# 2x2 Individual Channel Design MATLAB® Toolbox

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Abstract—In this paper a novel software for analysis and design of multivariable 2x2 control systems is presented. The 2x2 Individual Channel Design MATLAB® Toolbox is a valuable aid for analysing and designing multivariable control systems under the framework of the Multivariable Structure Function (MSF) and Individual Channel Design (ICD). Given a set of specifications for a 2-input 2-output multivariable control system the appropriate use of the toolbox can lead to successful and robust controllers. The process is based on an iterative procedure. Closed loop simulations (in SIMULINK®) are included so results can be tested. Final stability margins and robustness measures are also assessed. A design example is included for completeness. The development of the general m xm case is in progress.

#### I. INTRODUCTION

THE purpose of this paper is to introduce the novel Individual Channel Design MATLAB® Toolbox and to provide an explanation of how it should be used. At present the software considers only 2x2 control systems.

The toolbox is a valuable aid for analysing and designing multivariable control systems. Given a set of specifications for a  $2x^2$  multivariable control system the appropriate use of the toolbox can lead to successful controllers. The process is based on an iterative procedure. Closed loop simulations (in SIMULINK®) are included so results can be tested. Final stability margins and robustness measures are also assessed.

The toolbox is inspired on a new approach for multivariable control systems, referred as Individual Channel Design (ICD) [3,4]. ICD is a novel analytical framework that allows the analysis and synthesis of multivariable control systems under the context of the Multivariable Structure Function (MSF) by applying classical techniques based on the Bode and Nyquist plots. With the help of this framework it is possible to investigate the potential and limitations for feedback design of any multivariable linear time-invariant control system. Although ICD is in principle a feedback structure based on diagonal controllers, it can be applied to any cross coupled multivariable system [4,7]. It is based on the definition of individual transmission channels. In this context the control design is an interactive process that involves the required specifications, plant characteristics, and the multivariable feedback design process itself. Once the channels are

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J. Licéaga-Castro is with the Mechanic and Mechatronics Engineering Department, ITESM-CEM, Carretera Lago de Guadalupe Km.3.5, Atizapán de Zaragoza, C.P.52926, Edo.México, México (e-mail: jliceaga@itesm.mx). defined it is possible to form a feedback loop with the compensator specially designed to meet *customer* specifications. In this manner the multivariable control design problem is reduced to the design of a single–input single–output control for each channel [3,4].

ICD has been reported in some control strategies, such as in small scale power networks with embedded generation [1], in the automotive and the aerospace industry [5].

So far, this toolbox has been used in different control tasks, from induction motors [8,9], synchronous generators [10], to submarines [6]. It has also been employed to provide a design example in a family of plants that range from a simple and decoupled to a highly coupled and non-minimum phase one [7].

The document is organised as follows. Section II includes a brief review of ICD. In fact, it is highly recommended to have some expertise in the subject in order to exploit the toolbox successfully. In Section III the design procedure is explained. An introduction to the software and a design example using the toolbox is carried out in Section IV. It is shown in full detail in order to provide a clear demonstration of the toolbox potential. The example here presented corresponds to an interesting and rather challenging design case [7]. Finally, in Section V the conclusions end the paper.

#### II. AN OVERVIEW OF INDIVIDUAL CHANNEL DESIGN

In a typical control design task the performance is specified in terms of an output response to a given input. Meanwhile, in multivariable control, there are a number of inputs and outputs and, although it may be clear which inputs are intended to drive which outputs, the design task can be obscured by cross-coupling via the plant dynamics. Nevertheless, for clarity of both performance specification and design, it remains desirable to consider the inputs and outputs in pairs. The situation is depicted in Fig. 1, where **G** is the plant and **K** is the controller. Input  $r_i$  is paired with output  $y_i$  in accordance with specifications. An individual pairing is called a *channel*. Then, channel  $C_i$  is the pairing between  $r_i$  and  $y_i$  [3].



Fig. 1. MIMO multivariable system. Channel definition

When the plant cross-coupling is weak, the design task reduces to a set of SISO design tasks and a scalar controller can be designed separately for each channel. In such context, the most appropriate methodology is to apply classical Nyquist/Bode analysis and design to each channel.

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ICD is a framework in which Bode/Nyquist techniques can be applied directly to the channels not only when crosscoupling is weak but in all circumstances including when cross-coupling is strong. The multivariable system is decomposed into an equivalent set of SISO systems. Each SISO system is the open-loop channel transmittance between input  $r_i$  and output  $v_i$ , with the feedback loop between output  $y_i$  and input  $r_i$  open but all other feedback loops closed, for a particular choice of *i*. What is particular to Individual Channel Design is that the SISO channel transmittances are reformulated to make explicit the role of the plant structure. Scalar multivariable structure functions (MSFs) to which the individual channel transmittances are simply related encapsulate the significant aspects of the plant structure. The multivariable nature of the original plant is maintained in the equivalent SISO systems through the multivariable structure functions with no loss of information [3,4].

The ICD set up for a 2-input 2-output plant is shown next for completeness. Let a  $2x^2$  plant

$$\mathbf{Y}(s) = \mathbf{G}(s)\mathbf{u}(s) \tag{1}$$

be represented by

$$\begin{bmatrix} y_1(s) \\ y_2(s) \end{bmatrix} = \begin{bmatrix} g_{11}(s) & g_{12}(s) \\ g_{21}(s) & g_{22}(s) \end{bmatrix} \begin{bmatrix} u_1(s) \\ u_2(s) \end{bmatrix}$$
(2)

where  $g_{ij}(s)$  represents scalar transfer functions,  $y_i(s)$  the outputs, and  $u_i(s)$  the inputs of the system, with i,j = 1,2. If a diagonal controller is given by

$$\mathbf{u}(s) = \mathbf{K}(s)\mathbf{e}(s) \tag{3}$$

$$\begin{bmatrix} u_1(s) \end{bmatrix}_{-} \begin{bmatrix} k_{11}(s) & 0 \end{bmatrix} \begin{bmatrix} e_1(s) \end{bmatrix}$$
(4)

$$\begin{bmatrix} 1(r) \\ u_2(s) \end{bmatrix} = \begin{bmatrix} 1(r) \\ 0 \\ k_{22}(s) \end{bmatrix} \begin{bmatrix} 1(r) \\ e_2(s) \end{bmatrix}$$
(4)

with  $e_i(s) = r_i(s) - y_i(s)$ , where  $r_i(s)$  represents the plant references, then the open loop input–output channels are clearly defined from Figs. 2 and 3 as

$$C_i(s) = k_{ii}(s)g_{ii}(s)(1-\gamma_a(s)h_j(s))$$
(5)

where  $i \neq j$  and i, j = 1, 2. The complex valued function

$$y_a(s) = \frac{g_{12}(s)g_{21}(s)}{g_{11}(s)g_{22}(s)}$$
(6)

is referred to as the *multivariable structure function* (MSF). The functions  $h_i(s)$  are:

$$h_{i}(s) = \frac{k_{ii}(s)g_{ii}(s)}{1 + k_{ii}(s)g_{ii}(s)}$$
(7)

The interaction or cross coupling between the channels can be evaluated through a transfer function. For instance, the influence of channel–*j* on channel–*i* is (Fig. 3):

$$d_{i}(s) = \frac{g_{ij}(s)}{g_{jj}(s)} h_{j}(s) r_{j}(s) .$$
(8)

It is clear that the correct interpretation of the MSF (6) is of great importance because

- It determines the dynamical characteristics of each input–output configuration;
- It has an interpretation in the frequency domain;
- Its magnitude quantifies the coupling between the

channels (in the frequency domain);

- It is related to the plant transmission zeros (zeros of  $l-\gamma_a(s)$ ,  $|\mathbf{G}(s)| = g_{11}(s)g_{22}(s) g_{12}(s)g_{21}(s) = 0$ );
- $\gamma_a(s) = 1$  determines the non-minimum phase condition;
- Its closeness to (1,0) in the Nyquist plot indicates to what extent the plant is sensitive to uncertainty in terms of gain and phase margins. This fact plays a key role in order to obtain robust controllers.

A block diagram of the feedback system with the diagonal controller is shown in Fig. 2 and the equivalent scalar channels are shown in Fig. 3.



Fig. 2. The 2-input 2-output multivariable system with a diagonal controller



Fig. 3. Equivalent channels of a 2-input 2-output control system

It should be emphasised that in the individual channel representation of the multivariable system there is no loss of information [3]. The multivariable character and cross coupling of the plant are contained in the MSF and the cross coupling terms. That is, (5)–(8) are equivalent to the closed-loop matrix function

$$\mathbf{G}_{cl}(s) = \left(\mathbf{I} + \mathbf{G}(s)\mathbf{K}(s)\right)^{-1}\mathbf{G}(s)\mathbf{K}(s).$$
<sup>(9)</sup>

It can be proven that in order to stabilise (9) it is just necessary to stabilise the channels given by (5) [7,9]. In general stabilisation of the diagonal elements of G(s) is not required [9]. The open loop system dynamical structure with a diagonal controller is summarised in Table I [3].

Notice that the coupling can be expressed in decibels directly from the channels (5) by means of functions  $\gamma_a(s)h_j(s)$ . On the other hand, it is possible to determine the dynamical structure of the system using Table I and analysing the Nyquist plot of  $(1-\gamma_a(s)h_j(s))$ .

| Table I. Dynamical structure of open loop chan | nels |
|--|------|
|--|------|

| Channel    | Zeros                          | Poles   |
|------------|--------------------------------|---|
| $C_{l}(s)$ | Zeros of $(1-\gamma(s)h_2(s))$ | Poles of $g_{11}(s)$ , $g_{12}(s)$ , $g_{21}(s)$ , $h_2(s)$ |
| $C_2(s)$   | Zeros of $(1-\gamma(s)h_2(s))$ | Poles of $g_{22}(s)$ , $g_{12}(s)$ , $g_{21}(s)$ , $h_1(s)$ |

It is clear that the controller performance characteristics are determined by the MSF. If the transfer matrix G(s)possess a non-minimum phase transmission zero, some problems will arise while stabilising it especially if the value of the zero is smaller than the desired cut–off frequency. Moreover, the robustness of the channels can be established in terms of gain and phase margins as the Nyquist paths of the functions  $\gamma_a(s)h_i(s)$  do not pass near (1,0). Thus the design of  $k_{ii}(s)$ , which should provide adequate gain and phase margins for  $k_{ii}(s)g_{ii}(s)$ , can be obtained through an iterative process.

It should be noticed that the RHPPs of the channels are RHPPs of individual transfer functions as established in Table I. On the other hand, the RHPZs of the channels are RHPZs of  $(1-\gamma_a(s)h_i(s))$ . Moreover, the number of RHPZs of the previous function can be determined after applying the Nyquist Stability Criterion. In fact, the RHPZs of  $(1-\gamma_a(s)h_i(s))$  are given by

$$Z = N + P . \tag{10}$$

where *P* is the number of RHPPs of  $\gamma_a(s)h_i(s)$  and *N* is the number of encirclements in clockwise direction to (1,0) of the complex plane in the Nyquist diagram of  $\gamma_a(s)h_i(s)$ .

The dynamical structure of the  $2x^2$  plant is determined by the input–output channels defined by pairing each input to each output. For instance:

- (a)  $C_1(s)$ :  $u_1(s)-y_1(s)$  with  $\gamma_a(s) = g_{12}(s)g_{21}(s)/g_{11}(s)g_{22}(s)$  $C_2(s)$ :  $u_2(s)-y_2(s)$
- (b)  $C_1(s)$ :  $u_1(s)-y_2(s)$  with  $\gamma_b(s) = g_{11}(s)g_{22}(s)/g_{12}(s)g_{21}(s)$  $C_2(s)$ :  $u_2(s)-y_1(s)$

The coupling characteristic of each configuration is determined from  $\gamma_a(s)$  and  $\gamma_b(s)$  –their associated MSFs.

## III. THE DESIGN PROCEDURE

The dynamical characteristic of each input-output configuration is determined from their associated MSF. After choosing and analysing the desired MSF and following some steps, a successful design can be obtained using the ICD  $2x^2$  MATLAB® Toolbox.

The design approach follows the next methodology:

- 1. Define either a state space representation (time domain) or a transfer matrix (frequency domain)
- 2. Analyse the two possible MSFs and determine the existence of the controller for the desired case.
- 3. Define the controller matrix by choosing the inputoutput pairing associated to the desired MSF:

(a) 
$$\mathbf{K}(s) = \begin{bmatrix} k_{11}(s) & 0\\ 0 & k_{22}(s) \end{bmatrix} \Rightarrow \stackrel{r_1(s) \to y_1(s)}{r_2(s) \to y_2(s)} \Rightarrow \gamma_a(s)$$
  
(b) 
$$\mathbf{K}(s) = \begin{bmatrix} 0 & k_{12}(s)\\ k_{21}(s) & 0 \end{bmatrix} \Rightarrow \stackrel{r_1(s) \to y_2(s)}{r_2(s) \to y_1(s)} \Rightarrow \gamma_b(s)$$

If (a) is chosen, equations (3)–(8) apply. Otherwise, the channels are defined as

$$C_{i}(s) = k_{ij}(s)g_{ji}(s)(1 - \gamma_{b}(s)h_{j}(s))$$
(11)

where  $i \neq j$  and i,j = 1,2 and

$$\gamma_b(s) = \frac{g_{11}(s)g_{22}(s)}{g_{12}(s)g_{21}(s)}$$
(12)

$$h_{i}(s) = \frac{k_{ij}(s)g_{ji}(s)}{1 + k_{ij}(s)g_{ji}(s)}$$
(13)

$$d_{i}(s) = \frac{g_{jj}(s)}{g_{ij}(s)} h_{j}(s) r_{j}(s) .$$
<sup>(14)</sup>

4. First approximation in  $C_i(s)$  ( $h_j(s) = 1$ );  $i \neq j$ 

- 5. Controller design for channel  $C_i(s)$
- 6. Channel  $C_j(s)$  interaction defined ( $h_i(s)$  defined)
- 7. Structural robustness assessment ( $\gamma(s)h_i(s)$ )
- 8. Controller design for channel  $C_i(s)$
- 9. Channel  $C_i(s)$  interaction defined ( $h_i(s)$  defined)
- 10. Structural robustness assessment ( $\gamma(s)h_i(s)$ )
- 11. Controller redesign for channel  $C_i(s)$
- 12. Process repeated until a successful controller is achieved for both channels and robustness is assured in  $C_i(s)$ ,  $k_{ii}(s)g_{ii}(s)$  (or  $k_{ij}(s)g_{ji}(s)$ ), and  $\gamma(s)h_i(s)$  ( $i\neq j$ , i,j=1,2).

Notice that the design process is iterative.

## IV. THE SOFTWARE: A DESIGN EXAMPLE

The *Individual Channel Design 2x2 MATLAB® Toolbox* is a valuable aid for analysing and designing multivariable control systems. Nevertheless, the toolbox itself does not solve the control design problem. It is very important that the user has some expertise in ICD and classical control techniques (especially in Bode/Nyquist analysis). It is of special importance that prior to the control system design, the structure of the system is rigorously analysed.

The toolbox comprises a set of files which allow the analysis and design of the control system. Such files are programmed such that the design task is straightforward. After the design of each element of the diagonal controller, robustness of the channel itself and functions  $k_{ii}(s)g_{ii}(s)$  (or  $k_{ij}(s)g_{ji}(s)$ ), and  $\gamma_a(s)h_i(s)$  (or  $\gamma_b(s)h_i(s)$ ,  $i\neq j$ , i,j=1,2) can be studied.

The user-toolbox interaction is made through the MATLAB® command window. The MSFs, channels, and other useful information are presented in a sequential way depending on which channel is designed at the moment. When the user is satisfied with the whole design it can be tested in a SIMULINK® model, where the response of the closed-loop control system can be assessed.

The design example here presented corresponds to an interesting and rather challenging design case taken from literature [2,7]. Both input–output pairings are studied.

Let  $\mathbf{Y}(s) = \mathbf{G}(s)\mathbf{u}(s)$  be a 2x2 multivariable plant as in (2), where the elements of  $\mathbf{G}(s)$  are:

$$g_{11}(s) = \frac{2}{s^2 + 3s + 2}, \ g_{12}(s) = \frac{-2}{s + 1}$$

$$g_{21}(s) = \frac{-1}{s^2 + 2s + 1}, \ g_{22}(s) = \frac{6}{s^2 + 5s + 6}$$
(15)

The system is non-minimum phase.

The first step is to run the toolbox. Fig. 5 shows an example of the command window.

Plant (15) is introduced first. The user should follow carefully the instructions that appear in the command window after introducing any data. After the plant is defined the possible MSFs (depending on the pairing made) are

shown. The user then should choose which pairing use and which channel design first. It is important to mention that all the graphics shown here are obtained directly during the execution of the toolbox. Analysis and design for both pairings is made. It should be noticed that the system is non-minimum phase and in the pairing corresponding to  $\gamma_a(s)$  the coupling is strong, so the design task is quite challenging.



Fig. 5. MATLAB® Command Window. Using the toolbox.

1) Analysis with  $\gamma_a(s)$ : The corresponding MSF is

$$\gamma_a(s) = 0.16667 \cdot \frac{(s+3)(s+2)^2}{(s+1)^2}$$
(16)

and the input–output channels are defined as (5)-(8).

The corresponding Nyquist and Bode plots given by the toolbox are shown in Fig. 6. The user has to analyse the characteristics given in the plots, which are given next.

(a.1) The value of  $\gamma_a(0) > 1$  (in fact,  $\gamma_a(0)=2$ ) and the Nyquist diagram of  $\gamma_a(s)$  starts at the right side of (1,0). Also,  $\gamma_a(s)$  has no RHPPs and its Nyquist path encircles clockwise the point (1,0) twice. So, according to the Nyquist Stability Criterion,  $(1-\gamma_a(s))$  contains two RHPZs.

(a.2) If  $h_2(0) > 0.5$  and stable then  $(1-\gamma_a(0)h_2(0)) < 0$ . Thus, a stabilising controller  $k_{II}(s)$  is such that  $k_{II}(0) < 0$ . On the other hand,  $g_{II}(0) > 0$ ; therefore,  $h_I(s)$  is unstable with one RHPP. Also, if  $|k_{II}(0)g_{II}(0)| > 0$ , then  $h_I(0) > 0$ .

(a.3) If (a.1) and (a.2) hold, then  $g_{22}(0)(1-\gamma_a(0)h_1(0))>0$ ; thus, a stabilising controller  $k_{22}(s)$  is such that  $k_{22}(0)>0$ . Moreover, if the relative degree of  $h_1(s)$  is greater than the relative degree of  $\gamma_a(s)$ , then  $\gamma_a(s)h_1(s)$  encircles counter clockwise (1,0) once. Recall that  $\gamma_a(s)h_1(s)$  has one RHPP; thus, Channel 2 is minimum-phase.

(a.4) From (a.3) it is clear that the stabilising controller of Channel 2 stabilises both  $g_{22}(s)(1-\gamma_a(s)h_1(s))$  and  $g_{22}(s)$  simultaneously.

From the previous analysis, it can be concluded that no stabilisation of diagonal elements is required.

Considering the previous analysis, it can be concluded that the control for this configuration exists, but it presents performance limitations (Channel-*1* has 2 RHPZs) and it is not possible to arbitrarily reduce the sensitivity to uncertainty. With the use of the toolbox and following the steps given in Section III (which are followed in the same manner in the toolbox) the controller is designed. After performing the required iterations, the following controllers are obtained:



(17)

**Fig. 6.** Assessing  $\gamma_a(s)$  structure: (a) Nyquist diagram; (b) Bode diagram



Fig. 7. Control system structural robustness assessment with γ<sub>a</sub>(s):
(a) Channel 1 and Channel 2 Bode diagrams; (b) k<sub>11</sub>(s)g<sub>11</sub>(s) and k<sub>22</sub>(s)g<sub>22</sub>(s) Bode diagrams; (c) γ<sub>a</sub>(s), γ<sub>a</sub>(s)h<sub>2</sub>(s), and γ<sub>a</sub>(s)h<sub>1</sub>(s) Nyquist diagrams.

It should be mentioned that the controller (17) satisfies the analysis carried out before. Another possible controller may exist, but the one given before has an adequate performance (given the limitations pointed out before). Moreover, the associated structural robustness is summarised in Table II and Fig. 7.

**Table II.** Control system structural robustness assessment with  $\gamma_a(s)$ 

| Measure     | $C_{I}(s)$ | $k_{11}(s)g_{11}(s)$ | $\gamma_a(s)h_2(s)$ | $C_2(s)$ | $k_{22}(s)g_{22}(s)$ | $\gamma_a(s)h_1(s)$ |
|-------------|------------|----------------------|---------------------|----------|----------------------|---------------------|
| BW[rad/s]   | 0.103      | 0.1                  | _                   | 0.521    | 0.65                 | _                   |
| $G_M[dB]$   | 8.17       | $\infty$             | æ                   | 17.2     | 17                   | 21.82               |
| $P_M$ [deg] | 56.9       | 98                   | 124                 | 57.5     | 60                   | 41                  |

The diagrams of Fig. 7 are given directly by the toolbox. After the design for the previous channel pairing is done, it is possible to perform a simulation of the closed–loop system in SIMULINK®, as shown in Fig. 8.



Fig. 8. Closed–loop simulation associated with  $\gamma_a(s)$  in SIMULINK®

2) Analysis with  $\gamma_b(s)$ : This corresponds to the alternative pairing. The associated multivariable structure function is

$$\gamma_{b}(s) = 6 \frac{(s+1)^{2}}{(s+3)(s+2)^{2}}$$
(18)

and the channels are defined as in (11)–(14). The same procedure as in 1) is followed. The Nyquist and Bode plots of  $\gamma_b(s)$  obtained from the toolbox are shown in Fig. 9. The analysis of such diagrams is carried out next:



Fig. 9. Assessing  $\gamma_b(s)$  structure: (a) Nyquist diagram; (b) Bode diagram

**(b.1)** The magnitude of  $\gamma_b(s)$  while  $s \rightarrow \infty$  is less than one (in fact, zero), so  $1 - \lim_{s \to \infty} \gamma_b(s) > 0$ . In this case,  $\gamma_b(0) = 0.5$ . This

implies that its Nyquist plot starts at the left side of (1,0). Nevertheless, the Nyquist plot of  $\gamma_b(s)$  is not always located at the left side of (1,0) for any frequency value.

(b.2) Function  $\gamma_b(s)$  has no RHPPs and its Nyquist plot encircles the point (1,0) twice in clockwise direction. Then,  $(1-\gamma_b(s))$  contains 2 RHPZs.

**(b.3)** Notice that at low frequencies  $\gamma_b(s) < 1$ . Then, if  $k_{ij}(s)$  induce a roll–off at low frequencies  $\gamma_b(s)h_j(s)$  will not encircle (1,0), which in this case is desirable.

Thus, the existence of controllers reduces to the existence of  $k_{ij}(s)$  which simultaneously stabilise  $g_{ji}(s)(1-\gamma_b(s)h_j(s))$  and  $g_{ji}(s)$   $(i,j = 1,2, i \neq j)$ . Nevertheless, both channels should have limited bandwidth in order to avoid the 2 encirclements to the critical point by the functions  $\gamma_b(s)h_j(s)$ .

By means of the toolbox and after analysing the characteristics of  $\gamma_b(s)$  (and following the design steps given in Section III), the following controller was obtained:

$$k_{12}(s) = \frac{-6(s+2)^{2}(s+1)^{2}}{s(s+20)(s^{2}+0.9s+1.28)}$$

$$k_{21}(s) = \frac{-0.85(s+0.5882)}{s(s+1.667)}$$
(18)

The structural robustness of the channels and control system are summarised in Table III and Fig. 8. As in the case of  $\gamma_a(s)$ , a closed–loop simulation can be carried out.

**Table III.** Control system structural robustness assessment with  $\gamma_b(s)$ 

| Measure     | $C_{1}(s)$ | $k_{12}(s)g_{21}(s)$ | $\gamma_b(s)h_2(s)$ | $C_2(s)$ | $k_{21}(s)g_{12}(s)$ | $\gamma_b(s)h_1(s)$ |
|-------------|------------|----------------------|---------------------|----------|----------------------|---------------------|
| BW[rad/s]   | 1.28       | 1.33                 | _                   | 0.322    | 0.705                | _                   |
| $G_M[dB]$   | $\infty$   | $\infty$             | 6.02                | $\infty$ | $\infty$             | 6.02                |
| $P_M$ [deg] | 74.7       | 41.1                 | æ                   | 95.8     | 82                   | 34                  |



**Fig. 8.** Control system structural robustness assessment with  $\gamma_b(s)$ : Bode diagrams: (a) Channel 1 and Channel 2; (b)  $K_{12}(s)g_{21}(s)$  and  $K_{21}(s)g_{12}(s)$ 



**Fig. 8.** Control system structural robustness assessment with  $\gamma_b(s)$ : (c)  $\gamma_b(s)$ ,  $\gamma_b(s)h_2(s)$ , and  $\gamma_b(s)h_1(s)$  Nyquist diagrams.

### V. CONCLUSION

In this paper the novel 2x2 Individual Channel Design MATLAB® Toolbox is presented. The software is a friendly programme and a valuable aid for analysing and designing multivariable control systems. The control design for a challenging plant (non minimum phase and with strong coupling) found in literature is here developed.

Some designs for 2x2 MIMO systems have been actually carried out using this toolbox, proving its adequate performance. The toolbox saves time while designing, providing all the necessary information for every design step and allowing the assessment of controller performance throughout simulations. Structural robustness and stability margins can be considered while designing, so successful controllers can be obtained considering a possible parameter variation. Some tips are given during the design process and the instructions the designer should follow are also mentioned.

The generalisation for the *m* x *m* case is in progress.

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