Proceedings of the
44th IEEE Conference on Decision and Control, and
the European Control Conference 2005
Seville, Spain, December 12-15, 2005

Two dual classes of time-varying well-posed linear systems constructed from passive systems

George Weiss
Dept. of Electr. & Electronic Eng.
Imperial College London
Exhibition Rd, London SW7 2AZ
United Kingdom
G.Weiss@imperial.ac.uk

Roland Schnaubelt
Fachbereich Mathem. und Informatik
Universität Halle
06099 Halle (Saale)
Germany
schnaubelt@mathematik.uni-halle.de

***Abstract*— We introduce two classes of well-posed linear time-varying (LTV) systems. Each of these can be constructed easily, starting from a time-invariant scattering-passive system, by introducing a time-dependent inner product on the state space and modifying some of the generating operators. These classes of LTV systems are motivated by physical examples, such as an electromagnetic field around a moving object. To prove the well-posedness of these LTV systems, we use the Lax-Phillips semigroup induced by a well-posed linear system, as in scattering theory. We modify this semigroup to obtain a Lax-Phillips type evolution family.**

I. Introduction

The concept of a time-varying well-posed linear systems has emerged over the years as researchers studied partial differential equations with time-dependent coefficients and then abstracted certain properties. In the absence of inputs and outputs, we have the theory of evolution families, which is the natural generalization of the theory of strongly continuous semigroups. The relevant generation results have been developed by Kato [8], [9], but until today, this theory is much less complete than the theory of strongly continuous semigroups (see, for example, Fattorini [5], Tanabe [18] and the survey by Schnaubelt [14]).

Various classes of time-varying systems with inputs and outputs have been introduced by Acquistapace and Terreni [1], Curtain and Pritchard [3], Hinrichsen and Pritchard [6], Jacob [7], and Schnaubelt [13]. The most general definition is the one in [13], whose definition of a time-varying well-posed system mimicks the abstract definition of a (time-invariant) well-posed linear system from Weiss [19] (which is equivalent to the differently looking earlier definition in Salamon [12]). Unfortunately, for such systems, there is no complete representation theory available (unlike for time-invariant well-posed systems). It is difficult to verify that a given system of linear equations defines a time-varying well-posed system, and for this reason it is also difficult to construct non-trivial examples of such systems. The difficulties arise when we have unbounded control or observation operators and the evolution family is not of parabolic type.

In this paper we want to introduce two classes of time-varying well-posed linear systems. Each such system is constructed using a scattering passive time-invariant system and a family of time-dependent inner products on the state space. To state our main results, we need to recall some terminology and results concerning well-posed linear systems from Staffans and Weiss [17] and from [19] (see also Staffans [16]). We assume that the reader knows the definition of this class of systems.

Let Σ be a well-posed linear system. This implies that Σ can be described by

$$\begin{aligned} \dot{x}(t) &= Ax(t) + Bu(t), \\ y(t) &= \overline{C}x(t) + Du(t). \end{aligned} \tag{1.1}$$

Here $u(\cdot)$ is the *input function*, $x(t)$ is the *state* at time t and $y(\cdot)$ is the *output function*. We have $u(t) \in U$, $x(t) \in X$ and $y(t) \in Y$, where the Hilbert spaces U, X and Y are called the *input space, the state space* and the *output space*, respectively. The operator $A : \mathcal{D}(A) \to X$ is the generator of a strongly continuous semigroup on X, and it determines two new Hilbert spaces as follows: X_1 is $\mathcal{D}(A)$ with the norm $\|x\|_1 = \|(\beta I - A)x\|$ where $\beta \in \rho(A)$. The space X_{-1} is the completion of X with respect to the norm $\|(\beta I - A)^{-1}x\|$. We have $B \in \mathcal{L}(U, X_{-1})$ and $C \in \mathcal{L}(X_1, Y)$. We denote $Z = \mathcal{D}(A) + (\beta I - A)^{-1}BU$, and $\overline{C}$ is a bounded extension of C to Z. Finally, $D \in \mathcal{L}(U, Y)$. For details about this representation we refer to [17, Section 3]. Take $\tau \geq 0$. The equations (1.1) have classical solutions for $t \in [0, \tau]$ if

$$\mathbf{P}_\tau u \in \mathcal{H}^1(0, \tau; U), \quad Ax(0) + Bu(0) \in X. \tag{1.2}$$

Here, $\mathbf{P}_\tau u$ denotes the restriction of u to $[0, \tau]$. If (1.2) holds, then we can compute $x(\tau)$ and the restriction of y to $[0, \tau]$, denoted by $\mathbf{P}_\tau y$. There exist four families of operators $\mathbb{T}$, Φ, Ψ and $\mathbb{F}$, parametrized by $\tau \geq 0$, such that

$$\begin{bmatrix} x(\tau) \\ \mathbf{P}_\tau y \end{bmatrix} = \begin{bmatrix} \mathbb{T}_\tau & \Phi_\tau \\ \Psi_\tau & \mathbb{F}_\tau \end{bmatrix} \begin{bmatrix} x(0) \\ \mathbf{P}_\tau u \end{bmatrix}. \tag{1.3}$$

The 2×2 matrix appearing above is a bounded operator from $X \times L^2([0, \tau], U)$ to $X \times L^2([0, \tau], Y)$. In fact, in [17], [19] the system Σ is defined via the operator families $\mathbb{T}, \Phi, \Psi$ and $\mathbb{F}$. $\mathbb{T}$ is the strongly continuous semigroup generated by A.

Note that the bounded operators $\mathbb{T}_\tau$, Φ_τ, Ψ_τ and $\mathbb{F}_\tau$ are completely determined by their action on data pairs $(x(0), \mathbf{P}_\tau u)$ that satisfy (1.2), because such pairs are dense in $X \times L^2([0,\tau], U)$. Moreover, for such $(x(0), \mathbf{P}_\tau u)$, we have

$$\mathbf{P}_\tau y \in \mathcal{H}^1(0,\tau;Y), \quad Ax(\tau) + Bu(\tau) \in X. \tag{1.4}$$

However, the formula (1.3) defines the state trajectory and the output function of Σ for every $x(0) \in X$ and for every $u \in L^2([0,\infty), U)$.

The well-posed system Σ is called *scattering passive* if for every classical solution of (1.1) corresponding to initial data satisfying (1.2),

$$\frac{\mathrm{d}}{\mathrm{d}t}\|x(t)\|^2 \leq \|u(t)\|^2 - \|y(t)\|^2 \qquad \forall\, t \geq 0.$$

In particular, Σ is called *scattering energy preserving* if we always have equality in the above formula. Such systems occur often in mathematical physics, where $\frac{1}{2}\|x(t)\|^2$ is interpreted as the energy stored in the system at time t, $\frac{1}{2}\|u(t)\|^2$ is the incoming power and $\frac{1}{2}\|y(t)\|^2$ is the outgoing power. If we integrate the above inequality, we see that Σ is scattering passive if and only if

$$\left\| \begin{bmatrix} \mathbb{T}_\tau & \Phi_\tau \\ \Psi_\tau & \mathbb{F}_\tau \end{bmatrix} \right\| \leq 1$$

for some (hence, for every) $\tau > 0$. Similarly, Σ is scattering energy preserving if and only if the 2×2 matrix in (1.3) is isometric. Scattering passivity and scattering energy preservation can also be expressed in terms of the operators $A, B, \overline{C}, D$, see Arov and Nudelman [2], Malinen *et al* [11], Staffans [15] and [17] for details (in [17], scattering passive systems were called dissipative).

Let Σ be a time-invariant scattering passive system described by (1.1). Let J be a closed interval (usually $J = [0,\infty)$). Let $P : J \to \mathcal{L}(X)$ be a strongly continously differentiable function such that $P(t) = P(t)^* > 0$ and $P(t)^{-1}$ is bounded for every $t \in J$. We introduce two new systems on the time interval J, informally defined by the equations

$$\begin{aligned} \dot{x}(t) &= P(t)^{-1}Ax(t) + P(t)^{-1}Bu(t), \\ y(t) &= \overline{C}x(t) + Du(t), \end{aligned} \tag{1.5}$$

and

$$\begin{aligned} \dot{x}(t) &= AP(t)x(t) + Bu(t), \\ y(t) &= \overline{C}P(t)x(t) + Du(t). \end{aligned} \tag{1.6}$$

For the second system (described by (1.6)) we assume that the function $P(\cdot)$ is of class C^2 (instead of C^1).

At the first glance, it is not clear if these equations make sense, even for smooth u, because it is not clear what $P(t)^{-1}B$ means ($P(t)^{-1}$ has not been defined on X_{-1}) and it is not clear if $\overline{C}$ can be applied to $x(t)$ or to $P(t)x(t)$, respectively. It is even less clear if these equations have solutions for some subspace of initial conditions and inputs, and if these solutions depend continuously on the data (the initial state and the input function).

In this paper, we show that in fact, both of the above systems of equations define well-posed linear time-varying systems in the sense of Schnaubelt [13] (he actually used the name well-posed nonautonomous system). We denote by Σ_l the system generated by (1.5) and by Σ_r the system generated by (1.6) (here, l and r stand for left and right). In particular, if $\tau \in J$ and the pair $(x(\tau), u)$ satisfies

$$u \in \mathcal{H}^1(J;U), \quad Ax(\tau) + Bu(\tau) \in X, \tag{1.7}$$

then (1.5) has a classical solution for $t \geq \tau$, $t \in J$, which satisfies

$$\begin{aligned} \frac{\mathrm{d}}{\mathrm{d}t}\langle P(t)x(t), x(t)\rangle \leq\; & \|u(t)\|^2 - \|y(t)\|^2 \\ & + \langle \dot{P}(t)x(t), x(t)\rangle \end{aligned} \tag{1.8}$$

for every such t. If the pair $(x(\tau), u)$ satisfies

$$u \in \mathcal{H}^1(J;U), \quad AP(\tau)x(\tau) + Bu(\tau) \in X, \tag{1.9}$$

then (1.6) has a classical solution for $t \geq \tau$, $t \in J$, which satisfies the same balance inequality (1.8). In fact, the well-posedness of the systems Σ_l and Σ_r is a consequence of (1.8). If the original well-posed system Σ is energy preserving, then we have equality in (1.8) (for both time-varying systems).

II. Time-varying multiplicative perturbations of m-dissipative operators

The standing assumptions of this section are the following: $A : \mathcal{D}(A) \to X$ is a maximally dissipative (m-dissipative) operator on the Hilbert space X. As in Section I, J is a closed interval and the function $P : J \to \mathcal{L}(X)$ satisfies

(i) $P(t) = P(t)^* > 0$ and $P(t)^{-1}$ is bounded for each $t \in J$,

(ii) $P(\cdot)x \in C^1(J, X)$ for each $x \in X$.

For each $t \in J$, we equip X with the equivalent scalar product and norm

$$\langle x, z\rangle_t = \langle P(t)x, z\rangle, \qquad \|x\|_t = \|P(t)^{\frac{1}{2}}x\|.$$

In this section we study the initial value problems

$$\begin{gathered} \dot{x}(t) = P(t)^{-1}Ax(t), \\ t \geq \tau,\ t \in J, \qquad x(\tau) = x_0 \end{gathered} \tag{2.1}$$

and

$$\begin{gathered} \dot{x}(t) = AP(t)x(t), \\ t \geq \tau,\ t \in J, \qquad x(\tau) = x_0, \end{gathered} \tag{2.2}$$

where $\tau \in J$. Here the operators $P(t)^{-1}A$ and $AP(t)$ are endowed with their natural domains, namely,

$$\mathcal{D}(P(t)^{-1}A) = \mathcal{D}(A)$$

$$\mathcal{D}(AP(t)) = \{x \in X \mid P(t)x \in \mathcal{D}(A)\}.$$

Recall from Engel and Nagel [4, Section VI.9], Kato [8], Schnaubelt [14], Tanabe [18] that an *evolution family* $\mathbb{T}$ on X is a family of operators $\mathbb{T}(t,\tau) \in \mathcal{L}(X)$ defined for $t, \tau \in J$ with $t \geq \tau$ which satisfies

(i) $\mathbb{T}(t,\sigma)\mathbb{T}(\sigma,\tau) = \mathbb{T}(t,\tau)$ for every $t, \sigma, \tau \in J$ with $t \geq \sigma \geq \tau$,

(ii) $\mathbb{T}(t,t) = I$ for every $t \in J$,

(iii) $(t,\tau) \mapsto \mathbb{T}(t,\tau)$ is strongly continuous for $t, \tau \in J$, $t \geq \tau$.

The concept of an evolution family is the natural generalization of the concept of a strongly continuous semigroup to time-varying systems. When trying to generalize the concept of an infinitesimal generator, one runs into several difficulties, and as a result there is no standard notion of a time-varying infinitesimal generator. In this paper, we shall use the following definition:

Let $A(t) : \mathcal{D}(A(t)) \to X$, where $t \in J$, be a family of linear operators. We say that $A(\cdot)$ *generates* the evolution family $\mathbb{T}$ if $\mathbb{T}(t,\tau)\mathcal{D}(A(\tau)) \subset \mathcal{D}(A(t))$ for all $t, \tau \in J$ with $t \geq \tau$ and, for every $\tau \in J$ and $x_0 \in \mathcal{D}(A(\tau))$, the function $x(\cdot) = \mathbb{T}(\cdot,\tau)x_0$ is continuously differentiable and it is the unique solution of the Cauchy problem

$$\dot{x}(t) = A(t)x(t), \quad t \geq \tau, \; t \in J, \qquad x(\tau) = x_0 . \tag{2.3}$$

Proposition 2.1: Under the standing assumptions stated at the beginning of this section, $P(\cdot)^{-1}A$ generates an evolution family $\mathbb{T}_l$ on X. If, in addition, $P(\cdot)x \in C^2(J,X)$ for every $x \in X$, then $AP(\cdot)$ generates an evolution family $\mathbb{T}_r$ on X.

Proof. Let $t \in J$. It is clear that $P(t)^{-1}A$ is closed and that its adjoint is $A^*P(t)^{-1}$, with domain $\mathcal{D}(A^*P(t)^{-1}) = \{x \in X \mid P(t)^{-1}x \in \mathcal{D}(A^*)\}$. We have

$$\langle P(t)^{-1}Ax, x\rangle_t = \langle Ax, x\rangle \qquad \forall\, x \in \mathcal{D}(A),$$

so that $P(t)^{-1}A$ is dissipative on X with respect to the equivalent scalar product $\langle\cdot,\cdot\rangle_t$. Moreover, $A^*P(t)^{-1}$ is dissipative with respect to the equivalent scalar product $\langle P(t)^{-1}x, z\rangle$. Therefore $I - A^*P(t)^{-1}$ is injective, so that $I - P(t)^{-1}A$ has dense range. Since $P(t)^{-1}A$ is dissipative with respect to $\langle\cdot,\cdot\rangle_t$, $I - P(t)^{-1}A$ has closed range. This implies that $\mathrm{Ran}\,(I - P(t)^{-1}A) = X$, so that $P(t)^{-1}A$ is maximally dissipative with respect to $\langle\cdot,\cdot\rangle_t$. Take a compact interval $[a,b] \subset J$, $x \in X$, $t, \tau \in J$, and set

$$L = \max_{t\in[a,b]} \|\dot{P}(t)\|, \qquad M = \max_{t\in[a,b]} \|P(t)^{-1}\|.$$

We then estimate

$$\begin{aligned} \|x\|_t^2 &= \langle (P(t) - P(\tau) + P(\tau))x, x\rangle \\ &\leq L\,|t-\tau|\,\|x\|^2 + \|x\|_\tau^2 \\ &\leq LM\,|t-\tau|\,\|x\|_\tau^2 + \|x\|_\tau^2 \\ &\leq e^{LM|t-\tau|}\,\|x\|_\tau^2 . \end{aligned}$$

Due to this inequality and a result of Kato [8] (see also [18, Proposition 4.3.2]), $P(\cdot)^{-1}A$ is a stable family in the sense of [8]. In addition, $P(\cdot)^{-1}A$ is strongly continuously differentiable on the (time invariant) domain $\mathcal{D}(A)$. As a consequence, $P(\cdot)^{-1}A$ generates a unique evolution family $\mathbb{T}_l$ on the time interval $[a,b]$ by the corollary to Theorem 4.4.2 in [18]. The first assertion then follows since $a, b \in J$ with $a < b$ were arbitrary.

We now assume that $P(\cdot)$ is of class C^2. One can verify as above that $AP(t)$ is maximally dissipative with respect to the scalar product $\langle x, z\rangle_t$ for $t \in J$. However, it is now more difficult to prove that $AP(\cdot)$ generates an evolution family, since the domains of $AP(t)$ vary in time and the fundamental generation Theorem 6.1 from [8] does not apply. We consider the auxiliary operators $A_1(t) = P(t)A + \dot{P}(t)P(t)^{-1}$ with domains $\mathcal{D}(A_1(t)) = \mathcal{D}(A)$. Temporarily we restrict ourselves to a compact interval $[a,b] \subset J$. Arguing as in the first part of the proof and using the bounded perturbation theorem, we check that the operator $A_1(t) - cI$ is m-dissipative for the scalar product $\langle P(t)^{-1}x, z\rangle$ and a sufficiently large $c \geq 0$. As before we deduce from [18, Proposition 4.3.2] that the family $A_1(\cdot)$ is stable in sense of [8]. Moreover, $A_1(\cdot)x \in C^1([a,b],X)$ for all $x \in \mathcal{D}(A)$, since $P(\cdot)$ is of class C^2. The corollary to Theorem 4.4.2 in [18] now implies that $A_1(\cdot)$ generates an evolution family $\mathbb{S}$. We can now easily check that the operators

$$\mathbb{T}_r(t,\tau) = P(t)^{-1}\mathbb{S}(t,\tau)P(\tau),$$

defined for $t \geq \tau$, $t, \tau \in J$, are an evolution family $\mathbb{T}_r$ generated by $AP(\cdot)$. ■

III. Lax Phillips semigroups associated to well-posed linear systems

Starting from an arbitrary (time-invariant) well-posed linear system Σ, it is possible to define a strongly continuous semigroup which resembles those encountered in the scattering theory of Lax and Phillips [10], and which contains all the information about Σ. We recall this construction from Staffans and Weiss [17, Section 6].

For any $t \geq 0$, we denote by $\mathbf{L}_t^+$ the operator of left shift by t on $L^2([0,\infty),U)$. Similarly, $\mathbf{L}_t^-$ is the operator of left shift by t on $L^2((-\infty,0],Y)$, so that

$$(\mathbf{L}_t^- y)(\sigma) = \begin{cases} y(\sigma+t) & \text{for } \sigma + t \leq 0, \\ 0 & \text{for } \sigma + t > 0. \end{cases}$$

Like in Section I, we assume that Σ is a well-posed linear system with input space U, state space X, and output space Y. The operators $A, B, \overline{C}$ and D are as in (1.1) and the families of operators $\mathbb{T}, \Phi, \Psi$ and $\mathbb{F}$ are as in (1.3).

Proposition 3.1: Let $\mathcal{Y} = L^2((-\infty,0],Y)$ and $\mathcal{U} = L^2([0,\infty),U)$. For all $t \geq 0$ we define on $\mathcal{Y} \times X \times \mathcal{U}$ the operator $\mathfrak{T}_t$ by

$$\mathfrak{T}_t = \begin{bmatrix} \mathbf{L}_t^- & 0 & 0 \\ 0 & I & 0 \\ 0 & 0 & \mathbf{L}_t^+ \end{bmatrix} \begin{bmatrix} I & \Psi_t & \mathbb{F}_t \\ 0 & \mathbb{T}_t & \Phi_t \\ 0 & 0 & I \end{bmatrix}$$

Then $\mathfrak{T} = (\mathfrak{T}_t)_{t\geq 0}$ is a strongly continuous semigroup.

$\mathfrak{T}$ is called the *Lax-Phillips semigroup* induced by Σ. We see that this semigroup contains every operator from (1.3), so that it contains all the information about Σ. The intuitive interpretation of the space $\mathcal{Y} \times X \times \mathcal{U}$ and of $\mathfrak{T}_t$ acting on it is that the first component is the past output, the second component is the current state, while the third component is the future input. Indeed, let $y_0 \in \mathcal{Y}$, $x_0 \in X$, $u_0 \in \mathcal{U}$ and for $t \geq 0$ define

$$\begin{bmatrix} y_t \\ x_t \\ u_t \end{bmatrix} = \mathfrak{T}_t \begin{bmatrix} y_0 \\ x_0 \\ u_0 \end{bmatrix}.$$

Let x and y be the state trajectory and the output function of Σ, i.e., the solutions of (1.1) corresponding to the initial state $x(0) = x_0$ and the input function $u = u_0$. We extend y to $\mathbb{R}$ by putting $y(t) = y_0(t)$ for $t \leq 0$. Then

$$y(t-\xi) = y_t(-\xi), \quad x(t) = x_t, \quad u(t+\xi) = u_t(\xi)$$

for all $t \geq 0$ and for almost every $\xi \geq 0$.

Assume that x_0 and u_0 satisfy

$$u_0 \in \mathcal{H}^1(0, \infty; U), \quad Ax_0 + Bu_0(0) \in X.$$

Then for every $t \geq 0$ we have

$$\frac{\mathrm{d}}{\mathrm{d}t} \left\| \begin{bmatrix} y_t \\ x_t \\ u_t \end{bmatrix} \right\|^2 = \|y(t)\|^2 + \frac{\mathrm{d}}{\mathrm{d}t}\|x(t)\|^2 - \|u(t)\|^2.$$

Thus, Σ is scattering passive if and only if $\mathfrak{T}$ is a contraction semigroup. Similarly, Σ is scattering energy preserving if and only if $\mathfrak{T}$ is an isometric semigroup.

Proposition 3.2: Let $\mathfrak{T}$ be the Lax–Phillips semigroup induced by Σ. We denote the generator of $\mathfrak{T}$ by $\mathfrak{A}$.

The domain of $\mathfrak{A}$, $\mathcal{D}(\mathfrak{A})$, consists of all the vectors $\left[\begin{smallmatrix} y_0 \\ x_0 \\ u_0 \end{smallmatrix}\right] \in \mathcal{H}^1(-\infty, 0; Y) \times X \times \mathcal{H}^1(0, \infty; U)$ which satisfy $Ax_0 + Bu_0(0) \in X$ and $y_0(0) = \overline{C}x_0 + Du_0(0)$, and on $\mathcal{D}(\mathfrak{A})$, $\mathfrak{A}$ is given by

$$\mathfrak{A} \begin{bmatrix} y_0 \\ x_0 \\ u_0 \end{bmatrix} = \begin{bmatrix} y_0' \\ Ax_0 + Bu_0(0) \\ u_0' \end{bmatrix}. \tag{3.1}$$

This is proved in the paper [17].

In the journal version of this paper we shall apply the above concepts and results to show that each of the systems of equations (1.5) and (1.6) determine well-posed linear time-varying systems. The idea is to apply Proposition 2.1 to $\mathfrak{A}$ and $\mathcal{P}$, where

$$\mathcal{P} = \begin{bmatrix} I & 0 & 0 \\ 0 & P & 0 \\ 0 & 0 & I \end{bmatrix}.$$

References

[1] P. Acquistapace and B. Terreni, Classical solutions of nonautonomous Riccati equations arising in parabolic boundary control problems, *Appl. Math. & Optim.* **39** (1999), pp. 361–409.

[2] D.Z. Arov and M.A. Nudelman, Passive linear stationary dynamical scattering systems with continuous time, *Integral Equations and Operator Theory* **24** (1996), pp. 1–45.

[3] R.F. Curtain and A.J. Pritchard, *Infinite Dimensional Linear Systems Theory*, Springer-Verlag, Berlin, 1978.

[4] K.-J. Engel and R. Nagel, *One-Parameter Semigroups for Linear Evolution Equations*, Springer-Verlag, New York, 2000.

[5] H.O. Fattorini, *The Cauchy Problem*, Addison-Wesley, Reading, MA, 1983.

[6] D. Hinrichsen and A.J. Pritchard, Robust stability of linear evolution operators on Banach spaces, *SIAM J. Control & Optim.* **32** (1994), pp. 1503–1541.

[7] B. Jacob, *Time-Varying Infinite Dimensional State-Space Systems*, PhD thesis, Bremen, May 1995.

[8] T. Kato, Linear evolution equations of "hyperbolic" type, *J. Fac. Sci. Univ. Tokyo* **17** (1970), pp. 241–258.

[9] T. Kato, Linear evolution equations of "hyperbolic" type, II *J. Math. Soc. Japan* **25** (1973), pp. 648–666.

[10] P.D. Lax and R.S. Phillips, *Scattering Theory*, Academic Press, New York, 1967.

[11] J. Malinen, O.J. Staffans and G. Weiss, When is a linear system conservative? Submitted.

[12] D. Salamon, Infinite dimensional systems with unbounded control and observation: A functional analytic approach, *Trans. Amer. Math. Soc.* **300** (1987), pp. 383–431.

[13] R. Schnaubelt, Feedbacks for nonautonomous regular linear systems, *SIAM J. Control & Optim.* **41** (2002), pp. 1141–1165.

[14] R. Schnaubelt, Well-posedness and asymptotic behaviour of non-autonomous linear evolution equations, *Evolution equations, semigroups and functional analysis*, A. Lorenzi and B. Ruf (eds.), Birkhäuser, Basel, 2002, pp. 311–338.

[15] O.J. Staffans, Passive and conservative continuous-time impedance and scattering systems. Part I: well-posed systems, *Math. of Control, Signals and Systems* **15** (2002), pp. 291–315.

[16] O.J. Staffans, *Well-Posed Linear Systems*, Cambridge University Press, UK, 2004.

[17] O.J. Staffans and G. Weiss, Transfer functions of regular linear systems, Part II: The system operator and the Lax-Phillips semigroup, *Trans. Amer. Math. Society* **354** (2002), pp. 3329–3262.

[18] H. Tanabe, *Equations of Evolution*, Pitman, London, 1979.

[19] G. Weiss, Transfer functions of regular linear systems, Part I: Characterizations of regularity, *Trans. Amer. Math. Society* **342** (1994), pp. 827–854.