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Identification and Control of an Open-flow Canal using LPV Models

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Abstract— **In this paper, the identification and control of open-flow canals using linear parameter varying (LPV) models is proposed. A single reach open-flow canal has been modeled with an LPV first-order plus delay model relating upstream and downstream flows. The parameters of such model depend on the operating point and have been estimated using a parameter estimation algorithm for LPV models. The control is based on a gain-scheduling PI controller with a delay-scheduling Smith Predictor. Finally, the proposed LPV model, identification and control algorithms have been applied to a simulated open-canal: the Lunax gallery located at Gascogne (France).**

I. INTRODUCTION

OPEN-flow canals involve mass energy transport phenomena which behave as intrinsically distributed parameter systems. Their complete dynamics is represented by non-linear partial differential hyperbolic equations (PDE) that are function of time as well as of spatial coordinates: Saint-Venant's equations. This equation system has no known analytical solution in real geometry and it has to be solved numerically (characteristic method, Preissmann implicit scheme, etc.) [10]. The resulting simulation models are therefore suitable for scientific and time-consuming simulations but are too complex for on-line applications and control needs. Distributed parameter systems, considered as systems with a very large number of states could be approximated with low order linear time invariant (LTI) models in order to use classical linear control design tools, as it is usual in control engineering practice. However, simplified LTI parameter models lose any information about the spatial structure of the original system and cannot account for it although they can be satisfactory from an input-output point of view [6]. Some simplified model structures that still preserve some information about the non-linearity, the influence of the operating point and the spatial structure of system are needed. Such structure can be provided by linear parameter varying (LPV) models consisting of a linear lumped parameter model in which the system parameters are function of external parameters, or by quasi-LPV models when the parameters vary according to states and/or operating conditions of the system. Therefore, since a reach is a non-linear and distributed parameter system, an LPV control oriented model and its associated identification procedure will be proposed being this the first contribution of the paper.

LPV models are suitable for gain-scheduling (GS) control design. Using this approach, controllers for multiple industrial applications have been designed using conventional gain-scheduling (GS) control techniques [13] [16]. The conventional GS is an heuristic method that consists in dividing the parameter space into small regions (operating points), in which the plant is regarded as a LTI system and LTI controllers are designed for each of the fixed parameters to achieve a synthetic controller with the use of interpolation or other techniques as switching techniques [14] or fuzzy control [11] [19]. The heuristic gain scheduled controllers normally guarantee control system stability when the parameters are in a slow variation [16] but sometimes may lead to instability or chaotic behaviour [13]. In fact, the main known drawback of this technique is that it does not guarantee the stability and the performance rigorously [8] and does not provide a systematic design procedure. Latter, in [18], LPV systems are introduced. In this context, the synthesis problem can be formulated as a convex optimisation problem with linear matrix inequality (LMI) constraints wherein the controller is considered as a simple entity without the classical interpolation drawbacks [2] [5]. In this paper, using the obtained LPV model for a single reach canal, a conventional gain-scheduling controller coupled with a delay scheduling Smith Predictor will be designed and tested in real application. This is the second contribution of this paper. The rest of paper is organized as follows. In *Section II*, the motivation for using an LPV model in controlling open-flow canals is introduced. *Section III* presents a method for the identification of LPV models from input/output data. In *Section IV*, a gain-scheduling PI controller based on the proposed LPV models is designed. In

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Section V, the application of the proposed control law based on the LPV model to the control of the Lunax Gallery in France. Finally, in *Section VI* the main conclusions are presented.

II. LPV Model For a Single Reach Canal

A. Physical and control-oriented models

Given a single reach canal, the Saint-Venant equations describe accurately the dynamics in a one-dimensional free surface flow. They express the conservation of mass (1) and momentum principles (2) in a one-dimensional free surface flow:

$$\frac{\partial Q}{\partial x}+\frac{\partial A}{\partial t}=0 \qquad (1)$$

$$\frac{\partial Q}{\partial t}+\frac{\partial}{\partial x}\left(\frac{Q^2}{A}\right)+gA\frac{\partial y}{\partial x}-gA\left(I_0-I_f\right)=0 \qquad (2)$$

where $Q=Q(x,t)$ is the flow [m^3/s], $A=A(x,t)$ is the cross-sectional area [m^2], t is the time variable [s], x is the spatial variable [m], measured in the direction and the sense of the movement, g is the gravity [m/s^2], I_0 is the bottom slope, I_f is the friction slope. This pair of partial-differential equations constitutes a non-linear and hyperbolic system [2,3], that for an arbitrary geometry lacks of analytical solution. However, such equations are not useful for designing a controller.

B. LPV model motivation

Alternatively, a simplified model for control relating the upstream and downstream flow will be proposed. Applying a set of step inputs corresponding to different upstream flows and analyzing how the downstream flow changes it can be observed that:

- The system dynamics is different at each operating point.
- The dynamics of hydraulic wave can be approximated by a first order plus time delay at each operating point.
- The delay τ that corresponds to the time that the wave takes to arrive at downstream end after a gate opening varies according to the operating conditions.

From these observations, a quasi-LPV model with a first order differential equation with time delay (FOPTD) [9] seems to be suitable to describe canal dynamics at different operating points depending on gate position

$$Q_{dns}(t)+T\frac{dQ_{dns}(t)}{dt}=Q_{ups}(t-\tau) \qquad (3)$$

or equivalently:

$$Q_{dns}(s)=\frac{e^{-\tau s}}{Ts+1}Q_{ups}(s) \qquad (4)$$

where Q_{dns} and Q_{ups} are the downstream and upstream flow rate and s is the Laplace variable. All parameter of this equation: delay $\tau(p)$ and time constant $T(p)$ depend on the operating point p that will be described by measuring the upstream flow Q_{ups}.

Since such model is intended to be used to design a digital controller, it will be used in discrete-time. Using the zero-order hold discretisation method, the following discrete-time model is obtained

$$G(z,p)=\frac{b_o(p)}{z+a_1(p)}z^{-d(p)} \qquad (5)$$

where: $d(p)=\left\lceil\frac{\tau(p)}{T_s}\right\rceil$ is the delay in samples, $b_o(p)=1-e^{-\frac{T_s}{T(p)}}$ and $a_1(p)=-e^{-\frac{T_s}{T(p)}}$.

III. LPV Identification Algorithm

In order to estimate the parameters of model (5), the method proposed by Giarré (2002) is used. Such a method makes possible to estimate a discrete-time LPV model parameterized as follows:

$$A(q,p)y(k)=B(q,p)u(k) \quad (6)$$

where $y(k)$ is the output without delay, $u(k)$ is the input, q^{-1} is the delay operator and $p(k)$ is the operating point

$$B(q,p)=b_0(p)+b_1(p)q^{-1}+\ldots+b_{n_b}(p)q^{-n_b}$$
$$A(q,p)=1+a_1(p)q^{-1}+\ldots+a_{n_a}(p)q^{-n_a} \qquad (7)$$

where: $n=n_a+n_b+1$ is the number of parametric functions to be identified that are considered to be polynomials in $p(k)$ of order $N\text{-}1$, expressed as

$$a_i(p)=a_{i1}+a_{i2}p+\cdots+a_{iN}p^{N-1}$$
$$b_i(p)=b_{i1}+b_{i2}p+\cdots+b_{iN}p^{N-1} \qquad (8)$$

The parameter estimation problem can be easily formulated in linear regression form. According to Giarré (2002), least squares algorithm can be applied to this problem by introducing an extended regressor Ψ_k defined by

$$\Psi_k := \phi_k \pi_k = \begin{bmatrix} -y_{k-1} \\ \vdots \\ -y_{k-n_a} \\ u_k \\ \vdots \\ u_{k-n_b} \end{bmatrix} \begin{bmatrix} 1 & p_k & \cdots & p_k^{N-1} \end{bmatrix} \qquad (9)$$

Then,

$$y_k = \langle \Theta, \Psi_k \rangle \qquad (10)$$

where: $\langle A,B \rangle := trace(A^* B)$ is the inner product of the matrices A and B of equal dimensions, A^* is the complex conjugate transpose of the matrix A and the matrix Θ contains all the coefficients to be identified

$$\Theta := \begin{bmatrix} a_{11} & \cdots & a_{1N} \\ a_{21} & \cdots & a_{2N} \\ \vdots & \vdots & \vdots \\ a_{na1} & \cdots & a_{naN} \\ b_{01} & \cdots & b_{0N} \\ \vdots & \vdots & \vdots \\ b_{nb1} & \cdots & b_{nbN} \end{bmatrix} \qquad (11)$$

IV. CONTROL USING THE PROPOSED LPV MODEL

A. Gain-scheduling using LPV models

A straightforward way of using the LPV model proposed in this paper for control design is through ***gain-scheduling*** techniques (Åström and Wittenmark, 1989). As in this case it is known how the dynamics of the reach canal changes with the operating point, it is possible to change the parameters of the controller accordingly by monitoring such operating point described by a set of variables named as ***scheduling variables***. This idea is called gain scheduling since the scheme was originally used to accommodate changes in process gain only. Gain scheduling is a nonlinear feedback of special type because the parameters of a linear controller are changed as a function of operating conditions in a preprogrammed way (see Fig. 1). A main problem in the design of gain scheduling controllers is to find suitable scheduling variables. This is normally done based by knowledge of the system physics. The scheduling technique has a long history in control design (Shamma *et al.*, 1990) and has evolved along the time. Nowadays, there are mainly three types of approaches to design a gain scheduling control: classical GS, LPV GS and fuzzy GS. The first method is empirical, the second method is systematic and rigorous and the third is a combination of empirical knowledge and systematic procedure.

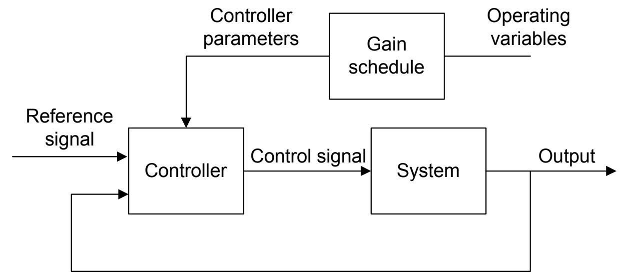

Figure.1. Gain-scheduling control

B. Design of a classical gain-scheduling PI + Smith Predictor

Recalling that the dynamics of the reach canal is modeled by FOPTD structure given by (3) a kind of "*delay scheduling*" Smith Predictor (Fig.2) updated using estimated delay given by *d(p)* is used to compensate the canal delay. This allow to design the GS PI controller without considering the delay as in the usual Smith Predictor scheme, then canal model without the delay is denoted by

$$G_m(z,p) = \frac{B(z,p)}{A(z,p)} = \frac{b_0(p)}{z + a_1(p)} \qquad (12)$$

with b_0 and a_1 are defined in (5) and where only one varying parameter (time constant) is considered in the design GS PI controller.

Considering that the GS PI controller to be tuned has the following form

$$G_{PI}(z,p) = \frac{c_0(p)z + c_1(p)}{z-1} = \frac{S(z,p)}{R(z,p)} \qquad (13)$$

the control system closed-loop transfer function is given by

$$\begin{aligned} G_{CL}(z,p) &= \frac{G_m(z,p)}{1 + G_{PI}(z,p)G_m(z,p)} \\ &= \frac{B(z,p)}{A(z,p)R(z,p) + B(z,p)S(z,p)} \end{aligned} \qquad (14)$$

Then, using the pole placement method (Landau, 1990) (Åström and Hägglund, 1995), the desired closed loop performance can be established by

$$A(z,p)R(z,p) + B(z,p)S(z,p) = (z + x_{cl})^2 \qquad (15)$$

where x_{cl} is a double pole of the closed loop to give a critical damping response ($\xi = 1$) without any overshoot and with a settling time of $t_{scl} = -4T_s / ln(-x_{cl})$. The design parameter x_{cl} has been chosen in order to achieve a closed loop response equal to the open loop response (5):

$t_{sol} = -4T_s / ln(-a_1)$. Thus $x_{cl} = a_1$. Moreover the steady-state error is zero in case of a step set-point/disturbance since the inclusion of an integrator in the controller. From equations (12), (13) and (15) the parameters of the GS PI controller are:

$$c_0 = \frac{1 + a_1(p)}{b_0(p)} \quad (16)$$

$$c_1 = \frac{a_1(p)^2 + a_1(p)}{b_0(p)} \quad (17)$$

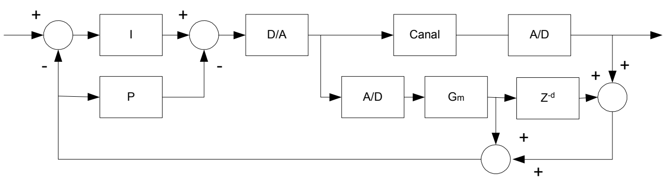

Figure 2. "Delay scheduling" Smith predictor scheme.

V. Application

A. Lunax gallery

The Lunax gallery is located at Gascogne in southwestern region of France. The dam-gallery is used to supply with water the river Gesse. As depicted in Figure 3, the plant consists in a single reach canal, the dam gate controls the upstream flow. The control objective aims at regulating the downstream flow even if the measurement point is far from the gate, at the output of the gallery.

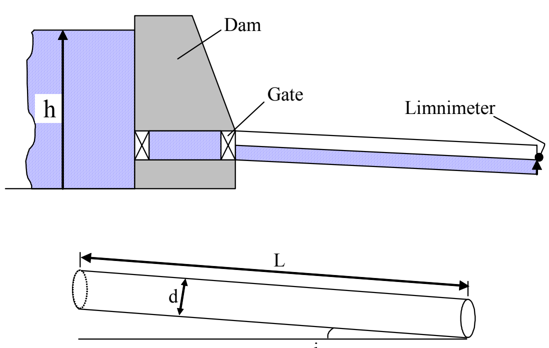

Figure 3. Lunax gallery.

The geometry of the gallery is circular. Geometrical data are displayed in Table 1.

Diameter d [m]	1.8
Length L [m]	946.65
Slope i [rad]	0.0026
Strickler coefficient	65

Table 1. Geometrical data of the Lunax gallery

The Simulation of Irrigation Canals (SIC) software is a mathematical model which can simulate the hydraulic behaviour of most of the irrigation canals or rivers, under steady and unsteady flow conditions [21]. Steady flow and unsteady flow computations can be performed on any type of hydraulic networks (linear, looped or branched). Any reach can be composed of a minor, a medium and a major bed. Storage pools can also be modelled. Only sub-critical flows can be modelled in unsteady flow, except at cross devices (gates, weirs). The model is built around three main computer programs that respectively carry out the topography and geometry generation, the steady flow computation and the unsteady flow computation. The evaluation of the control strategy is carried out by using Matlab/Simulink software communicating to the SIC software by a dynamic data exchange link. The SIC solver is based on a finite difference approximation of Saint-Venant equations by Preissmann's scheme. The downstream limit conditions are given by the limnimeter calibration data.

B. Model identification

A first order LPV model plus delay in discrete-time as those given by Eq. (5) is used to model the gallery. In order to apply the LPV identification method presented in *Section III*, model parameters are assumed to depend on the operating point *p(k)* described by the upstream flow through second order polynomials:

$$\begin{aligned} a_1(p) &= a_{11} + a_{12}p + a_{13}p^2 \\ b_0(p) &= b_{01} + b_{02}p + b_{03}p^2 \\ d(p) &= round(d_1 + d_2 p + d_3 p^2) \end{aligned} \quad (18)$$

The delay is estimated at each operating point using a set of step inputs (Fig. 4) and interpolated between operating points using a second order polynomial (Fig. 5). The parameters of this polynomial are presented in Table 2. Once the delay is identified, it is removed from the data in order to estimate the rest of parameters applying the identification algorithm described in *Section III*. As a result, two polynomials describing how parameters $a_1(k)$ and $b_0(k)$ evolve with the operating are obtained (Fig. 6). The parameters of these polynomials are also shown in Table 2.

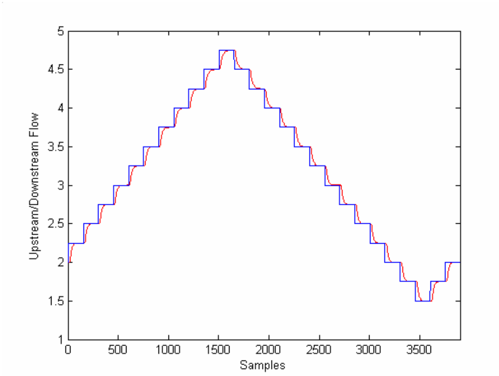

Figure 4. Set of step inputs used for delay and parameter estimation

i	0	1	2
a_{1i}	-0.9415	-0.0007	-0.0004
b_{0i}	0.0585	-0.0007	-0.0004
d_i	24.1072	-4.0721	0.2637

Table 2. Coefficients of the estimated LPV model.

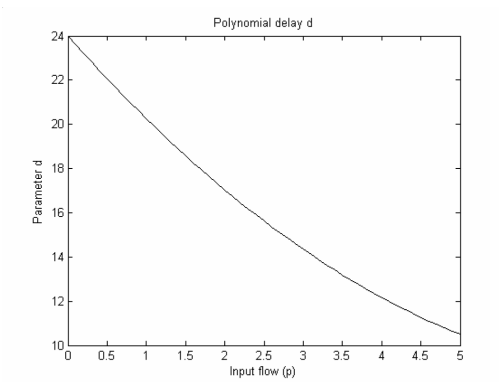

Figure 5. Polynomial for delay versus upstream flow

C. Control results

Figure 8 shows the behavior of the I-P Smith Predictor LPV controller. The first part of the figure presents the noise added to the output of the Lunax canal simulated non linear model and the second part of the figure shows an exponential signal for the set point in continuous line and the process variable in discontinuous line and in the third part the manipulated variable is drawn. The results are coherent with the specifications of the controller, any overshoot and the time response corresponds to approximately to the dynamic of assigned poles with the LPV model. Figure 9 shows the adaptation of the controller to different operating points keeping the design specifications.

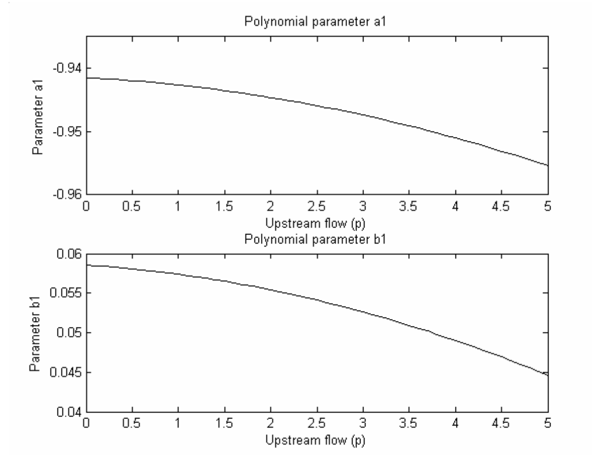

Figure 6. Polynomial for model parameters versus upstream flow

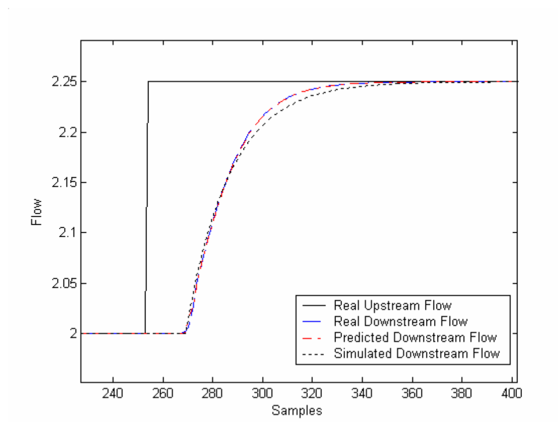

Figure 7. Estimated response provided by the LPV in simulation and prediction versus real response

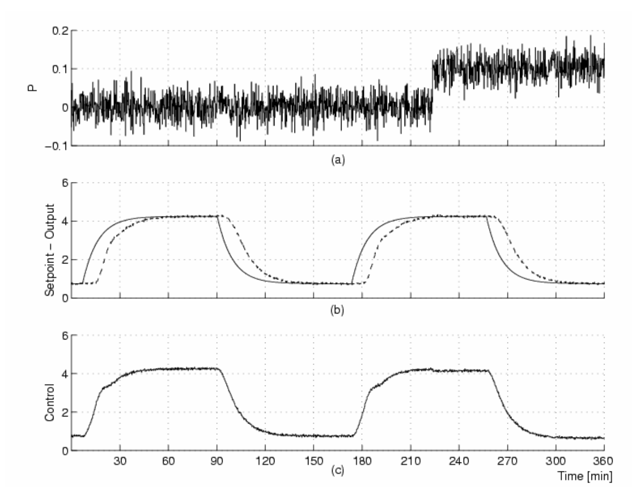

Figure 8. Validation of control specifications

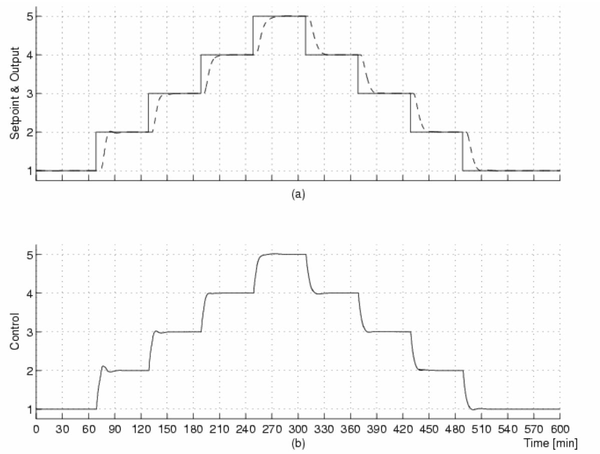

Figure 9. Controller adaptation to different operating points

VI. Conclusion

In this paper, a control oriented LPV model for a single reach open flow canal has been proposed. It has been modeled with an LPV first-order plus delay model relating upstream and downstream flows. The parameters of such model depend on the operating point and have been estimated using a parameter estimation algorithm for LPV models. The control is based on a gain-scheduling I-P controller with a delay-scheduling Smith Predictor. Finally, the proposed LPV model, identification and control algorithms have been successfully applied to a simulated open-canal corresponding to the Lunax gallery using the Saint's Vennant equations. As further research the controller will be redesigned using LPV control theory in order to guarantee robust stability and performance [2].

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