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Statistical Approach for Bias-free Identification of a Parallel Manipulator Affected by Large Measurement Noise

Houssem Abdellatif, Bodo Heimann and Martin Grotjahn

Abstract— **The problem of high measurement noise in identification issue is treated in this paper for an innovative parallel robotic manipulator. To consider the noise and the correlation across the system's output a complete statistical approach is presented. The Maximum-Likelihood estimator is used for the identification of the dynamics parameters. Furthermore the experiments were designed based on a statistical criterion, such that the resulting excitation trajectories minimize the uncertainty bounds of the estimation. The experimental results are consequently compared with those resulting from classic deterministic approaches. This comparison demonstrates that the presented methodology yields bias-free and asymptotic efficient estimation.**

I. INTRODUCTION

The background of this work is the innovative design of the parallel robotic manipulator PaLiDA ([1], Fig. 1). The machine has been developed by the Institute of Production Engineering and Machine Tools at the university of Hannover (IFW). The research is carried out in cooperation with the Hannover Center of Mechatronics. Beside high manoeuvrability and high dynamics ability, a main goal was the development of novel actuation principle for parallel kinematic machines (PKM). The idea was to qualify linear direct drives for PKM applications. A commercial electromagnetic linear motor originally designed for fast lifting movements, is improved for the use in length-variable struts [1]. This actuation principle has advantages compared to conventional ball screw drives: fewer mechanical components, no backlash, low inertia with a minimized number of wear parts. A disadvantage remains though. It is the lower accuracy of position measurement, because only the internal hall sensors are used. Beside an additional enhancement of the measurement process [1], the control design has to consider the output noise for appropriate control algorithms [2]. For PKM the use of model-based feedforward control is crucial for accuracy improvement because the nonlinear and coupled dynamics are dominant, especially in the range of high dynamics [2], [3]. Therefore it is of great importance to develop appropriate identification algorithms in order to get accurately parameterized models for control [2], [4], [5].

The challenge of this work is to provide bias-free estimation of the dynamic parameters despite important noise and disturbances. Inspired by the excellent theoretical background given in [6], [7] and former work on industrial robotics [5], [8] a statistical approach for identification is presented. This includes system analysis, experiment design and appropriate estimation technique. Traditionally, linear estimators are used for the identification of manipulator dynamics [2], [3], [9], [10]. These methods are available in a deterministic framework, when the assumption of low disturbed regression matrix and non correlated output noise is fulfilled. In a statistical framework it is advantageous to use Maximum-Likelihood (ML) estimator for consistent and asymptotically efficient and unbiased estimation [6], [7]. The ML estimator was already formulated for robotic applications in [8] and [11], but the non correlation of the output was still assumed, such that the ML-algorithm is reduced to the Gauss-Markov (GM) estimator. This simplification is removed in this work (section II) and the problem is formulated for the general form of covariance matrices. Additionally to the appropriate estimation technique, the experiment design is also based on the statistical properties of the estimator (section III). In stead of the conventional deterministic input design criterion given by the condition number of the regression matrix [9], [10], [12], the Cramér-Rao lower bound of the estimation covariance is used. Such design was referred by Ljung [6] as d-optimal design and it yields the minimization of the volume of the probability density region for the estimated parameters. Furthermore the parametrization of the excitation trajectories is presented according to the chosen criterion. It will be explained why the harmonic and periodic trajectories presented in [5], [8], [12] are more reliable and appropriate for PKM than conventional ones known from industrial robotics and recently proposed for parallel manipulators [9], [10]. In section IV a detailed analysis of PaLiDA is presented. Additionally, the results of an extensive experiment are presented to prove, that the statistical experiment design yields parameter esti-

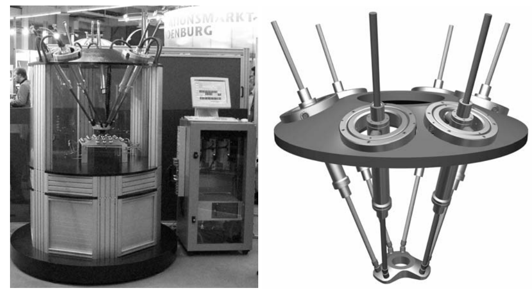

Fig. 1. PKM PaLiDA - left: Test bed at the Hannover Industrial Fair 2001, right: CAD-model.

H. Abdellatif and B. Heimann are with the Hannover Center of Mechatronics, University of Hannover, Appelstr. 11, 30167 Hannover, Germany. E-mail: abdellatif@mzh.uni-hannover.de

M. Grotjahn is with IAV GmbH, Gifhorn, Germany. E-mail: martin.grotjahn@iav.de

mates with smaller uncertainty bounds. Finally in section V the identification results of three LS, GM and ML estimators are presented and the verification of bias-free estimation is carried out experimentally.

II. Estimation of Dynamic Parameters

The presented approach is based on the formulation of the manipulator dynamics in a parameter linear form [2], [3], [8]

$$\boldsymbol{Q}_{\mathrm{a}} = \boldsymbol{A}(\boldsymbol{\lambda}, \dot{\boldsymbol{\theta}}, \ddot{\boldsymbol{\theta}})\,\boldsymbol{p}, \tag{1}$$

where $\boldsymbol{\lambda}$, $\dot{\boldsymbol{\theta}}$, and $\ddot{\boldsymbol{\theta}}$ are the n-dimensional minimal coordinates, velocities and accelerations, which are called kinematic information in the following. In the case of a 6-DOF parallel manipulator the vector of minimal coordinates contains the cartesian displacements and the orientation angles of the platform. The $n \times m$-matrix $\boldsymbol{A}$ refers to the regression matrix and $\boldsymbol{p}$ is the m-dimensional vector of minimal parameters [2], [12]. The measured variables are the actuator forces, presented by the vector $\boldsymbol{Q}_{\mathrm{a}}$ and the actuator lengths, presented by the vector $\boldsymbol{q}_{\mathrm{a}}$. Both are corrupted by disturbances $\boldsymbol{\eta}$ and $\boldsymbol{\delta}$

$$\begin{array}{ccccc} \boldsymbol{Q}_{\mathrm{a}} & = & \bar{\boldsymbol{Q}}_{\mathrm{a}} & + & \boldsymbol{\eta} \\ \boldsymbol{q}_{\mathrm{a}} & = & \bar{\boldsymbol{q}}_{\mathrm{a}} & + & \boldsymbol{\delta}, \end{array} \tag{2}$$

where $\bar{\boldsymbol{Q}}_{\mathrm{a}}$ and $\bar{\boldsymbol{q}}_{\mathrm{a}}$ denote the true values. Unlike serial mechanisms[1], the minimal coordinates can not be easily measured and have to be determined numerically from $\boldsymbol{q}_{\mathrm{a}}$ [2], [9], [12]. This is the most challenging issue in the identification of parallel manipulators. The disturbance $\boldsymbol{\delta}$ is overlapped with numerical disturbances due to the direct kinematics and differention, such that it is practically impossible to make any *a-priori* statement on the disturbances of the regression matrix. However, the conventional Least-Squares (LS) estimator is reliable and bias-free only for non disturbed matrix $\boldsymbol{A}$ [2], [6], [7], which makes its use here questionable. An experiment producing N noise corrupted data vectors yields

$$\underbrace{\begin{bmatrix} \boldsymbol{Q}_{\mathrm{a}_1} \\ \vdots \\ \boldsymbol{Q}_{\mathrm{a}_N} \end{bmatrix}}_{\boldsymbol{\Gamma}} = \underbrace{\begin{bmatrix} \boldsymbol{A}(\boldsymbol{\lambda}_1, \dot{\boldsymbol{\theta}}_1, \ddot{\boldsymbol{\theta}}_1) \\ \vdots \\ \boldsymbol{A}(\boldsymbol{\lambda}_N, \dot{\boldsymbol{\theta}}_N, \ddot{\boldsymbol{\theta}}_N) \end{bmatrix}}_{\boldsymbol{\Psi}} \boldsymbol{p} + \underbrace{\begin{bmatrix} \boldsymbol{e}_1 \\ \vdots \\ \boldsymbol{e}_N \end{bmatrix}}_{\boldsymbol{\eta}}. \tag{3}$$

A Maximum-Likelihood estimate $\hat{\boldsymbol{p}}_{\mathrm{ML}}$ maximizes the cost function $\boldsymbol{\pi}(\boldsymbol{\Gamma}|\boldsymbol{p})$, which is the probability density of the random vector $\boldsymbol{\Gamma}$ being generated by a model with parameters $\boldsymbol{p}$ [7]. To derive a numerically appropriate criterion, $\boldsymbol{\pi}$ is considered to be a normal distribution. The noise vectors are assumed to be gaussian with zero means. The covariance matrices of the measurement and information noise are denoted by $\boldsymbol{\Sigma}$ and $\boldsymbol{\Delta}$, respectively. Unlike common approaches, the covariances are not assumed to be diagonal, which is equivalent to the assumption of non-correlation between the different system outputs [8], [11]. Instead, cross covariance is taken into account. All the mentioned assumptions will be verified experimentally (see section IV). It was proven in fundamental works [6], [7], that the maximization of the likelihood or the log-likelihood under the mentioned assumptions yields

$$\hat{\boldsymbol{p}}_{\mathrm{ML}} = \arg\left(\max_{\boldsymbol{p}}\left(-\frac{1}{2}(\boldsymbol{\delta}^{\mathrm{T}}\boldsymbol{\Delta}^{-1}\boldsymbol{\delta} + \boldsymbol{\eta}^{\mathrm{T}}\boldsymbol{\Sigma}^{-1}\boldsymbol{\eta})\right)\right). \tag{4}$$

For the symmetric covariance matrices, a Cholesky factorization can be expressed by $\boldsymbol{\Delta}^{-1} = \boldsymbol{D}^{\mathrm{T}}\boldsymbol{D}$ and $\boldsymbol{\Sigma}^{-1} = \boldsymbol{S}^{\mathrm{T}}\boldsymbol{S}$. Equation (4) can be transformed to

$$\hat{\boldsymbol{p}}_{\mathrm{ML}} = \arg\left(\min_{\boldsymbol{p}}\left(\frac{1}{2}(\boldsymbol{\delta}^{\mathrm{T}}\boldsymbol{D}^{\mathrm{T}}\boldsymbol{D}\boldsymbol{\delta} + \boldsymbol{\eta}^{\mathrm{T}}\boldsymbol{S}^{\mathrm{T}}\boldsymbol{S}\boldsymbol{\eta})\right)\right), \tag{5}$$

and therefore to the formulation of a nonlinear Least-Squares optimization problem

$$\hat{\boldsymbol{p}}_{\mathrm{ML}} = \arg\left(\min_{\boldsymbol{p}}\frac{1}{2}\left(\|\boldsymbol{D}\boldsymbol{\delta}\|^2 + \|\boldsymbol{S}\boldsymbol{\eta}\|^2\right)\right) \tag{6}$$

that can be implemented with any commercial optimization tool [13]. If the information noise can be neglected ($\boldsymbol{\delta} \simeq \mathbf{0}$), the ML estimates simplifies and leads for models that are linear in the parameter vectors (1) to the Gauss-Markov (GM) estimator

$$\hat{\boldsymbol{p}}_{\mathrm{GM}} = \left(\boldsymbol{\Psi}^{\mathrm{T}}\boldsymbol{\Sigma}^{-1}\boldsymbol{\Psi}\right)^{-1}\boldsymbol{\Psi}^{\mathrm{T}}\boldsymbol{\Sigma}^{-1}\boldsymbol{\Gamma}, \tag{7}$$

which is the best linear unbiased estimator [6], [7]. The LS estimator can be derived from (7) by setting $\boldsymbol{\Sigma}$ as a diagonal matrix with equal entries.

III. Optimal Experiment Design

The optimal experiment design is the methodology to determine an experiment for collecting N measurement vector to get "best" identification results. Ljung suggested optimal input design, which is in our case the generation of excitation trajectories $\mathcal{T}(\boldsymbol{\xi})$. The vector of trajectory parameter $\boldsymbol{\xi}$ can be determined according a constrained nonlinear optimization [6], [8], [12], which requires again the definition of a criterion. It is sensitive to choose a trajectory that is most informative about the dynamics. The quantification of information amount can be given for the ML approach by the Fisher information matrix [6], [7]

$$\mathcal{F} = \mathrm{E}\left\{\frac{\partial}{\partial \boldsymbol{p}}\ln(\boldsymbol{\pi}(\boldsymbol{\Gamma}|\boldsymbol{p}))\frac{\partial}{\partial \boldsymbol{p}^{\mathrm{T}}}\ln(\boldsymbol{\pi}(\boldsymbol{\Gamma}|\boldsymbol{p}))\right\}. \tag{8}$$

For infinite $N \to \infty$ the distribution of $\hat{\boldsymbol{p}}_{\mathrm{ML}}$ tends to the gaussian unbiased one, denoted by $\mathcal{N}(\boldsymbol{p}, \mathcal{F}^{-1}(\boldsymbol{p}))$[7], that confirms the estimator property of asymptotical efficiency and unbiasness. For a finite measurement set the uncertainty given by the estimate covariance matrix $\boldsymbol{P}$ remains superior to the asymptotical Cramér-Rao bound

$$\boldsymbol{P} \geq \mathcal{F}^{-1}. \tag{9}$$

[1] for serial manipulators, the minimal coordinates are refereed to the joint angles, which can be accurately measured by encoder devices.

It is suggestive to design excitation trajectories that maximize the amount of information present in the investigated data and given by $\mathcal{F}$. This is equivalent to a minimization of the uncertainty of the parameter estimate or its lower bound which is given by $\mathcal{F}^{-1}$. Since a constrained optimization can not be expressed in a matrix sense, a scalar value is necessary. Many optimality criteria can be found in literature [6], [7] like the trace (A-optimality) of $\mathcal{F}$ or its minimal Eigenvalue (C-optimality). In a statistical framework it is advantageous to chose the d-optimality as criterion where the determinant of the information matrix is used. Minimizing $\det(\mathcal{F}^{-1})$ yields the minimization of the volume of the asymptotic confidence ellipsoids for the parameter [7]. The optimal experiment design can be expressed with respect to the optimal trajectory parameter set $\boldsymbol{\xi}_o$

$$\boldsymbol{\xi}_o = \arg\min_{\boldsymbol{\xi}} \left(\det(\mathcal{F}^{-1}(\boldsymbol{\xi}))\right). \tag{10}$$

For practical implementation, it is not recommended to invert numerically the disturbed information matrix. Furthermore, the determinant can become very small, so it is more efficient to minimize its logarithm [6], [8]

$$\boldsymbol{\xi}_o = \arg\min_{\boldsymbol{\xi}} \left(-\ln\det(\mathcal{F}(\boldsymbol{\xi}))\right). \tag{11}$$

The d-optimality criterion is independent from any non-singular scaling or reparametrization that does not depend on the experiment itself [7]. The chosen approach is based primarily on the use of ML for estimation. It is demonstrated in the next section that the consideration of statistical properties yields better parameter compared to the deterministic approach, where the condition of the information matrix is minimized.

Since the design of optimal input is a nonlinear constrained optimization process (10,11) it is sensitive to chose an appropriate input or trajectory form $\mathcal{T}(\boldsymbol{\xi})$ and to consider the system geometric properties and constraints to achieve efficient computation and to reach convergence. The trajectory can be described by

$$\mathcal{T}(\boldsymbol{\xi}) = \left\{\boldsymbol{\lambda} \in \mathbb{R}^6, \boldsymbol{f}_{\mathbf{t}}\left(\boldsymbol{\lambda}, t, \boldsymbol{\xi}\right) = \mathbf{0}, \forall t \in [t_0, t_e]\right\}. \tag{12}$$

Due to the limitation of the available workspace the task of defining the functional expression of the holonomic constraint $\boldsymbol{f}_{\mathbf{t}}\left(\boldsymbol{\lambda}, t, \boldsymbol{\xi}\right) = \mathbf{0}$ is more challenging for parallel manipulators. Any tilting of the end-effector platform yields considerable reduction of the available workspace. Bounded trajectories are more advantageous than classical approaches of polynomial functions, that have not led to any satisfactory results in our case [12]. In [8] a critical overview on known approaches for trajectory parametrization is given and a new approach is presented, where the excitation trajectories consist of a finite sum of harmonic sine and cosine functions in a form of a finite Fourier series. They are expressed in cartesian frame, since they present the minimal coordinates

$$\begin{aligned} &f_{\mathrm{t},i}(\lambda_i, t, \boldsymbol{\xi}^i) = \lambda_i(t) \\ &-\lambda_0^i - \textstyle\sum_{k=1}^{n} \left(\frac{\mu_k^i}{k\omega_f}\sin(k\omega_f t) - \frac{\nu_k^i}{k\omega_f}\cos(k\omega_f t)\right) \end{aligned} \tag{13}$$

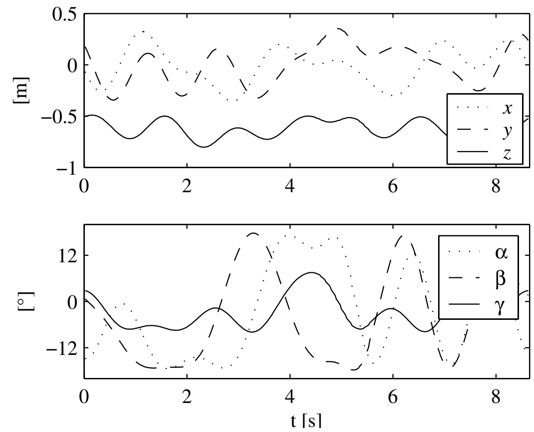

Fig. 2. Example of an optimized excitation trajectory in cartesian displacements (above) and tilting angles (below)

Each general coordinate λ_i corresponds an appropriate trajectory parameter vector

$$\boldsymbol{\xi}^i = \left[\begin{array}{ccccccc} \lambda_0^i, & \mu_1^i, & \ldots, & \mu_n^i, & \nu_1^i, & \ldots, & \nu_n^i \end{array}\right]^{\mathrm{T}}. \tag{14}$$

ω_f denotes the fundamental pulsation of the Fourier series that is the same for all dof. The trajectory is periodic with a period $T_f = 2\pi/\omega_f$ and bounded, which yields very efficient optimization process [12]. The vector of all trajectory parameters $\boldsymbol{\xi}$ contains all $\boldsymbol{\xi}^i$ and the fundamental pulsation ω_f. Its dimension is equal to $6 \times (2 \times n + 1) + 1 = 12n + 7$ and is then the number of the degrees of freedom of the optimization problem. Figure 2 depicts exemplarily one period of an optimized trajectory.

IV. Application to the Innovative Hexapod PaLiDA

This section presents the model of the considered parallel robot PaLiDA and experimental verification of the assumptions made for the system and on the measurement disturbances.

A. Dynamics Model Parameter

The robot consists in 6 struts and an end-effector platform (see Fig. 1 and 3). The actuators are integrated into the stators. Special custom cardan-rings allows high manoeuvrability a concentric rotation (passive joints α and β) [1]. PaLiDA is modelled with 19 bodies: the movable platform (index E), 6 identical movable cardan rings (index 1), 6 identical stators (index 2) and 6 identical sliders (index 3). The vector of minimal rigid-body parameters $\boldsymbol{p}_{\mathrm{rb}}$ is derived according to the methodologies known from robotics and mechanics [2], [8], [10]. They are presented in Table I. Moments of inertia are denoted by I_{xx}, I_{yy} and I_{zz}. The first order moments and masses are denoted respectively by s and m. Additional indices relate to the concerned body (1, 2, 3 or E) as defined above. Besides, the vectors $\boldsymbol{r}_{\mathrm{B}_j}^{E} = \left[r_{Bx_j}\ r_{By_j}\ r_{Bz_j}\right]^{\mathrm{T}}$ refer to those connecting the joints B_j to the Tool Center Point (TCP). Friction losses are modelled as a sum of dry and viscous friction. Optimal parametrization yields a common dry fiction coefficient r_α for all α-passive joints, a common coefficient r_β for all β-passive joints, 6 dry friction coefficients $r_{1\cdots6}$ for each actuator as well as 6 viscous damping coefficients $v_{1\cdots6}$. Viscous dumping in passive joints is demonstrably negligible [2], [4]. The integral model has consequently 10 rigid-body parameters and 14 friction coefficients (Table I).

TABLE I

RIGID-BODY AND FRICTION MODEL PARAMETERS.

rigid-body	friction
$p_1 = I_{zz_1} + I_{yy_2} + I_{zz_3}$ $[kgm^2]$	r_α $[N]$
$p_2 = I_{xx_2} + I_{xx_3} - I_{yy_2} - I_{zz_3}$ $[kgm^2]$	r_β $[N]$
$p_3 = I_{zz_2} + I_{yy_3}$ $[kgm^2]$	r_1 $[N]$
$p_4 = s_{y_2}$ $[kgm]$	r_2 $[N]$
$p_5 = s_{y_3}$ $[kgm]$	r_3 $[N]$
$p_6 = I_{xx_E} + m_3 \sum_{j=1}^{6}(r_{By_j}^2 + r_{Bz_j}^2)$ $[kgm^2]$	r_4 $[N]$
$p_7 = I_{yy_E} + m_3 \sum_{j=1}^{6}(r_{Bx_j}^2 + r_{Bz_j}^2)$ $[kgm^2]$	r_5 $[N]$
$p_8 = I_{zz_E} + m_3 \sum_{j=1}^{6}(r_{Bx_j}^2 + r_{By_j}^2)$ $[kgm^2]$	r_6 $[N]$
$p_9 = s_{z_E} + m_3 \sum_{j=1}^{6} r_{Bz_j}$ $[kgm]$	v_1 $[Nsm^{-1}]$
$p_{10} = m_E + 6\,m_3$ $[kg]$	v_2 $[Nsm^{-1}]$
	v_3 $[Nsm^{-1}]$
	v_4 $[Nsm^{-1}]$
	v_5 $[Nsm^{-1}]$
	v_6 $[Nsm^{-1}]$

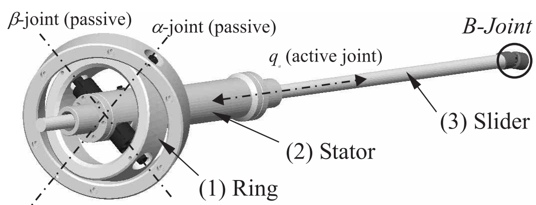

Fig. 3. Strut components of PaLiDA.

B. Analysis of Measurement Disturbances

The assumptions made on the noise vectors $\boldsymbol{\eta}$ and $\boldsymbol{\delta}$ are verified in the following. Practically the analysis and some experimental investigations are sufficient to deduce very good approximation or measurement of the output disturbances [1], [8]. The investigation of the output noise of the direct linear drives of PaLiDA yields the normal distribution given in Fig. 4. This shows that the output-noise can be assumed to be gaussian independent and with zero-mean. The application of ML is feasible. Furthermore the identified standard deviations vary from 20 N to 30 N. Considering that the actuation range covered by the linear direct drives is limited to ±230 N, the output measurement is considerably noise-corrupted. The entries of the covariances Σ and Δ show that both matrices are strongly different from the diagonal form, so that the algorithms proposed in [8], [11] can not be applied. A 2-dimensional visualization of Σ and Δ is presented by Fig. 5, where the isolines calculated from the matrices are presented. It shows especially that

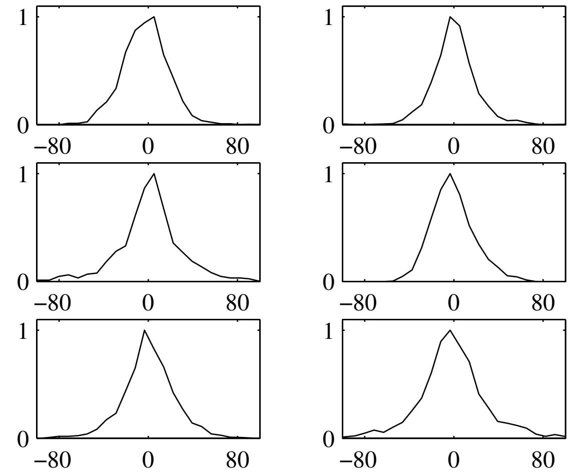

Fig. 4. experimentally investigated normalized output-noise distributions of the 6 actuators. All x-axes denote the noise amplitude in [N].

the entries of the kinematic information noise vector $\boldsymbol{\delta}$ are more or less significantly cross-correlated.

C. Deterministic vs. Statistical Experiment Design

As mentioned above, the assumption of noise-free information justifies the use of the condition number of the information matrix as a criterion for the input optimization [2], [8], [10], [12]. This approach is called a deterministic design. The statistical design, though, takes the estimator statistical properties into account to define the uncertainty volume of the parameter estimates as criterion for the optimization and was introduced (10,11). To compare both approaches the following extensive experiment was performed. Two trajectories $\mathcal{T}_d(\boldsymbol{\xi}_d)$ and $\mathcal{T}_s(\boldsymbol{\xi}_s)$ were optimized starting from the same initial parameter set until convergence and according to the deterministic and statistical design, respectively. Both trajectories were carried out 100 times. To have comparable and meaningful results, both trajectories were evaluated for identification by using LS-estimator. The statistical properties of the two parameter vectors $\hat{\boldsymbol{p}}_d$ and $\hat{\boldsymbol{p}}_s$ are compared in terms of the respective means and standard deviations. Figure 6 illustrates the experimental determined normal distributions of three exemplarily chosen parameter. It is obvious, that the statistically optimized trajectory $\mathcal{T}_s$ yields smaller standard deviations, which means smaller estimation uncertainty [6], [7]. This was observed for all estimated parameters (see Table II), except the very small dry friction coefficients in the passive joints. The results show also the excellent repeatability of the identification approach, which is justified by the chosen trajectory form (12) and by the method used for evaluation of the kinematic information presented in [12]. Furthermore a difference in the mean values of the estimation was observed. This issue is discussed in the next section, where the final identification step is fulfilled to get bias-free estimation.

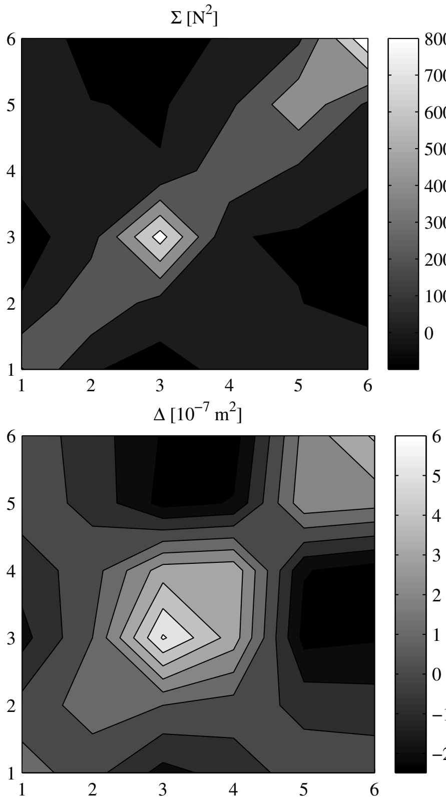

Fig. 5. 2-dimensional visualization of the entries of the covariance matrices. up: output-covariance, down: information covariance

V. Maximum-Likelihood Identification and Model Validation

The estimation results in the last section were based on the hypothesis of small disturbances in the kinematic information. This assumption has to be verified by applying a Maximum-Likelihood estimation. This consists in the iterative procedure of a nonlinear optimization, starting with the GM estimates [6], [7], [8], [11], [13]. The optimization formulated in (6) has to be though slightly modified by extending the argument vector to the parameter of the measured trajectory:

$$[\hat{\boldsymbol{p}}_{\mathrm{ML}}\,\hat{\boldsymbol{\xi}}] = \arg\left(\min_{\boldsymbol{p},\boldsymbol{\xi}} \frac{1}{2}\left(\|\boldsymbol{D}\boldsymbol{\delta}\|^2 + \|\boldsymbol{S}\boldsymbol{\eta}\|^2\right)\right). \qquad (15)$$

The modification is necessary for three reasons: to account for control errors and thus path deviations, to consider numerical errors resulting by the calculation of the kinematic variables $\boldsymbol{\lambda}$, $\dot{\boldsymbol{\theta}}$ and $\ddot{\boldsymbol{\theta}}$ from the measured actuator lengths $\boldsymbol{q}_{\mathrm{a}}$ [12] and because the model errors $\boldsymbol{\delta}$ and $\boldsymbol{\eta}$ can not be calculated based on the knowledge of $\boldsymbol{p}$ only [8]. In table III the estimation results are shown for different methods. By comparing the identified rigid-body parameters with their *a-priori* values given by CAD-Data [1], [2], it is obvious that the ML approach yields best (in some case exact) results. It is expected that the estimation of the friction coefficients has the same quality, although if no *a-priori* values are

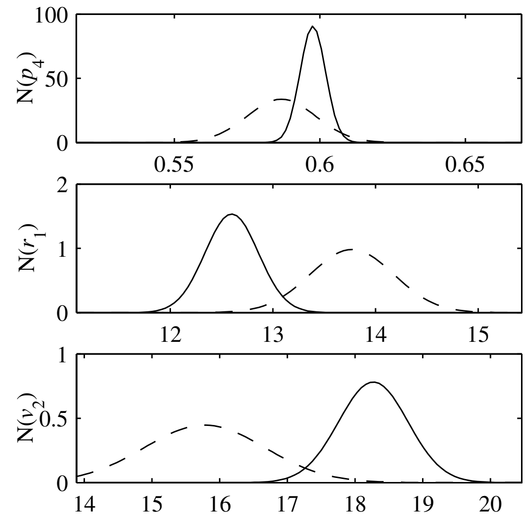

Fig. 6. Normal distribution of estimated parameters resulting from optimized experiments using deterministic (dotted line) and statistical criterion.

TABLE II

Standard deviations σ_d and σ_s of parameter estimates according respectively to deterministic and statistical experiment design

p_i	σ_{di}	σ_{si}	p_i	σ_{di}	σ_{si}
p_1	0.012	0.004	r_α	0.012	0.021
p_2	0.028	0.011	r_β	0.008	0.034
p_3	0.017	0.008	r_1	0.406	0.259
p_4	0.012	0.004	r_2	0.175	0.208
p_5	0.014	0.010	r_3	0.215	0.446
p_6	$1.8\ 10^{-3}$	$1.6\ 10^{-3}$	r_4	0.159	0.253
p_7	$1.0\ 10^{-3}$	$1.1\ 10^{-3}$	r_5	0.087	0.166
p_8	$1.5\ 10^{-3}$	$1.2\ 10^{-3}$	r_6	0.148	0.208
p_9	$1.7\ 10^{-3}$	$0.9\ 10^{-3}$	v_1	0.686	0.576
p_{10}	0.007	0.013	v_2	0.894	0.511
			v_3	0.794	0.530
			v_4	0.859	0.636
			v_5	1.259	0.520
			v_6	1.075	0.728

available for friction. The difference between the GM and ML estimates demonstrates that the information matrix is disturbed.

The estimated model can be validated for other trajectories that was not used for identification. In our work, a circular benchmark-motion is usually used [2], [4], [12]. The circle is located in the middle of the workspace and is inclined by 30° in respect to the x-axis. The maximal velocity is about 1 ms^{-1}. Figure 7 shows the model residua for 4 arbitrarily chosen actuators, where the differences of measured forces and those predicted with the identified model are depicted. The calculated covariance of the model estimation error yields for the 6 actuators $336.4\mathrm{N}^2$, 298.4, $992.0\mathrm{N}^2$, $594.1\mathrm{N}^2$, $118.4\mathrm{N}^2$ and $186.7\mathrm{N}^2$, respectively. The variances of the actuator measurement errors $\boldsymbol{\eta}_i$ are: $412.8\mathrm{N}^2$, $375.4\mathrm{N}^2$, $883.0\mathrm{N}^2$, $371.9\mathrm{N}^2$, $547.2\mathrm{N}^2$ and $970.9\mathrm{N}^2$. For the illustrated drives the model error variances

TABLE III

PARAMETER ESTIMATION RESULTS WITH LS, GM AND ML

p_i	LS	GM	ML	a-priori
p_1	−0.227	−0.033	−0.042	0.005
p_2	1.127	1.135	1.083	0.947
p_3	0.866	1.055	1.006	0.943
p_4	0.440	0.622	0.578	0.579
p_5	−1.201	−1.348	−1.300	−1.299
p_6	0.298	0.246	0.240	0.240
p_7	0.269	0.242	0.239	0.240
p_8	0.132	0.133	0.125	0.123
p_9	1.661	1.711	1.698	1.698
p_{10}	16.233	16.509	16.329	16.307
r_α	0.631	0.523	0.420	–
r_β	0.732	0.905	0.862	–
r_1	15.029	12.658	11.958	–
r_2	7.943	3.256	2.597	–
r_3	18.262	18.905	14.794	–
r_4	7.583	4.662	4.988	–
r_5	5.380	4.743	3.451	–
r_6	5.498	5.984	5.392	–
v_1	11.452	16.359	20.602	–
v_2	16.521	18.565	20.538	–
v_3	16.658	9.947	14.218	–
v_4	19.561	21.538	25.279	–
v_5	20.747	28.885	26.050	–
v_6	16.177	22.034	23.137	–

remain inferior to the output error variances. This verifies that the model is unbiased. However, this is not the case for actuator 3 and 4. It can be explained by the systematic disturbance known in electromagnetic linear drives and called cogging forces [1]. Their modelling and detection are quite delicate because cogging forces are dependent on machining tolerance of the single magnetic elements in the motor and are sensitive to initial control errors. For another trajectory with mirror-inverted geometry the model error variances are $318.5\mathrm{N}^2$, $168.5\mathrm{N}^2$, $324.0\mathrm{N}^2$, $235.3\mathrm{N}^2$, $562.8\mathrm{N}^2$ and $795.4\mathrm{N}^2$, respectively. In such a case the model is unbiased for all actuators. It can be concluded that the estimated model is very accurate and unbiased if the effect of the cogging force remains limited.

VI. CONCLUSIONS

Despite important measurement noise, the identification of unbiased dynamic models for a parallel manipulator with electromagnetic linear drives is demonstrated. The various conditions suggested by the estimation theory are verified experimentally, such that the Maximum-Likelihood approach is demonstrated to be the most efficient and unbiased. Furthermore the optimal experiment design in a statistical framework yields minimal uncertainty bounds of the estimates, which is discussed theoretically and substantiated experimentally. In addition, the chosen excitation trajectory form is well appropriate for parallel manipulators because it is suitable for the hard workspace constraints of such mechanisms. The application of the presented approach is very advantageous for the implementation of accurately parameterized and unbiased control models.

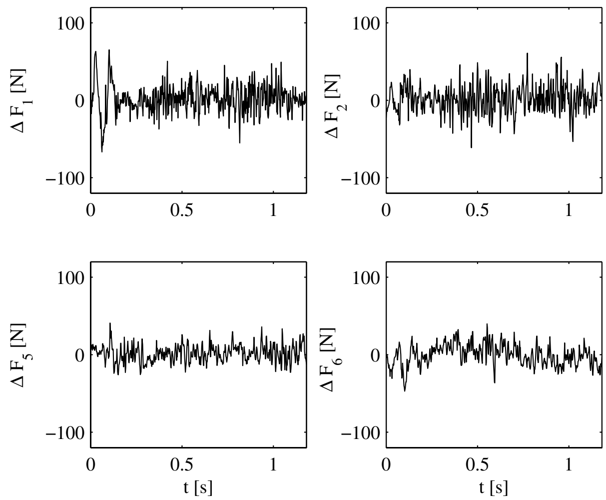

Fig. 7. Time history of model error for 4 arbitrarily chosen actuators

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