Sliding Motion and Direct Control of the Output Voltage in a Full Bridge Boost Converter

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Abstract— In this article a control scheme, that allows direct control of the output voltage in a full bridge boost converter by means of sliding modes, is proposed. This results in significant increase in the robustness of the converter in the presence of load disturbances. Furthermore, periodic signals may be tracked by the output voltage under appropriate restrictions.

I. INTRODUCTION

The possibility of using DC-to-DC switching power converters as source inverters has been thoroughly studied during the last twenty years. The first attempts with linear converters of the buck type [1], [2], [3] achieved robust AC voltage generation. However, for nonlinear converters, efforts are handicapped by the nonminimum phase character shown by these devices when direct control over the output voltage is exerted (see, for example, [4]). Hence, the unavoidable indirect control that has to be practised through the input current, leads to systems with high sensitivity to external perturbations and parameter uncertainties.

Several strategies have been proposed to overcome this problem. An algebraic method for the on-line identification of uncertain parameters is used in [5] for trajectory tracking by a double bridge buck converter. Dynamic sliding manifolds have been successfully used in [6] for the rejection of unmatched disturbances in output voltage tracking by nonlinear power converters. A Galerkin approximation-based dynamic compensator incorporating a perturbation observer has shown rapid speed of identification and good simulation results for

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A.S.I. Zinober is with the Department of Applied Mathematics, The University of Sheffield, Sheffield, UK a.zinober@sheffield.ac.uk tracking tasks of periodic references in load perturbed nonlinear converters [7].

On the other hand, the use of full bridge power converters increases the performance features of the devices and is the cornerstone of the already cited bucktype inverters, as well as those in [8]. In [9] direct tracking control of the output voltage was performed in a Čuk converter with magnetic coupling, while the input current was kept within a tolerance bandwidth by changing the polarity of the source. A similar structure has been used in [10] for robust regulation of the capacitor voltage of boost and buck-boost converters.

This scheme may be improved by allowing the new actuator, which commands the source polarity changes, to maintain the input current not within an interval but following a convenient signal. Meanwhile, the output voltage tracks the desired reference because of the action of the original switch. Among the advantages of direct control is the high benefit of the robustness of the converter.

Section II studies the sliding control of the nonperturbed full bridge boost converter. Section III contains the features associated with the robust tracking of an offset signal. An example of a sinusoidal reference with the simulation results are presented in Sections IV and V. Conclusions and suggestions for further research are presented in Section VI.

II. SLIDING CONTROL OF A FULL BRIDGE BOOST CONVERTER

The basic nonlinear switched boost converter has a general state-space representation in terms of a twodimensional bilinear system with the inductor current and the capacitor voltage as state variables and a control action q taking its values in the discrete set $\{0, 1\}$.

$$L\frac{di_{\rm L}}{d\tau} = V_{AB} - (1-q)v_{\rm C} \tag{1}$$

$$C\frac{dv_{\rm C}}{d\tau} = -\frac{v_{\rm C}}{R} + (1-q)i_{\rm L}.$$
 (2)

As pointed out in Section I, we use a full bridge of IGBT switches that allows the source to provide



Fig. 1. Full bridge boost converter.

 $V_{AB} = -V_g$ or $V_{AB} = V_g$ at will. This is equivalent to performing the substitution of V_{AB} by $V_g u_1$, u_1 being a control action that takes values in the discrete set $\{-1, 1\}$. The full bridge boost converter is depicted in Figure 1, while Table I contains the switching sequence of the IGBT that provides $u_1 = 1$ and $u_1 = -1$.

TABLE I Switching logic in the full bridge boost converter

	S_1	S_2	S_3	S_4
$u_1 = 1$	ON	OFF	ON	OFF
$u_1 = -1$	OFF	ON	OFF	ON

With the change of variables

$$x_1 = \frac{1}{V_g} \sqrt{\frac{L}{C}} i_L \qquad x_2 = \frac{1}{V_g} v_C \qquad t = \frac{1}{\sqrt{LC}} \tau$$

and the introduction of $\lambda = \frac{1}{R} \sqrt{\frac{L}{C}}$ and $u_2 = 1 - q$, the system becomes dimensionless:

$$\dot{x}_1 = u_1 - x_2 u_2 \tag{3}$$

$$\dot{x}_2 = -\lambda x_2 + x_1 u_2. \tag{4}$$

Denoting

$$A = \begin{pmatrix} 0 & 0 \\ 0 & -\lambda \end{pmatrix}, \quad B(x) = \begin{pmatrix} 1 & -x_2 \\ 0 & x_1 \end{pmatrix},$$

the multivariable system (3),(4) may be written as

$$\dot{x} = Ax + B(x)u,\tag{5}$$

with $x, u \in \mathbb{R}^2$.

The aim is to achieve the tracking of a desired reference $x_d(t)^{\top} = (x_{1d}(t), x_{2d}(t))$ by the state vector x by means of sliding control. Hence, the tracking error

 $e = x - x_d$ allows the following description for system (5):

$$\dot{e} = Ae + B\left(e + x_d\right)u + f(t),\tag{6}$$

with

$$f(t) = Ax_d(t) - \dot{x}_d(t).$$

Necessary restrictions for periodic reference signals are derived now. Consider the ideal steady state situation in which x is tracking the T-periodic reference $x_d(t)$, i.e., $x = x_d$ and the system is sliding over a certain switching surface. The actual equivalent control \bar{u}_{eq} , obtained from (6) after replacing e by 0, is:

$$\begin{pmatrix} \bar{u}_{1eq} \\ \bar{u}_{2eq} \end{pmatrix} = \frac{1}{x_{1d}} \begin{pmatrix} x_{1d}\dot{x}_{1d} + x_{2d}(\dot{x}_{2d} + \lambda x_{2d}) \\ \dot{x}_{2d} + \lambda x_{2d} \end{pmatrix}.$$
(7)

Notice that $x_{1d}(t), x_{2d}(t)$ have to be bounded and C^1 , as well as $x_{1d}(t) \neq 0$, $\forall t$. On the other hand $x_d(t)$ must also satisfy restrictions that require the equivalent control \bar{u}_{eq} to lie within $[-1, 1] \times [0, 1]$, i.e., inside the control unsaturated zone. Then, it suffices that

$$-1 < \frac{x_{1d}\dot{x}_{1d} + x_{2d}(\dot{x}_{2d} + \lambda x_{2d})}{x_{1d}} < 1, \qquad (8)$$

$$0 < \frac{\dot{x}_{2d} + \lambda x_{2d}}{x_{1d}} < 1.$$
⁽⁹⁾

A glance at (9) indicates that $\dot{x}_{2d} + \lambda x_{2d} \neq 0$ is required. Moreover, it is reasonable to impose $|x_{2d}(t)| > 1$, $\forall t \ge 0$ because we are dealing with a step-up converter. The next proposition studies these restrictions for *T*-periodic functions:

Proposition 1. Let x_{2d} be a C^1 , *T*-periodic function satisfying $|x_{2d}(t)| > 1$ and $\dot{x}_{2d}(t) + \lambda x_{2d}(t) \neq 0$. Then,

$$x_{2d}(\dot{x}_{2d} + \lambda x_{2d}) > 0.$$

Proof. (i) $\dot{x}_{2d} + \lambda x_{2d} > 0$ and $x_{2d} < -1$, lead to $x_{2d}(0)e^{-\lambda t} < x_{2d}(t) < -1$, $\forall t > 0$. However, $x_{2d}(0)e^{-\lambda t} < -1$ cannot be fulfilled $\forall t \geq \lambda^{-1} \log |x_{2d}(0)|$.

(ii) $\dot{x}_{2d} + \lambda x_{2d} < 0$ and $x_{2d} > 1$, lead to $1 < x_{2d}(t) < x_{2d}(0)e^{-\lambda t}$, $\forall t > 0$, which is again of impossible fulfillment.

In this situation the idea is to force the state variable x_1 to follow a constant reference $x_{1d} = x_{1d}^*$. Therefore, restrictions (8),(9) become

$$-1 < \frac{x_{2d}(\dot{x}_{2d} + \lambda x_{2d})}{x_{1d}^*} < 1, \quad 0 < \frac{\dot{x}_{2d} + \lambda x_{2d}}{x_{1d}^*} < 1.$$
(10)

Straightforward calculation allows us to prove the following result: **Proposition 2.** Let $x_d(t) = (x_{1d}^*, x_{2d}(t))^{\top}$, with $x_{1d} \in \mathbb{R} \setminus \{0\}$ and $x_{2d}(t)$ *T*-periodic, C^1 and such that $\dot{x}_{2d}(t) + \lambda x_{2d}(t) \neq 0$. When (*i*) $x_{1d}^* > \|x_{2d}(\dot{x}_{2d} + \lambda x_{2d})\|_{\infty}$ and $x_{2d}(t) > 1$, or (*ii*) $x_{1d}^* < 0$, $\|x_{2d}(\dot{x}_{2d} + \lambda x_{2d})\|_{\infty} > \|x_{1d}^*\|$ and $x_{2d}(t) < -1$, restrictions (10) are fulfilled. ■

The control target e = 0 will be achieved designing a switching logic capable of inducing a sliding motion over the surface s(e, t) = 0, with components

$$s_1 = e_1 s_2 = x_{1d}^* e_2 - x_{2d}(t)e_1.$$
(11)

Notice that, indeed, the intersection of the switching manifolds $s_1 = 0$, $s_2 = 0$ coincides with the intersection of $e_1 = 0$, $e_2 = 0$.

Proposition 3. Assume that the restrictions in Proposition 2 (i) are satisfied. Then, the control law

$$u_1 = \begin{cases} -1 & \text{if } s_1 > 0 \\ 1 & \text{if } s_1 < 0 \end{cases}, \quad u_2 = \begin{cases} 0 & \text{if } s_2 > 0 \\ 1 & \text{if } s_2 < 0 \end{cases}$$

produces a stable sliding regime of (5) on the intersection of the discontinuity surfaces $s_1 = 0$, $s_2 = 0$.

Proof. Let the matrix W(t) be defined as

$$W(t) = \begin{pmatrix} (x_{1d}^*)^2 + x_{2d}^2(t) & x_{2d}(t) \\ x_{2d}(t) & 1 \end{pmatrix}$$

and consider the continuously differentiable function

$$V(s,t) = \frac{1}{2(x_{1d}^*)^2} s^\top W(t)s.$$

It is straightforward that W(t) is positive definite, which makes V(s,t) positive definite. Furthermore, as W(t) is also symmetric, its eigenvalues are positive, real functions of t and, $\forall t \ge 0$ (see [11], for example),

$$0 \le \kappa_{\min}(t) \|s\|^2 \le 2(x_{1d}^*)^2 V(s,t) \le \kappa_{\max}(t) \|s\|^2,$$

where $\kappa_{min}(t), \kappa_{max}(t)$ are the smallest and largest eigenvalues of W(t), respectively. The continuity and *T*-periodicity of such eigenvalues allow us to conclude that they achieve a maximum and a minimum value in [0, T], i.e. their exist real, positive constants ρ_m , ρ_M fulfilling

$$2(x_{1d}^*)^2 \rho_m \leq \min_{t \in [0,T]} \{ \kappa_{min}(t) \} \\ 2(x_{1d}^*)^2 \rho_M \geq \max_{t \in [0,T]} \{ \kappa_{max}(t) \},$$

Therefore, V(s,t) is lower and upper bounded in each sphere ||s|| = R inside a neighborhood of s = 0by positive quantities depending only on R, and these lower and upper bounds $h_R = \rho_m R^2$, $H_R = \rho_M R^2$, respectively, are such that

$$\lim_{R \to 0} H_R = 0, \qquad \lim_{R \to \infty} h_R = \infty.$$

In order to evaluate the derivative of V(s,t) along the trajectories of (6) notice that, in fact,

$$V(s,t) = \frac{1}{2}e^{\top}e.$$

Thus,

$$V = e_1[u_1 - (e_2 + x_{2d})u_2] + + e_2[-\lambda e_2 + (e_1 + x_{1d}^*)u_2 - (\dot{x}_{2d} + \lambda x_{2d})] \le \le e_1[u_1 - x_{2d}u_2] + e_2[x_{1d}^*u_2 - (\dot{x}_{2d} + \lambda x_{2d})].$$

Realizing from (7) that

$$\bar{u}_{1eq} = x_{2d}\bar{u}_{2eq}, \quad \bar{u}_{2eq} = \frac{\dot{x}_{2d} + \lambda x_{2d}}{x_{1d}^*}$$

we can write

$$\dot{V} \leq e_1[u_1 - x_{2d}(u_2 - \bar{u}_{2eq} + \bar{u}_{2eq})] + + e_2(x_{1d}^*u_2 - x_{1d}^*\bar{u}_{2eq}) - \lambda e_2^2 \leq \leq e_1[u_1 - \bar{u}_{1eq}] + (x_{1d}^*e_2 - x_{2d}e_1)(u_2 - \bar{u}_{2eq}).$$

As $\bar{u}_{1eq}, \bar{u}_{2eq}$ are continuous and *T*-periodic and have been demanded to lay inside (-1, 1), (0, 1), respectively, they have maximum and minimum values therein. Let ϵ_1, ϵ_2 be such that

$$\begin{aligned} \bar{u}_{1eq} &\in [-\epsilon_1, \epsilon_1], \quad 0 < \epsilon_1 < 1, \\ \bar{u}_{2eq} &\in [\epsilon_2, 1 - \epsilon_2], \quad 0 < \epsilon_2 < \frac{1}{2}; \end{aligned}$$

then, the proposed switching logic yields

$$\dot{V}(s,t) \le -(1-\epsilon_1)|s_1| - \epsilon_2|s_2| \le -\alpha(|s_1| + |s_2|),$$

with $\alpha = \inf\{1-\epsilon_1, \epsilon_2\}$. From here it is straightforward that

$$\dot{V}(s,t) \le -\alpha \|s\|.$$

Then, a stable sliding mode motion along the intersection of the discontinuity surfaces s = 0 occurs [12].

Remark II.1. The requirement of continuous differentiability for the Lyapunov function is essential for the existence of sliding mode in multi-input systems. Chapter 4 of [12] contains clarifying examples.

III. ROBUST TRACKING

Let system (3),(4) suffer a load disturbance in the form of an additive term $R_p(t)$ to the nominal value R_N . Then, $R = R_N + R_p(t)$. From the definition of the parameter λ in Section II, $\lambda = \lambda_N + \lambda_p(t)$, with

$$\lambda_p(t) = -\frac{\lambda_N R_p(t)}{R_N + R_p(t)}.$$

System (5) may then be written

$$\dot{x} = Ax + B(x)u + p(x,t) \tag{12}$$

with $p(x,t) = (0, -\lambda_p(t)x_2)^{\top}$.

We can state that (12) exhibits a strong invariance property with respect to the disturbance vector field p(x,t) because the matching condition $p(x,t) \in$ span $\{B(x)\}$ [13] is trivially fulfilled. This guarantees the robustness of the converter in the sense that the induced sliding regimes are not affected by the presence of load disturbances: the system trajectories continue pointing towards the sliding manifold. However, λ is contained in the equivalent control expression. Hence, all the restrictions derived from the necessary satisfaction of (10) must be reviewed in order to prevent undesirable control saturations. Therefore, it is advisable to consider choosing a value for x_{1d}^* such that these restrictions are fulfilled for the load varying within an expected interval.

IV. TRACKING A SINUSOIDAL REFERENCE

The final aim that lays behind the study of the device we are dealing with is the possibility of developing a robust full bridge boost-based inverter. Following the proposal in [14], we should finally connect the load differentially across two converters in order to obtain a non-offset periodic signal. Therefore, it is actually interesting to exemplify the tracking procedure with a sinusoidal reference.

Let

$$x_{2d}(t) = A + B\sin\omega t, \qquad A, B > 0,$$

be a candidate to be tracked by the state variable x_2 . Therefore,

(i) $x_{2d} > 1$ yields A > 1 + B

(ii) From (i) and proposition 1, $\dot{x}_{2d} + \lambda x_{2d} > 0$ needs to be ensured. Since

$$\dot{x}_{2d} + \lambda x_{2d} = A\lambda + B\omega \cos \omega t + B\lambda \sin \omega t$$
$$= A\lambda + B\sqrt{\lambda^2 + \omega^2} \sin \left(\omega t + \arctan \frac{\omega}{\lambda}\right),$$



Fig. 2. The perturbed parameter λ .

then, taking into account the hypothesis on the signs of A and B, and assuming $\lambda > 0$, the requirement is

$$A > B \sqrt{1 + \left(\frac{\omega}{\lambda}\right)^2}$$

(iii) It suffices that $x_{1d}^* > ||x_{2d}||_{\infty} ||\dot{x}_{2d} + \lambda x_{2d}||_{\infty}$ for the fulfillment of $x_{1d}^* > ||x_{2d}(\dot{x}_{2d} + \lambda x_{2d})||_{\infty}$, demanded in Proposition 2 (i). This may be achieved with

$$x_{1d}^* > \lambda(A+B) \left[A + B\sqrt{1 + \left(\frac{\omega}{\lambda}\right)^2}\right]$$

In summary, the restrictions over A, B and x_{1d}^* are

$$A > \sup\left\{1 + B, B\sqrt{1 + \left(\frac{\omega}{\lambda}\right)^2}\right\},$$
 (13)

$$x_{1d}^* > \lambda(A+B) \left[A + B\sqrt{1 + \left(\frac{\omega}{\lambda}\right)^2} \right].$$
 (14)

In the presence of perturbations, the system is guaranteed to work in the unsaturated zone if (13),(14) are fulfilled for all λ within an expected interval of variation $[\lambda_{min}, \lambda_{max}]$.

V. SIMULATION RESULTS

A boost converter with parameters $V_{\rm g} = 10V$, $R_N = 100\Omega$, L = 4.79mH and $C = 47\mu F$ has been chosen to test the theory developed above. The output voltage reference for tracking is

$$v_{\rm Cd}(\tau) = 20 + 5\sin 2\pi\nu\tau \ V,$$



Fig. 3. Robust full bridge boost: the state variable x_1 .

with $\nu = 50 Hz$. The values of the corresponding dimensionless variables are $\lambda_N = 0.100953$, $\omega = 0.1508$ and

$$x_{2d}(t) = 2 + 0.5\sin\omega t.$$

Choosing $x_{1d}^* = 2$ for the boost converter, which corresponds to $i_{Ld} = 1.98A$, the system satisfies the restrictions (13),(14):

$$A = 2 > \sup \{1.5, 0.90\} = 1.5, \quad x_{1d}^* = 2 > 0.73.$$

Figure 2 portrays the effect of the disturbance on the parameter λ due to load perturbations up to 100% of the nominal value R_N , that is, $R_p = 100\Omega$, and a frequency $\omega_p = 4\omega$. Note that, in the presence of the perturbation, λ may achieve a minimum value of $\lambda = 0.050477$, but (13),(14) are still fulfilled:

$$A = 2 > \sup\{1.5, 1.57\} = 1.57, \quad x_{1d}^* = 2 > 0.45.$$

A SIMULINK model has been used in the simulations. A fixed step fifth-order Runge-Kutta algorithm is used to integrate the differential system, with an integration step of 0.002108 units in the dimensionless time variable, corresponding to $10^{-6}s$. The simulations are carried out for 150 scaled time units, corresponding to 0.0712s. The control actuators u_1 and u_2 switch at a maximum frequency of $\nu_{sM} = 20 \ kHz$. This effect is modelled through relays with hysteresis bandwidths of $\Delta S_{h1} = 0.1$ and $\Delta S_{h2} = 0.18$, respectively.

The simulation results show the system quickly reaches the reference and exhibits robust performance under this perturbation of the load. Figures 3 and 4 contain the responses of x_1 and x_2 state variables.



Fig. 4. Robust full bridge boost: the state variable x_2 .



Fig. 5. Robust full bridge boost: detail of the relative error erx_1 .

The relative errors in the steady state are plotted in Figures 5 and 6. Their values are below 3% for x_1 and 5% for x_2 .

VI. CONCLUSIONS AND FUTURE WORK

Sliding mode direct control of the output voltage in a full bridge boost converter has been studied. Periodic references have been tracked, while the unstable inductor current has been independently regulated at a prescribed level. Simulation results show that robustness to load disturbances is achieved.

Further research should address the behaviour of the proposed scheme under non-resistive loads. The final target may be the connection of the load differentially



Fig. 6. Robust full bridge boost: detail of the relative error erx_2 .

across two converters in order to achieve a robust inverter device, therefore improving the performance of the proposal in [14].

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