2), we have $n_1^d(\varepsilon, c) = 3$ and $n_1^e(\varepsilon, c) = 2$. Since $2 = n_1^e(\varepsilon, c) < n_1^d(\varepsilon, c) = 3$, we have $c_1(\varepsilon, c) = 1$ and $n_1(\varepsilon, c) = 2$, which implies that S_1 enables c initially with the ambiguity level 2.

TABLE I

LOCAL DECISIONS OF S_1 AND S_2.

$t \in M_1(L(G))$	$n_1^d(t,c)$	$n_1^e(t,c)$	$c_1(t,c)$	$n_1(t,c)$
ε	3	2	1	2
a	1	2	0	1
a'	1	0	1	0
d	2	3	0	2
da	2	1	1	1
da'	0	1	0	0
$t \in M_2(L(G))$	$n_2^d(t,c)$	$n_2^e(t,c)$	$c_2(t,c)$	$n_2(t,c)$
ε	3	2	1	2
b	1	2	0	1
b'	1	0	1	0
d	2	3	0	2
db	2	1	1	1
db'	0	1	0	0

Then, the global control decisions of the decentralized supervisor $\{S_i\}_{i\in I}$ on c are computed as shown in Table II. For example, $\{S_i\}_{i\in I}(a, c)$ is computed as follows. Since $1 = n_1(M_1(a), c) < n_2(M_2(a), c) = 2$ and $c_1(M_1(a), c) = 0$, we have $\{S_i\}_{i\in I}(a, c) = 0$. Also, for any $s \in L(G)$ and any $\sigma \in \Sigma_{uc}$, we have $\{S_i\}_{i\in I}(s, \sigma) = 1$. By Theorem 1, $\{S_i\}_{i\in I}$ is a nonblocking, admissible, and 2-inferring decentralized supervisor such that $L_m(\{S_i\}_{i\in I}/G) = K$.

Remark 1: We discuss how to verify N-inference-observability of a regular language $K \subseteq L(G)$ under the assumption that G is a finite automaton. The verification of N-inference-observability requires sequentially computing the language pair $(D_k(\sigma), E_k(\sigma))$ and checking their

TABLE II

GLOBAL DECISIONS OF $\{S_i\}_{i\in I}$.

$s \in L(G)$	$n(s,c)$	$\{S_i\}_{i\in I}(s,c)$
ε, c	2	1
a, ac	1	0
b, bc	1	0
ab', $ab'c$	0	1
ba', $ba'c$	0	1
d, dc	2	0
da, dac	1	1
db, dbc	1	1
dab', $dab'c$	0	0
dba', $dba'c$	0	0

emptiness for each $\sigma \in \Sigma_c$. Since checking emptiness of a language can be done linearly in the size of an acceptor of the language, we only discuss the computation of certain acceptors for the above sequence of language pairs. Let $R = (X, \Sigma, \xi, x_0, X_m)$ be a finite trim acceptor of K. Then, $L_m(R) = K$ and $L(R) = \overline{K}$ hold.

The language $E_0(\sigma)$ is computed by replacing the marked state set X_m with $X^\sigma := \{x \in X |\ \xi(\sigma, x) \text{ is defined}\}$. For the finite automaton $R^\sigma := (X, \Sigma, \xi, x_0, X^\sigma)$, we have $L_m(R^\sigma) = E_0(\sigma)$. Complexity of computing $E_0(\sigma)$ is $O(|X|)$. For computing $D_0(\sigma)$, we construct the synchronous composition $G \parallel R = (Z, \Sigma, \gamma, z_0, Z_m)$ of G and R. Then $L(G \parallel R) = \overline{K}$ holds. Let $Z^\sigma := \{(q, x) \in Z |\ \delta(\sigma, q) \text{ is defined, but } \xi(\sigma, x) \text{ is not}\}$ for each $\sigma \in \Sigma_c$. Then, for the finite automaton $(G \parallel R)^\sigma := (Z, \Sigma, \gamma, z_0, Z^\sigma)$, we have $L_m((G \parallel R)^\sigma) = D_0(\sigma)$. Complexity of computing $D_0(\sigma)$ is $O(|Q| \cdot |X|)$.

Let $R_{D_k(\sigma)}$ and $R_{E_k(\sigma)}$ be finite acceptors of $D_k(\sigma)$ and $E_k(\sigma)$, respectively. For each $i \in I$, a finite acceptor of $M_i^{-1}M_i(D_k(\sigma))$ is constructed as follows: Replicate each transition that exists in $R_{D_k(\sigma)}$ by a set of transitions on all M_i-indistinguishable events. Note that since an ε-transition is implicitly defined at each state as a self-loop, unobservable events will get added as self-loops at each state of $R_{D_k(\sigma)}$. Then, the resulting, possibly nondeterministic, automaton accepts $M_i^{-1}M_i(D_k(\sigma))$. It should be noted that this resulting automaton, denoted by $M_i^{-1}M_i(R_{D_k(\sigma)})$, has the same state set as $R_{D_k(\sigma)}$. In the same way, we can construct a finite automaton accepting $M_i^{-1}M_i(E_k(\sigma))$, denoted by $M_i^{-1}M_i(R_{E_k(\sigma)})$. Then, the synchronous compositions $R_{D_k(\sigma)} \parallel (\parallel_{i\in In(\sigma)} M_i^{-1}M_i(R_{E_k(\sigma)}))$ and $R_{E_k(\sigma)} \parallel (\parallel_{i\in In(\sigma)} M_i^{-1}M_i(R_{D_k(\sigma)}))$ accept $D_{k+1}(\sigma)$ and $E_{k+1}(\sigma)$, respectively. Let $X_{D_k(\sigma)}$ and $X_{E_k(\sigma)}$ be the state sets of $R_{D_k(\sigma)}$ and $R_{E_k(\sigma)}$, respectively. The languages $D_{k+1}(\sigma)$ and $E_{k+1}(\sigma)$ are computed from $D_k(\sigma)$ and $E_k(\sigma)$ in $O(|X_{D_k(\sigma)}| \cdot |X_{E_k(\sigma)}|^{|In(\sigma)|})$ and $O(|X_{E_k(\sigma)}| \cdot |X_{D_k(\sigma)}|^{|In(\sigma)|})$, respectively.

V. PROPERTIES OF N-INFERENCE-OBSERVABILITY

Since the sequence of pairs $(D_k(\sigma), E_k(\sigma))$ is monotonically decreasing, the following result is easily obtained.

Theorem 2: For any $N \in \mathcal{N}$, if a language $K \subseteq L(G)$ is N-inference-observable, then it is $(N+1)$-inference-observable.

The converse relation of Theorem 2 need not hold. For example, the language $K \subseteq L(G)$ of Example 1 is 2-inference-observable, but not 1-inference-observable.

We show that both coobservability and conditional coobservability defined in [13] and [14], respectively, are special cases of N-inference-observability.

Definition 3: [13] A language $K \subseteq L(G)$ is said to be

- *C&P-coobservable* with respect to $A \subseteq \Sigma_c$ if for any $s \in \overline{K}$ and any $\sigma \in A$ with $s\sigma \in L(G) - \overline{K}$, there exists $i \in In(\sigma)$ such that
$$(M_i^{-1}M_i(s) \cap \overline{K})\{\sigma\} \cap \overline{K} = \emptyset.$$
- *D&A-coobservable* with respect to $A \subseteq \Sigma_c$ if for any $s \in \overline{K}$ and any $\sigma \in A$ with $s\sigma \in \overline{K}$, there exists $i \in In(\sigma)$ such that
$$(M_i^{-1}M_i(s) \cap \overline{K})\{\sigma\} \cap L(G) \subseteq \overline{K}.$$
- *C&P$\vee$D&A-coobservable* if there exists a partition $\{\Sigma_{c,e}, \Sigma_{c,d}\}$ of Σ_c such that K is C&P-coobservable with respect to $\Sigma_{c,e}$ and D&A-coobservable with respect to $\Sigma_{c,d}$.

Definition 4: [14] A language $K \subseteq L(G)$ is said to be

- *conditionally C&P-coobservable* with respect to $A \subseteq \Sigma_c$ if for any $s \in \overline{K}$ and any $\sigma \in A$ with $s\sigma \in L(G) - \overline{K}$, there exists $i \in In(\sigma)$ such that
$$\begin{aligned}&(\forall s_i\sigma \in (M_i^{-1}M_i(s) \cap \overline{K})\{\sigma\} \cap \overline{K})\\ &\qquad \exists j \in In(\sigma); (M_j^{-1}M_j(s_i) \cap \overline{K})\{\sigma\} \cap L(G) \subseteq \overline{K}.\end{aligned}$$
- *conditionally D&A-coobservable* with respect to $A \subseteq \Sigma_c$ if for any $s \in \overline{K}$ and any $\sigma \in A$ with $s\sigma \in \overline{K}$, there exists $i \in In(\sigma)$ such that
$$\begin{aligned}&(\forall s_i\sigma \in (M_i^{-1}M_i(s) \cap \overline{K})\{\sigma\} \cap (L(G) - \overline{K}))\\ &\qquad \exists j \in In(\sigma); (M_j^{-1}M_j(s_i) \cap \overline{K})\{\sigma\} \cap \overline{K} = \emptyset.\end{aligned}$$
- *conditionally C&P$\vee$D&A-coobservable* if there exists a partition $\{\Sigma_{c,e}, \Sigma_{c,d}\}$ of Σ_c such that K is conditionally C&P-coobservable with respect to $\Sigma_{c,e}$ and conditionally D&A-coobservable with respect to $\Sigma_{c,d}$.

The following theorem establishes the equivalence of C&P$\vee$D&A-coobservability and zero-inference-observability.

Theorem 3: For any language $K \subseteq L(G)$, K is C&P$\vee$D&A-coobservable if and only if it is 0-inference-observable.

The next theorem establishes the equivalence of conditional C&P$\vee$D&A-coobservability and unity-inference-observability.

Theorem 4: For any language $K \subseteq L(G)$, K is conditionally C&P$\vee$D&A-coobservable if and only if it is 1-inference-observable.

VI. Conclusion

A key issue in decentralized decision-making is the "fusion" of the local decisions to arrive at a global decision. We present a new framework for such decision fusion, particularly applied to the setting of decentralized control of DESs. The main idea is to realize that the sensing limitations can create ambiguities of *differing gradations* at various local sites that are engaged in decision-making; use inferencing to assess the gradations of the ambiguities; and utilize this knowledge in forming a local decision. Each local decision is to be tagged with a certain grade of ambiguity. Then, a global decision is taken to be the same as a local decision carrying the minimum grade of ambiguity.

The proposed framework seems to be a more natural way of approaching decentralized control—It does not require the existence of any a-priori partition of the controllable events into permissive/anti-permissive sets with which certain default control decisions are required to be associated, nor does it require a fusion based on conjunction/disjunction taken over the set of local decisions. Also, the proposed framework is able to subsume the various decision fusion architectures examined in the prior works. It will be instructive to apply the proposed framework to other instances of decentralized decision-making problems. It should be noted that as the order of inferencing incorporated into decentralized decision-making is enhanced, the corresponding cost of computing the local decisions is also increased (as formalized in Remark 1). So the additional gain resulting from a higher order of inferencing comes at an additional computational cost.

References

[1] G. Barrett and S. Lafortune, "Decentralized supervisory control with communicating controllers," *IEEE Trans. Automat. Contr.*, vol. 45, no. 9, pp. 1620–1638, 2000.

[2] R. Cieslak, C. Desclaux, A. S. Fawaz, and P. Varaiya, "Supervisory control of discrete-event processes with partial observations," *IEEE Trans. Automat. Contr.*, vol. 33, no. 3, pp. 249–260, 1988.

[3] R. Kumar and M. A. Shayman, "Centralized and decentralized supervisory control of nondeterministic systems under partial observation," *SIAM J. Contr. Optim.*, vol. 35, no. 2, pp. 363–383, 1997.

[4] F. Lin and W. M. Wonham, "Decentralized supervisory control of discrete-event systems," *Inform. Sci.*, vol. 44, no. 3, pp. 199–224, 1988.

[5] A. Overkamp and J. H. van Schuppen, "Maximal solutions in decentralized supervisory control," *SIAM J. Contr. Optim.*, vol. 39, no. 2, pp. 492–511, 2001.

[6] J. H. Prosser, M. Kam, and H. G. Kwatny, "Decision fusion and supervisor synthesis in decentralized discrete-event systems," in *Proc. 1997 American Contr. Conf.*, pp. 2251–2255, 1997.

[7] P. J. Ramadge and W. M. Wonham, "Supervisory control of a class of discrete-event processes," *SIAM J. Contr. Optim.*, vol. 25, no. 1, pp. 206–230, 1987.

[8] S. L. Ricker and K. Rudie, "Know means no: Incorporating knowledge into discrete-event control systems," *IEEE Trans. Automat. Contr.*, vol. 45, no. 9, pp. 1656–1668, 2000.

[9] S. L. Ricker and K. Rudie, "Knowledge is a terrible thing to waste: Using inference in discrete-event control problems," in *Proc. 2003 American Contr. Conf.*, pp. 2246–2251, 2003.

[10] K. Rudie and W. M. Wonham, "Think globally, act locally: Decentralized supervisory control," *IEEE Trans. Automat. Contr.*, vol. 37, no. 11, pp. 1692–1708, 1992.

[11] W. Qiu, R. Kumar, and V. Chandra, "Decentralized control of discrete event systems using prioritized composition with exclusion," in *Proc. 2004 American Contr. Conf.*, pp. 4483–4487, 2004.

[12] Y. Willner and M. Heymann, "Supervisory control of concurrent discrete-event systems," *Int. J. Contr.*, vol. 54, no. 5, pp. 1143–1169, 1991.

[13] T.-S. Yoo and S. Lafortune, "A general architecture for decentralized supervisory control of discrete-event systems," *Discrete Event Dynamic Syst.: Theory and Appl.*, vol. 12, no. 3, pp. 335–377, 2002.

[14] T.-S. Yoo and S. Lafortune, "Decentralized supervisory control with conditional decisions: Supervisor existence", *IEEE Trans. Automat. Contr.*, vol. 49, no. 11, pp. 1886–1904, 2004.