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Stability Criteria of Networked Predictive Control Systems with Random Network Delay

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Abstract—**The stability criteria of closed-loop networked predictive control systems with random communication delay are presented in this paper, based on a novel networked predictive control scheme. The scheme mainly consists of a control prediction generator and a network delay compensator. The control prediction generator provides a control prediction sequence that can achieve the required control performance. The network delay compensator compensates the network communication delay. The stability criteria of closed-loop networked predictive control systems are analytically derived for both the constant and random communication delay.**

I. INTRODUCTION

A networked control system (NCS) is a feedback control system wherein the control loop is closed through a real-time network [1], which includes fieldbus control systems constructed on the base of bus technology (e.g., DeviceNet, ControlNet and LonWorks) and Internet based control systems using general computer networks. NCS is a completely distributed real-time feedback control system that is an integration of sensors, controllers, actuators and communication networks. It provides data transmission between devices in order that users of different sites can realize resource sharing and coordinate manipulation. The NCS reduces system wiring, makes system diagnosis and maintenance easier and increases system flexibility. But, the insertion of the communication network in the feedback control loop makes the analysis and design of an NCS complex. Conventional control theories with many ideal assumptions, such as synchronized control and non-delayed sensing and actuation, must be re-evaluated before they can be applied to NCSs. Specifically, the following issues need to be addressed [2-4]. 1) The network-induced delay (sensor-to-controller delay and controller-to-actuator delay) that occurs while exchanging data among devices connected to the shared network. This delay, either constant (with some jitter) or time varying, can degrade the performance of control systems designed without considering the delay and can even destabilize the system. 2) The network can be viewed as a web of unreliable transmission paths. Some packets not only suffer transmission delay but, even worse, can be lost during transmission. Thus, how such packet dropouts affect the performance of an NCS is an issue that must be considered. 3) The plant outputs may be transmitted using multiple network packets (so-called multiple-packet transmission), due to the bandwidth and packet size constraints of the network. Only a part or possible none of the packets could arrive on the controller side by the time of control calculation because of the arbitration of the network medium with other nodes on the network.

Recently, more and more attention has been paid to various issues of network-based control systems, for example, the stability problem [5,6] in the presence of network delays and data packet drops, the design and implementation problem of networked control systems [7, 8] and the network traffic congestion problem [9]. Although the network (for example, Internet) is being applied to the control system area, most control methods based on networks simply use the network as a data transmission device to transmit the control status of a system (to a website). This is not much different from the normal Internet information services. Much research work is needed to develop systematic design methods using this technology for the design of such network-based real-time distributed control systems. Much attention needs to be paid to network environment issues for control systems, such as the network communication delay and the stability of closed-loop networked control systems. This paper presents a novel networked predictive control strategy and studies its closed-loop stability, which are different from those in conventional (or non-networked) control systems [10].

II. NETWORKED PREDICTIVE CONTROL

In networked control systems (NCS), the control sequence can be transmitted at the same time through a network, which is not done in traditional control systems. Since there is an unknown network communication delay, the networked predictive control scheme is proposed. Generally speaking, it mainly consists of two parts: a network delay compensator and a control prediction generator. The former is used to compensate the unknown random communication delay. The latter is designed by conventional predictive control methods, for example, the general predictive control.

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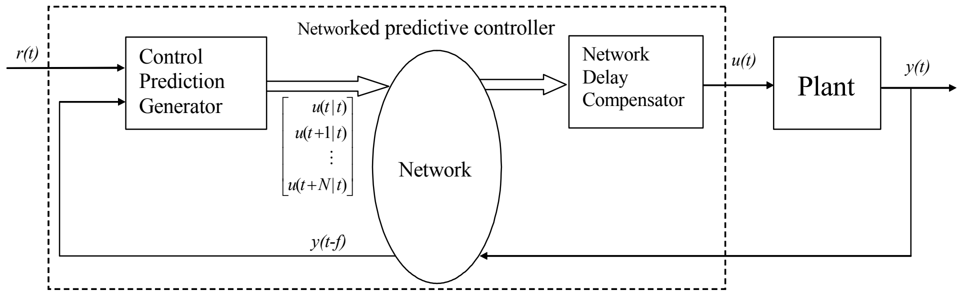

Figure 1. The networked predictive control system

A network has various locations in a networked control system, *e.g.,* between the sensor and controller, between the actuator and controller, between the reference and controller, etc. For example, in the case where the sensors are far away from the actuator, a communication network is needed either between the actuator and controller or between the sensors and controller.

In a practical NCS, if the data packet does not arrive at a destination in a certain transmission time, it means this data packet is lost, based on commonly used network protocols. From the physical point of view, it is natural to assume that only a finite number of consecutive data dropouts can be tolerated in order to avoid the NCS becoming open-loop.

The time of the data transmitted through a network is very important for networked control systems. This is because a control sequence of a control system is based on time.

For the sake of simplicity, the following assumptions are made:

a) A network is used in both the forward and feedback channels. The forward network delay is k and the feedback network delay is f.
b) The number of consecutive data package drops on the network is bounded;
c) The data transmitted through a network are with a time stamp.

Thus, the networked predictive control system (NPCS) structure is shown in Fig. 1.

Let $\Re[z^{-1},p]$ denote the set of polynomials in the indeterminate z^{-1} with coefficients in the field of real numbers and with the order p in a set of non-negative integer numbers. For example, the polynomial $A_k(z^{-1}) \in \Re[z^{-1},n]$, *i.e.,* $A_k(z^{-1}) = a_{k,0} + a_{k,1}z^{-1} + \ a_{k,2}z^{-2} + \cdots + a_{k,n}z^{-n}$.

Consider a single-input single-output discrete-time plant described by the following

$$A(z^{-1})y(t+d) = B(z^{-1})u(t) \tag{1}$$

where $y(t)$ and $u(t)$ are the output and control input of the plant, d is the time delay, and $A(z^{-1}) \in \Re[z^{-1},n]$ and $B(z^{-1}) \in \Re[z^{-1},m]$ are the system polynomials. If there is no network transmission delay, many control design methods are available for the plant (*1*), for example, PID, LQG, MPC etc. Here, it assumes that the controller of the system without network delay is given by

$$C(z^{-1})u(t) = D(z^{-1})e(t+d) \tag{2}$$

where the polynomials $C(z^{-1}) \in \Re[z^{-1},n_c]$ and $D(z^{-1}) \in \Re[z^{-1},n_d]$ and $e(t+d) = r(t+d) - \hat{y}(t+d)$ is the error between the future reference $r(t+d)$ and the output prediction $\hat{y}(t+d)$.

To compensate the network transmission delay, the control prediction sequence $u(t+k|t)$ at time t, for $i=0, 1, 2, ..., N,$ is generated by

$$C(z^{-1})u(t+i\,|\,t) = D(z^{-1})e(t+d+i\,|\,t) \tag{3}$$

and the error prediction $e(t+d+i|t)$ at time t is defined as

$$e(t+d+i\,|\,t) = r(t+d+i) - \hat{y}(t+d+i\,|\,t) \tag{4}$$

where $\hat{y}(t+d+i\,|\,t)$ is the output prediction at time t and $r(t+d+i)$ is the future reference input. For the sake of simplicity, define the following operations:

$$x(t-i\,|\,t-i) = z^{-1}x(t-i+1\,|\,t-i+1) \tag{5}$$

for $i=0, 1, 2, ..., t$

$$x(t+i\,|\,t) = z^{-1}x(t+i+1\,|\,t)\,, \tag{6}$$

for $i= 1, 2, ...,$ where $x(.)$ represents $\hat{y}(.)$ and $u(.)$.

For *i=0, 1, 2, ..., N*, there exists the following Diophantine equation:

$$A(z^{-1})E_i(z^{-1}) + z^{-i-f}F_i(z^{-1}) = 1 \tag{7}$$

where the polynomials $E_i(z^{-1}) \in \Re[z^{-1}, i+f-1]$ and $F_i(z^{-1}) \in \Re[z^{-1}, n-1]$. It is clear from assumption a) that the past outputs up to time *t-f* are available on the control prediction generator side. Combining the above and the controlled plant yields the following output predictions at *t*:

$$\begin{aligned}
\hat{y}(t+d\,|\,t) &= F_d(z^{-1})y(t-f) + B(z^{-1})E_d(z^{-1})u(t\,|\,t)\\
\hat{y}(t+d+1\,|\,t) &= F_{d+1}(z^{-1})y(t-f)\\
&\quad + B(z^{-1})E_{d+1}(z^{-1})u(t+1\,|\,t)\\
&\vdots\\
\hat{y}(t+d+N\,|\,t) &= F_{d+N}(z^{-1})y(t-f)\\
&\quad + B(z^{-1})E_{d+N}(z^{-1})u(t+N\,|\,t)
\end{aligned} \tag{8}$$

which can be compacted as

$$\begin{bmatrix} \hat{y}(t+d\,|\,t) \\ \hat{y}(t+d+1\,|\,t) \\ \vdots \\ \hat{y}(t+d+N\,|\,t) \end{bmatrix} = \begin{bmatrix} F_d(z^{-1}) \\ F_{d+1}(z^{-1}) \\ \vdots \\ F_{d+N}(z^{-1}) \end{bmatrix} y(t-f) + \begin{bmatrix} B(z^{-1})E_d(z^{-1})u(t\,|\,t) \\ B(z^{-1})E_{d+1}(z^{-1})u(t+1\,|\,t) \\ \vdots \\ B(z^{-1})E_{d+N}(z^{-1})u(t+N\,|\,t) \end{bmatrix} \tag{9}$$

The second term on the right side of the above can be separated into two parts: the first part contains the control sequence before time *t* and the second part the future control prediction sequence. So, let

$$\begin{bmatrix} B(z^{-1})E_d(z^{-1})u(t\,|\,t) \\ B(z^{-1})E_{d+1}(z^{-1})u(t+1\,|\,t) \\ \vdots \\ B(z^{-1})E_{d+N}(z^{-1})u(t+N\,|\,t) \end{bmatrix} = \begin{bmatrix} G_d(z^{-1}) \\ G_{d+1}(z^{-1}) \\ \vdots \\ G_{d+N}(z^{-1}) \end{bmatrix} u(t-1\,|\,t-1) + M_1 \begin{bmatrix} u(t\,|\,t) \\ u(t+1\,|\,t) \\ \vdots \\ u(t+N\,|\,t) \end{bmatrix} \tag{10}$$

where the polynomial $G_k(z^{-1}) \in \Re[z^{-1}, m+f+d-2]$ and the matrix $M_1 \in \Re^{(N+1)\times(N+1)}$. Thus,

$$\begin{aligned}\hat{Y}(t+d\,|\,t) &= F(z^{-1})y(t-f)\\ &\quad + G(z^{-1})u(t-1\,|\,t-1) + M_1U(t\,|\,t)\end{aligned} \tag{11}$$

where

$$\begin{aligned}\hat{Y}(t+d\,|\,t) = \big[&\hat{y}(t+d\,|\,t),\ \hat{y}(t+d+1\,|\,t),\\ &\cdots,\ \hat{y}(t+d+N\,|\,t)\big]^T\end{aligned} \tag{12}$$

$$U(t\,|\,t) = \big[u(t\,|\,t), \quad \cdots,\ u(t+N\,|\,t)\big]^T \tag{13}$$

$$G(z^{-1}) = \big[G_d(z^{-1}), \quad \cdots,\ G_{d+N}(z^{-1})\big]^T \tag{14}$$

$$F(z^{-1}) = \big[F_d(z^{-1}), \quad \cdots,\ F_{d+N}(z^{-1})\big]^T \tag{15}$$

From the controller designed for the system without network delay, it is clear that the future control sequence can be expressed by

$$C(z^{-1})U(t\,|\,t) = D(z^{-1})\big(R(t+d) - \hat{Y}(t+d\,|\,t)\big) \tag{16}$$

where $R(t+d) = \big[r(t+d), \quad \cdots,\ r(t+d+N)\big]^T$. The term $C(z^{-1})U(t\,|\,t)$ can also be separated into two parts: the first part contains the control sequence before time t and the second part the predicted future control sequence. Then, let

$$C(z^{-1})U(t\,|\,t) = H(z^{-1})u(t-1\,|\,t-1) + LU(t\,|\,t) \tag{17}$$

where $H(z^{-1}) = \big[H_0(z^{-1}),\ \ H_1(z^{-1}),\ \ \cdots,\ \ H_N(z^{-1})\big]^T$,the polynomial $H_i(z^{-1}) \in \Re[z^{-1}, \max\{n_c - i - 1, 0\}]$ and the matrix $L \in \Re^{(N+1)\times(N+1)}$. Combining (11), (16) and (17) gives

$$\begin{aligned}&H(z^{-1})u(t-1\,|\,t-1) + LU(t\,|\,t) = D(z^{-1})R(t+d)\\ &- D(z^{-1})F(z^{-1})y(t-f) - D(z^{-1})G(z^{-1})u(t-1\,|\,t-1)\\ &- D(z^{-1})M_1U(t\,|\,t)\end{aligned} \tag{18}$$

Let

$$\begin{aligned}&\Gamma(z^{-1})u(t-1\,|\,t-1) + MU(t\,|\,t)\\ &= D(z^{-1})\big(G(z^{-1})u(t-1\,|\,t-1) + M_1U(t\,|\,t)\big)\end{aligned} \tag{19}$$

where $\Gamma(z^{-1}) = [\Gamma_0(z^{-1}),\ \Gamma_1(z^{-1}),\ \cdots,\ \Gamma_N(z^{-1})]^T$, the polynomial $\Gamma_i(z^{-1}) \in \Re[z^{-1}, \max\{n_d + m + f + d - 2, 0\}]$ and the matrix $M \in \Re^{(N+1)\times(N+1)}$. As a result,

$$\begin{aligned}U(t\,|\,t) = (L+M)^{-1}\big(&D(z^{-1})R(t+d) -\\ &D(z^{-1})F(z^{-1})y(t-f) -\\ &\big(\Gamma(z^{-1}) + H(z^{-1})\big)u(t-1\,|\,t-1)\big)\end{aligned} \tag{20}$$

Therefore, the control prediction sequence can be determined by the following predictive controller:

$$\begin{bmatrix} u(t|t) \\ u(t+1|t) \\ \vdots \\ u(t+N|t) \end{bmatrix} = \begin{bmatrix} P_0(z^{-1}) \\ P_1(z^{-1}) \\ \vdots \\ P_N(z^{-1}) \end{bmatrix} r(t+d+N) - \begin{bmatrix} Q_0(z^{-1}) \\ Q_1(z^{-1}) \\ \vdots \\ Q_N(z^{-1}) \end{bmatrix} y(t-f) - \begin{bmatrix} S_0(z^{-1}) \\ S_1(z^{-1}) \\ \vdots \\ S_N(z^{-1}) \end{bmatrix} u(t-1|t-1) \tag{21}$$

where

$$\begin{aligned} &\left[P_0(z^{-1}) \quad P_1(z^{-1}) \quad \cdots \quad P_N(z^{-1})\right]^T \\ &\qquad = (L+M)^{-1}\left[z^{-N} \quad z^{-N+1} \quad \cdots \quad 1\right]^T D(z^{-1}) \\ &\left[Q_0(z^{-1}) \quad Q_1(z^{-1}) \quad \cdots \quad Q_N(z^{-1})\right]^T \\ &\qquad = (L+M)^{-1} F(z^{-1}) D(z^{-1}) \\ &\left[S_0(z^{-1}) \quad S_1(z^{-1}) \quad \cdots \quad S_N(z^{-1})\right]^T \\ &\qquad = (L+M)^{-1}\left(\Gamma(z^{-1}) + H(z^{-1})\right) \end{aligned}$$

and the polynomial $P_i(z^{-1}) \in \Re[z^{-1}, n_d + N]$, $Q_i(z^{-1}) \in \Re[z^{-1}, n_d + n - 1]$ and $S_i(z^{-1}) \in \Re[z^{-1}, \max\{n_c - i - 1, n_d + m + f + d - 2, 0\}]$

In order to compensate the network transmission delay, a network delay compensator is proposed. A very important characteristic of the network is that it can transmit a set of data at the same time. Thus, it is assumed that all predictive control sequences at time *t* are packed and sent to the plant side through the network. The networked delay compensator chooses the latest control value from the control prediction sequences available on the plant side. For example, if the following predictive control sequences are received on the plant side:

$$\begin{bmatrix} u(t-k_1|t-k_1) \\ u(t-k_1+1|t-k_1) \\ \vdots \\ u(t|t-k_1) \\ \vdots \\ u(t+N-k_1|t-k_1) \end{bmatrix}, \begin{bmatrix} u(t-k_2|t-k_2) \\ u(t-k_2+1|t-k_2) \\ \vdots \\ u(t|t-k_2) \\ \vdots \\ u(t+N-k_2|t-k_2) \end{bmatrix}, \ldots, \begin{bmatrix} u(t-k_t|t-k_t) \\ u(t-k_t+1|t-k_t) \\ \vdots \\ u(t|t-k_t) \\ \vdots \\ u(t+N-k_t|t-k_t) \end{bmatrix} \tag{22}$$

where the control values $u(t|t-k_i)$ for *i= 1, 2, .., t*, are available to be chosen as the control input of the plant at time *t*, the output of the network delay compensator will be

$$u(t) = u(t|t - \min\{k_1, \quad k_2, \quad \cdots, \quad k_t\}) \tag{23}$$

which is the latest predictive control value for time *t.*

As a sequence of control predictions is transmitted to the actuator each time through the network, it is clear that the proposed NPCS can cope with several consecutive data package dropouts if the number of control predictions in each package is greater than the assumed upper bound of the network delay.

III. STABILITY OF NETWORKED PREDICTIVE CONTROL SYSTEMS

In practical network communications, there exist a number of uncertainties which affect the transmission time during the data information exchange, for example, communication traffic congestion. This results in the communication delay being random. This section studies how this random communication delay affects the stability of closed-loop networked predictive control systems. Here, it is assumed that the random communication delay is bounded with d_N.

It assumes that the network transmission delay *k* in the forward channel is random but bounded, *i.e.*, $k \in \{0, 1, 2, \cdots, N\}$, where *N* is the upper bound, and the time delay *f* in the backward channel is constant. The plant can be written as

$$A(z^{-1})y(t) = z^{-d}B(z^{-1})u(t) = z^{-d}\sum_{i=0}^{m} b_i u(t-i) \tag{24}$$

Since the network transmission delay is random, to effectively compensate for this delay the networked control predictor is designed to be

$$u(t-i) = u(t-i|t-i-k_i), \text{for } i = 0, 1, 2, ..., m \tag{25}$$

subject to $k_i \le k_{i-1} + 1$, where $u(t-i|t-i-k_i)$ is the latest predictive control at time *t-i* which is available at the plant side and $k_i \in \{0, 1, 2, \cdots, N\}$ is a random number.

From the control prediction sequence derived in the previous section, it can be obtained that

$$\begin{aligned} u(t|t) &= P_0(z^{-1})r(t+d+N) \\ &\quad - Q_0(z^{-1})y(t-f) - S_0(z^{-1})u(t-1|t-1) \end{aligned} \tag{26}$$

Then

$$u(t|t) = \frac{P_0(z^{-1})r(t+d+N) - Q_0(z^{-1})y(t-f)}{1 + S_0(z^{-1})z^{-1}} \tag{27}$$

Using (21) and (27), the *k*-step ahead predictive control at time *t* is expressed by

$$
\begin{aligned}
&u(t+k\,|\,t) = P_k(z^{-1})r(t+d+N) - Q_k(z^{-1})y(t-f) \\
&- S_k(z^{-1})u(t-1\,|\,t-1) \\
&= \frac{P_k(z^{-1}) + P_k(z^{-1})S_0(z^{-1})z^{-1} - P_0(z^{-1})S_k(z^{-1})z^{-1}}{1+S_0(z^{-1})z^{-1}} r(t+d+N) \\
&- \frac{Q_k(z^{-1}) + Q_k(z^{-1})S_0(z^{-1})z^{-1} - Q_0(z^{-1})S_k(z^{-1})z^{-1}}{1+S_0(z^{-1})z^{-1}} y(t-f)
\end{aligned} \tag{28}
$$

Following (28), the control prediction $u(t-i\,|\,t-i-k_i)$ is calculated by

$$
\begin{aligned}
&u(t-i\,|\,t-i-k_i) \\
&= \frac{P_{k_i}(z^{-1}) + P_{k_i}(z^{-1})S_0(z^{-1})z^{-1} - P_0(z^{-1})S_{k_i}(z^{-1})z^{-1}}{1+S_0(z^{-1})z^{-1}} r(t+d+N-i-k_i) \\
&- \frac{Q_{k_i}(z^{-1}) + Q_{k_i}(z^{-1})S_0(z^{-1})z^{-1} - Q_0(z^{-1})S_{k_i}(z^{-1})z^{-1}}{1+S_0(z^{-1})z^{-1}} y(t-i-k_i-f)
\end{aligned} \tag{29}
$$

As a result, the closed-loop control system is

$$
\begin{aligned}
&A(z^{-1})y(t) = z^{-d}\sum_{i=0}^{m} b_i u(t-i\,|\,t-i-k_i) \\
&= \sum_{i=0}^{m} b_i \frac{P_{k_i}(z^{-1}) + P_{k_i}(z^{-1})S_0(z^{-1})z^{-1} - P_0(z^{-1})S_{k_i}(z^{-1})z^{-1}}{1+S_0(z^{-1})z^{-1}} z^{-i} r(t+d+N-k_i) \\
&- z^{-d}\sum_{i=0}^{m} b_i \frac{Q_{k_i}(z^{-1}) + Q_{k_i}(z^{-1})S_0(z^{-1})z^{-1} - Q_0(z^{-1})S_{k_i}(z^{-1})z^{-1}}{1+S_0(z^{-1})z^{-1}} z^{-i-k_i-f} y(t)
\end{aligned} \tag{30}
$$

Therefore, the closed-loop characteristic equation is

$$
\begin{aligned}
&A(z^{-1})\left(1+S_0(z^{-1})z^{-1}\right) + z^{-d-f}\sum_{i=0}^{m} b_i \Big(Q_{k_i}(z^{-1}) + \\
&Q_{k_i}(z^{-1})S_0(z^{-1})z^{-1} - Q_0(z^{-1})S_{k_i}(z^{-1})z^{-1}\Big) z^{-i-k_i} = 0
\end{aligned} \tag{31}
$$

subject to $k_i \le k_{i-1}+1$. As $k_i \in \{0, 1, 2, \cdots, N\}$ is a random number, there are a number of possible closed-loop characteristic equations. There exist the upper and lower bounds of each coefficient in the closed-loop characteristic polynomial.

Since the communication delay k_i varies randomly, the coefficients of the above characteristic polynomial also change randomly with the delay k_i. This makes it very difficult to determine the stability of the closed-loop networked predictive control system by the direct use of the above equation. Now, let $P(z^{-1})$ denote the closed-loop characteristic polynomial given in (31). Then

$$
P(z^{-1}) = P_1(z^{-1}) + z^{-d}P_2(z^{-1}) + z^{-d-f}\sum_{i=0}^{m} W_i(z^{-1}, k_i) \tag{32}
$$

where

$$
\begin{aligned}
&P_1(z^{-1}) + P_2(z^{-1})z^{-d-f} = A(z^{-1})\left(1+S_0(z^{-1})z^{-1}\right) \\
&W_i(z^{-1}, k_i) = b_i\left(Q_{k_i}(z^{-1}) + Q_{k_i}(z^{-1})S_0(z^{-1})z^{-1} - Q_0(z^{-1})S_{k_i}(z^{-1})z^{-1}\right)z^{-i-k_i}
\end{aligned}
$$

which can be expressed by

$$
\begin{aligned}
&P_1(z^{-1}) = 1 + p_{1,1}z^{-1} + p_{1,2}z^{-2} + \cdots + p_{1,d-1}z^{-d-f+1} \\
&P_2(z^{-1}) = p_{2,0}z^{-d-f} + p_{2,1}z^{-d-f-1} + p_{2,2}z^{-d-f-2} + \cdots \\
&W_i(z^{-1}, k_i) = w_{i,0}(k_i) + w_{i,1}(k_i)z^{-1} + w_{i,2}(k_i)z^{-2} + \cdots
\end{aligned}
$$

Clearly, the coefficients of the closed-loop characteristic polynomial

$$
P(z^{-1}) = 1 + p_1 z^{-1} + p_2 z^{-2} + \cdots + p_{\bar{n}} z^{-\bar{n}} \tag{33}
$$

are given by

$$
\begin{aligned}
&p_j = p_{1,j} && j = 1, 2, \cdots d+f-1 \\
&p_j = p_{2,j-d} + \sum_{i=0}^{m} w_{i,j-d}(k_i) && j = d+f, d+f+1, \cdots, \bar{n}
\end{aligned}
$$

where $\bar{n}$ is the highest order of the polynomial for $k_i \in \{0, 1, 2, \cdots, d_N\}$. Both the upper and lower bounds of the coefficients of the polynomial $P(z^{-1})$ are

$$
\begin{aligned}
&p_j^+ = p_j^- = p_j = p_{1,j}\,, \qquad for \quad j = 1, 2, \cdots d+f-1 \\
&p_j^+ = \max_{k_0, k_1, \cdots k_m} p_j = \max_{k_0, k_1, \cdots k_m}\left(p_{2,j-d} + \sum_{i=0}^{m} w_{i,j-d}(k_i)\right), \\
&\qquad for \;\; j = d+f, d+f+1, \cdots, \bar{n} \\
&p_j^- = \min_{k_0, k_1, \cdots k_m} p_j = \min_{k_0, k_1, \cdots k_m}\left(p_{2,j-d} + \sum_{i=0}^{m} w_{i,j-d}(k_i)\right), \\
&\qquad for \;\; j = d+f, d+f+1, \cdots, \bar{n}
\end{aligned}
$$

subject to $k_i \le k_{i-1}+1$ and $k_i \in \{0, 1, 2, \cdots, d_N\}$, which can be solved by a constrained optimisation method [11]. So, the stability of the closed-loop system is determined by the characteristic equation $P(z^{-1}) = 0$. This is equivalent to the stability of the following discrete system:

$$
x_{t+1} = T_t x_t \tag{34}
$$

where the state vector $x_t \in \Re^{\bar{n}}$ and

$$
T_t = \begin{bmatrix} 0 & 1 & & & \\ & 0 & 1 & & \\ & & \ddots & \ddots & \\ & & & \ddots & 1 \\ -p_{\bar{n}} & -p_{\bar{n}-1} & \cdots & \cdots & -p_1 \end{bmatrix}
$$

Since $p_i \in [p_i^-, p_i^+]$, for $i=1, 2, \ldots, \bar{n}$, system (34) is a subset of the following

$$
x_{t+1} = \left(\bar{T} + \bar{H}\;\bar{F}_t\right)x_t \tag{35}
$$

where

$$
\bar{T} = \begin{bmatrix} 0 & 1 & & & \\ & 0 & 1 & & \\ & & \ddots & \ddots & \\ & & & \ddots & 1 \\ -\frac{p_{\bar{n}}^+ + p_{\bar{n}}^-}{2} & -\frac{p_{\bar{n}-1}^+ + p_{\bar{n}-1}^-}{2} & \cdots & \cdots & -\frac{p_1^+ + p_1^-}{2} \end{bmatrix}
$$

$$\overline{H}=\begin{bmatrix} 0 & & & & \\ \vdots & 0 & & & \\ \vdots & \vdots & \ddots & & \\ 0 & 0 & \cdots & 0 & 0 \\ \frac{p_{\bar{n}}^{+}-p_{\bar{n}}^{-}}{2} & \frac{p_{\bar{n}-1}^{+}-p_{\bar{n}-1}^{-}}{2} & \cdots & \cdots & \frac{p_{1}^{+}-p_{1}^{-}}{2} \end{bmatrix}$$

$$\overline{F}_t=\begin{bmatrix} r_{\bar{n}}(t) & & & \\ & r_{\bar{n}-1}(t) & & \\ & & \ddots & \\ & & & r_1(t) \end{bmatrix}$$

and $r_i(t)\in[-1,1]$ is a random value. Clearly, $\overline{F}_t^T\overline{F}_t\leq I$. Following [12], system (35) is stable if and only if there exists a matrix $V=V^T>0$ satisfying the following matrix inequality:

$$\begin{bmatrix} -V^{-1} & \overline{T} & \overline{H} \\ \overline{T}^T & -V+\tau I & 0 \\ \overline{H}^T & 0 & -\tau I \end{bmatrix}\leq 0 \tag{36}$$

with constant $\tau\geq 0$.

Since system (34) is the subset of (35), inequality (36) is only a sufficient criterion rather than a necessary condition. Therefore, if inequality (36) holds the closed-loop networked predictive control system with a random communication delay is stable. In other words, the closed-loop system may be either stable or unstable if there does not exist a matrix V satisfying the above inequality.

In the case where the network communication delay is constant k, which is a special case of random network delay, it is clear from (31) that the closed-loop characteristic equation is

$$\begin{aligned} A(z^{-1})\left(1+S_0(z^{-1})z^{-1}\right)+z^{-d-f-k}B(z^{-1})\big(Q_k(z^{-1}) \\ +Q_k(z^{-1})S_0(z^{-1})z^{-1}-Q_0(z^{-1})S_k(z^{-1})z^{-1}\big)=0 \end{aligned} \tag{37}$$

So, the closed-loop networked predictive control system with constant network delay is stable if and only if the roots of the above polynomial is within the unit circle.

IV. CONCLUSIONS

This paper has presented a novel networked predictive control scheme and studied the stability of closed-loop networked predictive control systems. To compensate for the network communication delay in an active way, the networked predictive control scheme consists of two parts: a control prediction generator and a network delay compensator. The required control performance can be achieved using conventional control design methods. It has mainly considered the case of the network communication delay. The system stability criterion of closed-loop networked predictive control systems is expressed by a matrix inequality. For the case of constant network delay, a stability criterion of the closed-loop system is derived and is expressed by the stability of a single polynomial. These analytical results of system stability provide a fundamental requirement for the design of networked control systems.

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