Proceedings of the
44th IEEE Conference on Decision and Control, and
the European Control Conference 2005
Seville, Spain, December 12-15, 2005

Tracking and disturbance rejection for passive nonlinear systems

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Abstract— **In this paper, we apply internal model principle for plants that are passive nonlinear systems, to solve tracking of constant reference signal and disturbance rejection of a finite superposition of sine waves of arbitrary known frequencies. A desirable passivity property around an equilibrium (x_0,u_0) which generates y_0 is used to design the controller. The proposed controller is an LTI system, which assures that the state trajectories of the closed loop system is bounded and the error signal converges to zero.**

I. INTRODUCTION

The internal model principle for LTI systems suggests that the dynamic structure of the exosystem must be included in the controller (see also [7]). For example, to eliminate the steady-state error for step reference or disturbance signals, we need integrators in the loop. If an internal model with transfer function $s/(s^2+\omega^2)$ (with suitable multiplicity) is in the feedback loop and the closed-loop system is stable, then we obtain tracking and/or disturbance rejection for sinusoidal reference and disturbance signals of frequency ω. If the reference and disturbance signals are periodic, then the internal model principle leads to repetitive control (see for example [20]).

The idea of an internal model has been generalized for output regulation of nonlinear systems by Byrnes *et al.* [2]. Jayawardhana and Weiss [11] explore a simple LTI controller using an LTI internal model to solve a disturbance rejection problem for passive nonlinear plants, where the exosystem produces a finite superposition of sine waves of arbitrary known frequencies. In [11], the reference signal is taken to be zero and the solution of regulator equations is trivial.

In this paper, we propose a simple controller design method for nonlinear passive plants, which leads to an LTI controller (based on the internal model principle), to track a constant reference signal and to reject disturbance signals added to the control input. Here, a desirable passivity property around an equilibrium point (x_0,u_0), which generates the output y_0, is studied. If a storage function can be found such that the system is again passive with supply rate $\langle y-y_0,u-u_0\rangle$, then we solve the tracking problem for constant signals by recasting it into an input disturbance rejection problem for constant signal. Thus, the result in [11] can be applied directly to the new system to achieve the main objective.

The asymptotic tracking and disturbance rejection problem for constant signals using PI controllers for nonlinear plants has been discussed, for example, in Desoer and Lin [5] and Khalil [14]. Recently, a passivity-based PI controller for switched power converters has been introduced in Perez *et al* [16]. The local output regulation problem for signals generated by finite-dimensional exosystems (e.g., sinusoidal signals) for nonlinear plants has been investigated, for example, in Byrnes *et al* [2], Huang and Lin [9], using the internal model principle. The internal model is in fact an observer for the state of the exosystem. The proposed controller in [9] requires the solution of the regulator equations, which may be difficult to solve and it requires a precise model of the plant and the exosystem. In our work, we try to generalize the result of [16] for tracking a constant reference signal, while at the same time, the controller is able to reject disturbance signal added to the input without the need for a precise model of the plant.

Recent results in output regulation problem for nonlinear systems can be found in [3], [4], [6], [8], [10], [17] and references therein.

Passive systems have a $\mathscr{C}^1$ storage function H (defined on the state space) which has the intuitive meaning of stored energy. The input signal u and the output signal y take values in the same inner product space. We denote the state of the system at time t by $x(t)$. The defining property of a passive system is that

$$\dot{H} \le \langle y,u\rangle, \quad \text{where } \dot{H} = \left\langle \frac{\partial H(x)}{\partial x}^T, \dot{x} \right\rangle. \tag{1}$$

The function H is often used as a Lyapunov function in analyzing the system stability. The interconnection of several passive systems leads to a passive closed-loop system if the interconnection is neutral with respect to the power supply, see [18]. Many physical systems (electrical circuits, mechanical systems, etc.) are passive if the input and output variables are chosen carefully such that their product represents the flow of power into the system.

For nonlinear plants, passivity can be used for controller design, see for example [1], [11], [15] and [18].

II. PRELIMINARIES

Notation. Throughout this paper, the inner product on any Hilbert space is denoted by $\langle\cdot,\cdot\rangle$ and $\mathbb{R}_+ = [0,\infty)$. We refer to [14] and [18] for basic concepts on nonlinear systems and on passivity theory. For a finite-dimensional vector x, we use the norm $\|x\| = \left(\sum_n |x_n|^2\right)^{\frac{1}{2}}$ and for matrices, we use the operator norm induced by $\|\cdot\|$ (the largest singular value). For any $\varepsilon \ge 0$, we denote $\mathbf{B}_\varepsilon = \{x \in \mathbb{R}^n \mid \|x\| \le \varepsilon\}$. For a square matrix A, $\sigma(A)$ denotes the set of its eigenvalues.

This work is supported by the EPSRC, United Kingdom, under grant number GR/S61256/01.

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The space $\mathscr{C}^1(\mathbb{R}^l, \mathbb{R}^p)$ consists of continuously differentiable functions $f : \mathbb{R}^l \to \mathbb{R}^p$.

We consider a nonlinear plant $\mathbf{P}$ described by

$$\begin{array}{rcl} \dot{x} & = & f(x,u), \\ y & = & h(x), \end{array} \tag{2}$$

where the state x, the input u and the output y are functions of $t \geq 0$, such that $x(t) \in \mathbb{R}^n$, $u(t), y(t) \in \mathbb{R}^m$, $f \in \mathscr{C}^1(\mathbb{R}^n \times \mathbb{R}^m, \mathbb{R}^n)$ with $f(0,u) = 0 \Leftrightarrow u = 0$ and $h \in \mathscr{C}^1(\mathbb{R}^n, \mathbb{R}^m)$ with $h(0) = 0$. Let $\mathscr{Y} \subset \mathbb{R}^m$ be a domain of constant reference signal containing 0.

For the nonlinear plant $\mathbf{P}$ in (2) and $\mathscr{Y}$, we assume the following:

- **A1**. For any $y_0 \in \mathscr{Y}$, there exists a unique $x_0 \in \mathbb{R}^n$, a unique $u_0 \in \mathbb{R}^m$ and $H(x, y_0) \in \mathscr{C}^1(\mathbb{R}^n \times \mathscr{Y}, \mathbb{R}_+)$ such that

$$f(x_0, u_0) = 0, \quad h(x_0) = y_0, \tag{3}$$

and

$$\frac{\partial H(x,y_0)}{\partial x} f(x,u) \leq \langle y - y_0, u - u_0 \rangle. \tag{4}$$

Assumption **A1** shows that for any constant signal $y_0 \in \mathscr{Y}$, the plant $\mathbf{P}$ is passive with respect to storage function $H(x, y_0)$ and supply rate $\langle y - y_0, u - u_0 \rangle$, i.e., $\dot{H}(x, y_0) \leq \langle y - y_0, u - u_0 \rangle$. One particular class of port-controlled Hamiltonian systems, where the storage function $H(x, y_0)$ in Assumption **A1** can be constructed, is presented in [12].

Note that the Assumption **A1** shows that there exists an injective (one-to-one) mapping $\mathscr{Y} \mapsto \mathbb{R}^n$ and an injective mapping $\mathscr{Y} \mapsto \mathbb{R}^m$. For the plant $\mathbf{P}$, the trivia mapping of (x_0, u_0) for $y_0 = 0$ is $x_0 = 0$ and $u_0 = 0$.

Remark 2.1: The storage function $H(x, y_0)$ in Assumption **A1** can be used to show the passivity of $\mathbf{P}$ in (2) with supply rate $\langle y, u \rangle$. Indeed, by having $x_0 = 0$ and $u_0 = 0$, $H(x, 0)$ satisfies

$$\dot{H}(x,0) = \frac{\partial H(x,0)}{\partial x} f(x,u) \leq \langle y, u \rangle. \tag{5}$$

Remark 2.2: For affine systems described by

$$\dot{x} = f(x) + g(x)u, \qquad y = h(x), \tag{6}$$

the condition (4) is equivalent to the following conditions

$$\frac{\partial H(x,y_0)}{\partial x}[f(x) + g(x)u_0] \leq 0,$$
$$\frac{\partial H(x,y_0)}{\partial x} g(x) = h^T(x) - h^T(x_0).$$

Note that by taking $x_0 = 0$ and $u_0 = 0$, the above condition satisfies the *Hill-Moylan* condition (see also [18]).

Consider closed-loop system as in Fig. 1, with the plant $\mathbf{P}$ be as in (2), the controller $\mathbf{C}$ be given later, $y_0 \in \mathscr{Y}$ be constant reference signal and disturbance d be generated by exosystem $\mathbf{E}$

$$\begin{array}{rcl} \dot{w} & = & Sw \\ d(t) & = & C_w w(t) \end{array} \tag{7}$$

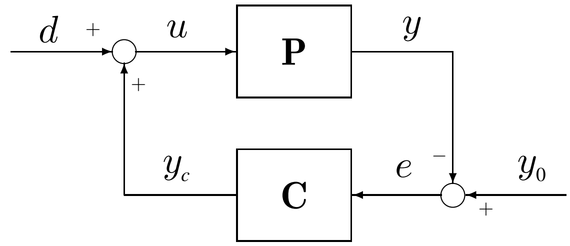

Fig. 1. The tracking and disturbance rejection problem for the plant $\mathbf{P}$ and a certain class of signals d and y_0 is to find a controller $\mathbf{C}$ such that the state trajectories of the closed-loop system $\mathbf{L}$ are bounded and $e(t) \to 0$ as $t \to \infty$.

where $C_w \in \mathbb{R}^{m \times p}$, $w(t) \in \mathbb{R}^p$ is the exosystem state, $S \in \mathbb{R}^{p \times p}$ has its eigenvalues on the imaginary axis and e^{St} is uniformly bounded for $t \geq 0$. An equivalent way of expressing our assumptions on S is the following: $\sigma(S) \subset i\mathbb{R}$ and all its Jordan blocks are of dimension 1 (i.e., there are no generalised eigenvectors for S).

Remark 2.3: Assumption **A1** implies that for any constant reference signal $y_0 \in \mathscr{Y}$, there exists solution to the regulator equation for the plant $\mathbf{P}$, to track reference signal y_0 and to reject disturbance signal d. More precisely, by choosing $x(t) = x_0$ and $y_c(t) = u_0 - d(t)$ where $h(x_0) = y_0$ and $f(x_0, u_0) = 0$, it follows that

$$\begin{array}{rcl} \dot{x} = 0 & = & f(x_0, y_c + d) = f(x_0, u_0), \\ 0 & = & y_0 - h(x_0). \end{array}$$

Remark 2.4: The storage function $H(x, y_0)$ which satisfies Assumption **A1** is natural in LTI systems. Suppose that an LTI system $\mathbf{P}$ with state x, input u and output y, is passive with respect to the quadratic storage function H. Let state x_0 and input u_0 give equilibrium for $\mathbf{P}$, generating the output y_0. Then, the same LTI system $\mathbf{P}$ is also passive with respect to the quadratic storage function $x \mapsto H(x - x_0)$ and supply rate $\langle y - y_0, u - u_0 \rangle$, i.e., $\dot{H}(x - x_0) \leq \langle y - y_0, u - u_0 \rangle$.

For any $y_0 \in \mathscr{Y}$, $H(x, y_0)$ is called *proper*, if $H(x, y_0) \to \infty$ whenever $\|x\| \to \infty$.

$\mathbf{P}$ is said to be *zero-state observable* if $u(t) = 0, y(t) = 0$ for all $t \geq 0$ implies that $x(t) = 0$ for all $t \geq 0$, and $\mathbf{P}$ is *zero-state detectable* if $u(t) = 0, y(t) = 0$ for all $t \geq 0$ implies that $\lim_{t \to \infty} x(t) = 0$.

Let us recall some definitions and results from [11] for the control system as in Figure 1, with the plant $\mathbf{P}$ be as in (2), $r = 0$ and disturbance d be as in (7).

Let $\chi(s) = s^q + a_{q-1}s^{q-1} \ldots + a_1 s + a_0$ be the minimal polynomial of $S \in \mathbb{R}^{p \times p}$, so that

$$S^q + a_{q-1}S^{q-1} + \ldots + a_2 S^2 + a_1 S + a_0 = 0, \tag{8}$$

where $a_{q-1}, \ldots, a_0 \geq 0$, $q \leq p$ and χ has only simple zeros, all on $i\mathbb{R}$.

Suppose that $S_{min} \in \mathbb{R}^{q \times q}$ is such that $S_{min} + S_{min}^T = 0$ and its characteristic polynomial is χ. If $0 \in \sigma(S)$, then the simplest choice would be

$$S_{min} = \begin{bmatrix} 0 & 0 & 0 & \cdots & 0 \\ 0 & \Omega_1 & 0 & \cdots & 0 \\ 0 & 0 & \Omega_2 & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \cdots & \Omega_\nu \end{bmatrix}, \tag{9}$$

where for each $k = 1\ldots,\nu$, $\Omega_k = \begin{bmatrix} 0 & -\omega_k \\ \omega_k & 0 \end{bmatrix}$ for some $\omega_k \in \mathbb{R}\backslash\{0\}$ and $\omega_k \neq \omega_j$ for $k \neq j$. The set $\sigma(S_{min}) = \sigma(S)$ contains 0 and $\pm i\omega_k$ ($k = 1,\ldots,\nu$) (0 and ω_k are the known frequencies of the disturbance signal). If $0 \notin \sigma(S)$, then we omit the first line and the first column in (9), so that $\sigma(S_{min})$ contains only $\pm i\omega_k$.

For $i = 1\ldots,m$, let $\Gamma_i \in \mathbb{R}^{q\times 1}$ be such that (Γ_i^T, S_{min}) is observable (the m vectors Γ_i may be taken equal). Consider the controller **C** described in state space as follows:

$$\begin{aligned} \dot{x}_c &= Ax_c + Be, \\ y_c &= B^T x_c + De, \end{aligned} \tag{10}$$

where $x_c \in \mathbb{R}^{qm}$ (q is as in (8)), $e \in \mathbb{R}^m$, $y_c \in \mathbb{R}^m$, the matrices $A \in \mathbb{R}^{qm\times qm}$ and $B \in \mathbb{R}^{qm\times m}$ are given by

$$A = \begin{bmatrix} S_{min} & 0 & \cdots & 0 \\ 0 & S_{min} & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & S_{min} \end{bmatrix}, B = \begin{bmatrix} \Gamma_1 & 0 & \cdots & 0 \\ 0 & \Gamma_2 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & \Gamma_m \end{bmatrix}, \tag{11}$$

and $D = kI^{m\times m}$ where $k > 0$.

In the closed-loop system **L** shown in Fig. 1, the controller **C** solves the *output regulation problem locally* for the plant **P**, the exosystem **E** and the constant reference signal set $\mathscr{Y}$, if for any constant reference signal $y_0 \in \mathscr{Y}$, any initial conditions $(x(0), w(0))$ in the neighborhood of $(x_0, 0)$ and $x_c(0) \in \mathscr{X}_c$ (which depends on u_0), all state trajectories of the closed-loop system are bounded and $e(t) \to 0$ as $t \to \infty$. **C** solves *output regulation problem globally* for **P**, **E** and $\mathscr{Y}$, if for any constant reference signal $y_0 \in \mathscr{Y}$, for any initial conditions $(x(0), w(0), x_c(0)) \in \mathbb{R}^n \times \mathbb{R}^p \times \mathbb{R}^{qm}$, all state trajectories of the closed-loop system are bounded and $e(t) \to 0$ as $t \to \infty$.

Lemma 2.5: [11] Suppose that the plant **P** defined by (2) is zero-state detectable. Let the controller **C** be given by (10) and consider the control system **L** as in Figure 1, with $y_0 = 0$. Then, the following two conditions are equivalent.

(1) **L** is zero-state detectable (with output y).

(2) For any $x_{co} \in \mathbb{R}^{qm}$, $x_{co} \neq 0$ and for any $x(0) = x_0 \in \mathbb{R}^n$, the plant **P** satisfies $u(t) = B^T e^{At} x_{co} \Rightarrow \exists\, t \geq 0$ such that $y(t) \neq 0$.

Proof: See also [11].

(1) $\Rightarrow$ **(2)**. By using contradiction, suppose there exist x_{co} and x_0 such that $u(t) = B^T e^{At} x_{co} \Rightarrow y(t) = 0$ for all $t \geq 0$. By taking $x_c(0) = x_{co}$, $x(0) = x_0$ and $d = 0$, we have $y = 0$ and hence $x_c(t) = e^{At} x_{co} \nrightarrow 0$ as $t \to \infty$. Then **L** is not zero-state detectable (with output y), a contradiction.

(2) $\Rightarrow$ **(1)**. From Figure 1, if $y = 0$ and $d = 0$, then we have

$$u(t) = y_c(t) = B^T e^{At} x_c(0) \qquad \text{where } x_c(0) \in \mathbb{R}^l.$$

This together with the condition **(2)** implies that $x_c(0) = 0$, hence $x_c(t) = 0$ for all $t \geq 0$ and $u = 0$. By the zero-state detectability of **P**, $u = 0$ and $y = 0$ implies that $x(t) \to 0$ as $t \to \infty$. Hence, $\begin{bmatrix} x(t) \\ x_c(t) \end{bmatrix} \to 0$ as $t \to \infty$. □

Theorem 2.6: [11] Suppose that the plant **P** defined by (2) satisfies (5) (passivity) with a storage function $H(x,0)$ such that $H(x,0) > 0$ for $x \neq 0$, $H(0,0) = 0$. Assume that **P** is zero-state detectable. Let the set of reference signal $\mathscr{Y} = \{0\}$. Let **E** be the exosystem from (7) and denote by χ the minimal polynomial of S. Let the controller **C** be given by (10) – (11) where S_{min} has the characteristic polynomial χ and satisfies $S_{min} + S_{min}^T = 0$. Consider the control system **L** as in Figure 1, with $y_0 = 0$. We assume that **P** has property **(2)** from Lemma 2.5.

Then **C** solves the output regulation problem locally for **P**, **E** and $\mathscr{Y}$. Moreover, if $H(x,0)$ is proper, then **C** solves the output regulation problem globally.

Proof: See also [11]. Let

$$\Sigma = -(\phi_c)^{-1}\phi_w, \tag{12}$$

where

$$\phi_c = \begin{bmatrix} B^T \\ B^T A \\ \vdots \\ B^T A^{q-1} \end{bmatrix}, \quad \phi_w = \begin{bmatrix} C_w \\ C_w S \\ \vdots \\ C_w S^{q-1} \end{bmatrix}. \tag{13}$$

For conciseness, it can be shown that Σ satisfies

$$\Sigma S = A\Sigma \quad \text{and} \quad B^T\Sigma + C_w = 0. \tag{14}$$

The proof follows from the fact that $\phi_c \in \mathbb{R}^{qm\times qm}$ has full rank and invertible by the observability of (Γ_i^T, S_{min}). It can be checked that $0 = \phi_c\Sigma + \phi_w$ satisfies the second equation in (14). Moreover, by some simple algebraic manipulations and by using (8), it can be checked that the first equation in (14) holds.

Let us denote $\rho = x_c - \Sigma w$. Then using (14), the closed-loop system, with $\left[\begin{smallmatrix} x \\ \rho \end{smallmatrix}\right]$ as the state variables, can be written as follows

$$\begin{aligned} \dot{x} &= f(x, B^T[\rho + \Sigma w] - kh(x) + C_w w) \\ &= f(x, B^T\rho - kh(x)), & (15) \\ \dot{\rho} &= A[\rho + \Sigma w] - Bh(x) - \Sigma S w \\ &= A\rho - Bh(x), & (16) \\ y &= h(x). & (17) \end{aligned}$$

Note that this corresponds to the closed-loop equations of (2), (10) with $y_0 = 0, d = 0$ and with ρ in place of x_c.

Consider the storage function $H_{cl}(x,\rho) = H(x,0) + \frac{1}{2}\|\rho\|^2$. Then, using (5) and (15) – (17), $\dot{H}_{cl}$ is given by

$$\begin{aligned}\dot{H}_{cl} &= \left\langle \left(\frac{\partial H(x,0)}{\partial x}\right)^T, f(x, B^T\rho - kh(x)) \right\rangle \\ &\quad + \langle \rho, A\rho - Bh(x)\rangle \\ &\leq \langle h(x), B^T\rho - kh(x)\rangle - \langle \rho, Bh(x)\rangle = -k\|y\|^2.\end{aligned}$$

By the assumptions of the theorem and using Lemma 2.5, the system described by (15), (16) and (17) is zero-state detectable.

Since $\dot{H}_{cl} \leq -k\|y\|^2 \leq 0$ for all $t \geq 0$, this implies (using H_{cl} as a Lyapunov function) that $(x,\rho) = (0,0)$ is a stable equilibrium point. It follows that for any $\varepsilon > 0$, there exists $\delta > 0$ such that $\begin{bmatrix} x(0) \\ \rho(0)\end{bmatrix} \in \mathbf{B}_\delta \Rightarrow \begin{bmatrix} x(t) \\ \rho(t)\end{bmatrix} \in \mathbf{B}_\varepsilon$ for all $t \geq 0$. According to the La-Salle invariance principle [18], such a state trajectory $\begin{bmatrix} x \\ \rho\end{bmatrix}$ converges to the largest invariant set Ω contained in $\{z \in \mathbf{B}_\varepsilon \mid \dot{H}_{cl}(z) = 0\}$. On the invariant set Ω, H_{cl} is constant along state trajectories, and $y = 0$ along such trajectories. By the zero-state detectability of (15) – (17), all these trajectories converge to 0, hence $H_{cl}(z) = H_{cl}(0) = 0$ for all $z \in \Omega$. Since $H_{cl}(z) > 0$ for all $z \neq 0$, we obtain $\Omega = \{0\}$. Thus, there exists $\delta > 0$ such that $\begin{bmatrix} x(0) \\ \rho(0)\end{bmatrix} \in \mathbf{B}_\delta \Rightarrow \begin{bmatrix} x(t) \\ \rho(t)\end{bmatrix} \to 0$ as $t \to \infty$.

If $H(x,0)$ is proper, then H_{cl} is proper. It implies that every state trajectory of $\mathbf{L}$ remains bounded, as it is easy to see. Thus, for any state trajectory $\begin{bmatrix} x \\ \rho\end{bmatrix}$, we can apply the preceding argument with $\mathbf{B}_\varepsilon$ that contains this state trajectory. Then, we conclude that $\lim_{t\to\infty} \begin{bmatrix} x(t) \\ \rho(t)\end{bmatrix} = 0$. □

III. MAIN RESULT

Let us consider the following motivating example for the plant $\mathbf{P}$, which is an integrator with a saturated output.

$$\dot{x} = u, \qquad y = \tanh(x), \tag{18}$$

where x, u and y is a function of t. Consider the control block in Fig. 1 with $y_0 \in (-1,1)$ be a constant reference signal. It can be evaluated that by using a proportional gain feedback $y_c = Ke$, where $K > 0$ and $e = y_0 - y$, the tracking objective can be achieved whenever $d = 0$. However, if d is generated by the exosystem (7), where $0 \in \sigma(S)$, the closed loop system is only locally Input-to-State Stable (ISS, for definition see also [14],[19]), $e(t) \nrightarrow 0$ as $t \to \infty$ and the closed loop system can become unstable for large constant disturbance.

The system (18) satisfies the Assumption **A1**. Indeed, for any $y_0 \in (-1,1)$, $x_0 = \tanh^{-1}(y_0)$, $u_0 = 0$ and the storage function $H(x,y_0) = \int_{\tanh^{-1}(y_0)}^{x} \tanh(\sigma)\mathrm{d}\sigma - y_0(x - \tanh^{-1}(y_0))$ satisfy Assumption **A1**. It is easy to see that for any $y_0 \in (-1,1)$, $H(x,y_0) > 0$ for $x \neq x_0$, $H(x_0,y_0) = 0$ and $H(x,y_0)$ is proper. It can be evaluated that the controller, which will be described in Proposition 3.1, can be used for (18), to globally track constant reference signal $y_0 \in (-1,1)$ and to globally reject disturbance d generated by (7).

Proposition 3.1: Suppose that for the plant $\mathbf{P}$ defined by (2) and for the set of constant reference signal $\mathscr{Y}$, the Assumption **A1** is satisfied. We assume that for any $y_0 \in \mathscr{Y}$, $H(x,y_0) > 0$ for $x \neq x_0$ and $H(x_0,y_0) = 0$.

Consider the control system $\mathbf{L}$ as in Figure 1. Suppose that $y_0 \in \mathscr{Y}$ is a constant reference signal, the exosystem $\mathbf{E}$ is as in (7) with $0 \in \sigma(S)$ and denote by χ the minimal polynomial of S. Let the controller $\mathbf{C}$ be given by (10) – (11) where S_{min} has characteristic polynomial χ and satisfies $S_{min} + S_{min}^T = 0$.

Let the system $\tilde{\mathbf{P}}$ be defined by

$$\begin{aligned}\dot{\eta} &= f(\eta + x_0, v + u_0), \\ \tilde{y} &= h(\eta + x_0) - y_0,\end{aligned} \tag{19}$$

where the state $\eta(t) \in \mathbb{R}^n$, the input signal $v(t) \in \mathbb{R}^m$, the output $\tilde{y}(t) \in \mathbb{R}^m$, the mappings f and h are as in (2). Suppose that for any pair of (x_0,u_0,y_0) satisfying Assumption **A1**, $\tilde{\mathbf{P}}$ is zero-state detectable with input v and output $\tilde{y}$, and $\tilde{\mathbf{P}}$ has property **(2)** from Lemma 2.5.

Then $\mathbf{C}$ solves the output regulation problem locally for $\mathbf{P}$, $\mathbf{E}$ and $\mathscr{Y}$.

Moreover, if for any $y_0 \in \mathscr{Y}$, $H(x,y_0)$ is proper, then $\mathbf{C}$ solves the output regulation problem globally for $\mathbf{P}$, $\mathbf{E}$ and $\mathscr{Y}$.

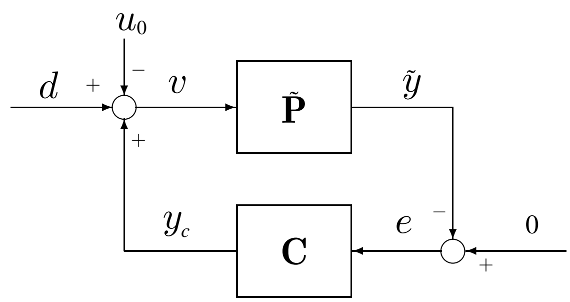

Fig. 2. The disturbance rejection problem for the plant $\tilde{\mathbf{P}}$.

Proof: The original plant $\mathbf{P}$ in (2) with output $\tilde{y} = h(x) - y_0$, can be seen as the input of plant $\tilde{\mathbf{P}}$ being disturbed by constant disturbance $-u_0$ (see Fig. 2). Indeed, if we substitute $v = u - u_0$ into (19) and let $x = \eta + x_0$, we get the original plant $\mathbf{P}$ in (2) with output $\tilde{y} = y - y_0$.

By denoting $\eta = x - x_0$, it can be evaluated that the storage function $H_{\tilde{P}}(\eta) = H(\eta + x_0, y_0)$ defines the passivity of $\tilde{\mathbf{P}}$, i.e., $\dot{H}_{\tilde{P}} \leq \langle \tilde{y}, v\rangle$ (by Assumption **A1**). $H_{\tilde{P}}(\eta)$ satisfies the condition in Theorem 2.6, i.e., $H_{\tilde{P}}(\eta) > 0$ for $\eta \neq 0$ and $H_{\tilde{P}}(0) = 0$.

Let $\tilde{d}(t) = d(t) - u_0$ be the input disturbance signal to the passive plant $\tilde{\mathbf{P}}$ and $\tilde{d}(t)$ is generated by exosystem $\tilde{\mathbf{E}}$

$$\begin{aligned}\dot{\tilde{w}} &= \tilde{S}\tilde{w}, \\ \tilde{d}(t) &= \begin{bmatrix} C_w & I^{m\times m}\end{bmatrix} \tilde{w},\end{aligned}$$

where $\tilde{w}(t) = \begin{bmatrix} w(t) \\ w_{yo}(t)\end{bmatrix}$, $w(t) \in \mathbb{R}^p$, $w_{yo}(t) \in \mathbb{R}^m$,

$$\tilde{S} = \begin{bmatrix} S & 0 \\ 0 & 0\end{bmatrix}, \tag{20}$$

$C_w \in \mathbb{R}^{m\times p}$, $S \in \mathbb{R}^{p\times p}$ are as in (7) and $w_{yo}(0) = -u_0$. Note that since we assume $0 \in \sigma(S)$, the minimal polynomial of $\tilde{S}$ is also given by χ.

Then, using the assumptions of the proposition, the storage function $H_{\tilde{P}}(\eta)$ and Theorem 2.6, the controller **C** solves the output regulation problem locally for $\tilde{\mathbf{P}}$, $\tilde{\mathbf{E}}$ and $\{0\}$. This implies that there exists $\delta > 0$ such that $\begin{bmatrix} \eta(0) \\ x_c(0)-\Sigma\tilde{w}(0) \end{bmatrix} \in \mathbf{B}_\delta \Rightarrow \begin{bmatrix} \eta(t) \\ x_c(t)-\Sigma\tilde{w}(t) \end{bmatrix} \to 0$ as $t \to \infty$ and $\tilde{y}(t) \to 0$ as $t \to \infty$, where $\Sigma = -\phi_c^{-1}\phi_w$, ϕ_c is as in (13) and

$$\phi_w = \begin{bmatrix} [C_w \quad I] \\ [C_w \quad I]\tilde{S} \\ \vdots \\ [C_w \quad I]\tilde{S}^{q-1} \end{bmatrix}.$$

The same arguments can be carried out for any $y_0 \in \mathscr{Y}$. Thus, the controller **C** solves the output regulation problem locally for **P**, **E** and $\mathscr{Y}$, i.e., there exists $\delta > 0$ such that $\begin{bmatrix} x(0)-x_0 \\ x_c(0)-\Sigma\tilde{w}(0) \end{bmatrix} \in \mathbf{B}_\delta \Rightarrow \begin{bmatrix} x(t)-x_0 \\ x_c(t)-\Sigma\tilde{w}(t) \end{bmatrix} \to 0$ as $t \to \infty$ and $e(t) \to 0$ as $t \to \infty$.

If for any $y_0 \in \mathscr{Y}$, $H(x,y_0)$ is proper, then the corresponding $H_{\tilde{P}}(\eta)$ is also proper. Thus, using a similar method as above and by Theorem 2.6, the controller **C** solves the output regulation problem globally for **P**, **E** and $\mathscr{Y}$. □

Remark 3.2: If $0 \notin \sigma(S)$ in (7), we can assume a fictitious constant disturbance signal into the exosystem dynamics, so that $0 \in \sigma(S)$.

In many passive systems, such as mechanical and electrical systems, we may already know the storage function $H_0(x) = H(x,0)$ which defines the passivity of the plant **P** with supply rate $\langle y,u\rangle$, i.e. $\dot{H}_0 \leq \langle y,u\rangle$. Thus, the storage function $H(x,y_0)$ which satisfies Assumption **A1** may be constructed using the storage function $H_0(x)$ as presented in [12].

IV. EXAMPLE

Consider the electrical circuit in Figure 3, where the voltage V on the nonlinear load **P** should track a constant reference voltage y_0. We can only control the current source I_c. The main current I_d comes from an external power supply, for example, from an AC/DC converter with power factor precompensation, as discussed in [13], and we treat I_d as a disturbance. The current source I_d has a DC component which is approximately equal to the desired current through **P**, but any small deviation of the DC component, as well as the AC components of I_d, should be compensated by controlling I_c. The AC components of I_d correspond to the fundamental frequency of the power grid and its harmonics.

The nonlinear load **P** in Fig. 3 can be described in state space as follows:

$$\begin{aligned} \dot{I}_2 &= \left(\frac{\mathrm{d}\phi(I_2)}{\mathrm{d}I_2}\right)^{-1}(-\alpha(I_2)+V), \\ \dot{V} &= \left(\frac{\mathrm{d}q(V)}{\mathrm{d}V}\right)^{-1}(-I_2+I), \qquad (21) \\ y &= V, \quad e = y_0 - y, \qquad (22) \end{aligned}$$

where $x(t) \equiv \begin{bmatrix} I_2(t) \\ V(t) \end{bmatrix} \in \mathbb{R}^2$ is the state of **P**, $I \in \mathbb{R}$ is the input current, $y \in \mathbb{R}$ is the output voltage, $y_0 \in \mathbb{R}$ is a constant reference voltage. Here, ϕ, q and α are in $\mathscr{C}^1(\mathbb{R},\mathbb{R})$,

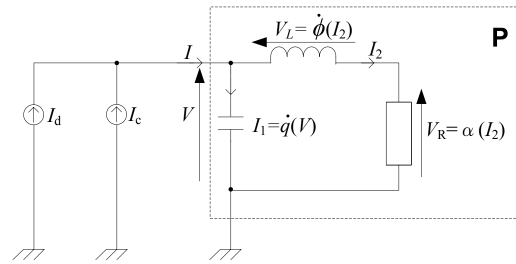

Fig. 3. Electrical circuit of voltage regulation, where the load consists of a nonlinear resistor $V_R = \alpha(I_2)$, a nonlinear inductor and a nonlinear capacitor.

$\phi(0) = 0$, $q(0) = 0$, $\alpha(0) = 0$ and these functions are *strictly monotone increasing*. (The fact that ϕ is strictly monotone increasing means that $(\phi(a) - \phi(b))(a-b) > 0$ for any $a \neq b$.)

The physical meaning of $\phi(I_2)$ is the magnetic flux of the inductor, so that the voltage across the inductor is $V_L = \dot{\phi}(I_2)$. The meaning of $q(V)$ is the electric charge in the capacitor, so that the current flowing through the capacitor is $I_1 = \dot{q}(V)$. Note that for a linear resistor, $\alpha(I_2) = RI_2$, where $R > 0$ is the resistance, for a linear inductor, $\phi(I_2) = LI_2$, where $L > 0$ is the inductance, and for a linear capacitor, $q(V) = CV$, where $C > 0$ is the capacitance.

With the storage function

$$H_0(x) = \phi(I_2)I_2 - \int_0^{I_2}\phi(\lambda)\mathrm{d}\lambda + q(V)V - \int_0^V q(\lambda)\mathrm{d}\lambda,$$

P is passive with input I and output V, i.e. $\dot{H}_0 \leq \langle V,I\rangle$. Let us denote by $\alpha^{-1}(\cdot)$ the inverse function of $\alpha(\cdot)$, such that $\alpha \circ \alpha^{-1}(a) = a$ for any $a \in \mathbb{R}$.

It can be checked that for any $y_0 \in \mathbb{R}$,

$$x_0 = \begin{bmatrix} \alpha^{-1}(y_0) \\ y_0 \end{bmatrix}, \quad u_0 = \alpha^{-1}(y_0), \qquad (23)$$

satisfy the existence of x_0 and u_0 in Assumption **A1**.

The storage function

$$\begin{aligned} H(x,y_0) &= \phi(I_2)\left(I_2 - \alpha^{-1}(y_0)\right) - \int_{\alpha^{-1}(y_0)}^{I_2}\phi(\lambda)\mathrm{d}\lambda \\ &\quad + q(V)(V - y_0) - \int_{y_0}^{V} q(\lambda)\mathrm{d}\lambda, \qquad (24) \end{aligned}$$

satisfies Assumption **A1** and for any pair of (x_0,u_0,y_0), $H(x,y_0) > 0$ for $x \neq x_0$, $H(x_0,y_0) = 0$. Indeed,

$$\begin{aligned} &\frac{\partial H(x,y_0)}{\partial x}\begin{bmatrix} \left(\frac{\mathrm{d}\phi(I_2)}{\mathrm{d}I_2}\right)^{-1}(-\alpha(I_2)+V) \\ \left(\frac{\mathrm{d}q(V)}{\mathrm{d}V}\right)^{-1}(-I_2+u_0+I-u_0) \end{bmatrix} \\ &= -\left(I_2 - \alpha^{-1}(y_0)\right)(\alpha(I_2) - y_0) + \langle V - y_0, I - u_0\rangle \\ &\leq \langle V - y_0, I - u_0\rangle, \end{aligned}$$

where the last inequality is due to the monotonicity of α.

Let us consider the control block for the voltage regulation problem as in Fig. 4. We assume that I_d is the disturbance

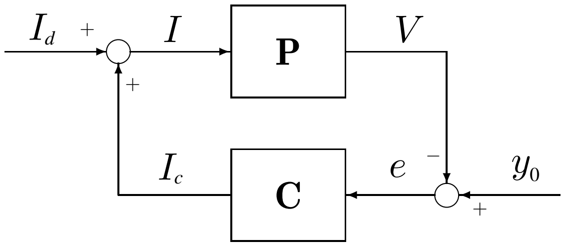

Fig. 4. The voltage regulation control block where the objective of controller **C** is to reject disturbance signal I_d, to track constant reference voltage y_0 and to keep the closed-loop state trajectories bounded.

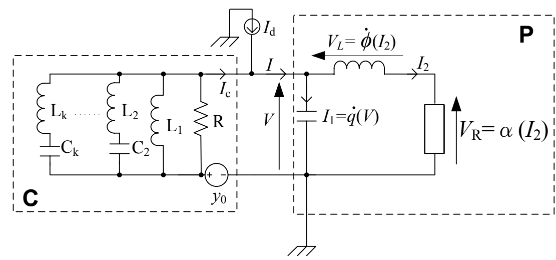

Fig. 5. One realization of controller **C** for voltage regulation as in Fig. 4, where L_1 determines the gain of the integrator and the pair of inductor L_k and capacitor C_k, $k = 2, ..., \nu$, determine the frequencies of the signal to be rejected from the current I_d, i.e., $\omega_k = \sqrt{\frac{1}{L_k C_k}}$.

signal which is generated by the exosystem (7) with $0 \in \sigma(S)$. By denoting $\eta = x - x_0$, it is easy to verify that $\tilde{\mathbf{P}}$

$$\begin{aligned}
\dot{\eta}_1 &= \left(\frac{\mathrm{d}\phi}{\mathrm{d}\eta_1}\right)^{-1} \left(-\alpha(\eta_1 + \alpha^{-1}(y_0)) + \eta_2 + y_0\right), \\
\dot{\eta}_2 &= \left(\frac{\mathrm{d}q}{\mathrm{d}\eta_2}\right)^{-1} \left(-(\eta_1 + \alpha^{-1}(y_0)) + u_0 + v\right), \\
-e &= \tilde{y} = \eta_2,
\end{aligned}$$

is zero state-observable with input v and output $\tilde{y}$, i.e., $\tilde{y} = 0$ and $v = 0 \Rightarrow \eta = 0$.

It can be checked that the conditions in Proposition 3.1 with the controller **C** as in (10) are satisfied. Thus, the controller **C** can be used to control I_c, for solving output (voltage) regulation problem locally for **P**, **E** and $\mathbb{R}$ where I_d is seen as the exosystem **E**. If the controller for I_d is able to produce desired current $I_d(t) = \alpha^{-1}(y_0)$ for all $t \geq 0$, then $I_c(t) \to 0$ and $e(t) \to 0$ as $t \to \infty$. One such realization of the controller **C** is shown in Fig. 5.

Moreover, if ϕ and q is such that $\frac{\mathrm{d}\phi(I_2)}{\mathrm{d}I_2} \geq \varepsilon > 0$ and $\frac{\mathrm{d}q(V)}{\mathrm{d}V} \geq \varepsilon > 0$ for all $I_2 \in \mathbb{R}$ and $V \in \mathbb{R}$, then for any $y_0 \in \mathbb{R}$, $H(x, y_0)$ in (24) is proper. Thus, the same controller **C** solves the output regulation problem globally.

Since the controller we have discussed exploits the passivity of nonlinear plant and induces an output strictly passive closed-loop system, it can be expected that the closed-loop system possesses L_2-stability property (see also [18]) and has some robustness property with respect to parameter uncertainties. Indeed, in the voltage regulation example above, for any additive parameter uncertainties in the inductance $\Delta\phi(I_2)$, in the capacitance $\Delta q(V)$ and/or in the resistance $\Delta\alpha(I_2)$ such that $(\phi + \Delta\phi)$, $(q + \Delta q)$ and/or $(\alpha + \Delta\alpha)$ are strictly monotone increasing, the same controller still assures tracking error e to converge to zero, while the plant state x and the controller state x_c converge to different values.

V. CONCLUSIONS

A method to track a constant reference signal and to reject disturbance signal for passive nonlinear systems using a simple LTI controller is discussed. The controller assures that the tracking error converges to zero and state trajectories of the closed-loop system are bounded. An example to the control of voltage regulation is presented.

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