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Differential Game based Safe Controller Design for Intelligent Cruise Control

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***Abstract*— The paper presents an approach to safe controller design for intelligent cruise control applications using differential game theory. The design problem is formulated as a zero-sum differential game, where the objective of the lead vehicle is to provoke a collision and the objective of the following vehicle is to prevent a collision from happening regardless of the motion of the leader. A control strategy is safe if it can guarantee that no collision will occur between vehicles. The approach is applicable to Adaptive Cruise Control (ACC) and Cooperative ACC systems.**

I. INTRODUCTION

Adaptive Cruise Control (ACC), an expansion of the conventional cruise control, is a driver assistance system designed to provide ameliorated efficiency, convenience and safety. ACC distinguishes itself from cruise control in its use of sensors that measure the headway distance and a controller which adjusts the velocity and distance to the vehicle in front. When there is no "leader" vehicle present, ACC defaults to conventional cruise control and reverts to the driver set speed. A variant of ACC is Cooperative ACC (CACC), where the forward-looking sensor is complemented by a wireless communication link that provides hop-by-hop, leader to follower updates of critical information. Such a system can be designed to follow vehicles with higher accuracy and faster response than traditional ACC systems, and should allow for freeway throughput capacity increases. ACC and CACC are areas of active research, and different methodologies for controller design can be found in [1], [2], [3], [4], [5], [6], [7], [8]. Recently, with the prospect of increasing levels of automation and of a lesser role played by the human driver, safety guarantees become of paramount importance, and ACC and CACC make safe controller design more practicable.

In this paper we are concerned with the design of safe control laws for ACC and CACC applications. Our approach proposes two nested controllers. The inner controller consists of an ACC or CACC control law. The outer controller may override the operation of the inner controller to ensure safety. This approach provides safety guarantees as well as the flexibility to select any approach that does not violate the safety conditions. The safety conditions are propagated through the vehicle dynamics to derive the conditions under which the outer controller overrides the operation of the inner controller. In the remainder of the paper we will be concerned with the design of the outer controller.

Theoretic work related to our safe controller design for hybrid systems can be found in [10], [11], [12]. The authors in [11], [12] consider a two-vehicle (lead and follower vehicles) merge maneuver with the following specifications. Denote the inter-vehicle displacement Δx, the lead vehicle velocity v_l, the follower vehicle velocity v_f, the relative velocity $\Delta v = \Delta \dot{x}$ and the maximum recommended velocity for a vehicle to travel on the highway v_{max}. The objective is to design an efficient and comfortable control law that decreases the initial relative displacement $\Delta x(0)$ to a desired inter-vehicle spacing Δx_{join} subject to the safety constraint

$$\forall t : \Delta x(t) \leq 0 \Rightarrow \Delta v(t) \geq -\bar{v} \text{ and } v_l > 0 \quad (1)$$

The quantity $\bar{v}$ is the maximum tolerated impact velocity in case of a collision. The following theorem, from [11], states the existence of safe control laws and provides a class of maximum braking safe control laws.

Theorem 1: Suppose that the acceleration of each vehicle is bounded by $[-\underline{a}, \bar{a}]$ and that the maximum deceleration $-\underline{a}$ is achieved and maintained at most d seconds after a maximum braking command is issued. Let X_{MS}, X_{safe} and X_{bound} denote the set of all triples $X = (\Delta x, \Delta v, v_l)$ with $v_l \geq 0$ that satisfy (1), (2) and (3) respectively:

$$\Delta v \geq Ad - \max\{\sqrt{2\underline{a}\Delta x + {v_l}^2 + \bar{v}^2 + \underline{a}Ad^2} - v_l, \bar{v}\} \quad (2)$$

$$\Delta v \geq \max\{\sqrt{2\underline{a}\Delta x + {v_l}^2 + \bar{v}^2} - v_l, \bar{v}\} \quad (3)$$

where $A = \underline{a} + \bar{a}$. Then, provided that $X(t) \in X_{safe}$:
1. There exists a control law that is safe for all times $s > t$. i.e., $\forall s > t$, $X(s) \in X_{MS}$.
2. Any control law that applies maximum braking whenever $X(t) \notin X_{safe}$ is safe for all times $s > t$. Furthermore, $\forall s > t$, $X(s) \in X_{bound}$.
3. $X_{safe} \subset X_{bound} \subset X_{MS}$.

Observe that the maximum braking safe control law naturally defines a class of two-state hybrid automaton. In the *Max Braking* state Theorem 1 asserts that $X \in X_{bound}$ provided that maximum braking is applied. In the *Control* state we impose the invariant $X \in X_{safe}$ and do not put any restrictions on the control law. The conditions of Theorem 1 are satisfied as soon as we deploy the transition from *Control* to *Max Braking* which is specified to occur as soon as $X \notin X_{safe}$. See also [10] where a three-state regulation layer controller is designed as a hybrid automaton and its safety properties are verified using game-theoretic methods.

Our approach also uses game-theoretic models. It is based on the control framework for differential games described in [9]: we use the "theorem of the alternative" to derive the theoretic foundations of our design; and a construction method for safe sets (also called stable bridges in this framework) to construct safe sets for intelligent cruise control applications. We use a simple dynamic model to illustrate the approach. The advantage of this construction method is that it can be used with more general non-linear models under mild assumptions for cruise control applications, namely that the "small-game condition" is satisfied [9]. This is usually the case since the disturbance and the control terms are additive.

The paper is organized as follows. In section II we introduce the problem statement and the underlying assumptions. In section III we describe our approach. In section IV we draw some conclusions.

II. Problem statement

Consider two vehicles (V_1 and V_2 respectively) moving on a single-lane highway. V_1 travels in front of V_2 as depicted in Fig. 1.

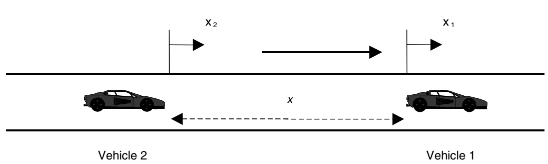


Fig. 1. Problem setup: two vehicles run on a single-lane highway.

The dynamics of both vehicles are as follows ($i = 1, 2$):

$$\dot{x_i}(t) = f_i(t, x_i, u_i), \qquad u_i \in [u_{min_i}, u_{max_i}] \tag{4}$$

where

- $f_i : \Re \times \Re^2 \times \Re \rightarrow \Re^2$ is a continuous function in all of its arguments.
- $u_{min_i} < 0$ and $u_{max_i} > 0$ represent, respectively, the maximum braking and acceleration capabilities of V_i.

With respect to the notation in Fig. 1 we redefine x_1 to designate the position of the rear point of V_1 (v_1 is the corresponding velocity); x_2 and v_2 designate the position and velocity of the front point of V_2.

We consider a simple model to illustrate our approach. However, the approach is applicable to more general non-linear models.

$$\begin{aligned} \dot{v_i}(t) &= \left\{ \begin{array}{ll} u_i & \text{if } v_i \geq 0 \\ 0 & \text{otherwise} \end{array} \right., & v_i(0) &= v_{i_0} \\ \dot{x_i}(t) &= \qquad v_i, & x_i(0) &= x_{i_0} \end{aligned} \tag{5}$$

In this model we consider that negative accelerations can only result from braking, i.e., vehicles cannot move backward. Hence, the hybrid model (5) with speed-dependent modes of operation.

We are interested in the relative position x and velocity v of both vehicles defined as follows:

$$\begin{aligned} v(t) &= v_1(t) - v_2(t) \\ x(t) &= x_1(t) - x_2(t) \end{aligned} \tag{6}$$

The corresponding dynamics are given by:

$$\begin{aligned} \dot{v}(t) &= u_1 - u_2, & v(0) &= v_0 \\ \dot{x}(t) &= v_1 - v_2, & x(0) &= x_0 \end{aligned} \tag{7}$$

We study the problem of synthesizing collision avoidance strategies for V_2. The setting is as follows:

- The controller of V_2 does not know u_1, the control selection from V_1.
- V_2 is capable of measuring its relative position and velocity with respect to V_1.

To formalize the specification for the collision avoidance problem define the collision set as:

$$C := \{(x, v_1, v_2) \in \Re^3 : x < 0 \wedge (v_1 - v_2) < 0, \\ v_1 \geq 0, v_2 \geq 0\}$$

This specification considers as safe the situation where both vehicles are in contact at zero relative speed.

Definition 1 (Safe set): The safe set S is the set of all initial relative positions x and velocities v_1 and v_2 such that there exists a controller for V_2 ensuring that the collision set C is never attained by the motions of both vehicles.

In order to model the fact that the V_2 controller does not know the current control setting u_1, we formulate this problem as a zero-sum differential game: the objective of V_1 is to provoke a collision; and the objective of V_2 is to prevent it. Stated otherwise, the objective of the V_1 controller is to steer the state of the system to enter C; the objective of the V_2 controller is to keep this state outside C. More formally, consider the following cost functional:

$$\gamma(x(x_0, v_{10}, v_{20}, U, V)) = \left\{ \begin{array}{ll} 1 & \text{if } x(\cdot) \text{ intersects } C \\ 0 & \text{otherwise} \end{array} \right.$$

where

- $U(\cdot)$ and $V(\cdot)$ are control functions respectively for V_2 and V_1.
- $x(\cdot)$ is the trajectory of the system starting at (x_0, v_{10}, v_{20}) under control functions $U(\cdot)$ and $V(\cdot)$.

The adversarial aspect of control is captured in an optimization problem where $U(\cdot)$ seeks to minimize γ and $V(\cdot)$ seeks to maximize it. Now consider the following expression.

$$V(x_0, v_{10}, v_{20}) = \inf_U \sup_V \gamma(x(x_0, v_{10}, v_{20}, U, V)) \tag{8}$$

In general if we switch the inf and the sup in an expression such as (8) we obtain different results. In the terminology of differential games, the game is said to have a value if the following condition is satisfied.

$$\inf_U \sup_V \gamma(x(x_0, v_{10}, v_{20}, U, V)) = \sup_V \inf_U \gamma(x(x_0, v_{10}, v_{20}, U, V))$$

The game has a saddle point, if there exist strategies U^*, V^* such that:

$$\gamma(x(x_0, v_{10}, v_{20}, U^*, V)) \leq \gamma(x(x_0, v_{10}, v_{20}, U^*, V^*)) \leq \gamma(x(x_0, v_{10}, v_{20}, U, V^*))$$

If the game has a value our design problem is as follows.

Problem 1 (Design): Solve the following sub-problems:

- Determine the value function V.
- Determine S from the 0-level set of the value function V (i.e. the set of all points where $V(x_0, v_{10}, v_{20}) = 0$).
- Derive an optimal safe controller for V_2.

We explain the problem setup in more detail in the following section.

III. Approach

Here, we are concerned with the computation of the safe set. Generally, this is a difficult task. However, when the equations of motion are linear and separable in the controls there is a constructive procedure to determine this safe set (see section 2.5 of [9]). We adapt this construction to our problem.

A. Background

Consider the motion of point $x \in \Re$ described by the following equation with two adversarial control inputs (u, v):

$$\dot{x} = f(t, x, u, v), u \in P, v \in Q \tag{9}$$

where the following hypotheses hold

H1) f is continuous in all variables and $t \in T = (-\infty, \theta]$.

H2) For any bounded region D in $\Re \times \Re$, f satisfies the following Lipschitz condition:

$$\|f(t, x_1, u, v) - f(t, x_2, u, v)\| \leq \lambda(D)\|x_1 - x_2\|$$

for any $(t, x_i) \in D, (u, v) \in P \times Q$

H3) For any $(t, x, u, v) \in T \times \Re \times P \times Q$ the following inequality, where σ is a constant, is valid:

$$x f(t, x, u, v) \leq \sigma(1 + \|x\|^2)$$

H4) For any $(t, x) \in T \times \Re$ and $s \in \Re$ the so-called "saddle point condition in a small game" is valid:

$$\min_{u \in P} \max_{v \in Q} s f(t, x, u, v) = \max_{v \in Q} \min_{u \in P} s f(t, x, u, v)$$

Consider the set M:

$$M = \{(t, x) \in T \times \Re : t = \theta, m_1 \leq x \leq m_2\}$$

The objective of u is to control x, the motion of the system, to enter M at time θ or, equivalently, is to have $x(\theta) \in [m_1, m_2]$. The objective of v is exactly the opposite. More formally, consider the following cost functional (note that this functional is not the same as the one mentioned previously):

$$\gamma(x(t_0, x_0, U, V)) = \begin{cases} 0 & \text{if } x(\cdot) \text{ intersects } M \\ 1 & \text{otherwise} \end{cases}$$

where

- U is a control function for the first player and V is a control function for the second player.
- $\gamma(x(t_0, x_0, U, V)) = 0$ means that the trajectory $x(\cdot)$ of the system departing from (t_0, x_0) under controls $U(\cdot)$ and $V(\cdot)$ enters the target set M at time θ.

Under the hypotheses (H1-4) this game has a value. Moreover, the game has a saddle point, i.e., there exist strategies U^*, V^* such that:

$$\gamma(x(t_0, x_0, U^*, V)) \leq \gamma(x(t_0, x_0, U^*, V^*)) \leq \gamma(x(t_0, x_0, U, V^*))$$

The main result, the "theorem on an alternative" from [9], is stated next:

Theorem 2 (Theorem on an alternative): For any closed set M and for any initial position (t_0, x_0), one and only one of the following assertions is valid:

- The value of the game is 0. Any (U^*, V) is a saddle point (V is any feedback strategy).
- The value of the game is 1.

The optimal strategies (U^* and V^*) are feedback strategies.

This is an important result. It states that we cannot obtain improvements in the performance if we consider additional information (other than the state information) in the optimal strategy.

The notion of u-stable bridge is a very important one in this setting. Informally, a u-stable bridge W is the set of all points (t_0, x_0) such that there exists an optimal strategy U^* that keeps the motions of the system departing from (t_0, x_0) inside W until M is reached.

In this problem setup it is possible to derive a closed form for the u-stable bridge. First set:

$$f_1(t, x) := \max_{u \in P} \min_{v \in Q} f(t, x, u, v) \tag{10}$$

$$f_2(t, x) := \min_{u \in P} \max_{v \in Q} f(t, x, u, v) \tag{11}$$

Consider solutions w_1 and w_2 to the following differential equations:

$$\dot{w_1}(t) = f_1(t, w_1(t)), \qquad w_1(\theta) = m_1 \tag{12}$$

$$\dot{w_2}(t) = f_2(t, w_2(t)), \qquad w_2(\theta) = m_2 \tag{13}$$

The u-stable bridge is the set:

$$W := \{(t, x) \in T \times \Re : t \in T_*, x \in [w_1(t), w_2(t)]\}$$

where

$$T_* = [\tau_*, \theta], \tau_* = sup\{t \in T : w_2(t) > w_1(t)\}$$

Remark 1: This construction has a simple and appealing geometric interpretation (see Fig. 2). Keep in mind that the state evolves in $\Re$. Equations (12,13) describe the evolution (in reverse time) of the boundaries (m_1, m_2) of the target set M when each vehicle (player) adopts the best possible control strategy (given by the argmax and argmin in (10, 11)). Now, consider an initial state (t, x) in the relative interior of W (if the interior is empty this means that the initial state is at the boundary of W). Then, apply any control strategy U until the state reaches the boundary of W. From this time onwards each player applies the control settings given by the argmax and argmin in (10,11). Then, from the construction of w_1 and w_2 we conclude that the state slides along one of the boundaries of W (w_1 or w_2) until it reaches m_1 or m_2, respectively, at time θ.

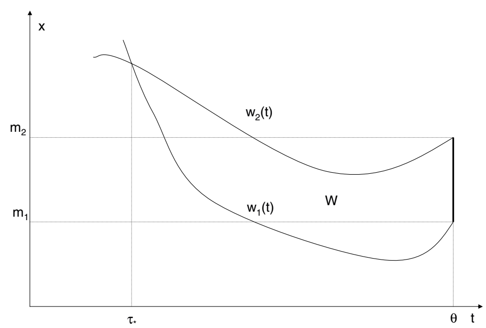


Fig. 2. Construction of Krasovskii u-stable bridge.

Remark 2: A generalization of the method for the case where the state evolves in $\Re^n$ is also described in [9]. In the latter case, equations (10,11) are replaced by equations on the norm of the state.

B. Optimal strategy

The optimal strategy U^* for the following car is expressed in terms of $x(t)$, as shown in [9].

$$U^*(t,x) = \begin{cases} -\underline{a}\dfrac{x(t)}{\|x(t)\|} & \text{for } \|x(t)\| \neq 0 \\ 0 & \text{for } \|x(t)\| = 0 \end{cases} \tag{14}$$

C. Construction of the safe set

Here, we discuss the approach to solve *Problem 1* and describe the construction of the u-stable bridge for this differential game.

Depending on the relative braking capabilities of both vehicles we will get different results for the safe set S. f in (4) does not depend on time. We expect the safe set to depend only on the initial velocities (v_{10}, v_{20}) and relative position x_0 of both vehicles.

Here, we adapt the construction method from the previous section. We have a two-state (x, v) dynamic model. With respect to the previous example, we are just interested in the boundary of the safe set. Moreover, we have to consider the state constraint $(x \geq 0)$ in the differential game formulation. This means that the vehicles are not allowed to collide before coming to a stop.

The construction proceeds backwards in time integrating the equations of motion using, as final condition, the boundary of the collision set. If it is true that the state evolves in $\Re^2$, it is also true that we have scalar control inputs for both players as before. This is why we are able to relate the evolution of the boundary in the velocity coordinates to the evolution of the corresponding boundary in the position coordinates. In our case, the maximization and minimization in (10), depend on the 2-state model of both vehicles. This results in having both vehicles using maximum braking capabilities (while there is a forward motion with nonzero velocity).

Let's just consider the case when $u_{min_1} > u_{min_2}$, since automated systems typically have much lower braking capabilities, as the human driver is, nowadays, still considered responsible for the ultimate safety of the vehicle.

$$x_i = \begin{cases} x_{i0} + v_{i0}t + \dfrac{u_{min_i}}{2}t^2, & 0 \leq t \leq t_{if} \\ x_{if}, & t > t_{if} \end{cases} \tag{15}$$

where

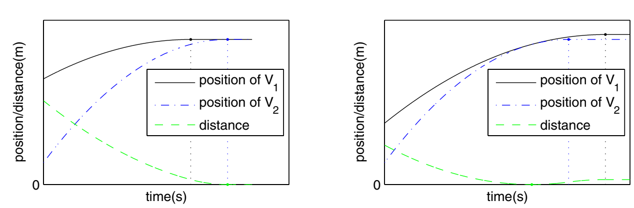
$$t_{if} = -\frac{v_{i0}}{u_{min_i}}, \qquad x_{if} = x_{i0} - \frac{v_{i0}^2}{2u_{min_i}}, \qquad i = 1, 2$$

1) $t_{1f} \leq t_{2f}$: The distance between the lead vehicle and the follower is:

$$\begin{aligned} x(t) &= x_1(t) - x_2(t) \\ &= \begin{cases} x_1(t) - x_2(t), & 0 \leq t < t_{1f} \\ x_{1f} - x_2(t), & t_{1f} \leq t < t_{2f} \\ x_{1f} - x_{2f}, & t \leq t_{2f} \end{cases} \end{aligned} \tag{16}$$

Since $-u_{min_1} < -u_{min_2}$ and $t_{1f} \leq t_{2f}$, $v_{10}(= -t_{1f}u_{min_1}) < v_{20}(= -t_{2f}u_{min_2})$. As shown in Fig. 3($a$), the minimum of x is reached at t_{2f}:

$$x(t_{2f}) = x_{1f} - x_{2f} = x(0) - (\frac{v_{10}^2}{2u_{min_1}} - \frac{v_{20}^2}{2u_{min_2}})$$



(a) V_1 stops first. (b) V_2 stops first, and $v_1 \leq v_2$.

Fig. 3. Positions and the relative positions of both vehicles as they apply full brakes simultaneously

Therefore, the safe set is:

$$\frac{v_1^2}{2u_{min_1}} - \frac{v_2^2}{2u_{min_2}} \leq x \tag{17}$$

2) $t_{1f} > t_{2f}$: The relative position of two vehicles is:

$$\begin{aligned} x(t) &= x_1(t) - x_2(t) \\ &= \begin{cases} x_1(t) - x_2(t), & 0 \le t < t_{2f} \\ x_1(t) - x_{2f}, & t_{2f} \le t < t_{1f} \\ x_{1f} - x_{2f}, & t \le t_{1f} \end{cases} \end{aligned} \tag{18}$$

(i) $v_{10} < v_{20}$ As shown in Fig. 3(b), the minimum of x is

$$x(-\frac{v_{10} - v_{20}}{u_{min_1} - u_{min_2}}) = x(0) - \frac{(v_{10} - v_{20})^2}{2(u_{min_1} - u_{min_2})}$$

Thus, the safe set should be:

$$\frac{(v_1 - v_2)^2}{2(u_{min_1} - u_{min_2})} \le x \tag{19}$$

(ii) $v_{10} \ge v_{20}$ In this case, the follower would have enough time to stop before a collision.

In a sum, the characterization of the safe set:

$$\begin{aligned} S = \{(x, v_1, v_2) \in \Re^3_+ : ((\frac{v_1^2}{2u_{min_1}} - \frac{v_2^2}{2u_{min_2}} \le x) \wedge \\ (t_{1f} < t_{2f})) \vee ((\frac{(v_1 - v_2)^2}{2(u_{min_1} - u_{min_2})} \le x) \wedge \\ (v_1 < v_2) \wedge (t_{1f} \ge t_{2f}))\} \end{aligned} \tag{20}$$

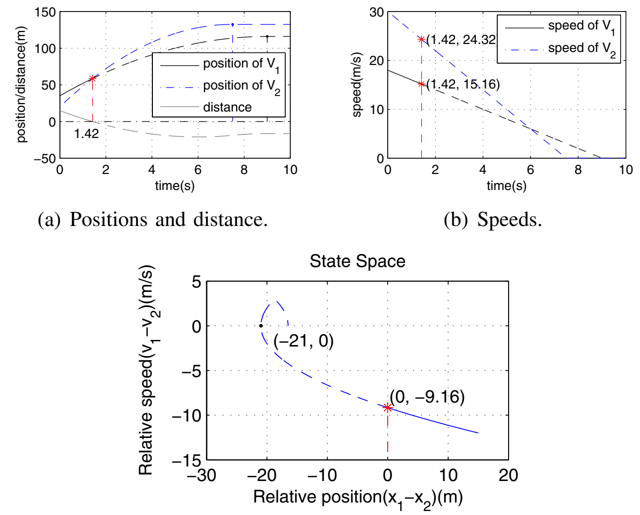
D. Safe controller

This derivation also gives conditions for synthesizing a safe controller for vehicle V_2. When $(x, v_1, v_2) \in int(S)$ use any control (int designates the relative interior). When (x, v_1, v_2) reaches the boundary of S set $u_2 = u_{min_2}$. Note that this control law prevents the trajectories of the system for crossing the boundary of S.

As an illustration, consider the following example with two vehicles in a single lane highway, with initial positions $x_{10} = 35$, $x_{20} = 20$, and initial velocities $v_{10} = 18$, $v_{20} = 30$. We consider all units to be metric, that is, both vehicles are 15 meters apart initially, with the leader going roughly 65km/hr and the follower going roughly 108km/hr. The lead vehicle has a deceleration capability of $-2m/s^2$, corresponding to the capabilities of a vehicle under automated braking control. The follower vehicle has a braking capacity of $-4m/s^2$, which corresponds to a typical braking ability for a human driver. Fig. 4 shows the positions and velocities of both vehicles for this set of initial conditions and maximum braking conditions. Obviously, this is a case where the system is not safe due to the short initial spacing and great mismatch in initial speed between the two vehicles, despite the added braking capacity for the following vehicle. A collision will occur 1.42 seconds after the lead vehicle hits maximum brakes.

Fig. 4(c) shows a state space view of the design, using spacing as the abscissa and derivative of spacing as the ordinate. Among other things, this diagram indicates that if vehicle 2 had detected the lead vehicle at least 36 meters ahead (instead of $15m$), the collision could have been avoided despite the speed and acceleration differences.

Remark 3: The implementation of this approach does not require communications between the vehicles. However, it



(a) Positions and distance.

(b) Speeds.

(c) State-Space View.

Fig. 4. Positions, the relative positions and speeds of both vehicles after two players apply full brakes at the same time while the safe set boundary is reached.

requires the knowledge of the braking capabilities of the front vehicle. A conservative design would take the maximum over the set of all possible braking capabilities of the vehicles in the road. If the braking capabilities of the front vehicle were known in real-time (i.e., sensed and communicated) this would lead to a less conservative design, and to adaptation to the road and environmental conditions.

Remark 4: The structure of the safe controller allows for another controller to control V_2 while it is safe to do so i.e., while the relative velocities and positions are inside S. This makes it possible to have a nested control structure where the inner controller implements an ACC or CACC control law and the outer controller is the safe controller, which overrides the operation of the former when the state reaches the boundary of the safe set.

IV. Conclusions

We consider the design of safe control laws for intelligent cruise control applications. The problem is one of collision avoidance, and is phrased as a zero-sum differential game where the lead vehicle tries to provoke a collision while the following vehicle tries to avoid it.

The approach provides the structure of controllers and a closed-form solution for the safe set. We used simple dynamics in this paper to illustrate the approach. This approach can accommodate systems with complete non-linear dynamics as long as they are affine in the controls and more general types of specifications of safe sets. Other types of uncertainty can be considered, namely parametric uncertainty, etc.. In more general cases it may not be possible to find a closed-form safe set, where we may need to resort to numerical techniques to integrate the ordinary differential equations (12, 13).

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