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The Iterative Learning Control Based Local Traffic Volume Control Approach via Ramp Metering

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Abstract—**In this work, we apply the iterative learning control approach to address the traffic volume control via ramp metering in a macroscopic level freeway environment. By formulating the original ramp metering problem into a volume output tracking and disturbance rejection problem, iterative learning control method can be successfully applied in the ramp metering problem. The effectiveness of the new approach is verified through rigorous theoretical analysis and intensive simulations.**

I. Introduction

IN increasing important area in the field of traffic engineering is freeway traffic control which becomes feasible owing to the freeway infrastructure development in metropolitan areas both in developed and developing countries. Ramp metering, when properly applied, is a valuable tool for efficient traffic management on freeways and freeway networks [1]-[2]. The purpose of ramp metering is to regulate the amount of traffic entering a given freeway at its entry ramps, so that the freeway can operate at a desired level of service. Ramp metering is only useful when traffic is not too light (otherwise ramp metering is not needed) and not too dense (otherwise breakdown will happen anyway). Ramp metering is implemented by placing a traffic light at the on-ramp that allows the vehicles to enter the freeway in a controlled way and thus reduces the disturbance of the traffic in the mainline.

Ramp metering strategies may be classified into two catalogs. One is the local, the other is the coordinated. Local strategies make use of traffic measurements in the vicinity of each ramp, to calculate the corresponding individual ramp metering values, whereas coordinate strategies may use available traffic measurements from greater portions of a freeway. Local strategies are far easy to design and implement. Nevertheless, they have been proven to be non-inferior to more sophisticated coordinated approaches under recurrent traffic congestion conditions [1][2].

The most well-known local ramp metering strategies are the demand-capacity (DC) strategy [3], the occupancy (OCC) strategy [3] and ALINEA [4]. ALINEA has been found to lead to significantly better performance as compared to DC and OCC strategies in several comparative field-evaluations [1][4]. According to a recent report, ALINEA and its variations, e.g., flow based ALINEA strategy (FL-ALINEA), upstream-flow- based ALINEA strategy (UP-ALINEA) and their combination of above strategy *X*-ALINEA/Q, has been implemented in various sites in European countries[1][5] [6].

It is worth to point out that the macroscopic traffic flow patterns are in general repeated every day although they may vary slightly in the time-of-day. For instance, the traffic flow will start from a very low level in the midnight, and increase gradually up to the first peak during morning rush hour, which is often from 7-9 AM, and the second one from 5-7 PM. Ruling out the occasional occurrence of accidents, the routine traffic flow on freeway in the macroscopic level will show inherent repeatability everyday. We may easily find that, the traffic flow patterns in the same weekday of two consecutive weeks are very close. Likely we can find the similarities on a monthly basis. In fact, the traffic repeatability is implicitly assumed in all fixed-time (time-of-day/pre-time) traffic control methods.

A limitation of all existing traffic control methods, whether predominant by feedback or feed-forward, is the lack of capability to learn and improve the control performance from a repeated traffic process. Without learning, a control system can only produce the same performance without improvement even the process is repeated once again.

Iterative learning control (ILC) was first proposed by Arimoto [7] for the control of a system that repeats the same task in a finite interval. Since then, ILC has been extensively studied and achieved significant progress in both theory and applications [8]-[11], etc. ILC has a very simple structure, i.e., an integration along the iteration axis; and is a memory based learning process. It requires very little system knowledge, and it is almost a model-free method. Thus this is a very desirable feature in traffic control, as the traffic model and the exogenous quantities may not be well known in practice [12][13].

The goal of this paper is to design an iterative learning control law, generating a sequence of control input profiles

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that drives the traffic flow volume converge to the desired level in the presence of the modeling uncertainty and exogenous disturbances (exogenous quantities that are not measurable). The reasons why we use the volume as the control objective, rather than the occupancy, have been clearly stated in literature [5].

This paper is organized as follows. In Section 2, the discrete traffic flow model is given. In Section 3, the convergence analysis of the proposed ILC controllers is presented. Simulation results are provided in Section 4. Section 5 gives the conclusions.

II. TRAFFIC FLOW MODEL AND PROBLEM FORMULATION

A. Traffic Flow Model

The space and time discretized traffic flow model [14] or a single freeway lane with one on-ramp and one off-ramp is given by (1)-(4) below.

$$\rho_i(k+1)=\rho_i(k)+\frac{T}{L_i}[q_{i-1}(k)-q_i(k)+r_i(k)-s_i(k)], \quad (1)$$

$$q_i(k)=\omega\rho_i(k)v_i(k)+(1\text{-}\omega)\rho_{i+1}(k)v_{i+1}(k), \quad (2)$$

$$\begin{aligned} v_i(k+1)=v_i(k)&+\frac{T}{\tau}[V(\rho_i(k))-v_i(k)]+\\ &\frac{T}{L_i}v_i(k)[v_{i-1}(k)-v_i(k)]-\\ &\frac{\nu T}{\tau L_i}\frac{[\rho_{i+1}(k)-\rho_i(k)]}{[\rho_i(k)+\kappa]}, \end{aligned} \quad (3)$$

$$V(\rho_i(k))=v_{free}(1-[\frac{\rho_i(k)}{\rho_{jam}}]^l)^m, \quad (4)$$

where T is the sample time interval in hour, $k=\{0,1,\cdots,K\}$ is the k-th time interval; $i=\{1,2,\cdots,N\}$ is the i-th section of a freeway; and N is the total section number. Model variables are listed below. $\rho_i(k)$: density in section i at time kT, (veh/lane/km); $v_i(k)$: space mean speed in section i at time kT, (km/h); $q_i(k)$: traffic flow leaving section i entering section $i+1$ at time kT, (veh/h); $r_i(k)$: on-ramp traffic volume for section i at time kT (veh/h); $s_i(k)$: off-ramp traffic volume for section i at time kT (veh/h), which is regarded as an unknown disturbance; L_i: Length of freeway in section i, (km); v_{free} and ρ_{jam} are the free speed and maximum possible density per lane, respectively; $\tau,\nu,\kappa,l,m,\omega$ are constant parameters characterizing a given traffic system in terms of the street geometry, vehicle characteristics, drivers' behaviors, etc.

B. Boundary

Boundary conditions are summarized as follows:

$$\rho_0(k)=q_0(k)/v_1(k), \quad (5)$$

$$v_0(k)=v_1(k), \quad (6)$$

$$\rho_{N+1}(k)=\rho_N(k), \quad (7)$$

$$v_{N+1}(k)=v_N(k), \qquad \forall k. \quad (8)$$

C. Control Objective

The control objective is to seek an appropriate control profile which specifies the on-ramp traffic flow, $r_i(k)$, that drives traffic volume $q_i(k)$ of sections i at time k to converge to the desired traffic volume $q_{i,desired}(k)$ for $k\in\{0,1,\cdots,K\}$, despite the modeling uncertainties and disturbances occurred at some off-ramps.

III. ILC BASED TRAFFIC VOLUME CONTROL VIA RAMP METERING

A. State Space Representation and Assumptions

The macroscopic traffic flow model described by equations (1) (2) of n-th iteration can be written in the following form

$$x_n(k+1)=f(x_n(k),u_n(k),k)+\varphi_n(k), \quad (9a)$$

$$y_n(k)=g(x_n(k),k)+\psi_n(k), \quad (9b)$$

where

$x(k)=[\rho_1(k),\rho_2(k),\cdots,\rho_N(k),v_1(k),v_2(k),\cdots,v_N(k)]^T$

$u(k)=[r_1(k),r_2(k),\cdots,r_N(k)]^T$,

$y(k)=[q_1(k),q_2(k),\cdots,q_N(k)]^T$,

$f(\cdot,\cdot,\cdot),g(\cdot,\cdot)$ are two nonlinear functions corresponding to the traffic flow model, and $\varphi(k),\psi(k)$ are the repeated system noise and output disturbance vectors with bounds b_φ,b_ψ defined as $b_\varphi=\sup_{k\in[1,K]}\|\varphi(k)\|,b_\psi=\sup_{k\in[1,K]}\|\psi(k)\|$. The off-ramp flow $s_i(k)$ is included in the system noise. The subscript n denotes the iteration number.

Before showing the main results of the proposed discrete ILC system, we define the λ norm of a vector $u(k)$ as

$$\|u(k)\|_\lambda=\sup_{k\in[0,K]}a^{-\lambda k}\|u(k)\|,$$

where $\lambda>0$ and $a>1$.

Assumption 1: Functions $f(x_n(k),u_n(k),k)$, and $g(x(k),k)$ are uniformly globally Lipschitz on a compact set $\Omega=X\times Y$ or $\Omega=X$ with respect to their arguments for $k\in[0,K]$, i.e.,

$$\begin{aligned}&\|f(x_1(k),y_1(k),k)-f(x_2(k),y_2(k),k)\|\\ &\qquad\le k_x\|x_1(k)-x_2(k)\|+k_y\|y_1(k)-y_2(k)\|,\end{aligned}$$

$$\|g(x_1(k),k)-g(x_2(k),k)\|\le k_g\|x_1(k)-x_2(k)\|,$$

where k_x,k_y,k_g are Lipschitz constants. X and Y are the ranges of speed and density, and that of the traffic volume of the traffic flow on the freeway respectively.

Assumption 2: The re-initialization condition is satisfied throughout the repeated iterations, i.e.,

$$\|x_d(0) - x_n(0)\| \le b_{x_0}, \forall n,$$

where $x_d(0)$ is the initial value of the desired state, n is the iteration number for the ILC.

Assumption 3: There exists a control profile $u_d(k)$ that can exactly drive the system output to track the desired trajectory $y_d(k)$ for the systems (9) over the finite time interval [0,*K*].

Assumption 1 requests the traffic model be globally Lipschitz continuous, which is satisfied in our case because the traffic flow model (1-4) is continuous differential in all arguments on any compact set Ω . Moreover, the system states (density and mean speed) cannot be infinite in practice. In addition, the time interval is also finite. This leads to the compact set Ω .

Assumption 2 demands the initial state values to be consistent with the desired one. In practice, if this condition is not met, we can always revise the target trajectory aligned with the actual one at the initial stage of tracking [11].

Assumption 3 is a quite reasonable assumption that the task should be solvable.

B. Iterative Learning Traffic Volume Control Strategy

The iterative learning control law for system (9) is constructed as below

$$u_{n+1}(k) = u_n(k) + \beta e_n(k+1), \tag{10}$$

where n indicates the iteration number, and β is an iterative learning gain matrix. $e_n(k+1) = y_d(k+1) - y_n(k+1)$ and $y_d(k)$ is the desired output signal (volume) at the time k.

Theorem: Under Assumptions 1-3, choosing the learning gain matrix β such that $\|I - \beta g_x(\zeta_n) f_u(\xi_n)\| < 1$ in the ILC law (10), the mapped output of the traffic system (10) will converge to the desired output along the iteration axis, i.e. if $\lim_{n\to\infty} b_{x_0} = 0, \lim_{n\to\infty} b_\varphi = 0, \lim_{n\to\infty} b_\psi = 0$ for all $k = 1,2,\cdots,K$, then

$$\lim_{n\to\infty} y_n(k) = y_d(k).$$

Proof: See the Appendix.

Remark 1: According to the traffic model, the scope of gain β can be easily determined. Note that β matrix is diagonal, and all parameters L_i and T are known a priori. The learning gain matrix becomes $\beta = diag(\beta_1, \beta_2, \cdots, \beta_N)$ where β_i satisfying relation $0 < \beta_i < \frac{2L_i}{Tv_i}$.

Remark 2: It is interesting to note that, the learning convergence is solely depending on the known parameters L_i , T and v_i . Other system parameters, such as $\tau, \nu, \kappa, l, m, \omega$ whose exact values may not be available, will not affect the learning convergence. Therefore ILC is suitable for traffic control when the model mismatching exists. Moreover, the exogenous disturbances $s(k)$, including in the unknown system noise, as far as repeatable, will be eliminated entirely by the learning control, thereafter does not affect the learning convergence.

IV. SIMULATION STUDIES

In order to verify the effectiveness of the ILC based approach, we simulate a freeway traffic flow process around the desired volume $y_d = 1700 veh/km$, in the presence of a large exogenous disturbance (modeled by an exiting flow in an off-ramp during a period). It is worthwhile pointing out that, model based optimal control or pure error feedback is not able to completely reject the influence from such an unknown exogenous disturbance. Further, even if the disturbance is known, it is still a difficult job to find an appropriate control profile, due to the highly nonlinear and uncertain factors in the traffic model.

Consider a long segment of freeway that is subdivided into 12 sections. The length of each section is 500m. The initial traffic volume entering section 1 is 1500 vehicles per hour. In order to show the proposed scheme's robustness, the initial density and mean speed of each section are set as shown in Table 1.

TABLE 1: INITIAL VALUES ASSOCIATED WITH THE TRAFFIC MODEL

SECTION	1	2	3	4	5	6	7	8	9	10	11	12
$\rho_i(0)$	30	30	30	30	30	30	30	30	30	30	30	30
$v_i(0)$	50	50	50	50	50	50	50	50	50	50	50	50

The parameters used in this model are also listed here. $\rho_{jam} = 80$(veh/lane/km) , $v_{free} = 80(km/h)$, $l = 1.8$, $m = 1.7$, $\kappa = 13(veh/km)$, $\tau = 0.1(h)$, , $T = 0.00417(h)$, $\upsilon = 35(km^2/h)$, $\alpha = 0.95$.

From this table, we can see that the initial values of density and speed are not in their equilibrium position according to the initial flow volume, that is, the initial ILC convergence conditions $\delta x(0) = 0$ is not satisfied strictly.

There are two on-ramps in the segment, located in sections 2 and 9 respectively. There is one off-ramp located in section 7. The traffic demand patterns (on-ramp) and outflow pattern (off-ramp) are shown in Fig.1. They were chosen to simulate a traffic scenario during rush hour. Under these settings, we can find that Assumption 3 is partly satisfied via the following simulation, that is there exists a control profile $u_d(k) = [r_2(k), r_9(k)]^T$ that can exactly drive the system output to track the desired trajectory $y_d(k) = [1700,1700]^T$ for the systems (9) over a finite time interval of [0,*K*].

Note that the queuing demands actually impose a constraint on the control inputs of ramp metering, i.e. the on-ramp volumes cannot exceed the current demands plus the existing waiting queues at on-ramps $i \in I_{on}$ at time k, thus $r_i(k) \le d_i(k) + \frac{l_i(k)}{T}, i \in I_{on}$, where $l_i(k)$ denotes the length (in vehicles) of a possibly existing waiting queue at time k at i^{th} on-ramp, $d_i(k)$ is the demand flow at time k at i^{th} on-ramp (veh/h), and $I_{on} = \{2,9\}$ denotes the set of indices of the sections where an on-ramp exists. On the other hand, the waiting queue is the accumulation of the difference between the demand and actual on-ramp, i.e.

$$l_i(k+1) = l_i(k) + T[d_i(k) - r_i(k)],\ i \in I_{on}.$$

Three cases are simulated in this part. Case I is the no control case; Case II is the FL-ALINEA control; Case III is the ILC based control. Without any losing of generality, we do not consider any practical maximum and minimum ramp metering limits in this part.

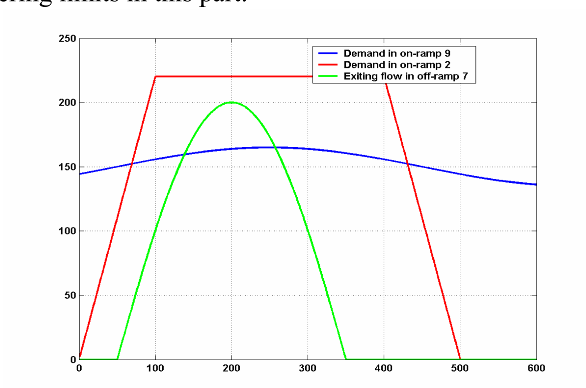

Fig.1 Known traffic demands in on-ramps and unknown exiting flow in off-ramp

Case I. No Control

Without any control, the traffic on the mainstream entering from the traffic demands in on-ramp 2 and 9, and exiting in off-ramp 7, which shown by Fig. 2, are so heavy that there exists a traffic congestion, which is represented by the downstream densities after section 9 getting higher and exceeding the critical density $\rho_{cr} = 37.5$, in the sequel results in slow traffic speed.

Case II. FL-ALINEA Control

FL-ALINEA strategy is used in sections 2 and 9 for the ramp metering. The FL-ALINEA gains for sections 2 and 9 are chosen to be 1 as suggested in literatures [5]. The simulation results are shown in Fig 3. From Fig. 3 (a), we can see that the traffic jam has vanished under the control of FL-ALINEA, and the tracking performance is acceptable. However, the deviations from the desired volume in sections 2 and 9 can be observed, due to the nature of the integral action of ALINEA (lack of damping), and the time delay of the process.

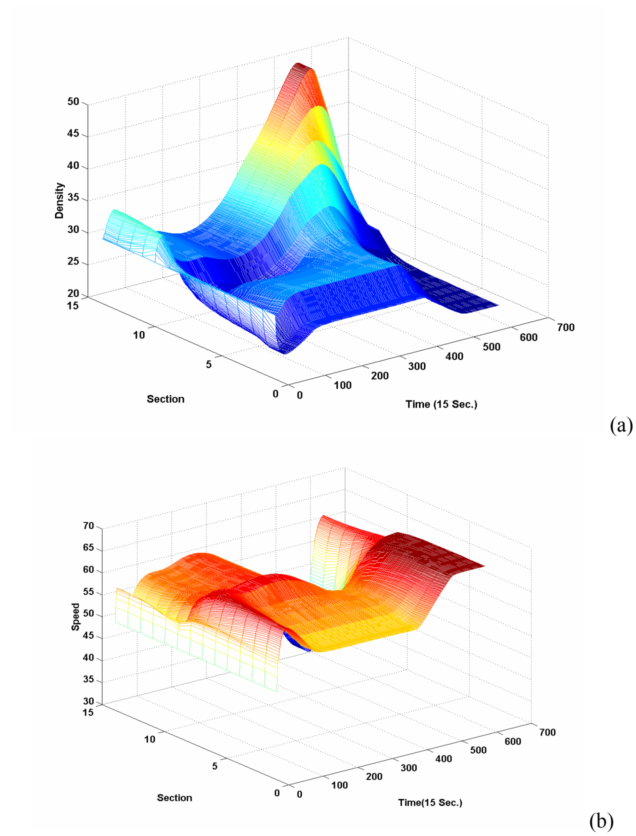

Fig.2: (a) Density profile with no control. (b) Speed Profile with no control

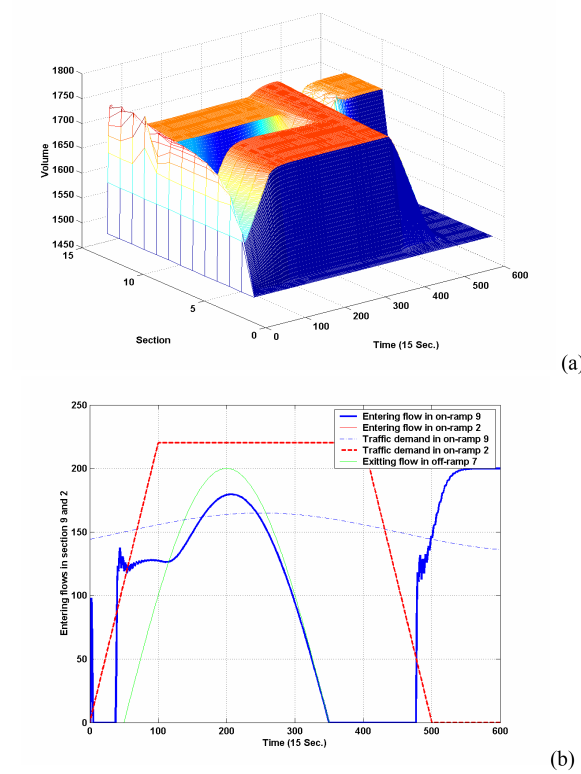

Fig.3: (a) Traffic volume performances in on-ramp 2 and 9. (b) Entering flows in on-ramp 2 and 9.

Case III. ILC Based Control

The ILC gains β_i ($i=1,2$) are set to be 1. The theoretically feasible range for β_i is (0, 2.997) according to the learning convergence condition with the maximum speed. The learning process is iterated for 10 cycles, simulation results are shown in Fig. 4.

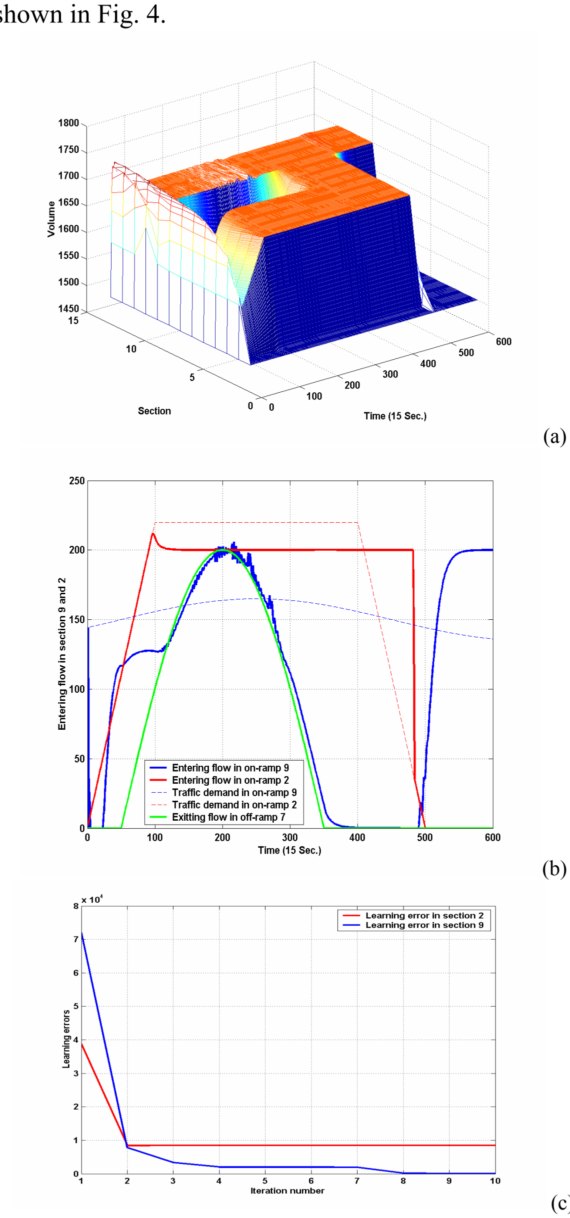

Fig.4: (a) Traffic volume performances in on-ramp 2 and 9 at the 10th iteration. (b) Entering flows in on-ramp 2 and 9. (c) Iterative errors in sections 2 and 9.

Comparing Fig. 3 (a) to Fig. 4 (a), we can clearly see the significant performance improvement in volume control. The deviation in section 2 from the beginning to k=100, and from k=500 to k=600, which cannot be eliminated through learning, is due to the lack of queuing traffic demand during that period, as shown in Fig.4 (b). In fact, we can observe from Fig.4 (b) that the on-ramp demand profile is fully used up by the learning controller before time instant 500, hence the performance cannot be any better. Fig. 4 (c) shows the learning error in sections 2 and 9. From it, we can see the learning error in section 9 converges to zero after only a few iterations. Note that the horizon part of the learning error of section 2 is caused by the insufficient traffic demand in range of sample time interval of 500 to 600. Here the learning error is defined as the maximum absolute error between the real volume and the desired one over the whole period of 600 sampling instant concerned.

V. Conclusion

In this paper, ILC based control approach has been successfully applied to solve the traffic volume control problem at the macroscopic level in a freeway environment. The learning control approach is able to guarantee the asymptotic convergence of the traffic volume to the desired one, despite the presence of the system modeling uncertainties, nonlinearities and exogenous disturbances. The simulation results show satisfactory responses and confirm the efficacy of the proposed approach.

Appendix

Proof of Theorem: The iterative learning controller is

$$u_{n+1}(k)=u_n(k)+\beta e_n(k+1). \tag{11}$$

Defining $\delta x_n(k)=x_d(k)-x_n(k), \delta u_n(k)=u_d(k)-u_n(k)$, then, from (9a)

$$\begin{aligned}\delta x_n(k+1) &= x_d(k+1)-x_n(k+1)\\ &= f(x_d(k),u_d(k),k)-f(x_n(k),u_d(k),k)+\\ &\quad f(x_n(k),u_d(k),k)-f(x_n(k),u_n(k),k)-\varphi_n(k)\\ &= f_x(\xi_n)\delta x_n(k)+f_u(\xi_n)\delta u_n(k)-\varphi_n(k),\end{aligned} \tag{12}$$

where

$\xi_n=[(x_n(k)+\tau\delta x_n(k))^T,(u_n(k)+\tau\delta u_n(k))^T,k]^T\in\Omega,\tau\in[0,1]$.

Taking norm for (12) yields

$$\begin{aligned}\|\delta x_n(k+1)\| &= \|f_x(\xi_n)\delta x_n(k)+f_u(\xi_n)\delta u_n(k)-\varphi_n(k)\|\\ &\le \sigma_1\|\delta x_n(k)\|+\sigma_2\|\delta u_n(k)\|+b_\varphi,\end{aligned} \tag{13}$$

where

$\sigma_1=\sup_{k\in[0,K]}\|f_x(\xi_n)\|$, $\sigma_2=\sup_{k\in[0,K]}\|f_u(\xi_n)\|$, $b_\varphi=\sup_{k\in[0,K]}\|\varphi_n(k)\|$.

Applying recursion formula:

$$z_{n+1}=a_1z_n+a_2z_n^*+a_3=a_1^{n+1}z_0+\sum_{j=0}^{n}a_1^{n-j}(a_2z_j^*+a_3)$$

from (13) we get

$$\|\delta x_n(k+1)\|\le\sigma_1^{k+1}b_{x_0}+\sum_{j=0}^{k}\sigma_1^{k-j}(\sigma_2\|\delta u_n(j)\|+b_\varphi), \tag{14}$$

then, taking the $\lambda-$norm ($\lambda>1$) operation of (14) gives

$$\begin{aligned}&\sup_{k\in[1,K]}\sigma_1^{-\lambda k}\|\delta x_n(k)\|\\ &\le \sup_{k\in[1,K]}\sigma_1^{-\lambda k}[\sigma_1^k b_{x_0}+\sum_{j=0}^{k-1}\sigma_1^{k-j-1}(\sigma_2\|\delta u_n(j)\|+b_\varphi)],\end{aligned} \tag{15}$$

that is

$$\|\delta x_n(k)\|_\lambda \le b_{x_0} + \sigma_2 \frac{1-\sigma_1^{-(\lambda-1)K}}{\sigma_1^\lambda - \sigma_1}\|\delta u_n(k)\|_\lambda + b_\varphi \sup_{k\in[0,K]} \frac{\sigma_1^{-(\lambda-1)k}(1-\sigma_1^{-k})}{\sigma_1 - 1}. \tag{16}$$

From (9a), the tracking error at the n-th iteration is

$$\begin{aligned} e_n(k+1) &= y_d(k+1) - y_n(k+1) = \\ &g(x_d(k+1),k+1) - g(x_n(k+1),k+1) \\ &-\psi_n(k+1) = g_x(\zeta_n)\delta x_n(k+1) - \psi_n(k+1), \end{aligned} \tag{17}$$

where $\zeta_n = [(x_n(k+1)+\tau\delta x_n(k+1))^T, k]^T \in \Omega, \tau\in[0,1]$.

Inserting (12) into (17) gives

$$\begin{aligned} e_n(k+1) &= g_x(\zeta_n)\delta x_n(k+1) - \psi_n(k+1) = \\ &g_x(\zeta_n)[f_x(\xi_n)\delta x_n(k) + f_u(\xi_n)\delta u_n(k) \\ &-\varphi_n(k)] - \psi_n(k+1). \end{aligned} \tag{18}$$

From ILC updating law (11), we have

$$\begin{aligned} \delta u_{n+1}(k) &= u_d(k) - u_{n+1}(k) = u_d(k) - u_n(k) - \beta e_n(k+1) \\ &= \delta u_n(k) - \beta e_n(k+1). \end{aligned} \tag{19}$$

Substituting (19) by (18) yields

$$\begin{aligned} \delta u_{n+1}(k) &= \delta u_n(k) - \beta[g_x(\zeta_n)f_x(\xi_n)\delta x_n(k) + \\ &g_x(\zeta_n)f_u(\xi_n)\delta u_n(k) - g_x(\zeta_n)\varphi_n(k) - \psi_n(k+1)]. \end{aligned} \tag{20}$$

Tanking norm operation of (20) gives

$$\begin{aligned} \|\delta u_{n+1}(k)\|_\lambda &\le \|(1-\beta g_x(\zeta_n)f_u(\xi_n))\|\|\delta u_n(k)\|_\lambda \\ &+ \beta\sigma_1\sigma_3\|\delta x_n(k)\|_\lambda + \beta\sigma_3 b_\varphi + \beta b_\psi, \end{aligned} \tag{21}$$

where

$\sigma_3 = \sup_{k\in[1,K]}\|g_x(\zeta_n)\|$, $b_\varphi = \sup_{k\in[1,K]}\|\varphi_n(k)\|$, $b_\psi = \sup_{k\in[1,K]}\|\psi_n(k)\|$.

Inserting (16) into (21) yields

$$\begin{aligned} \|\delta u_{n+1}(k)\|_\lambda &\le \|(1-\beta g_x(\zeta_n)f_u(\xi_n))\|\|\delta u_n(k)\|_\lambda + \\ &\beta\sigma_1\sigma_3\|\delta x_n(k)\|_\lambda + \beta\sigma_3 b_\varphi + \beta b_\psi \\ &\le (\|(1-\beta g_x(\zeta_n)f_u(\xi_n))\| + \beta\sigma_1\sigma_2\sigma_3\frac{1-\sigma_1^{-(\lambda-1)K}}{\sigma_1^\lambda-\sigma_1})\|\delta u_n(k)\|_\lambda \\ &+\beta\sigma_1\sigma_3[b_{x_0} + b_\varphi \sup_{k\in[1,K]}\frac{\sigma_1^{-(\lambda-1)k}(1-\sigma_1^{-k})}{\sigma_1-1}] + \beta\sigma_3 b_\varphi + \beta b_\psi. \end{aligned} \tag{22}$$

Let $\rho = \|(1-\beta g_x(\zeta_n)f_u(\xi_n))\| + \beta\sigma_1\sigma_2\sigma_3\frac{1-\sigma_1^{-(\lambda-1)K}}{\sigma_1^\lambda-\sigma_1})$,

$$\varepsilon = \beta\sigma_1\sigma_3[b_{x_0} + b_\varphi \sup_{k\in[1,K]}\frac{\sigma_1^{-(\lambda-1)k}(1-\sigma_1^{-k})}{\sigma_1-1}] + \beta\sigma_3 b_\varphi + \beta b_\psi,$$

choosing β satisfying $\|(1-\beta g_x(\zeta_n)f_u(\xi_n))\| < 1$, and sufficiently large λ to ensure $\rho < 1$, therefore, (22) can be rewritten as follows

$$\|\delta u_{n+1}(k)\|_\lambda \le \rho\|\delta u_n(k)\|_\lambda + \varepsilon, \tag{23}$$

thus we can conclude that

$$\lim_{n\to\infty}\|\delta u_{n+1}(k)\| \le \frac{\varepsilon}{1-\rho}, \tag{24}$$

so it is immediate result that

$$\lim_{n\to\infty}\|\delta u_{n+1}(k)\| \to 0, \tag{25}$$

if all $b_{x_0}, b_\varphi, b_\psi$ converge uniformly to zero for $k = 1,2,\cdots,K$ as $n\to\infty$ in absence of disturbance and initialization error, i.e., $b_{x_0}, b_\varphi, b_\psi \to 0$.

Taking λ _norm operation of (17), and replacing the $\|\delta x_n(k)\|_\lambda$ by (16), we get

$$\|e_n(k)\|_\lambda \le \sigma_3[b_{x_0} + \sigma_2\frac{1-\sigma_1^{-(\lambda-1)K}}{\sigma_1^\lambda-\sigma_1}\|\delta u_n(k)\|_\lambda + b_\varphi \sup_{k\in[0,K]}\frac{\sigma_1^{-(\lambda-1)k}(1-\sigma_1^{-k})}{\sigma_1-1}] + b_\psi, \tag{26}$$

then, from (26) and (25), we can conclude that the $y_n(k)\to y_d(k)$ as $n\to\infty$ if $b_{x_0}, b_\varphi, b_\psi \to 0$. □□□.

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