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Robust sliding mode observer-based actuator fault detection and isolation for a class of nonlinear systems

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Abstract— **In this paper, an actuator fault detection and isolation scheme for a class of nonlinear systems with uncertainty is considered. The uncertainty is allowed to have a nonlinear bound which is a general function of the state variables. A sliding mode observer is established first based on a constrained Lyapunov equation. Then, the equivalent output error injection signal is employed to reconstruct the fault signal using the characteristics of the sliding mode observer and the structure of the uncertainty. Finally, a simulation study of the HIRM aircraft system is presented to show the effectiveness of the scheme.**

I. INTRODUCTION

For several decades, fault detection and isolation (FDI) has been studied extensively (see e.g the survey paper [7] and the references therein). Advances in control theory have greatly sped up the development of FDI of dynamical systems, and various approaches have been proposed (see e.g [5], [6], [1]). Among these approaches, observer-based FDI is an effective one and has been widely studied especially in recent decades. Some control inspired approaches, for instance, sliding mode techniques [3], modern differential geometric approaches [10] and adaptive control ideas [18] have been successfully incorporated with the observer-based FDI approach, and many fruitful results have been achieved but most of the work has focused on 'certain' systems. Recently, systems with parametric uncertainty [18] or unknown inputs [15] have been considered where it is required that the uncertainty has a linear structure and is bounded by a constant or a linear function of the norm of the states. Notably, sliding mode techniques have good robustness and are completely insensitive to so-called matched uncertainty [2], [14]. The reduced-order characteristic of the sliding motion makes it possible to improve the robustness with respect to mismatched uncertainty and it has been shown that sliding mode techniques can be used to deal with structural uncertainty [16]. Therefore the application of sliding mode ideas to systems with uncertainty offers good potential in the field of FDI.

Although a sliding mode observer was first used in FDI more than ten years ago [11], important progress has been made in recent years. Edwards *et al* [3] proposed an approach based on the equivalent output injection where the sliding motion is maintained even in the presence of faults which can be reproduced faithfully under certain conditions. Later it was extended by Tan and Edwards in [12] where sensor faults were considered. In these papers, uncertainty was not considered. More recently, Tan and Edwards [13] proposed a FDI scheme for a class of linear systems with uncertainty which focused on minimizing the $\mathcal{L}_2$ gain between the uncertainty and the fault reconstruction signal by using LMIs. A robust fault detection method for nonlinear systems with disturbances was considered in [4] where strict geometric conditions are exploited and the disturbance can effectively be considered as linear parametric uncertainty. It should be emphasised that the "precise" fault reconstruction approach which has been proposed in [3], [12] was in the absence of uncertainty. This problem is challenging when the system considered suffers from uncertainty. In all the existing robust FDI schemes, much effort has been devoted to fault estimation [8] or the reduction of the error between the real fault signal and the reconstructed signal [13]. However, the "precise" reconstruction scheme is still not generally available in the presence of uncertainty. Therefore, it is meaningful to explore under what conditions the fault can be reconstructed with arbitrary accuracy in the presence of uncertainty.

In this paper, an actuator FDI scheme for a class of nonlinear uncertain systems is considered where the uncertainty is allowed to have nonlinear bounds. Based on a constrained Lyapunov equation, a robust sliding mode observer is established in the presence of uncertainties and the faults. An actuator fault reconstruction instead of just detection is presented based on the equivalent output injection approach proposed by Edwards et al. [3]. Unlike the existing robust FDI results based on sliding mode techniques [13], [4], [8], the bound on the uncertainty has a more general form and the scheme generates not just an estimation of the fault – the reconstructed signal converges to the fault with arbitrary accuracy by exploiting the features of the sliding motion and the limitations on the structure of the uncertainty.

Notation: For a square matrix A, $A > 0$ denotes a symmetric positive definite matrix, and $\lambda_{min}(A)$ denotes the minimum eigenvalue of A. The symbol I_n represents the nth order unit matrix and $\mathcal{R}^+$ represents the set of non-negative real numbers. The Lipschitz constant of a function f will be written as $\mathcal{L}_f$. Finally, $\|\cdot\|$ denotes the Euclidean norm or its induced norm.

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II. SYSTEM DESCRIPTION AND PRELIMINARIES

Consider a system described by

$$\dot{x} = Ax + G(x,u) + \Delta\Phi(x,t) + Df(u,t) \quad (1)$$
$$y = Cx, \quad (2)$$

where $x \in \mathcal{R}^n$, $u \in \mathcal{R}^m$ and $y \in \mathcal{R}^p$ are the state variable, the input and the output respectively, $A \in \mathcal{R}^{n\times n}$, $D \in \mathcal{R}^{n\times q}$ and $C \in \mathcal{R}^{p\times n}$ ($q \le p < n$) are constant matrices with D and C both being of full rank, the nonlinear term $G(x,u)$ is assumed to be known and Lipschitz about x uniformly for $u \in \mathcal{U}$ (an admissible control set), $\Delta\Phi(x,t)$ is the uncertainty which affects the system and the unknown function $f(u,t) \in \mathcal{R}^q$ represents the actuator fault which satisfies

$$\|f(u,t)\| \le \rho(u,t) \quad (3)$$

where the bounding function $\rho(u,t)$ is known. All the functions are assumed to be continuous in their arguments.

Assumption 1. The matrix pair (A,C) is detectable.

It follows from Assumption 1 that there exists a matrix L such that $A - LC$ is stable, and thus for any $Q > 0$ the Lyapunov equation

$$(A - LC)^T P + P(A - LC) = -Q \quad (4)$$

has an unique solution $P > 0$.

Assumption 2. The uncertainty $\Delta\Phi(x,t)$ has a decomposition

$$\Delta\Phi(x,t) = E\Delta\Psi(x,t) \quad (5)$$

where $E \in \mathcal{R}^{n\times r}$ and $\|\Delta\Psi(x,t)\| \le \xi(x,t)$ where $\xi(x,t)$ is known and Lipschitz about x uniformly for $t \in \mathcal{R}^+$.

Remark 1. The matrix E in (5) is called the structural matrix which is employed to characterize the structure of the uncertainty $\Delta\Phi(x,t)$. Here the bound on $\Delta\Psi(x,t)$ takes a more general nonlinear form (as in [16]) compared with the work in [13], [8].

Assumption 3. There exist matrices $F_1 \in \mathcal{R}^{r\times p}$ and $F_2 \in \mathcal{R}^{q\times p}$ such that the solution P to the Lyapunov equation (4) satisfies the constraint:

$$\begin{bmatrix} E^T \\ D^T \end{bmatrix} P = \begin{bmatrix} F_1 \\ F_2 \end{bmatrix} C \quad (6)$$

Remark 2. Equation (6) is a structural property associated with system $(A, [\,E \quad D\,], C)$ and is independent of the choice of coordinate system. A similar limitation has been imposed by many authors (see e.g, [2], [17] and references therein). It should be pointed out that Assumption 3 implies that rank$\,[\,E \quad D\,] \le p$.

Without loss of generality, it is assumed that the output matrix C of system (1)–(2) has the following form

$$C = [\,0 \quad I_p\,] \quad (7)$$

Then system (1)–(2) can be rewritten by

$$\dot{x}_1 = A_1x_1 + A_2x_2 + G_1(x,u) + E_1\Delta\Psi + D_1 f(u,t) \quad (8)$$
$$\dot{x}_2 = A_3x_1 + A_4x_2 + G_2(x,u) + E_2\Delta\Psi + D_2 f(u,t) \quad (9)$$
$$y = x_2 \quad (10)$$

where $x = \mathrm{col}(x_1, x_2)$ with $x_1 \in \mathcal{R}^{n-p}$; $G_1(x,u) \in \mathcal{R}^{n-p}$ and $G_2(x,u) \in \mathcal{R}^p$ are respectively the first $n-p$ and the last p components of $G(x,u)$; and

$$\begin{bmatrix} A_1 & A_2 \\ A_3 & A_4 \end{bmatrix} = A, \quad \begin{bmatrix} E_1 \\ E_2 \end{bmatrix} = E, \quad \begin{bmatrix} D_1 \\ D_2 \end{bmatrix} = D \quad (11)$$

where $A_1 \in \mathcal{R}^{(n-p)\times(n-p)}$, $E_1 \in \mathcal{R}^{(n-p)\times r}$ and $D_1 \in \mathcal{R}^{(n-p)\times q}$. Introduce partitions of P and Q which are conformable with the decomposition in (8)–(10):

$$P = \begin{bmatrix} P_1 & P_2 \\ P_2^T & P_3 \end{bmatrix}, \qquad Q = \begin{bmatrix} Q_1 & Q_2 \\ Q_2^T & Q_3 \end{bmatrix} \quad (12)$$

It follows from $P > 0$ and $Q > 0$ that $P_1 > 0$, $P_3 > 0$, $Q_1 > 0$ and $Q_3 > 0$.

Lemma 1: If P and Q have the partition in (12), then

i) $P_1^{-1}P_2E_2 + E_1 = 0$ and $P_1^{-1}P_2D_2 + D_1 = 0$ if (6) is satisfied;

ii) the matrix $A_1 + P_1^{-1}P_2A_3$ is stable if Lyapunov equation (4) is satisfied.

Proof: i). From the matrix partitions, it follows that

$$\begin{aligned} &[\,E_1^T \quad E_2^T\,]\,P \\ &= [\,E_1^TP_1 + E_2^TP_2^T \quad E_1^TP_2 + E_2^TP_3\,] \\ &= [\,\big(P_1(E_1 + P_1^{-1}P_2E_2)\big)^T \quad E_1^TP_2 + E_2^TP_3\,] \end{aligned} \quad (13)$$

Similarly

$$\begin{aligned} &[\,D_1^T \quad D_2^T\,]\,P = \\ &\quad [\,\big(P_1(D_1 + P_1^{-1}P_2D_2)\big)^T \quad D_1^TP_2 + D_2^TP_3\,] \end{aligned} \quad (14)$$

From (6) and (7), it follows that

$$\begin{bmatrix} 0 & F_1 \\ 0 & F_2 \end{bmatrix} = \begin{bmatrix} F_1 \\ F_2 \end{bmatrix} C = \begin{bmatrix} [E_1^T \quad E_2^T]P \\ [D_1^T \quad D_2^T]P \end{bmatrix} \quad (15)$$

Hence conclusion i) follows by comparing (13) and (14) with (15).

ii). Applying block multiplication to (4):

$$A_1^TP_1 + A_3^TP_2^T + P_1A_1 + P_2A_3 = -Q_1$$

This implies that

$$(A_1 + P_1^{-1}P_2A_3)^TP_1 + P_1(A_1 + P_1^{-1}P_2A_3) = -Q_1 \quad (16)$$

Hence the conclusion follows from the fact that $Q_1 > 0$ and $P_1 > 0$. #

III. SLIDING MODE OBSERVER DESIGN

In this section, a sliding mode observer will be presented which guarantees that the state estimation error can be driven to a pre-designed sliding surface in finite time, and a sliding motion takes place thereafter.

Consider system (8)–(10). Introduce a linear coordinate transformation $z = Tx$ where

$$T :\equiv \begin{bmatrix} I_{n-p} & P_1^{-1}P_2 \\ 0 & I_p \end{bmatrix} \tag{17}$$

In the new coordinate system z, system (8)–(10) has the following form

$$\begin{aligned} \dot{z}_1 &= (A_1 + P_1^{-1}P_2A_3)z_1 + \big(A_2 - A_1P_1^{-1}P_2 \\ &\quad + P_1^{-1}P_2(A_4 - A_3P_1^{-1}P_2)\big)z_2 + G_1(T^{-1}z, u) \\ &\quad + P_1^{-1}P_2G_2(T^{-1}z, u) \end{aligned} \tag{18}$$

$$\begin{aligned} \dot{z}_2 &= A_3z_1 + \left(A_4 - A_3P_1^{-1}P_2\right)z_2 + G_2(T^{-1}z, u) \\ &\quad + E_2\Delta\Psi(T^{-1}z, t) + D_2f(u, t) \end{aligned} \tag{19}$$

$$y = z_2 \tag{20}$$

where $z = \mathrm{col}(z_1, z_2)$ with $z_1 \in \mathcal{R}^{n-p}$ and conclusion i) of Lemma 1 is used above.

For system (18)–(20), consider a dynamical system

$$\begin{aligned} \dot{\hat{z}}_1 &= (A_1 + P_1^{-1}P_2A_3)\hat{z}_1 + \big(A_2 - A_1P_1^{-1}P_2 \\ &\quad + P_1^{-1}P_2(A_4 - A_3P_1^{-1}P_2)\big)y + G_1(T^{-1}\hat{z}, u) \\ &\quad + P_1^{-1}P_2G_2(T^{-1}\hat{z}, u) \end{aligned} \tag{21}$$

$$\dot{\hat{z}}_2 = A_3\hat{z}_1 + \left(A_4 - A_3P_1^{-1}P_2\right)\hat{z}_2 + G_2(T^{-1}\hat{z}, u) + \nu \tag{22}$$

$$\hat{y} = \hat{z}_2 \tag{23}$$

where $\hat{z} :\equiv \mathrm{col}(\hat{z}_1, y)$, $\hat{y}$ is the output of the dynamical system, and ν is defined by

$$\begin{aligned} \nu = &\Big(\|E_2\|\xi(T^{-1}\hat{z}, t) + \|A_4 - A_3P_1^{-1}P_2\|\,\|y - \hat{y}\| \\ &+ \|D_2\|\rho(u, t) + k\Big)\mathrm{sgn}(y - \hat{y}) \end{aligned} \tag{24}$$

where ξ is given in Assumption2, sgn denotes the usual sign vector function and k is a positive constant to be determined later.

Let $e_1 = z_1 - \hat{z}_1$, and $e_y = y - \hat{y}$. Then from (18)–(20) and (21)–(23), the error dynamical equation is described by

$$\begin{aligned} \dot{e}_1 &= (A_1 + P_1^{-1}P_2A_3)e_1 + G_1(T^{-1}z, u) - G_1(T^{-1}\hat{z}, u) \\ &\quad + P_1^{-1}P_2\Big(G_2(T^{-1}z, u) - G_2(T^{-1}\hat{z}, u)\Big) \end{aligned} \tag{25}$$

$$\begin{aligned} \dot{e}_y &= A_3e_1 + \left(A_4 - A_3P_1^{-1}P_2\right)e_y + G_2(T^{-1}z, u) - \\ &\quad G_2(T^{-1}\hat{z}, u) + E_2\Delta\Psi(T^{-1}z, t) + D_2f(u, t) - \nu \end{aligned} \tag{26}$$

where $\hat{z} = \mathrm{col}(\hat{z}_1, y)$ and ν is defined by (24).

Now, consider system (25). It follows that

$$\begin{aligned} e_1(t) &= \exp\left\{(A_1 + P_1^{-1}P_2A_3)t)\right\}e_1(0) + \int_0^t \exp\big\{(A_1 \\ &\quad + P_1^{-1}P_2A_3)(t-\tau)\big\}\Big\{G_1(T^{-1}z(\tau), u(\tau)) - \\ &\quad G_1(T^{-1}\hat{z}(\tau), u(\tau)) + P_1^{-1}P_2\Big(G_2(T^{-1}z(\tau), u(\tau)) \\ &\quad - G_2(T^{-1}\hat{z}(\tau), u(\tau))\Big)\Big\}d\tau \end{aligned}$$

where $\hat{z}(\tau) = \mathrm{col}(\hat{z}_1(\tau), y(\tau))$. From conclusion ii) of Lemma 1, there exist positive constants a_0 and c_0 such that for any $t > 0$

$$\begin{aligned} \|e_1(t)\| &\le c_0\exp\{-a_0t\}\|e_1(0)\| + c_0\exp\{-a_0t\}\|T^{-1}\| \\ &\quad \cdot\left(\mathcal{L}_{G_1} + \|P_1^{-1}P_2\|\mathcal{L}_{G_2}\right)\int_0^t \exp\{a_0\tau\}\|e_1(\tau)\|d\tau \end{aligned}$$

where $\mathcal{L}_{G_1}$ and $\mathcal{L}_{G_2}$ are both well defined since G is assumed to be Lipschitz. Multiplying by $\exp\{a_0t\}$ on both sides of the inequality above, it follows that

$$\begin{aligned} \exp\{a_0t\}\|e_1(t)\| &\le c_0\|e_1(0)\| + c_0\|T^{-1}\| \\ &\quad \cdot\left(\mathcal{L}_{G_1} + \|P_1^{-1}P_2\|\mathcal{L}_{G_2}\right)\int_0^t \exp\{a_0\tau\}\|e_1(\tau)\|d\tau \end{aligned}$$

Then, from the Gronwall-Bellman inequality (see, [9]),

$$\begin{aligned} \|e_1(t)\| \le c_0\|e_1(0)\|\exp\Big\{\Big(&c_0\|T^{-1}\| \\ &+ \left(\mathcal{L}_{G_1}\|P_1^{-1}P_2\|\mathcal{L}_{G_2}\right) - a_0\Big)t\Big\} \end{aligned} \tag{27}$$

The following conclusion can be obtained directly:

Lemma 2: Consider the error system (25)–(26). Then, under Assumption 1, $e_1(t)$ is bounded and its bound is independent of the system input u and output y if

$$c_0\|T^{-1}\|\left(\mathcal{L}_{G_1} + \|P_1^{-1}P_2\|\mathcal{L}_{G_2}\right) \le a_0 \tag{28}$$

where a_0 and c_0 are positive constants.

Proof: The proof can be obtained directly from (27). #

For convenience, assume that

$$\|e_1(t)\| \le \beta, \qquad t \ge 0 \tag{29}$$

Next, the stability of the error dynamic equation (25)–(26) is shown using sliding mode theory, which implies that the dynamical system (21)–(22) is an asymptotic observer of system (18)–(20).

For system (25)–(26), consider a sliding surface

$$S = \{(e_1, e_y) \mid e_y = 0\} \tag{30}$$

The following conclusion is ready to be presented:

Proposition 1: Under Assumption 1, the sliding motion of system (25)–(26) associated with the sliding surface (30) is asymptotically stable if

$$\lambda_{min}(Q_1) > 2\|P_1\|\,\|T^{-1}\|\left(\mathcal{L}_{G_1} + \|P_1^{-1}P_2\|\mathcal{L}_{G_2}\right) \tag{31}$$

where P_1 and Q_1 are from (12), and T is given in (17).

Proof: It is only needed to prove that (25) is asymptotically stable. Consider as a Lyapunov function $V = e_1^T P_1 e_1$. It follows that the time derivative of V along the trajectories of system (25) is given by

$$\dot{V} = e_1^T \left(P_1(A_1 + P_1^{-1}P_2A_3) + (A_1 + P_1^{-1}P_2A_3)^T P_1 \right) e_1 + 2e_1^T P_1 \Big(G_1(T^{-1}z, u) - G_1(T^{-1}\hat{z}, u) + P_1^{-1}P_2 \big(G_2(T^{-1}z, u) - G_2(T^{-1}\hat{z}, u) \big) \Big)$$

Then, from (16), it is observed that

$$\dot{V} \leq 2\|e_1\| \, \|P_1\| (\mathcal{L}_{G_1} + \|P_1^{-1}P_2\| \mathcal{L}_{G_2}) \left\| T^{-1}(z - \hat{z}) \right\| - e_1^T Q_1 e_1$$

and so

$$\dot{V} \leq \Big(2\|P_1\| \, \|T^{-1}\| (\mathcal{L}_{G_1} + \|P_1^{-1}P_2\| \mathcal{L}_{G_2}) - \lambda_{min}(Q_1) \Big) \|e_1\|^2$$

Hence the conclusion follows from (31). #

Proposition 1 has shown that the sliding mode associated with the sliding surface S given in (30) is stable. The objective now is to determine the gain k in (24) such that the system can be driven to S in finite time and a sliding motion can be obtained. From (26)

$$e_y^T \dot{e}_y = e_y \Big(A_3 e_1 + (A_4 - A_3 P_1^{-1} P_2) e_y + G_2(T^{-1}z, u) - G_2(T^{-1}\hat{z}, u) + E_2 \Delta\Psi(T^{-1}z, t) + D_2 f(\cdot) - \nu \Big) \quad (32)$$

By applying (3) and (24) to (32), it follows from Assumption 2 that

$$\begin{aligned} & e_y^T \dot{e}_y \\ \leq \ & \left(\|A_3\| + \|T^{-1}\| \mathcal{L}_{G_2} \right) \|e_1\| \, \|e_y\| + \Big(\|E_2\| \xi(T^{-1}z, t) \\ & + \|D_2\| \rho(u, t) \Big) \|e_y\| + \|A_4 - A_3 P_1^{-1} P_2\| \, \|e_y\|^2 \\ & - \Big(\|A_4 - A_3 P_1^{-1} P_2\| \, \|e_y\| + \|E_2\| \xi(T^{-1}\hat{z}, t) \\ & + \|D_2\| \rho(u, t) + k \Big) e_y^T \operatorname{sgn}(e_y) \\ \leq \ & \left(\|A_3\| + \|T^{-1}\| (\mathcal{L}_{G_2} + \|E_2\| \mathcal{L}_{\xi}) \|e_1\| - k \right) \|e_y\| \end{aligned} \quad (33)$$

From the analysis above, the following can be obtained directly.

Proposition 2: Under Assumptions 1 and 2, system (25)–(26) is driven to the sliding surface (30) in finite time and remains on it if

$$k \geq \|A_3\| + \|T^{-1}\| (\mathcal{L}_{G_2} + \|E_2\| \mathcal{L}_{\xi}) \beta + \eta \quad (34)$$

where β is determined by (29) and η is a positive constant.

Proof: If Assumption 1 holds, then from Lemma 2 the state error e_1 is bounded, and (29) is true. Applying (34) and (29) to (33), it follows that

$$e_y^T \dot{e}_y \leq -\eta \|e_y\|$$

This shows that the reachability condition is satisfied. Hence the conclusion follows. #

By combining Proposition 1 with Proposition 2, it follows from sliding mode control theory that system (25)–(26) is asymptotically stable. Therefore, (21)–(22) is a sliding mode observer of system (18)–(20), and $\hat{y}$ defined by (23) is called the observer output.

Remark 3. The terms $G_1(T^{-1}\hat{z}, u)$ and $G_2(T^{-1}\hat{z}, u)$ in observer (21)–(22) can be replaced by $G_1(y, u)$ and $G_2(y, u)$ if $G(x, u) = G(y, u)$. In this case, the Lipschitz constants $\mathcal{L}_{G_1}$ and $\mathcal{L}_{G_2}$ can both be chosen as zero, and thus condition (31) is satisfied automatically. This implies that the sliding mode will always be stable no matter whether $G(x, u)$ is matched or mismatched.

IV. RECONSTRUCTION OF ACTUATOR FAULTS

The objective now is to use the so-called equivalent output injection (see [3]) to reconstruct the actuator fault. It is assumed that the sliding mode observer given in section 3 has been designed to satisfy (31) and (34).

Comparing system (8)–(9) with (18)–(19):

$$\begin{bmatrix} E_1 \\ E_2 \end{bmatrix} = T^{-1} \begin{bmatrix} 0 \\ E_2 \end{bmatrix}, \quad \begin{bmatrix} D_1 \\ D_2 \end{bmatrix} = T^{-1} \begin{bmatrix} 0 \\ D_2 \end{bmatrix}$$

where T is defined by (17). It follows that D_2 is full rank since D is full rank.

Assumption 4. There exists a nonsingular matrix $M \in \mathcal{R}^{p \times p}$ such that

$$M \left[E_2 \quad D_2 \right] = \begin{bmatrix} H_1 & H_2 \\ 0 & H_3 \end{bmatrix}$$

where $H_1 \in \mathcal{R}^{(p-q) \times r}$ and $H_3 \in \mathcal{R}^{q \times q}$ is nonsingular.

Remark 4: Assumption 4 guarantees that the fault can be separated from the uncertainty, which makes "precise" fault reconstruction possible. This condition is equivalent to the fact that the matrix $\left[E_2 \quad D_2 \right]$ can be transformed to a special block-diagonal matrix only using elementary row operations. Thus M can be obtained easily by matrix theory.

Consider the error dynamics (25)–(26). Multiplying (26) by M, it follows from Assumption 4 that

$$\begin{aligned} M\dot{e}_y = \ & M A_3 e_1 + M \left(A_4 - A_3 P_1^{-1} P_2 \right) e_y \\ & + M \left(G_2(T^{-1}z, u) - G_2(T^{-1}\hat{z}, u) \right) \\ & + \begin{bmatrix} H_1 & H_2 \\ 0 & H_3 \end{bmatrix} \begin{bmatrix} \Delta\Psi(T^{-1}z, t) \\ f(u, t) \end{bmatrix} - M\nu \end{aligned} \quad (35)$$

where M is given in Assumption 4.

Whilst sliding, $e_y = 0$ and $\dot{e}_y = 0$, and thus from (35)

$$\begin{aligned} 0 = \ & M A_3 e_1 + M \left(G_2(T^{-1}z, u) - G_2(T^{-1}\hat{z}, u) \right) \\ & + \begin{bmatrix} H_1 & H_2 \\ 0 & H_3 \end{bmatrix} \begin{bmatrix} \Delta\Psi(T^{-1}z, t) \\ f(u, t) \end{bmatrix} - M\nu_{eq} \end{aligned} \quad (36)$$

where ν_{eq} is the equivalent output error injection signal which represents the average behaviour of the discontinuous function ν defined by (24), which keeps the motion on the sliding surface. It follows that

$$\left\|G_2(T^{-1}z,u) - G_2(T^{-1}\hat{z},u)\right\| \leq \mathcal{L}_{G_2}\|T^{-1}\|\,\|e_1\| \quad (37)$$

The analysis in section 3 has shown that $\lim_{t\to\infty} e_1 = 0$. Therefore from (36) and (37)

$$M\nu_{eq} \rightarrow \begin{bmatrix} H_1 & H_2 \\ 0 & H_3 \end{bmatrix} \begin{bmatrix} \Delta\Psi(T^{-1}z,t) \\ f(u,t) \end{bmatrix}, \qquad (t\rightarrow\infty)$$

This implies that

$$f(u,t) \rightarrow H_3^{-1}M_2\nu_{eq}, \qquad (t\rightarrow\infty) \quad (38)$$

where M_2 denotes the last q rows of M.

In order to reconstruct the actuator fault, it is required to recover the equivalent output error injection signal ν_{eq}. In the work described in [14], ν_{eq} was obtained using a low-pass filter. Here an approach given by Edwards et al [3] will be employed. From (24), the equivalent output error injection signal in (38) can be described by

$$\nu_{eq} = \Big(\|E_2\|\xi(T^{-1}\hat{z},t) + \|A_4 - A_3P_1^{-1}P_2\|\,\|e_y\| + \|D_2\|\rho(u,t) + k\Big)\zeta(e_y) \quad (39)$$

where $\hat{z} = \text{col}(\hat{z}_1, y)$ and $\zeta(\cdot)$ is defined by

$$\zeta(e_y) = \frac{e_y}{\|e_y\| + \delta_1\exp\{-\delta_2 t\}} \quad (40)$$

where δ_1 and δ_2 are positive constants: the former is normally small and the latter large.

The analysis above shows that the fault $f(u,t)$ can be reconstructed by

$$f(u,t) \approx H_3^{-1}M_2\nu_{eq} \quad (41)$$

where ν_{eq} is defined by (39), M_2 is the last q rows of M, and M and H_3 are both given in Assumption 4.

V. SIMULATION

Consider the simplified dynamics of the HIRM aircraft at the trim values Mach: 0.8, Height: 5000ft (see [17]):

$$\mathcal{A} = \begin{bmatrix} -0.0318 & 0.0831 & -0.0008 & -0.0367 \\ -0.0716 & -1.4850 & 0.9848 & 0 \\ -0.2797 & -5.6725 & -1.0253 & 0 \\ 0 & 0 & 1.0000 & 0 \end{bmatrix} \quad (42)$$

$$\mathcal{B} = \begin{bmatrix} 0.0120 & -0.0071 \\ -0.3058 & -0.0223 \\ -22.4293 & 7.8777 \\ 0 & 0 \end{bmatrix} \quad (43)$$

$$\mathcal{G}(x,u) = \mathcal{B}u + \begin{bmatrix} 0 \\ \frac{F_e}{M}(\sin\bar{x}_2)/(1+\bar{x}_1) \\ 0 \\ 0 \end{bmatrix} \quad (44)$$

$$\mathcal{C} = [\,I_3 \quad 0\,] \quad (45)$$

where the parameters F_e and M are the engine thrust and the aircraft mass respectively. This system has four states $\bar{x} = \text{col}(\bar{x}_1, \bar{x}_2, \bar{x}_3, \bar{x}_4)$: $(v - v_0)/v_0$, with v and $v_0 = 267.51$ respectively the current airspeed (m/s) and the desired airspeed (m/s); angle of attack (rad); pitch rate (rad/s) and pitch angle (rad). The two inputs $u = \text{col}(u_1, u_2)$ represent symmetrical tailplane deflection (rad) and symmetrical canard deflection (rad). The three outputs $y = \text{col}(y_1, y_2, y_3)$ are $(v - v_0)/v_0$, angle of attack (rad) and pitch rate (rad/s). It is assumed that the system suffers from uncertainty of the form

$$\Delta\phi(\bar{x},t) = [0 \ 2.0275\Delta\psi(\bar{x},t) \ 10\Delta\psi(\bar{x},t) \ \Delta\psi(\bar{x},t)]^T$$

where $|\Delta\psi(\bar{x},t)| \leq 0.001\|y\|\sin^2\bar{x}_4$ which is caused by the aerodynamic drag and the modelling error from the lift term. As in the work described in [17], the system is considered in the domain

$$\Omega = \left\{\bar{x} \,\middle|\, |\,\bar{x}_1\,| < 0.18,\ |\,\bar{x}_2\,|, |\,\bar{x}_3\,| < 0.17,\ |\,\bar{x}_4\,| < 0.52\right\}$$

In the following simulations, for demonstration purposes, a linear state feedback controller has been used to assign closed-loop system poles at $\{-4.5, -4, -3, -2.5\}$. In this example any actuator fault is assumed to occur in the input channel and thus the fault distribution matrix is assumed to be $\mathcal{D} = \mathcal{B}$. Let

$$T_c = \begin{bmatrix} 0 & I_3 \\ 1 & 0 \end{bmatrix}$$

Under the coordinate transformation $x = T_c^{-1}\bar{x}$, the system (42)–(45) in the form of (8)–(10) is as follows

$$A = \left[\begin{array}{c|ccc} 0 & 0 & 0 & 1.0000 \\ \hline -0.0367 & -0.0318 & 0.0831 & -0.0008 \\ 0 & -0.0716 & -1.4850 & 0.9848 \\ 0 & -0.2797 & -5.6725 & -1.0253 \end{array}\right]$$

$$G = \left[\begin{array}{c} 0 \\ \hline 0.0120u_1 - 0.0071u_2 \\ -0.3058u_1 - 0.0223u_2 + \frac{F_e}{M}(\sin x_3)/(1+x_2) \\ -22.4293u_1 + 7.8777u_2 \end{array}\right]$$

$$E = \left[\begin{array}{c} 1 \\ \hline 0 \\ 2.0275 \\ 10 \end{array}\right] \quad D = \left[\begin{array}{cc} 0 & 0 \\ \hline 0.0120 & -0.0071 \\ -0.3058 & -0.0223 \\ -22.4293 & 7.8777 \end{array}\right]$$

and

$$C = [\,0 \quad I_3\,]$$

Choose

$$L = \begin{bmatrix} -27.2784 & 0.9086 & 0.9544 \\ 1.9682 & 0.0609 & 0.0000 \\ -0.0716 & 0.0150 & 0.9848 \\ -0.2797 & -5.6725 & 0.9747 \end{bmatrix}$$

Then for

$$Q = \begin{bmatrix} -1.0000 & -27.2968 & 0.6058 & -0.0228 \\ -27.2968 & -746.1127 & 16.5354 & -0.6229 \\ 0.6058 & 16.5354 & -1.3669 & 0.0138 \\ -0.0228 & -0.6229 & 0.0138 & -1.0005 \end{bmatrix} > 0$$

the Lyapunov equation (4) has a unique solution

$$P = \left[\begin{array}{c|ccc} 0.5000 & 13.6392 & -0.3029 & 0.0114 \\ \hline 13.6392 & 372.5563 & -8.2621 & 0.3113 \\ -0.3029 & -8.2621 & 0.5168 & -0.0069 \\ 0.0114 & 0.3113 & -0.0069 & 0.2503 \end{array}\right]$$

Let

$$F_1 = [0 \quad 0.6758 \quad 2.5], \quad \xi = 0.001\|y\| \sin^2 x_1$$

and

$$F_2 = \begin{bmatrix} 0.0060 & -0.1019 & -5.6073 \\ -0.0035 & -0.0074 & 1.9694 \end{bmatrix}$$

By direct computation, it can be shown that Assumptions 1-3 and Propositions 1-2 are all satisfied in Ω. Therefore, the observer (25)–(26) is well-defined. Finally, let

$$M = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & -0.2028 \\ 1 & 0 & 0 \end{bmatrix}$$

Then it can be shown that Assumption 4 is satisfied with

$$H_3 = \begin{bmatrix} 4.2418 & -1.6196 \\ 0.0120 & -0.0071 \end{bmatrix}$$

From (39), ν_{eq} can be computed directly online, and thus the actuator fault $f(u,t)$ can be reconstructed from (41) where $\delta_1 = 0.001$ and $\delta_2 = 1$. For the fault signals given in Figure 1, the reconstruction signals in Figure 2 show that the developed result is very effective and indeed visually Figure 1 and Figure 2 are identical. The simulations show that the fault signals can be reconstructed faithfully using the proposed scheme.

VI. Conclusion

An approach for robust actuator fault detection and isolation for a class of nonlinear uncertain systems has been proposed based on a sliding mode observer. Unlike the associated existing work where only an estimation of the fault is generated, the fault can be precisely reconstructed even in the presence of uncertainties by using sliding mode techniques. The reconstructed signal is obtained on-line and thus it is easy to implement in a real system. A simulation based on a HIRM aircraft model has demonstrated its effectiveness in achieving robust fault detection and isolation.

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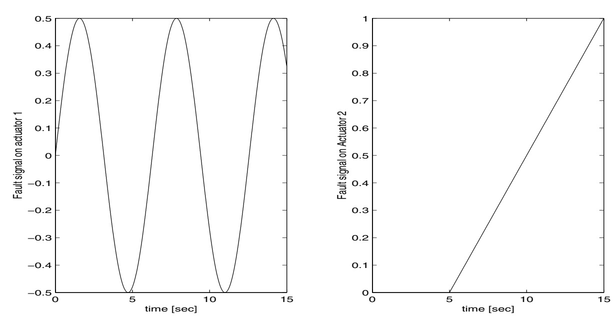

Fig. 1. The fault signals

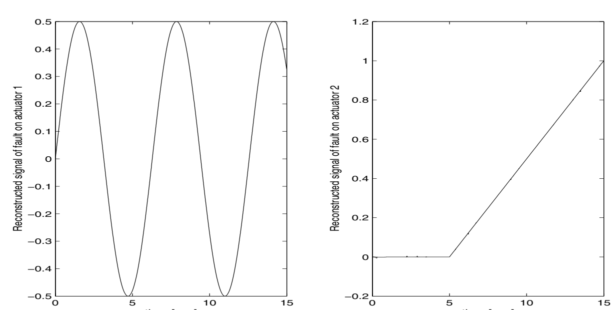

Fig. 2. The reconstructed faults