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Adaptive way-point tracking control for underactuated autonomous vehicles

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Abstract— **This paper presents a control strategy for global κ-exponential way-point tracking control of a class of underactuated mechanical systems with movement in six degrees of freedom. We use a cascaded approach and find sufficient conditions for global κ-exponential convergence to the desired way-point. A backstepping-based method for synthesizing controllers that satisfy the developed dynamical constraints is presented, and it is shown how adaption can be included to counteract environmental disturbances in all six degrees of freedom while guaranteeing global asymptotic way-point tracking. A case study presenting simulation results for the method applied to a model of the HUGIN AUV is presented.**

I. INTRODUCTION

The desired trajectory of an autonomous vehicle is commonly expressed in terms of way-points. The way-point description is attractive since the reference trajectory can be compactly stored onboard the vehicle and the reference trajectory generation does not put any significant burden on the onboard computer. The way-points can be pre-mission programmed or generated at run-time by using maps and sensors. Once the way-points are decided, it is usually desirable that the autonomous vehicle tracks the way-points as close as possible, even in the presence of unknown environmental disturbances.

Way-point tracking has been studied by numerous authors in recent years. Systems studied include wheeled mobile robots, surface ships and autonomous underwater vehicles. Most of the available results consider planar way-point tracking, leaving the altitude (or depth) of the vehicle constant. For a vehicle with movement in six degrees of freedom, the motion is highly coupled and nonlinear. Thus decoupling the altitude control and the planar way-point tracking control may result in poor performance. For systems experiencing quick maneuvers and high speed movement in three dimensional space, such as autonomous underwater vehicles and unmanned aerial vehicles, a 3-dimensional way-point tracking controller is preferable.

Review [1] considers way-point tracking control of underactuated autonomous underwater vehicles (AUVs) in the horizontal plane. Using a 3-DOF model model of the AUV, integrator backstepping is used to design a controller that guarantees asymptotic convergence to the way-points. The paper also propose a strategy for counteracting constant ocean currents. An exponentially stable observer for the current velocities is presented and the trajectories of the resulting observer-controller system is shown to be globally bounded. Experimental results with the Sirene AUV are presented.

Line Of Sight (LOS) based control of underactuated surface vessels are considered in [2] and [3]. Using a 3-DOF model of a surface vessel, a LOS guidance law and a *dynamic-state* backstepping controller is proposed. The LOS tracking error and the surge speed tracking error is proven to be GUAS and the internal controller dynamics are shown to be globally bounded.

LOS based control of marine surface vessels is further studied in [4], where positional convergence to straight lines and circles are proven for an ideal particle. Using this as theoretical justification for the LOS guidance principle, the controllers of [2] and [3] are used in a case study with a model ship.

[5] considers way-point tracking control of ships using yaw torque control. A full state feedback control law is developed using a cascaded approach, and global asymptotical stability of the heading and cross-track error is proven.

A similar approach to [5] is also taken in [6], where LOS based way-point maneuvering for underactuated surface ships is studied. A controller is synthesized using integrator backstepping and designed to obtain a cascaded structure of the closed loop system. Using nonlinear cascaded systems theory, global κ-exponential stability of the surge speed, the heading and the cross-track error is proven.

[7] consider position regulation of nonholonomic mobile robots. A discontinuous, adaptive, full state feedback controller is derived and the closed loop system is shown to be globally convergent. Even though the problem studied in [7] is a regulation problem, the proposed control strategy is directly applicable to way-point tracking control by using the concept of *circle-of-acceptance*.

Control of nonholonomic robots are also studied in [8], where trajectory tracking for nonholonomic mobile robots are studied. The paper presents an algorithm for generating way-points from witch smooth reference trajectories are generated. A nonlinear feedback control law is presented that asymptotically stabilizes the closed loop system.

The purpose of this paper is to present a solution to the 3D way-point tracking problem for a class of underactuated mechanical systems. Using a simple LOS-based guidance law, we find conditions on the closed loop dynamics that must be satisfied in order to guarantee convergence to the desired way-point. Our cascaded approach gives the system a *separation property* where the convergence properties of

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the overall system are independent of the particular choice of controller, as long as the dynamical conditions are satisfied. This separation property gives a better understanding of what contributes to the convergence of the overall system. Moreover, it makes the result valid for a whole class of controllers and not only the particular controller presented in this paper.

In this paper, we address the problem of disturbance compensation for an underactuated vehicle. It has long been an open question whether it is possible to counteract the effects of environmental disturbances in all degrees of freedom for an underactuated vehicle. [1] and [9] gives some preliminary results for marine vehicles, where a nonlinear controller-observer scheme is used to estimate and counteract unknown ocean currents. [10] present an adaptive approach for marine vehicles with unknown damping coefficient. Another adaptive approach, valid for a larger class of disturbances, is considered in [11]. Finally, an interesting approach is taken by [12], where bounded, but unknown, environmental disturbances are estimated like unknown model parameters.

In this paper, we propose a Lyapunov-based adaptive scheme for estimating unknown (constant) disturbances. Different from [11], we show that by exploiting the intrinsic structure of the system, we can impose some indirect control over the underactuaed degrees of freedom and thereby counteract constant environmental disturbances. This is done by a special *twist* to the usual backstepping procedure.

This paper is organized as follows: Section II presents the vehicle model and the control objective. Section III presents the main stability proofs and Section IV presents an adaptive control strategy based on the backstepping procedure. Section V presents a case study with the HUGIN AUV.

II. THE VEHICLE MODEL AND CONTROL OBJECTIVE

We consider a class of underactuated mechanical systems that can be described by the 6-DOF model

$$\dot{\boldsymbol{\eta}} = \boldsymbol{J}(\boldsymbol{\eta})\boldsymbol{\nu} \tag{1}$$

$$\boldsymbol{M}\dot{\boldsymbol{\nu}} + \boldsymbol{C}(\boldsymbol{\nu})\boldsymbol{\nu} + \boldsymbol{D}(\boldsymbol{\nu})\boldsymbol{\nu} + \boldsymbol{g}(\boldsymbol{\eta}) = \boldsymbol{\tau} + \boldsymbol{J}^{-1}(\boldsymbol{\eta})\boldsymbol{w} \tag{2}$$

where

$$\boldsymbol{\eta} = \begin{pmatrix} x & y & z & \phi & \theta & \psi \end{pmatrix}^T$$
$$\boldsymbol{\nu} = \begin{pmatrix} u & v & w & p & q & r \end{pmatrix}^T$$

The vector $\boldsymbol{\eta} \in \boldsymbol{R}^6$ describes the position and orientation of the system in an inertial reference frame, and the vector $\boldsymbol{\nu} \in \boldsymbol{R}^6$ describes the linear and angular velocities of the system in the body frame. The matrix $\boldsymbol{J}(\boldsymbol{\eta})$ is the kinematic transformation matrix, $\boldsymbol{M}$ is the mass and inertia matrix, $\boldsymbol{C}(\boldsymbol{\nu})$ is the Coriolis and centripetal matrix, $\boldsymbol{D}(\boldsymbol{\nu})$ is the damping and friction matrix and $\boldsymbol{g}(\boldsymbol{\eta})$ accounts for forces and moments caused by gravitation (and possibly buoyancy). Finally, the vector $\boldsymbol{w}$ describes constant, but unknown, environmental forces acting on the system and the vector $\boldsymbol{\tau}$ is the control input. The vehicle is assumed to have independent control in surge, pitch and yaw (i.e. τ_1, τ_5 and τ_6 can be controlled independently), and the dynamical model (2) is assumed to satisfy the following properties:

$$\dot{\boldsymbol{M}} = \boldsymbol{0}_{6\times 6} \text{ and } \boldsymbol{x}^T\boldsymbol{M}\boldsymbol{x} > 0 \quad , \boldsymbol{x} \in \boldsymbol{R}^6\backslash\{\boldsymbol{0}\} \tag{3}$$

$$\dot{\boldsymbol{w}} = \boldsymbol{0} \tag{4}$$

The differential vehicle kinematics (1) relates the body-fixed velocities $\boldsymbol{\nu}$ to the inertial velocities $\dot{\boldsymbol{\eta}}$ through the following six nonlinear differential equations ([13]):

$$\begin{aligned} \dot{x} = &\cos\psi\cos\theta u + (\cos\psi\sin\theta\sin\phi - \sin\psi\cos\phi)v \\ &+ (\sin\psi\sin\phi + \cos\psi\cos\phi\sin\theta)w \end{aligned} \tag{5}$$

$$\begin{aligned} \dot{y} = &\sin\psi\cos\theta u + (\cos\psi\cos\phi + \sin\phi\sin\theta\sin\psi)v \\ &+ (\sin\theta\sin\psi\cos\phi - \cos\psi\sin\phi)w \end{aligned} \tag{6}$$

$$\dot{z} = -\sin\theta u + \cos\theta\sin\phi v + \cos\theta\cos\phi w \tag{7}$$

$$\dot{\phi} = p + \sin\phi\tan\theta q + \cos\phi\tan\theta r \tag{8}$$

$$\dot{\theta} = \cos\phi q - \sin\phi r \tag{9}$$

$$\dot{\psi} = \frac{\sin\phi}{\cos\theta}q + \frac{\cos\phi}{\cos\theta}r \tag{10}$$

The objective of this paper is to design a control system that force the underactuated vehicle to converge to the the desired way-point while satisfying a, possible time-varying, surge velocity assignment. In particular, we seek to develop a state feedback control law $\boldsymbol{\tau} = \boldsymbol{\tau}(\boldsymbol{\eta}, \boldsymbol{\nu})$ such that

$$(x, y, z) \rightarrow (x_d, y_d, z_d) \text{ and } u(t) \rightarrow u_d(t) \text{ as } t \rightarrow \infty$$

where (x_d, y_d, z_d) is the position of the desired way-point and $u_d(t)$ is the desired surge velocity.

III. GLOBAL κ-EXPONENTIAL WAY-POINT TRACKING

The way-points are assumed to be a set of fixed points in space, given in Cartesian coordinates. Let the position of the desired way-point, the way-point the vehicle is currently tracking, be denoted by (x_d, y_d, z_d). Inspired by [1], we choose the Euclidian metric as a measure of distance and define the way-point tracking error according to

$$e \triangleq \sqrt{(x - x_d)^2 + (y - y_d)^2 + (z - z_d)^2} \tag{11}$$

With reference to Figure 1 we then have the following relationship between the position variables and the way-point tracking error e:

$$x - x_d = -e\cos\theta_d\cos\psi_d \tag{12}$$

$$y - y_d = -e\cos\theta_d\sin\psi_d \tag{13}$$

$$z - z_d = e\sin\theta_d \tag{14}$$

where θ_d and ψ_d are defined as

$$\theta_d = \tan^{-1}\left(\frac{z - z_d}{\sqrt{(x - x_d)^2 + (y - y_d)^2}}\right) \tag{15}$$

$$\psi_d = \tan^{-1}\left(\frac{y - y_d}{x - x_d}\right) \tag{16}$$

Eq. (15) and (16) are referred to as the LOS angles. We define the LOS angle tracking errors and the surge speed tracking error as

$$\tilde{\theta} \triangleq \theta - \theta_d \qquad \tilde{\psi} \triangleq \psi - \psi_d \qquad \tilde{u} \triangleq u - u_d \tag{17}$$

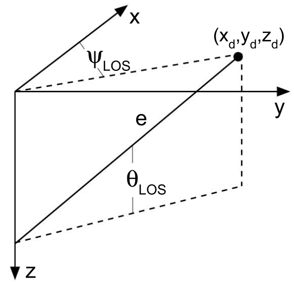


Fig. 1. LOS angles θ_d and ψ_d.

where $u_d(t)$ is the desired surge speed.

In order to obtain the way-point tracking error dynamics, we differentiate (11) with respect to time and use (12-14) and (17):

$$\begin{aligned}\dot{e} &= \frac{1}{e}\left(-e\cos\theta_d\cos\psi_d\dot{x} - e\cos\theta_d\sin\psi_d\dot{y} + e\sin\theta_d\dot{z}\right)\\ &= (-\cos\tilde{\psi}\cos\theta\cos\theta_d - \sin\theta\sin\theta_d)(\tilde{u}+u_d) +\\ &\quad(\sin\tilde{\psi}\cos\phi\cos\theta_d - \cos\tilde{\psi}\cos\theta_d\sin\theta\sin\phi\\ &\qquad+\sin\theta_d\cos\theta\sin\phi)v +\\ &\quad(-\sin\tilde{\psi}\cos\theta_d\sin\phi - \cos\tilde{\psi}\cos\theta_d\sin\theta\cos\phi\\ &\qquad+\sin\theta_d\cos\theta\cos\phi)w \end{aligned} \tag{18}$$

Factoring (18) with respect to $\tilde{\theta}$, $\tilde{\psi}$ and $\tilde{u}$, then gives:

$$\begin{aligned}\dot{e} &= -u_d\\ &\quad+\left(-\frac{\sin\tilde{\theta}}{\tilde{\theta}}(\sin\phi v + \cos\phi w) + \frac{1-\cos\tilde{\theta}}{\tilde{\theta}}u_d\right)\tilde{\theta}\\ &\quad+\left(\frac{\sin\tilde{\psi}}{\tilde{\psi}}(\cos\phi\cos\theta_d v - \cos\theta_d\sin\phi w)\right.\\ &\qquad+(\cos\theta\cos\theta_d u_d + \cos\theta_d\sin\theta\sin\phi v\\ &\qquad\left.+\cos\theta_d\sin\theta\cos\phi w)\frac{1-\cos\tilde{\psi}}{\tilde{\psi}}\right)\tilde{\psi}\\ &\quad+(-\cos\tilde{\psi}\cos\theta\cos\theta_d - \sin\theta\sin\theta_d)\tilde{u}\end{aligned} \tag{19}$$

Note that $\lim_{x\to 0}\frac{\sin x}{x} = 1$ and $\lim_{x\to 0}\frac{1-\cos x}{x} = 0$. Hence, (19) is well defined $\forall\ \tilde{\theta}, \tilde{\psi}, \tilde{u} \in \boldsymbol{R}\times\boldsymbol{R}\times\boldsymbol{R}$.

For now we assume that the time-development of $\tilde{\theta}(t)$, $\tilde{\psi}(t)$ and $\tilde{u}(t)$ is governed by the time-varying nonlinear system

$$\dot{\boldsymbol{\chi}} = \boldsymbol{f}(t,\boldsymbol{\chi}) \qquad \boldsymbol{\chi}\in\boldsymbol{R}^m, m\geq 3 \tag{20a}$$
$$\tilde{\boldsymbol{x}} = \boldsymbol{S}\boldsymbol{\chi} \tag{20b}$$

where $\tilde{\boldsymbol{x}} = \begin{pmatrix}\tilde{\theta} & \tilde{\psi} & \tilde{u}\end{pmatrix}^T$ and $\boldsymbol{S}\in\boldsymbol{R}^{3\times m}$ is a constant selection matrix. The system (20) represent the closed loop error dynamics of a controller to be designed. Based on (19), we state our proposition.

Proposition 1. *Let $u_d(t) > 0$ and $u_d(t)\in\mathcal{L}_\infty$, $\forall\ t\geq t_0$. If there exists a controller which makes the origin $\boldsymbol{\chi} = \mathbf{0}$ a globally κ-exponentially stable equilibrium point of (20), while guaranteeing $v(t), w(t)\in\mathcal{L}_\infty$, $\forall\ t\geq t_0$, the way-point tracking error e converges κ-exponentially to zero for all initial conditions $e(t_0)\in\mathbf{R}^+$.*

Proof. We view the signals $(\tilde{\theta},\tilde{\psi},\tilde{u})$ as perturbing inputs to the way-point tracking error dynamics (19), and rewrite (19) to obtain a cascaded structure:

$$\dot{e} = f_e(t) + \boldsymbol{g}(t,\boldsymbol{\chi})\boldsymbol{S}\boldsymbol{\chi} \tag{21a}$$
$$\dot{\boldsymbol{\chi}} = \boldsymbol{f}(t,\boldsymbol{\chi}) \tag{21b}$$

where $f_e(t)$ and $\boldsymbol{g}(t,\tilde{\boldsymbol{x}})$ are defined as

$$f_e(t) \triangleq -u_d(t)$$
$$\boldsymbol{g}(t,\boldsymbol{\chi}) \triangleq (g_1 \quad g_2 \quad g_3)$$

and

$$\begin{aligned}g_1 &= \frac{1-\cos\tilde{\theta}}{\tilde{\theta}}u_d - \frac{\sin\tilde{\theta}}{\tilde{\theta}}(\sin\phi v + \cos\phi w)\\ g_2 &= \frac{1-\cos\tilde{\psi}}{\tilde{\psi}}\Big(\cos(\tilde{\theta}+\theta_d)\cos\theta_d u_d\\ &\qquad+\cos\theta_d\sin(\tilde{\theta}+\theta_d)\sin\phi v\\ &\qquad+\cos\theta_d\sin(\tilde{\theta}+\theta_d)\cos\phi w\Big)\\ &\qquad+\frac{\sin\tilde{\psi}}{\tilde{\psi}}(\cos\phi\cos\theta_d v - \cos\theta_d\sin\phi w)\\ g_3 &= -\cos\tilde{\psi}\cos(\tilde{\theta}+\theta_d)\cos\theta_d - \sin(\tilde{\theta}+\theta_d)\sin\theta_d\end{aligned}$$

The way-point tracking error dynamics (21a) is not defined at the origin $e = 0$, since (15) and (16) are both undefined at $e = 0$. Due to this, we cannot show *stability* of the way-point tracking dynamics to the origin. We can however show *convergence* to the origin. That is, we can show that given $\epsilon > 0$, $\exists\ T = T(\epsilon) > 0$ such that

$$||e(t)|| \leq \epsilon \qquad \forall t_0 + T \leq t < t_e$$

where $e(t)$ is defined on the interval $t\in[t_0, t_e)$.

The cascaded system (21) can be seen as the nominal system

$$\Sigma_1: \qquad \dot{e} = f_e(t)$$

perturbed by the output of the globally κ-exponentially stable system

$$\Sigma_2: \qquad \dot{\boldsymbol{\chi}} = \boldsymbol{f}(t,\boldsymbol{\chi}) \tag{22}$$

through the interconnection $\boldsymbol{g}(t,\boldsymbol{\chi})\boldsymbol{S}$.

To show convergence of the way-point tracking error to zero, we note that for any finite $r > 0$ and $r > \epsilon > 0$, the upper right hand derivative of $e(t)$ of the nominal system Σ_1 satisfy

$$D^+e(t) \leq \begin{cases} -k & e\geq r\\ -\gamma e & \epsilon < e\leq r\end{cases} \tag{23}$$

where $0 < k \leq \min u_d(t)$, $\gamma = \frac{k}{r} > 0$ and $D^+e(t)$ denotes the upper right hand derivative of $e(t)$. Since $\epsilon > 0$ can be arbitrarily small, we define a new continuous and locally Lipschitz function

$$f_2(e) \triangleq \begin{cases} -k & e \geq r \\ -\gamma e & e \leq r \end{cases} \tag{24}$$

and analyze the *stability* of the modified cascaded system

$$\dot{e}_2 = f_2(e_2) + \boldsymbol{g}(t,\boldsymbol{\chi})\boldsymbol{S}\boldsymbol{\chi} \tag{25a}$$

$$\dot{\boldsymbol{\chi}} = \boldsymbol{f}(t,\boldsymbol{\chi}) \tag{25b}$$

where $e_2(t_0) = e(t_0) > 0$. The idea is then to use the Comparison Lemma ([14], Lemma 3.4) to prove the proposition for the original cascaded system (21).

We start by analyzing the stability of the modified nominal system:

$$\bar{\Sigma}_1 : \dot{e}_2 = f_2(e_2) \tag{26}$$

We take the positive definite and radially unbounded Lyapunov Function Candidate (LFC) $V = \frac{1}{2}e_2^2$ and differentiate V along the solution of $\bar{\Sigma}_1$:

$$\dot{V} = e\dot{e} = \begin{cases} -ke_2 & , e_2 > r \\ -\gamma e_2^2 & , e_2 \leq r \end{cases} < 0 \quad , \forall\ e_2 \in \mathbf{R}^+$$

Note that the tracking error $e \in \boldsymbol{R}^+$ and $e_2(t_0) = e(t_0)$. From (26) and (24) we then have that $e_2(t) \in \boldsymbol{R}^+$, $\forall t \geq t_0$, implying that $\dot{V}$ is negative definite along the trajectories of $\bar{\Sigma}_1$. In particular, $e_2 = 0$ is a globally uniformly asymptotically stable (GUAS) equilibrium point of $\bar{\Sigma}_1$.

Moreover, for any finite $r > 0$ such that $e_2(t_0) \in \{0 \leq e_2(t_0) \leq r\}$, the nominal system is given by

$$\dot{e}_2 = -\gamma e_2 \tag{27}$$

where $\gamma > 0$, and it follows immediately that the nominal system $\bar{\Sigma}_1$ is locally uniformly exponentially stable (LUES). Then, since $\bar{\Sigma}_1$ is both GUAS and LUES, $\bar{\Sigma}_1$ is globally κ-exponentially stable (see [15]).

In order to show that the origin $(e_2, \boldsymbol{\chi}) = (0, \mathbf{0})$ of the cascaded system (25) is globally κ-exponentially stable, we first apply ([16], Th. 7) to show that the origin is GUAS and then ([16], Lemma 8) to show that the origin is globally κ-exponentially stable. We proceed to verify the three assumptions of ([16], Th. 7):

- *Assumption on $\bar{\Sigma}_1$:* The system $\bar{\Sigma}_1$ is GUAS with Lyapunov Function $V(e_2) = \frac{1}{2}e_2^2$ satisfying

$$\dot{V} = \frac{\partial V}{\partial e_2} f_2(e_2) = \begin{cases} -ke_2 & , e_2 > r \\ -\gamma e_2^2 & , e_2 \leq r \end{cases} < 0 \quad ,\forall e_2 \in \boldsymbol{R}^+$$

$$\left|\left|\frac{\partial V}{\partial e_2}\right|\right| \cdot ||e_2|| = ||e_2||^2 \leq 2V(e_2) \qquad ,\forall e_2 \in \boldsymbol{R}^+$$

  Hence, the first assumption of ([16], Th. 7) is satisfied with $c \geq 2$ and any $\eta > 0$.

- *Assumption on the interconnection*: The norm of the interconnection term $\boldsymbol{g}(t,\boldsymbol{\chi})\boldsymbol{S}$ is globally bounded:

$$\begin{aligned} ||\boldsymbol{g}(t,\boldsymbol{\chi})\boldsymbol{S}||_1 &\leq (|g_1| + |g_2| + |g_3|)\, ||\boldsymbol{S}||_1 \\ &= (2(|u_d(t)| + 1) + 3(|v(t)| + |w(t)|))\, ||\boldsymbol{S}||_1 \\ &\leq c_g \end{aligned}$$

  where $c_g > 0$ is a suitable constant. The constant c_g exists since $\boldsymbol{S}$ is a constant matrix and $u_d(t), v(t), w(t) \in \mathcal{L}_\infty$, $\forall t \geq t_0$, by assumption. By taking $\theta_2 = 0$ and $\theta_1 = c_g$, the second assumption of ([16], Th. 7) is satisfied.

- *Assumption on Σ_2:* The perturbing system Σ_2 is a globally κ-exponentially stable and there exists a class $\mathcal{K}$ function $\alpha_1(\cdot)$ and a constant $\beta_1 > 0$ such that:

$$\begin{aligned} \int_{t_0}^{\infty} ||\boldsymbol{\chi}(\tau)|| d\tau &\leq \int_{t_0}^{\infty} \alpha_1(||\boldsymbol{\chi}(t_0)||) e^{-\beta_1(\tau - t_0)} d\tau \\ &= \frac{1}{\beta_1}\alpha_1(||\boldsymbol{\chi}(t_0)||) \end{aligned}$$

  Hence, the third and final assumption of ([16], Th. 7) is satisfied with the class $\mathcal{K}$ function $\frac{1}{\beta_1}\alpha_1(\cdot)$.

We have verified all the assumptions of ([16], Th. 7) and we conclude that the origin $(e_2, \boldsymbol{\chi}) = (0, \mathbf{0})$ is a GUAS equilibrium point of the cascaded system (25). Moreover, since $\bar{\Sigma}_1$ and Σ_2 are both globally κ-exponentially stable, we conclude from ([16], Lemma 8) that $(e_2, \boldsymbol{\chi}) = (0, \mathbf{0})$ is a globally κ-exponentially stable equilibrium point of the cascaded system (25).

It remains to show that global κ-exponential stability of (25), implies global κ-exponential convergence of the way-point tracking error $e(t)$ to zero. Let the solution of (21a) be defined on the interval $t \in [t_0, t_e)$, where $e(t_e) = 0$, and let $e(t_0) = e_2(t_0)$. Then from (23) and the Comparison Lemma ([14], Lemma 3.4), it follows that $e(t) \leq e_2(t)\ \forall\ t \in [t_0, t_e)$. Moreover, since the origin of the system (25) is globally κ-exponentially stable, there exists a class $\mathcal{K}$ function $\alpha_2(\cdot)$ and a constant $\beta_2 > 0$ such that

$$\left|\left| \begin{array}{c} e(t) \\ \boldsymbol{\chi}(t) \end{array} \right|\right| \leq \left|\left| \begin{array}{c} e_2(t) \\ \boldsymbol{\chi}(t) \end{array} \right|\right| \tag{28}$$

$$\leq \alpha_2\left(\left|\left| \begin{array}{c} e_2(t_0) \\ \boldsymbol{\chi}(t_0) \end{array} \right|\right|\right) e^{-\beta_2(t-t_0)} \tag{29}$$

$$= \alpha_2\left(\left|\left| \begin{array}{c} e(t_0) \\ \boldsymbol{\chi}(t_0) \end{array} \right|\right|\right) e^{-\beta_2(t-t_0)} \tag{30}$$

$\forall\ t \in [t_0, t_e)$. Since $e = 0$ is the only point where the solution of (21a) is undefined, we know that $e(t_e) = 0$. From (30) and for any $\epsilon > 0, \exists\ T = T(\epsilon) > 0$ such that

$$\left|\left| \begin{array}{c} e(t) \\ \boldsymbol{\chi}(t) \end{array} \right|\right| < \epsilon\ ,\ \ \forall\ t \in [t_0 + T, t_e) \Rightarrow |e(t)| < \epsilon$$

Thus we can conclude that the tracking error e converges to the origin. Since the origin of (25) is globally κ-exponentially stable, the tracking error $e(t)$ converges κ-exponentially to zero for all $e(t_0) \in \boldsymbol{R}^+$. This completes the proof. □

IV. Controller Design

In the last section we showed that if we can make the origin $\boldsymbol{\chi} = \mathbf{0}$, where $\tilde{\boldsymbol{x}} = (\tilde{\theta}\ \tilde{\psi}\ \tilde{u})^T$ is a subvector of $\boldsymbol{\chi}$, of the system (20) globally κ-exponentially stable, while assuring that $v(t), w(t) \in \mathcal{L}_\infty\ \forall t \geq t_0$, the way-point tracking error $e(t)$ converges globally and κ-exponentially to zero. We now proceed to design a controller that satisfies the sufficient conditions of Proposition 1 using a backstepping approach inspired by [2]. In particular we present a adaptive scheme for counteracting constant environmental forces in all six degrees of freedom using only three available controls.

We define the first controller error as

$$\boldsymbol{z}_1 \triangleq \begin{pmatrix} \int_{t_0}^{t} \tilde{u}(s)ds \\ \tilde{\theta} \\ \tilde{\psi} \end{pmatrix}$$

where the integral of the surge speed error is included to achieve integral action in the surge mode. The integral action gives increased robustness to unmodeled dynamics and environmental disturbances, something which is favorable in many applications.

We define a second controller error variable

$$\boldsymbol{z}_2 \triangleq \boldsymbol{\nu} - \boldsymbol{\alpha} \tag{31}$$

where $\boldsymbol{\alpha} \in \boldsymbol{R}^6$ is a vector of stabilizing functions to be determined, and compute the $\boldsymbol{z}_1$ error dynamics using (9)-(10) and (31):

$$\dot{\boldsymbol{z}}_1 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \mathrm{c}\phi & -\mathrm{s}\phi \\ 0 & \frac{\mathrm{s}\phi}{\mathrm{c}\theta} & \frac{\mathrm{c}\phi}{\mathrm{c}\theta} \end{pmatrix} (\mathcal{F}\boldsymbol{z}_2 + \begin{pmatrix} \alpha_1 \\ \alpha_5 \\ \alpha_6 \end{pmatrix}) - \begin{pmatrix} u_d \\ \dot{\theta}_d \\ \dot{\psi}_d \end{pmatrix}$$

where $\mathrm{s}(\cdot) = \sin(\cdot)$, $\mathrm{c}(\cdot) = \cos(\cdot)$ and where

$$\mathcal{F} \triangleq \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

Choosing α_1, α_5 and α_6 according to

$$\begin{pmatrix} \alpha_1 \\ \alpha_5 \\ \alpha_6 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \mathrm{c}\phi & \mathrm{c}\theta\mathrm{s}\phi \\ 0 & -\mathrm{s}\phi & \mathrm{c}\theta\mathrm{c}\phi \end{pmatrix} \left[\begin{pmatrix} u_d \\ \dot{\theta}_d \\ \dot{\psi}_d \end{pmatrix} - \boldsymbol{K}\boldsymbol{z}_1 \right] \tag{32}$$

where $\boldsymbol{K} \in \boldsymbol{R}^{3\times3}$ and $\boldsymbol{K} = \boldsymbol{K}^T > 0$, results in the following closed loop $\boldsymbol{z}_1$-system:

$$\dot{\boldsymbol{z}}_1 = -\boldsymbol{K}\boldsymbol{z}_1 + \bar{\mathcal{F}}(\boldsymbol{\eta})\boldsymbol{z}_2 \tag{33}$$

where

$$\bar{\mathcal{F}}(\boldsymbol{\eta}) \triangleq \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos\phi & -\sin\phi \\ 0 & \frac{\sin\phi}{\cos\theta} & \frac{\cos\phi}{\cos\theta} \end{pmatrix} \mathcal{F}$$

We choose the positive definite and radially unbounded LFC $V_1 = \frac{1}{2}\boldsymbol{z}_1^T\boldsymbol{z}_1$ and differentiate V_1 along the solutions of (33):

$$\dot{V}_1 = -\boldsymbol{z}_1^T\boldsymbol{K}\boldsymbol{z}_1 + \boldsymbol{z}_1^T\bar{\mathcal{F}}(\boldsymbol{\eta})\boldsymbol{z}_2$$

For the next step of backstepping, we differentiate $\boldsymbol{z}_2$ with respect to time and pre-multiply the result by $\boldsymbol{M}$:

$$\begin{aligned} \boldsymbol{M}\dot{\boldsymbol{z}}_2 = &-\boldsymbol{C}(\boldsymbol{\nu})\boldsymbol{\nu} - \boldsymbol{D}(\boldsymbol{\nu})\boldsymbol{\nu} - \boldsymbol{g}(\boldsymbol{\eta}) \\ &+ \boldsymbol{\tau} + \boldsymbol{J}(\boldsymbol{\eta})^{-1}\boldsymbol{w} - \boldsymbol{M}\dot{\boldsymbol{\alpha}} \end{aligned} \tag{34}$$

We take $V_2 = \frac{1}{2}\boldsymbol{z}_2^T\boldsymbol{M}\boldsymbol{z}_2 + V_1$ as our second LFC and differentiate V_2 along the solutions of (33) and (34):

$$\begin{aligned} \dot{V}_2 &= \dot{V}_1 + \boldsymbol{z}_2^T(-\boldsymbol{C}(\boldsymbol{\nu})\boldsymbol{\nu} - \boldsymbol{D}(\boldsymbol{\nu})\boldsymbol{\nu} - \boldsymbol{g}(\boldsymbol{\eta}) \\ &\qquad + \boldsymbol{\tau} + \boldsymbol{J}(\boldsymbol{\eta})^{-1}\boldsymbol{w} - \boldsymbol{M}\dot{\boldsymbol{\alpha}}) \\ &= \dot{V}_1 + \boldsymbol{z}_2^T(\boldsymbol{\tau} - \boldsymbol{N}(\boldsymbol{\eta},\boldsymbol{\nu}) + \boldsymbol{J}(\boldsymbol{\eta})^{-1}\boldsymbol{w} - \boldsymbol{M}\dot{\boldsymbol{\alpha}}) \end{aligned} \tag{35}$$

where $\boldsymbol{N}(\boldsymbol{\eta},\boldsymbol{\nu})$ is defined as

$$\boldsymbol{N}(\boldsymbol{\eta},\boldsymbol{\nu}) \triangleq \boldsymbol{C}(\boldsymbol{\nu})\boldsymbol{\nu} + \boldsymbol{D}(\boldsymbol{\nu})\boldsymbol{\nu} + \boldsymbol{g}(\boldsymbol{\eta}) = [n_i(\boldsymbol{\eta},\boldsymbol{\nu})]$$

The independent controls τ_1, τ_5 and τ_6 are chosen to stabilize the surge, pitch and yaw modes respectively:

$$\tau_1 = n_1(\boldsymbol{\eta},\boldsymbol{\nu}) + \boldsymbol{e}_1^T\boldsymbol{M}\dot{\boldsymbol{\alpha}} - c_1 z_{2,1} - z_{1,1} - \hat{w}_1^b \tag{36}$$

$$\begin{aligned} \tau_5 = &\, n_5(\boldsymbol{\eta},\boldsymbol{\nu}) + \boldsymbol{e}_5^T\boldsymbol{M}\dot{\boldsymbol{\alpha}} - c_5 z_{2,5} \\ &- (\cos\phi z_{1,2} + \frac{\sin\phi}{\cos\theta} z_{1,3}) - \hat{w}_5^b \end{aligned} \tag{37}$$

$$\begin{aligned} \tau_6 = &\, n_6(\boldsymbol{\eta},\boldsymbol{\nu}) + \boldsymbol{e}_6^T\boldsymbol{M}\dot{\boldsymbol{\alpha}} - c_6 z_{2,6} \\ &- (-\sin\phi z_{1,2} + \frac{\cos\phi}{\cos\theta} z_{1,3}) - \hat{w}_6^b \end{aligned} \tag{38}$$

where $c_i > 0$, $i \in \{1,5,6\}$, are controller gains and $\hat{w}_1^b, \hat{w}_5^b, \hat{w}_6^b$ are body-frame estimates of the unknown environmental force components w_1^b, w_5^b, w_6^b respectively. We have yet to stabilize the sway, heave and roll modes. The elements τ_2-τ_4 cannot be used for this purpose, since they are functions of the already chosen controls τ_1, τ_5 and τ_6. However, the stabilizing functions α_2-α_4 have yet to be determined, and a closer look at (35) reveals that $\dot{\alpha}_2$-$\dot{\alpha}_4$ can be used as *virtual independent controls* to render $\dot{V}_2$ negative semi-definite. We choose $\dot{\alpha}_2$, $\dot{\alpha}_3$ and $\dot{\alpha}_4$ according to

$$\begin{pmatrix} m_{22} & m_{23} & m_{24} \\ m_{32} & m_{33} & m_{34} \\ m_{42} & m_{43} & m_{44} \end{pmatrix} \begin{pmatrix} \dot{\alpha}_2 \\ \dot{\alpha}_3 \\ \dot{\alpha}_4 \end{pmatrix} = \begin{pmatrix} -n_2(\boldsymbol{\eta},\boldsymbol{\nu}) + \tau_2 \\ -n_3(\boldsymbol{\eta},\boldsymbol{\nu}) + \tau_3 \\ -n_4(\boldsymbol{\eta},\boldsymbol{\nu}) + \tau_4 \end{pmatrix.$$
$$\begin{pmatrix} +\hat{w}_2^b + c_2 z_{2,2} - m_{21}\dot{\alpha}_1 - m_{25}\dot{\alpha}_5 - m_{26}\dot{\alpha}_6 \\ +\hat{w}_3^b + c_3 z_{2,3} - m_{31}\dot{\alpha}_1 - m_{35}\dot{\alpha}_5 - m_{36}\dot{\alpha}_6 \\ +\hat{w}_4^b + c_4 z_{2,4} - m_{41}\dot{\alpha}_1 - m_{45}\dot{\alpha}_5 - m_{46}\dot{\alpha}_6 \end{pmatrix} \tag{39}$$

where $c_i > 0$, $i \in \{2,3,4\}$, are controller gains and $\hat{w}_2^b, \hat{w}_3^b, \hat{w}_4^b$ are body-frame estimates of the unknown environmental force components w_2^b, w_3^b, w_4^b respectively. Note that (39) can be rewritten as

$$\begin{pmatrix} m_{22} & m_{23} & m_{24} \\ m_{32} & m_{33} & m_{34} \\ m_{42} & m_{43} & m_{44} \end{pmatrix} \begin{pmatrix} \dot{\alpha}_2 \\ \dot{\alpha}_3 \\ \dot{\alpha}_4 \end{pmatrix} = \begin{pmatrix} -c_2\alpha_2 + h_2(t) \\ -c_3\alpha_3 + h_3(t) \\ -c_4\alpha_4 + h_4(t) \end{pmatrix}$$

where $h_i(t)$, $i \in \{2,3,4\}$, are defined as

$$\begin{aligned} h_i(t) = &-n_i(\boldsymbol{\eta}(t),\boldsymbol{\nu}(t)) + \tau_i(t) + \hat{w}_i^b(t) + c_i\nu_i(t) \\ &- m_{i1}\dot{\alpha}_1(t) - m_{i5}\dot{\alpha}_5(t) - m_{i6}\dot{\alpha}_6(t) \end{aligned}$$

Equation (39) is clearly globally Lipschitz and provided that $\boldsymbol{\eta}(t)$ and $\boldsymbol{\nu}(t)$ exist for all t, the functions α_2-α_4 exists and are unique. Note that the sub-matrix of $\boldsymbol{M}$ in (39) is non-singular, since $\boldsymbol{M}$ is positive definite. Applying the control laws (36-38), where α_1, α_5 and α_6 are given by (32) and $\dot{\alpha}_2$-$\dot{\alpha}_4$ are given by (39), gives the following Lyapunov Function derivative $\dot{V}_2$:

$$\begin{aligned} \dot{V}_2 &= -\boldsymbol{z}_1^T\boldsymbol{K}\boldsymbol{z}_1 + \boldsymbol{z}_1^T\bar{\mathcal{F}}(\boldsymbol{\eta})\boldsymbol{z}_2 - \boldsymbol{z}_2^T\boldsymbol{\mathcal{C}}\boldsymbol{z}_2 - \boldsymbol{z}_2^T\hat{\boldsymbol{w}}^b \\ &\quad - \boldsymbol{z}_2^T\bar{\mathcal{F}}^T(\boldsymbol{\eta})\boldsymbol{z}_1 + \boldsymbol{z}_2^T(\boldsymbol{J}^{-1}(\boldsymbol{\eta})\boldsymbol{w}) \\ &= -\boldsymbol{z}_1^T\boldsymbol{K}\boldsymbol{z}_1 - \boldsymbol{z}_2^T\boldsymbol{\mathcal{C}}\boldsymbol{z}_2 + \boldsymbol{z}_2^T(\boldsymbol{J}^{-1}(\boldsymbol{\eta})\boldsymbol{w} - \hat{\boldsymbol{w}}^b) \end{aligned}$$

where $\hat{\boldsymbol{w}}^b = (\hat{w}_1^b\ \hat{w}_2^b\ \hat{w}_3^b\ \hat{w}_4^b\ \hat{w}_5^b\ \hat{w}_6^b)^T$ and $\boldsymbol{\mathcal{C}} = \mathrm{diag}\{c_i\} > 0$.

For the ideal case of no environmental disturbances, $\boldsymbol{w} = \hat{\boldsymbol{w}} = \mathbf{0}$, V_2 is a Lyapunov Function for the $\boldsymbol{z}_1, \boldsymbol{z}_2$-system and $\dot{V}_2$ is bounded from above by a quadratic and negative definite function. Then, since V_2 is quadratic and positive definite, it follows that $(\boldsymbol{z}_1, \boldsymbol{z}_2) = (\mathbf{0}, \mathbf{0})$ is a globally uniformly exponentially stable (GUES) equilibrium point of (33) and (34). In particular, the closed loop system is

$$\begin{pmatrix} \dot{\boldsymbol{z}}_1 \\ \boldsymbol{M}\dot{\boldsymbol{z}}_2 \end{pmatrix} = \begin{pmatrix} -\boldsymbol{K} & \bar{F}(\boldsymbol{\eta}) \\ -\bar{F}^T(\boldsymbol{\eta}) & -\boldsymbol{\mathcal{C}} \end{pmatrix} \begin{pmatrix} \boldsymbol{z}_1 \\ \boldsymbol{z}_2 \end{pmatrix} \tag{40}$$

where the characteristic skew-symmetric structure resulting from backstepping is apparent.

Moving to the case of constant environmental disturbances, we define the disturbance estimation error as

$$\tilde{\boldsymbol{w}} \triangleq \hat{\boldsymbol{w}} - \boldsymbol{w}$$

where $\hat{\boldsymbol{w}} = \boldsymbol{J}(\boldsymbol{\eta})\hat{\boldsymbol{w}}^b$ is the disturbance estimate. We expand the current $\boldsymbol{z}_1, \boldsymbol{z}_2$-statespace with $\tilde{\boldsymbol{w}}$ and choose $V_3 = V_2 + \frac{1}{2}\tilde{\boldsymbol{w}}^T\boldsymbol{\Gamma}^{-1}\tilde{\boldsymbol{w}}$, $\boldsymbol{\Gamma} = \boldsymbol{\Gamma}^T > 0$, as our final LFC. Differentiating V_3 along the solutions of the extended system then gives:

$$\dot{V}_3 = -\boldsymbol{z}_1^T\boldsymbol{K}\boldsymbol{z}_1 - \boldsymbol{z}_2^T\boldsymbol{\mathcal{C}}\boldsymbol{z}_2 + \tilde{\boldsymbol{w}}^T\boldsymbol{\Gamma}^{-1}(\dot{\hat{\boldsymbol{w}}} - \boldsymbol{\Gamma}\boldsymbol{J}^{-T}\boldsymbol{z}_2)$$

Choosing the adaptive update law for $\dot{\hat{\boldsymbol{w}}}$ as

$$\dot{\hat{\boldsymbol{w}}} = \boldsymbol{\Gamma}\boldsymbol{J}^{-T}(\boldsymbol{\eta})\boldsymbol{z}_2 \tag{41}$$

renders the Lyapunov Function derivative $\dot{V}_3$ globally negative semi-definite:

$$\dot{V}_3 = -\boldsymbol{z}_1^T\boldsymbol{K}\boldsymbol{z}_1 - \boldsymbol{z}_2^T\boldsymbol{\mathcal{C}}\boldsymbol{z}_2 \le 0 \tag{42}$$

and the origin $(\boldsymbol{z}_1, \boldsymbol{z}_2, \tilde{\boldsymbol{w}}) = (\mathbf{0}, \mathbf{0}, \mathbf{0})$ is thus globally uniformly stable (GUS). Finally, application of Barbalats Lemma gives that $(\boldsymbol{z}_1, \boldsymbol{z}_2)$ converges to the origin globally and asymptotically with time.

Note that since $\boldsymbol{z}_2$ is not persistently exciting (PE), $\tilde{\boldsymbol{w}}$ will generally not converge to zero. However, since the closed loop system is GUS, $\tilde{\boldsymbol{w}}(t)$ is bounded for all t. The actual value of the disturbance is however not interesting for the stabilization.

The convergence of $\boldsymbol{z}_1 \to \mathbf{0}$ as $t \to \infty$, implies that

$$\tilde{\theta} \to 0 \quad \tilde{\psi} \to 0 \quad \int_{t_0}^{t} \tilde{u}(s)ds \to 0 \quad \text{as } t \to \infty.$$

Moreover, on the manifold $(\boldsymbol{z}_1, \boldsymbol{z}_2) = (\mathbf{0}, \mathbf{0})$, it follows from (32) that $\alpha_1 = u_d$ and $\boldsymbol{z}_2 = \mathbf{0} \Rightarrow \boldsymbol{\nu} = \boldsymbol{\alpha} \Rightarrow u \to u_d$.

Result: For the ideal case of no environmental disturbances, the non-adaptive control laws (36-38) renders the origin $(\boldsymbol{z}_1, \boldsymbol{z}_2) = (\mathbf{0}, \mathbf{0})$ of (40) GUES. Moreover, since $\boldsymbol{z}_2(t) \in \mathcal{L}_\infty$, $\forall t \ge t_0$, the controller guarantees that $v(t), w(t) \in \mathcal{L}_\infty$, $\forall t \ge t_0$. Then, if $u_d(t) > 0$ and $u_d(t) \in \mathcal{L}_\infty$, $\forall t \ge t_0$, and θ_d and ψ_d are chosen according to (15) and (16), all the assumptions of *Proposition 1* are satisfied, and the global κ-exponential convergence to the desired way-point is guaranteed.

With the presence of unknown and constant environmental disturbances, the adaptive control laws (36)-(38) together with the adaptive update law (41), guarantees that $(\boldsymbol{z}_1, \boldsymbol{z}_1) \to (\mathbf{0}, \mathbf{0})$. However, in this case, the convergence rate is not necessarily *uniform* with respect to t_0. This prevents the use of the same cascaded systems results as in the non-adaptive case. However, it can be verified that since the adaptive closed loop system is GUS and the interconnection term $\boldsymbol{g}(t, \boldsymbol{\chi})\boldsymbol{S}$ is globally bounded, (21a) is forward complete, meaning that $e(t)$ does not escape to infinity in finite time. Moreover, as $(\boldsymbol{z}_1, \boldsymbol{z}_1) \to (\mathbf{0}, \mathbf{0})$, there exists an $0 < \epsilon < \frac{\min u_d}{c_g}$ and a $T = T(\epsilon, t_0) > 0$ such that

$$||\boldsymbol{\chi}(t)|| \triangleq ||(\boldsymbol{z}_1, \boldsymbol{z}_1)|| \le \epsilon \quad \forall t \ge t_0 + T$$

Then, with $V = \frac{1}{2}e^2$ and taking the time derivative of V along (21a), we have that

$$\dot{V} \le -c_g \left(\frac{\min u_d}{c_g} - \epsilon \right) |e| < 0 \quad t \ge t_0 + T$$

Since $e(t)$ is does not blow up on $t \in (t_0, t_0 + T)$, we have that $e(t) \to 0$ globally and asymptotically with time.

V. CASE STUDY: HUGIN AUV

In this section we present some simulation results for the proposed control stratgy applied to a model of the HUGIN AUV[1]. HUGIN is a slender AUV equipped with rudders for steering and diving, and a propeller to provide forward thrust.

The motion of the HUGIN AUV can be described by the model (1-2) where $\boldsymbol{M}$ is constant, symmetric and positive definite and hence satisfies the assumptions (3). Moreover, HUGIN is equipped with three independent controls, and the control vector $\boldsymbol{\tau}$ is given by

$$\boldsymbol{\tau} = \begin{pmatrix} T(n) \\ 0 \\ 0 \\ Q(n) \\ 0 \\ 0 \end{pmatrix} + \begin{pmatrix} 0 & 0 \\ Y_{\delta_S} & 0 \\ 0 & Z_{\delta_D} \\ 0 & 0 \\ 0 & Z_{\delta_S} l_x \\ Y_{\delta_S} l_x & 0 \end{pmatrix} \begin{pmatrix} \delta_D \\ \delta_D \end{pmatrix} u^2$$

where the propeller thrust $T(n)$ and the rudder deflections δ_D and δ_S are the independent controls. The thrust $T(n)$ the moment $Q(n)$ are both nonlinear functions of the propeller revolution n, dependent on the propeller characteristics. The following actuator limitations were used in the simulations:

$$|n| \le 230 \text{ [rpm]} \quad |\delta| \le 20 \; [^\circ] \quad |\dot{\delta}| \le 10 \; [^\circ/\text{s}]$$

and the controller gain matrices were chosen as

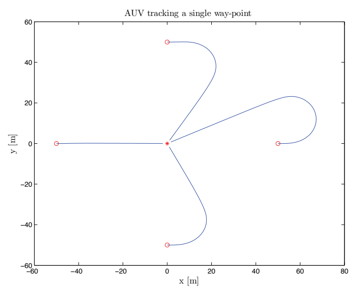
$$\boldsymbol{K} = 35 \cdot \boldsymbol{I}_{3\times3} \quad \boldsymbol{\mathcal{C}} = 50 \cdot \boldsymbol{I}_{6\times6} \quad \boldsymbol{\Gamma} = 10 \cdot \boldsymbol{I}_{6\times6}$$

The simulation results for a way-point tracking scenario is shown in Figure 2(a)-2(c). Figure 2(a) shows the the xy-trajectory of the AUV when tracking a single way-point located at the origin from different initial positions. The atan2 function was used to calculate the LOS angles. To avoid the discontinuity of the atan2 function at the $-\pi \backslash \pi$ junction, the computed LOS angles were mapped from the interval $(-\pi, \pi)$ to the interval $(0, \infty)$ before being used as reference inputs to the control system.

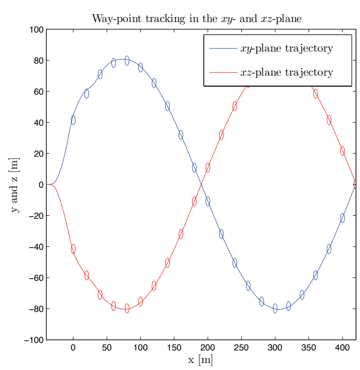
[1]HUGIN is developed by the Norwegian Defense Research Establishment (FFI). Model parameters will not be presented

Figure 2(b) shows the xy- and xz- trajectories of the AUV when tracking a set of way-points with a constant disturbance $\boldsymbol{w} = (100\ 100\ 100\ 0\ 0\ 0)^T$. The AUV was given the initial position $(x, y, z) = (-50, 0, 0)$ [m] and the initial orientation $(\phi, \theta, \psi) = (0°, 0°, 0°)$. The initial surge speed of the AUV was $u = 1.5$ [m/s] and the desired surge speed was chosen as $u_d = 2.0$ [m/s]. The surge speed of the AUV during the way-point tracking is shown in Figure 2(c). A *sphere of acceptance* with radius 4 [m], centered at each way-point, was used to perform way-point switches.

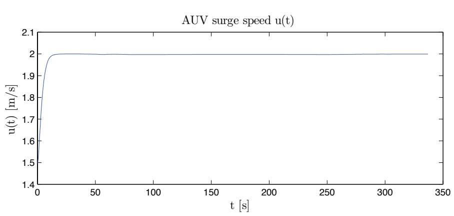
The simulations show the effectiveness of the proposed way-point tracking scheme. The AUV is able to follow the way-points closely and maintain the desired surge speed. The surge speed quickly converge to the desired value, much thanks to the integral action provided by the controller.



(a) Tracking a single way-point.



(b) xy- and xz-trajectory.



(c) Surge speed $u(t)$.

Fig. 2.

VI. Conclusions

In this paper, we have proposed a control scheme for global κ-exponential way-point tracking of underactuated autonomous vehicles. A controller with integral action in the surge mode was synthesized using integrator backstepping and global κ-exponential convergence to the desired way-point was proven using nonlinear cascaded systems theory. Furthermore, it was shown how the structure of the system could be exploited to include disturbance adaption to counteract constant and unknown environmental disturbances in all degrees of freedom, while achieving global asymptotic convergence to the desired way-point.

It has long been an open question whether it is possible for an underactuated system to counteract environmental disturbances in all degrees of freedom. In the proposed approach we used dynamic stabilizing functions in the backstepping controller, and exploited the structure of the system to counteract environmental disturbances in all six degrees of freedom, using only three available controls. The proposed control strategy was simulated on a model of the HUGIN AUV and simulation results showed good way-point tracking capability even in the presence of constant and unknown environmental disturbances.

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