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Delay-dependent State Feedback Guaranteed Cost Control for Uncertain Singular Time-delay Systems

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***Abstract*— This paper considers the problem of state feedback guaranteed cost controller design for singular time-delay systems with norm-bounded parameter uncertainty. First, in terms of the Lyapunov technique and linear matrix inequalities (LMIs), a delay-dependent stability criterion for the normal singular time-delay system is established, which guarantees the system to be regular, impulse free and asymptotically stable. Then, based on the concept of generalized quadratic stability, we obtain a delay-dependent sufficient condition for the existence of the state feedback guaranteed cost controller in the forms of linear matrix inequalities. Finally, one numerical example is given to illustrate the validity of the arithmetic provided in this paper.**

I. INTRODUCTION

The study of guaranteed cost control about uncertain linear systems and uncertain linear time-delay systems has becoming perfect day by day. References [2]-[7] discuss both delay-independent and delay-dependent conditions for the existences of the guaranteed cost controllers, which have covered state feedback and output feedback controller designs. As to the guaranteed cost control for uncertain singular time-delay systems, many problems still need us to work out. Reference [9] discusses delay-independent state feedback conditions. Obviously, the delay-independent results are more conservative. To the best of our knowledge, it seems that there are few results on delay-dependent guaranteed cost control for uncertain singular time-delay systems. This has motivated our research.

In this paper, the problem of the delay-dependent robust stability criterion and the design of delay-dependent state feedback guaranteed cost controller for singular time-delay systems with norm-bounded parameter uncertainties is discussed. we consider the case of single constant time-delay, whose value is not required to be known. Delay-dependent stability criterion for the normal singular time-delay systems is established in terms of Lyapunov technique and LMIs. Since the stability criterion is delay-dependent, it is less conservative than those obtained in [8] and [9]. Based on the criterion, we present a sufficient condition for the existence of the state feedback guaranteed cost controller in terms of matrix inequalities and then change it into LMIs.

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II. PROBLEM FORMULATION AND PRELIMINARIES

We consider a class of uncertain singular time-delay system represented by

$$\begin{cases} E\dot{x}(t) = (A + \triangle A)x(t) + (A_\tau + \triangle A_\tau)x(t-\tau) \\ \qquad\quad + (B + \triangle B)u(t) \\ \ \ x(t) = \phi(t), \quad t \in [-\tau, 0] \end{cases} \tag{1}$$

where $x(t) \in R^n$, $u(t) \in R^m$ are the state and control input, respectively. E, A, A_τ and B are known real constant matrices with appropriate dimensions and $0 <$ rank $E = p < n$. τ is an unknown constant delay and satisfies $0 < \tau \le \tau_m.\tau_m$ is a known constant. $\phi(t) \in C_{n,\tau}$ is a compatible initial function. $\triangle A, \triangle A_\tau$ and $\triangle B$ are time-invariant matrices representing norm-bounded parametric uncertainties which are of the following form:

$$\begin{bmatrix} \triangle A & \triangle A_\tau & \triangle B \end{bmatrix} = DF \begin{bmatrix} E_1 & E_\tau & E_2 \end{bmatrix} \tag{2a}$$

$$F^T F \le I_j, \quad F \in R^{i\times j} \tag{2b}$$

where $D \in R^{n\times i}$, $E_1 \in R^{j\times n}$, $E_\tau \in R^{j\times n}$, $E_2 \in R^{j\times m}$ are known real constant matrices and F is an uncertain real matrix. $\triangle A, \triangle A_\tau$ and $\triangle B$ are said to be admissible if (2) is satisfied.

Given positive definite symmetric real matrices R and S, we will consider the cost functional

$$J = \int_0^\infty (x^T(t)Sx(t) + u^T(t)Ru(t))dt \tag{3}$$

The objective of this paper is to design guaranteed cost controller of the system (1)

$$u(t) = Kx(t), \quad K \in R^{m\times n} \tag{4}$$

So we will first present the definition of the guaranteed cost controller[4]

Definition 1 : Consider uncertain system (1). If there exist a controller in the form of (4) and a positive scalar J^* such that for all admissible uncertainties (2), the closed-loop system is regular, impulse free, zero solution asymptotically stable and the closed-loop value of the cost functional (3) satisfies $J \le J^*$, the controller (4) is said to be a guaranteed cost controller and J^* is said to be a guaranteed cost.

Firstly, we will give some definitions and lemmas about the nominal unforced singular time-delay system of (1):

$$\begin{cases} E\dot{x}(t) &= Ax(t) + A_\tau x(t-\tau) \\ \ \ x(t) &= \phi(t), \quad t \in [-\tau, 0] \end{cases} \tag{5}$$

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For this purpose, the following notations are needed:

$S_0 := \{\phi(t) \mid \phi(t) \in C_{n,\tau}, \phi(t)$ is the compatible initial function of system (5) $\}$;

$S := \{\phi(t) \mid \phi(t) \in S_0$ and there exists a unique continuous solution of system (5) on $[0, +\infty)$ for $\phi(t)\}$;

$B(0, \delta) := \{\phi(t) \mid \phi(t) \in C_{n,\tau}, \| \phi \|_c \leq \delta, \delta > 0\}$.

Definition 2 [11]:

1) The pair (E, A) is said to be regular if $\det(sE-A) \not\equiv 0$;

2) The pair (E, A) is said to be impulse free, if degree $\{\det(sE - A)\}$=rank$E$.

Lemma 1[8]*:* If the pair (E, A) is regular and impulse free, there exists a unique continuous solution on $[0, +\infty)$ of system (5) for all compatible initial function $\phi(t)$. That is, if the pair (E, A) is regular and impulse free, $S = S_0$.

Definition 3: The singular time-delay system (5) is said to be regular and impulse free if the pair (E, A) is regular and impulse free.

Definition 4:

1) If for any $\epsilon > 0$, there exists a $\delta(\epsilon) > 0$ such that $\| x(t) \| \leq \epsilon, \forall t \in [0, \infty)$ when the initial function $\phi \in B(0, \delta(\epsilon)) \cap S$, the zero solution of system (5) is said to be stable.

2) If the zero solution of system (5) is stable and furthermore, there is a scalar $b_0 > 0$ such that the initial function $\phi \in B(0, b_0) \cap S$ implies that $\lim_{t\to\infty} x(t) = 0$, the zero solution of system (5) is said to be asymptotically stable.

Remark 1: From $0 < rankE = p < n$, there exist nonsingular matrices M, N such that

$$\bar{E} = MEN = \begin{bmatrix} I_p & 0 \\ 0 & 0 \end{bmatrix} \tag{6}$$

Denote

$$\bar{A} = MAN = \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix}, \bar{B} = MB = \begin{bmatrix} B_1 \\ B_2 \end{bmatrix},$$

$$\bar{A}_\tau = MA_\tau N = \begin{bmatrix} A_{\tau 11} & A_{\tau 12} \\ A_{\tau 21} & A_{\tau 22} \end{bmatrix}, \bar{D} = MD = \begin{bmatrix} D_1 \\ D_2 \end{bmatrix},$$

$$\bar{K} = KN = \begin{bmatrix} K_1 & K_2 \end{bmatrix},$$

$$\begin{bmatrix} \triangle\bar{A} & \triangle\bar{A}_\tau & \triangle\bar{B} \end{bmatrix} = \bar{D}F \begin{bmatrix} \bar{E}_1 & \bar{E}_\tau & E_2 \end{bmatrix},$$

$$\bar{E}_1 = E_1 N = \begin{bmatrix} E_{11} & E_{12} \end{bmatrix},$$

$$\bar{E}_\tau = E_\tau N = \begin{bmatrix} E_{\tau 1} & E_{\tau 2} \end{bmatrix}. \tag{7}$$

Under coordinate transformation

$$y(t) = N^{-1}x(t) = \begin{bmatrix} y_1^T(t) & y_2^T(t) \end{bmatrix}^T \tag{8}$$

here $y_1(t) \in R^p, y_2(t) \in R^{n-p}$. Hence, the system (1) and (5) are respectively r.s.e.(restricted system equivalence)[10] equivalent to:

$$\left\{ \begin{array}{rcl} \bar{E}\dot{y}(t) & = & (\bar{A} + \triangle\bar{A})y(t) + (\bar{A}_\tau + \triangle\bar{A}_\tau)y(t-\tau) \\ & & +(\bar{B} + \triangle\bar{B})u(t) \\ y(t) & = & N^{-1}\phi(t), \quad t \in [-\tau, 0] \end{array} \right. \tag{1'}$$

$$\left\{ \begin{array}{rcl} \bar{E}\dot{y}(t) & = & \bar{A}y(t) + \bar{A}_\tau y(t-\tau) \\ y(t) & = & N^{-1}\phi(t), \quad t \in [-\tau, 0] \end{array} \right. \tag{5'}$$

The controller (4) and the cost functional (3) become respectively

$$u(t) = \bar{K}y(t), \quad \bar{K} \in R^{m\times n} \tag{4'}$$

$$J = \int_0^\infty [y^T(t)N^T SNy(t) + u^T(t)Ru(t)]dt \tag{3'}$$

Lemma 2[11]*:* Assume that $a(\cdot) \in R^{n_a}, b(\cdot) \in R^{n_b}$, and $N(\cdot) \in R^{n_a\times n_b}$ are defined on the interval Π. Then, for any matrices $X \in R^{n_a\times n_a}, Y \in R^{n_a\times n_b}, Z \in R^{n_b\times n_b}$, if $\begin{bmatrix} X & Y \\ * & Z \end{bmatrix} \geq 0$, the following inequality holds: $-2\int_\Pi a^T(\alpha)N(\alpha)b(\alpha)d\alpha \leq \int_\Pi \begin{bmatrix} a(\alpha) \\ b(\alpha) \end{bmatrix}^T \begin{bmatrix} X & Y - N(\alpha) \\ * & Z \end{bmatrix} \begin{bmatrix} a(\alpha) \\ b(\alpha) \end{bmatrix} d\alpha$

III. MAIN RESULTS

To predigest computation, we will first present the delay-dependent criterion, which will guarantee the equivalent system (5') to be regular, impulse free and asymptotically stable. Then we will offer a guaranteed cost controller by dealing with the equivalent system (1').

Theorem 1: The system (5') is regular, impulse free and zero solution asymptotically stable for any constant τ, satisfying $0 < \tau \leq \tau_m$, if there exist matrices $\bar{Q} \in R^{n\times n}$, $\bar{X} \in R^{2n\times 2n}$, $\bar{Z} \in R^{n\times n}$, $\bar{Q} > 0$, $\bar{X} \geq 0$, $\bar{Z} > 0$, and $\bar{P} \in R^{n\times n}$, $\bar{Y} \in R^{2n\times n}$ satisfying:

$$\bar{P}\bar{E} = \bar{E}^T\bar{P}^T \geq 0 \tag{9a}$$

$$\begin{bmatrix} \bar{X} & \bar{Y} \\ * & \bar{Z} \end{bmatrix} = \begin{bmatrix} \bar{X}_{11} & \bar{X}_{12} & \bar{Y}_1 \\ * & \bar{X}_{22} & \bar{Y}_2 \\ * & * & \bar{Z} \end{bmatrix} \geq 0 \tag{9b}$$

$$\bar{\Theta} = \begin{bmatrix} \Theta_{11} & \Theta_{12} & \Theta_{13} \\ * & \Theta_{22} & \Theta_{23} \\ * & * & -\bar{Q} \end{bmatrix} < 0 \tag{9c}$$

where

$$\Theta_{11} = \bar{A}^T\bar{P}^T + \bar{P}\bar{A} + \bar{Y}_1\bar{E} + \bar{E}^T\bar{Y}_1^T + \tau_m\bar{X}_{11} + \bar{Q}$$

$$\Theta_{12} = (\bar{A}^T\bar{P}^T + \bar{E}^T\bar{Y}_2^T + \tau_m\bar{X}_{12}) \begin{bmatrix} I_p \\ 0 \end{bmatrix}$$

$$\Theta_{13} = \bar{P}\bar{A}_\tau - \bar{Y}_1\bar{E}, \quad \Theta_{23} = \begin{bmatrix} I_p & 0 \end{bmatrix} (\bar{P}\bar{A}_\tau - \bar{Y}_2\bar{E})$$

$$\Theta_{22} = \begin{bmatrix} I_p & 0 \end{bmatrix} [-(\bar{P} + \bar{P}^T) + \tau_m(\bar{X}_{22} + \bar{Z})] \begin{bmatrix} I_p \\ 0 \end{bmatrix}$$

Proof : 1. To prove that the system (5') is regular and impulse free. Refer to Definition 3, it means to prove that the pair $(\bar{E}, \bar{A})$ is regular and impulse free.

Corresponding to the blocks of $\bar{E}$, we get

$$\bar{X}_{jj} = \begin{bmatrix} X_{jj11} & X_{jj12} \\ X_{jj12}^T & X_{jj22} \end{bmatrix}, j = 1, 2; \bar{X}_{12} = \begin{bmatrix} X_{1211} & X_{1212} \\ X_{1221} & X_{1222} \end{bmatrix}$$

$$\bar{Y}_i = \begin{bmatrix} Y_{i11} & Y_{i12} \\ Y_{i21} & Y_{i22} \end{bmatrix}, i = 1, 2; \bar{Z} = \begin{bmatrix} Z_{11} & Z_{12} \\ Z_{12}^T & Z_{22} \end{bmatrix},$$

$$\bar{P} = \begin{bmatrix} P_{11} & P_{12} \\ P_{21} & P_{22} \end{bmatrix}, \bar{Q} = \begin{bmatrix} Q_{11} & Q_{12} \\ Q_{12}^T & Q_{22} \end{bmatrix}. \tag{10}$$

Substituting $\bar{E}$ and $\bar{P}$ into (9a), it is clear that $P_{21} = 0, P_{11} \geq 0$. In the other hand ,from a Schur Complement argument and (9c) we can get $\bar{A}^T\bar{P}^T + \bar{P}\bar{A} + \bar{Y}_1\bar{E} + \bar{E}^T\bar{Y}_1^T + \tau_m\bar{X}_{11} + \bar{Q} + (\bar{P}\bar{A}\tau - \bar{Y}_1\bar{E})\bar{Q}^{-1}(\bar{P}\bar{A}\tau - \bar{Y}_1\bar{E})^T < 0$, that is $\bar{P}(\bar{A} - \bar{A}\tau\bar{Q}^{-1}\bar{E}^T\bar{Y}_1^T) + (\bar{A} - \bar{A}\tau\bar{Q}^{-1}\bar{E}^T\bar{Y}_1^T)^T\bar{P}^T + \tau_m\bar{X}_{11} + \bar{P}\bar{A}\tau\bar{Q}^{-1}\bar{A}\tau^T\bar{P}^T + (\bar{Y}_1\bar{E} + \bar{Q})\bar{Q}^{-1}(\bar{Y}_1\bar{E} + \bar{Q})^T < 0$. So we have $\bar{P}(\bar{A} - \bar{A}\tau\bar{Q}^{-1}\bar{E}^T\bar{Y}_1^T) +$

$(\bar{A}-\bar{A}\tau\bar{Q}^{-1}\bar{E}^T\bar{Y}_1^T)^T\bar{P}^T<0$, which implies that $\bar{P}$ is nonsingular. Hence $P_{11}>0$. Substituting (6) and (7) into (9c), we get

$$\begin{bmatrix} P_{22}A_{22}+A_{22}^TP_{22}^T+Q_{22} & P_{22}A_{\tau 22} \\ * & -Q_{22}\end{bmatrix}<0 \qquad (11)$$

which implies that A_{22} and P_{22} are nonsingular. While the nonsingularity of A_{22} implies that the pair $(\bar{E},\bar{A})$ is regular and impulse free.

That the pair $(\bar{E},\bar{A})$ is regular and impulse free means that there exist nonsingular matrices $\bar{M},\bar{N}$ such that $(\bar{E},\bar{A})$ is r. s. e. equivalent to the Weierstrass standard form $(\breve{E},\breve{A})$:

$$\breve{E}=\bar{M}\bar{E}\bar{N}=\begin{bmatrix} I_p & 0 \\ 0 & 0\end{bmatrix},\ \breve{A}=\bar{M}\bar{A}\bar{N}=\begin{bmatrix} A_1 & 0 \\ 0 & I_{n-p}\end{bmatrix}$$

For simplicity, we still use the marks in (6) and (7), here we only need to stress the form of $\bar{A}$ as:

$$\bar{A}=\begin{bmatrix} A_1 & 0 \\ 0 & I_{n-p}\end{bmatrix} \qquad (12)$$

Now, system (5') can be equivalently transformed into:

$$\begin{cases} \dot{y}_1(t) = A_1y_1(t)+A_{\tau 11}y_1(t-\tau)+A_{\tau 12}y_2(t-\tau) \\ 0 = y_2(t)+A_{\tau 21}y_1(t-\tau)+A_{\tau 22}y_2(t-\tau) \\ y(t) = N^{-1}\phi(t)=\begin{bmatrix}\phi_1(t)\\ \phi_2(t)\end{bmatrix},\quad t\in[-\tau,0]\end{cases} \qquad (13)$$

where $\phi_1(t)\in R^p,\phi_2(t)\in R^{n-p}$. Note that $y_1(t)-y_1(t-\tau)=\int_{t-\tau}^t\dot{y}_1(\alpha)d\alpha,\ t\geq\tau$. So system (13) can be rewritten as:

$$\begin{cases}\bar{E}\dot{y}(t)=\begin{bmatrix}A_1+A_{\tau 11}\\ A_{\tau 21}\end{bmatrix}y_1(t)-\begin{bmatrix}A_{\tau 11}\\ A_{\tau 21}\end{bmatrix}\int_{t-\tau}^t\dot{y}_1(\alpha)d\alpha \\ \qquad +\begin{bmatrix}0\\ I_{n-p}\end{bmatrix}y_2(t)+\begin{bmatrix}A_{\tau 12}\\ A_{\tau 22}\end{bmatrix}y_2(t-\tau),t\geq\tau \\ y(t)=\psi(t)=\begin{bmatrix}\psi_1(t)\\ \psi_2(t)\end{bmatrix},t\in[-\tau,\tau]\end{cases} \qquad (14)$$

where $\psi_1(t)\in R^p,\psi_2(t)\in R^{n-p}$, which satisfies

$$\begin{cases}\psi(t)=N^{-1}\phi(t)=\begin{bmatrix}\phi_1(t)\\ \phi_2(t)\end{bmatrix},\quad t\in[-\tau,0] \\ \dot{\psi}_1(t)=A_1\psi_1(t)+A_{\tau 11}\phi_1(t-\tau)+A_{\tau 12}\phi_2(t-\tau), \\ \qquad t\in[0,\tau] \\ \psi_2(t)=-A_{\tau 21}\phi_1(t-\tau)-A_{\tau 22}\phi_2(t-\tau),\quad t\in[0,\tau]\end{cases} \qquad (15)$$

Obviously, $\psi(t)$ is the compatible initial function of system (14). From (15) we can easily see that if $\phi(t)\in B(0,\delta)\cap S$,

$$\begin{aligned}\|\psi_1(t)\|\leq\|e^{A_1t}\phi_1(0)\|+\|\textstyle\int_0^t e^{A_1(t-s)}[A_{\tau 11}\phi_1(s-\tau) \\ +A_{\tau 12}\phi_2(s-\tau)]ds\|\leq M_1\delta,t\in[0,\tau]\end{aligned} \qquad (16a)$$

$$\begin{aligned}\|\psi_2(t)\|\leq\|A_{\tau 21}\|\|\phi_1(t-\tau)\|+\|A_{\tau 22}\|\|\phi_2(t-\tau)\| \\ \leq M_2\delta,t\in[0,\tau]\end{aligned} \qquad (16b)$$

where $M_1=(1+\tau(\|A_{\tau 11}\|+\|A_{\tau 12}\|))\cdot\max_{t\in[0,\tau]}\|e^{A_1t}\|$, $M_2=\|A_{\tau 21}\|+\|A_{\tau 22}\|$. That is $\psi(t)\in B(0,M_0\delta)\cap S,\ M_0=M_1+M_2+1,\ t\in[-\tau,\tau]$. It means that if $\phi(t)$ is the compatible initial function of system (13) and $\phi(t)\in B(0,\delta)$,$\psi(t)$ defined by (15) must be the compatible initial function of system (14) and $\psi(t)\in B(0,M_0\delta)$. Although system (13) and (14) are not equivalent to each other, the solution of system (13) must be the solution of system (14). In addition, system (5') is r.s.e. equivalent to system (13). Therefore, we can consider the asymptotical stability of the zero solution of system (14) instead of system (5').

2.To prove that system (5') is asymptotically stable. As mentioned above, we only need to prove that system (14) is asymptotically stable. The proof will be given as follows: First, prove that the first p-dimensional component of the zero solution of system (14) is asymptotically stable. Then prove that the system (14) is asymptotically stable. For this purpose, the following auxiliary lemma is introduced.

Lemma 3[9]*:* If there exist a continuous functional $V(y_t):C_{n,\tau}\rightarrow R$ and continuous nondecreasing functions $u,v,w:R^+\rightarrow R^+$, with $u(0)=v(0)=0,\ u(s)>0,v(s)>0,\forall s>0,V(y_t)$ satisfies:
i)$u(\|y_1(t)\|^2)\leq V(y_t)\leq v(\|y_t\|_c^2)$;
ii)$D^+(V(y_t))\leq -w(\|y_1(t)\|^2)$,
where $y_t:=y(t+\theta),\quad\theta\in[-2\tau,0]$, the first p-dimensional component of the zero solution of system (14) is stable, i. e., for any $\epsilon>0$, there exists a $\delta(\epsilon)>0$ such that $\|y_1(t)\|\leq\epsilon,t\geq\tau$ when the initial function $\psi(t)\in B(0,\delta(\epsilon))\cap S$.

Furthermore, if $w(s)>0$ for $s>0$, and there exist constant scalars l_0,m_0 such that $\|\dot{y}_1(t)\|\leq m_0,t\geq\tau$ when $\|y_1(t)\|\leq l_0,t\geq\tau$, the first p-dimensional component of the zero solution of system (14) is asymptotically stable, i. e.: i) the the first p-dimensional component of the zero solution is stable; ii) there exists a positive scalar δ_0 which is sufficiently small, such that $\lim_{t\rightarrow\infty}y_1(t)=0$ when the initial function $\psi(t)\in B(0,\delta_0)\cap S$.

Define[13]

$$z(t)=\bar{E}\dot{y}(t)=\begin{bmatrix}\dot{y}_1^T(t) & 0\end{bmatrix}^T=\begin{bmatrix}z_1^T(t) & 0\end{bmatrix}^T$$
$$\eta^T(t)=\begin{bmatrix}y^T(t) & z^T(t)\end{bmatrix}=\begin{bmatrix}y^T(t) & z_1^T(t) & 0\end{bmatrix} \qquad (17)$$

and quote Lyapunov-Krasovskii functional

$$\begin{aligned}V(y_t)&=y^T(t)\bar{P}\bar{E}y(t)+\int_{t-\tau}^t y^T(s)\bar{Q}y(s)ds \\ &+\int_{-\tau}^0\int_{t+\beta}^t z^T(t)\bar{Z}z(t)d\alpha d\beta,\quad t\geq\tau\end{aligned} \qquad (18)$$

where P_{11} and Z_{11} can be known in (10). Noting that

$$\begin{aligned}&\int_{-\tau}^0\int_{t+\beta}^t z^T(t)\bar{Z}z(t)d\alpha d\beta=\int_{-\tau}^0\int_{t+\beta}^t\dot{y}^T(\alpha)\bar{E}^T\bar{Z}\bar{E}\dot{y}(\alpha)d\alpha d\beta \\ &\leq\tau\int_{t-\tau}^t(\bar{A}y(\alpha)+\bar{A}_\tau y(\alpha-\tau))^T\bar{Z}(\bar{A}y(\alpha)+\bar{A}_\tau y(\alpha-\tau))d\alpha \\ &\leq\tau^2(\|\bar{A}^T\bar{Z}\bar{A}\|+2\|\bar{A}^T\bar{Z}\bar{A}_\tau\|+\|\bar{A}_\tau^T\bar{Z}\bar{A}_\tau\|)\|y_t\|_c^2\end{aligned} \qquad (19)$$

where $t\geq\tau,\|y_t\|_c=\sup_{\theta\in[-2\tau,0]}\|y(t+\theta)\|$. Then we can deduce

$$\begin{aligned}&\lambda_{min}(P_{11})\|y_1(t)\|^2\leq V(y_t)\leq[\tau_m^2(\|\bar{A}^T\bar{Z}\bar{A}\|+2\|\bar{A}^T\bar{Z}\bar{A}_\tau\| \\ &+\|\bar{A}_\tau^T\bar{Z}\bar{A}_\tau\|)+\lambda_{max}(P_{11})+\tau_m\|\bar{Q}\|]\|y_t\|_c^2,\quad t\geq\tau.\end{aligned} \qquad (20)$$

The time-derivative of $V(y_t)$ along with the solution of (14) is

$$\begin{aligned}\dot{V}(y_t)\mid_{(14)}=&\ y^T(t)\bar{P}\bar{E}\dot{y}(t)+\dot{y}^T(t)\bar{E}\bar{P}y(t) \\ &-y^T(t-\tau)\bar{Q}y(t-\tau)+\tau z^T(t)\bar{Z}z(t) \\ &+y^T(t)\bar{Q}y(t)-\textstyle\int_{t-\tau}^t z^T(\alpha)\bar{Z}z(\alpha)d\alpha\end{aligned} \qquad (21)$$

Let

$$G^T=\begin{bmatrix}\bar{P} & \bar{P}\\ 0 & \bar{P}\end{bmatrix}. \qquad (22)$$

From (9a) (9B) (14)and (17),using Lemma 2, we have

$$\dot{y}^T(t)\bar{P}\bar{E}y(t)+y^T(t)\bar{P}\bar{E}\dot{y}(t)=2y^T(t)\bar{P}z(t)$$

$$\leq 2\eta^T(t)G^T\{\begin{bmatrix}0\\ ----\\ A_1+A_{\tau 11}\\ A_{\tau 21}\end{bmatrix}y_1(t)+\begin{bmatrix}0\\ --\\ 0\\ I_{n-p}\end{bmatrix}y_2(t)$$

$$+\begin{bmatrix} I_n \\ -I_n \end{bmatrix} z(t) + \begin{bmatrix} 0 \\ -- \\ A_{\tau 12} \\ A_{\tau 22} \end{bmatrix} y_2(t-\tau)\} + \tau\eta^T(t)\bar{X}\eta(t)$$

$$+2\eta^T(t)\{\bar{Y}\begin{bmatrix} I_p \\ 0 \end{bmatrix} - G^T \begin{bmatrix} 0 \\ -- \\ A_{\tau 11} \\ A_{\tau 21} \end{bmatrix}\} y_1(t) + 2\eta^T(t)\{G^T$$

$$\cdot \begin{bmatrix} 0 \\ -- \\ A_{\tau 11} \\ A_{\tau 21} \end{bmatrix} - \bar{Y}\begin{bmatrix} I_p \\ 0 \end{bmatrix}\} y_1(t-\tau) + \int_{t-\tau}^{t} z^T(\alpha)\bar{Z}z(\alpha)d\alpha. \quad (23)$$

Substituting (23) into (21), we have

$$\dot{V}(y_t)\mid_{(14)} \leq \eta^T(t)\{G^T \begin{bmatrix} 0 & I_n \\ \bar{A} & -I_n \end{bmatrix} + \begin{bmatrix} 0 & \bar{A}^T \\ I_n & -I_n \end{bmatrix} G$$

$$+ \tau_m \bar{X} + \bar{Y}\begin{bmatrix} \bar{E} & 0 \end{bmatrix} + \begin{bmatrix} \bar{E} & 0 \end{bmatrix}^T \bar{Y}^T + \begin{bmatrix} \bar{Q} & 0 \\ 0 & \tau_m \bar{Z} \end{bmatrix}\}$$

$$\times \eta(t) + 2\eta^T(t)\{G^T \begin{bmatrix} 0 \\ \bar{A}_\tau \end{bmatrix} - \bar{Y}\bar{E}\} y(t-\tau)$$

$$- y^T(t-\tau)\bar{Q}y(t-\tau) = \zeta^T(t)\Phi\zeta(t) = \xi^T(t)\bar{\Theta}\xi(t)$$

$$\leq \begin{bmatrix} y(t) \\ z_1(t) \end{bmatrix}^T \begin{bmatrix} \Omega_{11} & \Omega_{12} \\ * & -\Omega_{22} \end{bmatrix} \begin{bmatrix} y(t) \\ z_1(t) \end{bmatrix} \leq y^T(t)Wy(t) \quad (24)$$

where

$$\zeta^T(t) = \begin{bmatrix} y^T(t) & z^T(t) & y^T(t-\tau) \end{bmatrix},$$

$$\xi^T(t) = \begin{bmatrix} y^T(t) & z_1^T(t) & y^T(t-\tau) \end{bmatrix},$$

$$W = \Omega_{11} + \Omega_{12}\Omega_{22}^{-1}\Omega_{12}^T,$$

$$\Omega_{11} = \bar{P}\bar{A} + \bar{A}^T\bar{P}^T + \bar{Y}_1\bar{E} + \bar{E}^T\bar{Y}_1^T + \tau_m\bar{X}_{11} + \bar{Q} + (\bar{P}\bar{A}_\tau - \bar{Y}_1\bar{E})\bar{Q}^{-1}(\bar{P}\bar{A}_\tau - \bar{Y}_1\bar{E})^T$$

$$\Omega_{12} = (\bar{P}\bar{A} + \bar{Y}_2\bar{E} + \tau_m\bar{X}_{12}^T + (\bar{P}\bar{A}_\tau - \bar{Y}_2\bar{E})\bar{Q}^{-1}(\bar{P}\bar{A}_\tau - \bar{Y}_1\bar{E})^T)^T \begin{bmatrix} I_p \\ 0 \end{bmatrix}$$

$$\Omega_{22} = \begin{bmatrix} I_p & 0 \end{bmatrix}[(\bar{P} + \bar{P}^T) - \tau_m(\bar{X}_{22} + \bar{Z}) - (\bar{P}\bar{A}_\tau - \bar{Y}_2\bar{E})\bar{Q}^{-1}(\bar{P}\bar{A}_\tau - \bar{Y}_2\bar{E})^T] \begin{bmatrix} I_p \\ 0 \end{bmatrix}$$

$$\Phi = \begin{bmatrix} \bar{P}\bar{A} + \bar{A}^T\bar{P}^T + Y_1\bar{E} + \bar{E}^T Y_1^T + \tau_m\bar{X}_{11} + \bar{Q} & \bar{A}^T\bar{P}^T + \bar{E}^T\bar{Y}_2^T + \tau_m\bar{X}_{12} & \bar{P}\bar{A}_\tau - \bar{Y}_1\bar{E} \\ * & -(\bar{P} + \bar{P}^T) + \tau_m\bar{X}_{22} + \tau_m\bar{Z} & \bar{P}\bar{A}_\tau - \bar{Y}_2\bar{E} \\ * & * & -\bar{Q} \end{bmatrix} \quad (25)$$

From (9c) we know $\bar{\Theta} < 0$. So $\begin{bmatrix} \Omega_{11} & \Omega_{12} \\ * & -\Omega_{22} \end{bmatrix} < 0$, further $W < 0$. Thus, we can have

$$\dot{V}(y_t)\mid_{(14)} \leq \lambda_{min}(W)y^T(t)y(t) \leq \lambda_{min}(W)y_1^T(t)y_1(t), t \geq \tau. \quad (26)$$

By Lemma 3, the first p-dimensional component of the zero solution of system (14) is asymptotically stable.

Next we will prove the zero solution of system (14) is asymptotically stable. Pre-multiplying $\begin{bmatrix} -A_{\tau 22}^T & I \end{bmatrix}$ and post-multiplying $\begin{bmatrix} -A_{\tau 22}^T & I \end{bmatrix}^T$ on both sides of (11) and noticing (12), we have $A_{\tau 22}^T Q_{22} A_{\tau 22} - Q_{22} < 0$ which implies that $\rho(A_{\tau 22}) < 1$ since $Q_{22} > 0$. So we can obtain

$$\parallel A_{\tau 22}^k \parallel \leq M\alpha^k, \quad k = 0, 1, ... \quad (27)$$

where M, α are constant. $M \geq 1, \alpha \in (0, 1)$. Noting that for any $t \geq 0$, there exists a positive integer k such that $k\tau \leq t \leq (k+1)\tau, k = 0, 1, \cdots$, calculating $y_2(t)$ from (13), we get

$$y_2(t) = (-A_{\tau 22})^k y_2(t - k\tau) - \sum_{i=1}^{k} (-A_{\tau 22})^{i-1} A_{\tau 21} y_1(t - i\tau). \quad (28)$$

Recalling that the first p-dimensional component of the zero solution of system (14) is asymptotically stable, we know that for any $\epsilon > 0$, there exists $0 < \delta(\epsilon) < \epsilon$ such that $\parallel y_1(t) \parallel \leq \epsilon, t \geq -\tau$ when the initial function $\psi(t) \in B(0, \delta(\epsilon)) \cap S$. Hence, when $\psi(t) \in B(0, \delta(\epsilon))$, from (27) and (28) we can evaluate $y_2(t)$:

$$\parallel y_2(t) \parallel \leq M \cdot (1 + \frac{1}{1-\alpha} \parallel A_{\tau 21} \parallel)\epsilon, \; t \geq -\tau \quad (29)$$

Next we will show that $y_2(t) \to 0, t \to \infty$. Since the first p-dimensional component of the zero solution of system (14) is asymptotically stable, there exists a sufficiently small scalar $\delta_0 > 0, \delta_0 \leq \delta(\epsilon_0) < \epsilon_0$ such that $\parallel y_1(t) \parallel \leq \epsilon_0, t \geq -\tau$ and $y_1(t) \to 0, t \to \infty$, when the initial function $\psi(t) \in B(0, \delta_0) \cap S$. In addition, we know $0 < \alpha < 1$ from (27). Thus for given positive scalar μ, it is obvious that there exists a positive integer $T_0(\mu)$ such that $\alpha^{T_0(\mu)} < \mu$ and $\parallel y_1(t) \parallel \leq \mu, t \geq T_0(\mu)\tau$. Let $T(\mu) = 2T_0(\mu)\tau$. Then when $t \geq T(\mu)$, we have

$$\begin{aligned} \parallel y_2(t) \parallel \quad & \leq \quad M\alpha^{2T_0(\mu)}\epsilon_0 + M \parallel A_{\tau 21} \parallel (\sum_{i=1}^{T_0(\mu)} \alpha^{i-1}\mu \\ & \quad + \sum_{i=T_0(\mu)+1}^{2T_0(\mu)} \alpha^{i-1}\epsilon_0) \\ & \leq \quad M(\epsilon_0 + \tfrac{1+\epsilon_0}{1-\alpha} \parallel A_{\tau 21} \parallel)\mu, \quad t \geq T(\mu) \end{aligned} \quad (30)$$

That is $y_2(t) \to 0, t \to \infty$, which combined with $y_1(t) \to 0, t \to \infty$ implies that $y(t) \to 0, t \to \infty$. Therefore the zero solution of system (14) is asymptotically stable.

As mentioned above, the asymptotical stability of the zero solution of system (5') can be obtained from that of system (14). It completes the proof.

In view of this, we will present the delay-dependent sufficient condition for the existence of the state feedback guaranteed cost controller via Theorem 1. The following assumption is needed.

Theorem 2: Consider the uncertain singular time-delay system (1') and the cost functional (3'). Given a positive scalar ϵ_1, if there exist a controller of form (4') and matrices with appropriate dimensions $\bar{Q} > 0, \bar{X} \geq 0, \bar{Z} > 0, \bar{P}, \bar{Y}$ that satisfy (9a),(9b) and the following matrix inequality:

$$\Lambda = L\{\Psi + \epsilon_1\tilde{D}\tilde{D}^T + \epsilon_1^{-1}\tilde{E}^T\tilde{E}\}L^T < 0 \quad (31)$$

where $L = blockdiag\{I_n, \begin{bmatrix} I_p & 0 \end{bmatrix}, I_n\}$

$\Psi =$

$$\begin{bmatrix} \Psi_{11} & (\bar{A} + \bar{B}\bar{K})^T\bar{P}^T + \bar{E}^T\bar{Y}_2^T + \tau_m\bar{X}_{12} & \bar{P}\bar{A}_\tau - \bar{Y}_1\bar{E} \\ * & -(\bar{P} + \bar{P}^T) + \tau_m(\bar{X}_{22} + \bar{Z}) & \bar{P}\bar{A}_\tau - \bar{Y}_2\bar{E} \\ * & * & -\bar{Q} \end{bmatrix}$$

$$\Psi_{11} = \bar{P}(\bar{A} + \bar{B}\bar{K}) + (\bar{A} + \bar{B}\bar{K})^T\bar{P}^T + \bar{Y}_1\bar{E} + \bar{E}^T\bar{Y}_1^T + \tau_m\bar{X}_{11} + \bar{Q} + N^T SN + \bar{K}^T R\bar{K}$$

$$\tilde{D} = \begin{bmatrix} \bar{P}\bar{D} \\ \bar{P}\bar{D} \\ 0 \end{bmatrix}, \; \tilde{E} = \begin{bmatrix} \bar{E}_1 + E_2\bar{K} & 0 & \bar{E}_\tau \end{bmatrix} \quad (32)$$

the closed-loop system is regular, impulse free, zero solution asymptotically stable and the cost functional (3') satisfies the following inequality

$$J \leq \phi^T(0)(N^{-1})^T \bar{P}\bar{E}N^{-1}\phi(0)+\int_{-\tau}^{0}\phi^T(s)(N^{-1})^T\bar{Q}N^{-1}\phi(s)ds$$
$$+\int_{-\tau}^{0}\int_{\theta}^{0}\dot{\phi}^T(\alpha)(N^{-1})^T\bar{E}^T\bar{Z}\bar{E}N^{-1}\dot{\phi}(\alpha)d\alpha d\theta \tag{33}$$

Proof : Noting $y_1(t)-y_1(t-\tau)=\int_{t-\tau}^{t}\dot{y}_1(\alpha)d\alpha \quad t\geq 0$, from (1') and (4') we get the closed-loop system:

$$\begin{cases} \bar{E}\dot{y}(t) = \{\begin{bmatrix} A_{11}+B_1K_1+D_1F(E_{11}+E_2K_1) \\ A_{21}+B_2K_1+D_2F(E_{11}+E_2K_1)\end{bmatrix} \\ \quad +\begin{bmatrix} A_{\tau 11}+D_1FE_{\tau 1} \\ A_{\tau 21}+D_2FE_{\tau 1}\end{bmatrix}\}y_1(t) \\ \quad +\begin{bmatrix} A_{12}+B_1K_2+D_1F(E_{12}+E_2K_2) \\ A_{22}+B_2K_2+D_2F(E_{12}+E_2K_2)\end{bmatrix}y_2(t) \\ \quad -\begin{bmatrix} A_{\tau 11}+D_1FE_{\tau 1} \\ A_{\tau 21}+D_2FE_{\tau 1}\end{bmatrix}\int_{t-\tau}^{t}\dot{y}_1(\alpha)d\alpha \\ \quad +\begin{bmatrix} A_{\tau 12}+D_1FE_{\tau 2} \\ A_{\tau 22}+D_2FE_{\tau 2}\end{bmatrix}y_2(t-\tau), \quad t\geq 0 \\ y(t) = \psi(t)=\begin{bmatrix}\psi_1^T(t) & \psi_2^T(t)\end{bmatrix}^T, \ t\in[-\tau,0] \end{cases} \tag{34}$$

where $\psi(t)=N^{-1}\phi(t),\ \psi_1(t)\in R^p$. Now we let $z(t),\eta(t)$, and G have the same form as (17) and (22), and define Lyapunov-Krasovskii functional $V(y_t), t\geq 0$ as (18). Dealing with the time-derivative of $V(y_t)$ along with the solution of (34) via the same way as Theorem 1, we can get

$$\dot{V}(y_t)\mid_{(34)}$$
$$\leq \begin{bmatrix}\eta(t) \\ y(t-\tau)\end{bmatrix}^T\{\epsilon_1^{-1}\begin{bmatrix}(\bar{E}_1+E_2\bar{K})^T \\ 0 \\ \bar{E}_\tau^T\end{bmatrix}\begin{bmatrix}\bar{E}_1+E_2\bar{K} & 0 & \bar{E}_\tau\end{bmatrix}+\epsilon_1\begin{bmatrix}\bar{P}\bar{D} \\ \bar{P}\bar{D} \\ 0\end{bmatrix}\begin{bmatrix}\bar{D}^T\bar{P}^T & \bar{D}^T\bar{P}^T & 0\end{bmatrix}+\Psi\}\begin{bmatrix}\eta(t) \\ y(t-\tau)\end{bmatrix}$$
$$-y^T(t)(N^TSN+\bar{K}^TR\bar{K})y(t) \tag{35}$$

where ε_1 is an uncertain positive scalar. Ψ can be known in (32). For simplicity, let $\xi^T(t)=\begin{bmatrix}y^T(t) & z_1^T(t) & y^T(t-\tau)\end{bmatrix}$ and quote a matrix Λ as shown in (31). Then (35) can be written as

$$\dot{V}(y_t)\mid_{(34)}\leq \xi^T(t)\Lambda\xi(t)-y^T(t)(N^TSN+\bar{K}^TR\bar{K})y(t) \tag{36}$$

By Theorem 1,(9a)(9b) and (31) suggest that the closed-loop system (34) is regular, impulse free and zero solution asymptotically stable.

In the other hand, using (3'), $J_T \leq \int_0^T y^T(t)(N^TSN+\bar{K}^TR\bar{K})y(t)+\dot{V}(y(t)))dt - V(y(T))+V(y(0)) < V(y(0))$, let $T\to\infty$. We can get (33). It completes the proof.

The result in terms of LMIs is stated as the following:

Theorem 3. Given positive scalars ϵ_1,ϵ_2, if there exist $2n\times 2n$ matrix $\tilde{X}\geq 0$, $n\times n$ matrices $\tilde{Z}>0,\tilde{Q}>0$, $2n\times n$ matrix $\tilde{Y}$, $m\times n$ matrix $\tilde{W}$, $p\times(n-p)$ matrix U and $n\times n$ nonsingular matrix $\tilde{P}$ satisfying :

$$\begin{bmatrix} \Sigma_{11} & * & * & * & * & * & * & * & * \\ \Sigma_{21} & \Sigma_{22} & * & * & * & * & * & * & * \\ \Sigma_{31} & \Sigma_{32} & -\tilde{Q} & * & * & * & * & * & * \\ \Sigma_{41} & 0 & \bar{E}_\tau\tilde{P} & -\epsilon_1 I & * & * & * & * & * \\ \Sigma_{51} & 0 & \bar{A}_\tau\tilde{P} & 0 & -\epsilon_2 I & * & * & * & * \\ \Sigma_{61} & \Sigma_{62} & 0 & 0 & 0 & -\epsilon_1^{-1}I & * & * & * \\ \Sigma_{71} & \Sigma_{72} & 0 & 0 & 0 & 0 & -\epsilon_2^{-1}I & * & * \\ N\tilde{P} & 0 & 0 & 0 & 0 & 0 & 0 & \tilde{S} & * \\ \tilde{W} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \tilde{R} \end{bmatrix} < 0 \tag{37a}$$

$$\begin{bmatrix}\tilde{X} & \tilde{Y} \\ * & \tilde{Z}\end{bmatrix}=\begin{bmatrix}\tilde{X}_{11} & \tilde{X}_{12} & \tilde{Y}_1 \\ * & \tilde{X}_{22} & \tilde{Y}_2 \\ * & * & \tilde{Z}\end{bmatrix}\geq 0 \tag{37b}$$

$$\bar{E}\tilde{P}=\tilde{P}^T\bar{E}^T\geq 0, \begin{bmatrix}0 & I_{n-p}\end{bmatrix}\tilde{P}\begin{bmatrix}I_p & 0\end{bmatrix}^T=0 \tag{37c}$$

where $\tilde{S}=-S^{-1}, \tilde{R}=-R^{-1}$

$$\Sigma_{11}=\bar{A}\tilde{P}+\tilde{P}^T\bar{A}^T+\bar{B}\tilde{W}+\tilde{W}^T\bar{B}^T+\tilde{Y}_1\bar{E}+\bar{E}^T\tilde{Y}_1^T+\tau_m\tilde{X}_{11}+\tilde{Q}$$
$$\Sigma_{21}=\begin{bmatrix}I_p & 0\end{bmatrix}[(\bar{A}\tilde{P}+\bar{B}\tilde{W})+\tilde{Y}_2\bar{E}+\tau_m\tilde{X}_{12}^T]$$
$$\Sigma_{22}=-(\tilde{P}_{11}+\tilde{P}_{11}^T)+\tau_m(\tilde{X}_{2211}+\tilde{Z}_{11}),\ \ \Sigma_{31}=\tilde{P}^T\bar{A}_\tau^T-\bar{E}^T\tilde{Y}_1^T$$
$$\Sigma_{32}=\tilde{P}^T\bar{A}_\tau^T\begin{bmatrix}I_p \\ 0\end{bmatrix}-\bar{E}^T\begin{bmatrix}\tilde{Y}_{211}^T \\ \tilde{Y}_{212}^T\end{bmatrix},\ \Sigma_{41}=\bar{E}_1\tilde{P}+E_2\tilde{W},$$
$$\Sigma_{51}=\bar{A}\tilde{P}+\bar{B}\tilde{W},\ \ \Sigma_{61}=\bar{D}^T\begin{bmatrix}I_p & 0 \\ U^T & I_{n-p}\end{bmatrix},$$
$$\Sigma_{62}=\bar{D}^T\begin{bmatrix}I_p \\ U^T\end{bmatrix},\Sigma_{71}=\begin{bmatrix}0 & 0 \\ U^T & 0\end{bmatrix},\Sigma_{72}=\begin{bmatrix}0 \\ U^T\end{bmatrix} \tag{38}$$

there exists a state feedback controller $u(t)=\bar{K}y(t),\bar{K}=\tilde{W}\tilde{P}^{-1}$, such that the closed-loop system (34) is regular, impulse free, zero solution asymptotically stable and the cost functional satisfy

$$J\leq J^*=\phi^T(0)(N^{-1})^T\begin{bmatrix}(\tilde{P}_{11}^{-1})^T & (\tilde{P}_{11}^{-1})^TU \\ 0 & (\tilde{P}_{22}^{-1})^T\end{bmatrix}\bar{E}N^{-1}\phi(0)$$
$$+\int_{-\tau}^{0}\int_{\theta}^{0}\dot{\phi}^T(\alpha)(N^{-1})^T\bar{E}^T(\tilde{P}^{-1})^T\tilde{Z}\tilde{P}^{-1}\bar{E}N^{-1}\dot{\phi}(\alpha)d\alpha d\theta$$
$$+\int_{-\tau}^{0}\phi^T(s)(N^{-1})^T(\tilde{P}^{-1})^T\tilde{Q}\tilde{P}^{-1}N^{-1}\phi(s)ds \tag{39}$$

Proof: From Theorem 2, given a positive scalar ϵ_1, if there exist a controller of form (4') and matrices with appropriate dimensions $\bar{Q}>0,\bar{X}\geq 0,\bar{Z}>0,\bar{P},\bar{Y}$ that satisfy (9a),(9b) and (31), the closed-loop system is regular, impulse free, zero solution asymptotically stable and the cost value has the evaluation of (33). By the same way of Theorem 1, we can prove that $P_{11}>0$ and P_{22} is nonsingular.

Now we set $\tilde{P}_{11}=(P_{11}^{-1})^T, \tilde{P}_{22}=(P_{22}^{-1})^T, \tilde{P}=\begin{bmatrix}\tilde{P}_{11} & 0 \\ 0 & \tilde{P}_{22}\end{bmatrix}$, $\tilde{X}_{11}=\tilde{P}^T\bar{X}_{11}\tilde{P},\ \tilde{X}_{12}=\tilde{P}^T\bar{X}_{12}\tilde{P},\ \tilde{X}_{22}=\tilde{P}^T\bar{X}_{22}\tilde{P},\ \tilde{Y}_1=\tilde{P}^T\bar{Y}_1\tilde{P},\ \tilde{Y}_2=\tilde{P}^T\bar{Y}_2\tilde{P},\ \tilde{Z}=\tilde{P}^T\bar{Z}\tilde{P},\ \tilde{Q}=\tilde{P}^T\bar{Q}\tilde{P},\ \tilde{W}=\bar{K}\tilde{P},\ U=\tilde{P}_{11}^TP_{12}$. Pre-multiplying and post-multiplying (9a) by $\tilde{P}^T$ and $\tilde{P}$, respectively, and noting $\begin{bmatrix}I_p & U \\ 0 & I_{n-p}\end{bmatrix}\bar{E}=\bar{E}$, we can get $\bar{E}=\bar{E}^T=\bar{E}$. Because $\tilde{P}_{11}>0$ and $\tilde{P}=\begin{bmatrix}\tilde{P}_{11} & 0 \\ 0 & \tilde{P}_{22}\end{bmatrix}$ must be satisfied, we need (37c); Pre-multiplying and post-multiplying (9b) by $diag\{\tilde{P}^T,\tilde{P}^T,\tilde{P}^T\}$ and $diag\{\tilde{P},\tilde{P},\tilde{P}\}$, respectively, we can get (37b); Pre-multiplying and post-multiplying (31) by $diag\{\tilde{P}^T,\tilde{P}_{11}^T,\tilde{P}^T\}$ and $diag\{\tilde{P},\tilde{P}_{11},\tilde{P}\}$, respectively, and noting $\tilde{P}\bar{E}=\bar{E}\tilde{P}$ we can get

$$\begin{bmatrix}\Gamma_{11} & * & * \\ \Sigma_{21} & \Sigma_{22} & * \\ \Sigma_{31} & \Sigma_{32} & -\tilde{Q}\end{bmatrix}+\begin{bmatrix}\Sigma_{71}^T\Sigma_{51}+\Sigma_{51}^T\Sigma_{71} & * & * \\ \Sigma_{72}^T\Sigma_{51} & 0 & * \\ \tilde{P}^T\bar{A}_\tau^T\Sigma_{71} & \tilde{P}^T\bar{A}_\tau^T\Sigma_{72} & 0\end{bmatrix}+\epsilon_1\begin{bmatrix}\Sigma_{61}^T \\ \Sigma_{62}^T \\ 0\end{bmatrix}\begin{bmatrix}\Sigma_{61} & \Sigma_{62} & 0\end{bmatrix}+\epsilon_1^{-1}\begin{bmatrix}\Sigma_{41}^T \\ 0 \\ \tilde{P}^T\bar{E}_\tau^T\end{bmatrix}$$

$$\times \left[\begin{array}{ccc} \Sigma_{41} & 0 & \bar{E}_\tau \tilde{P} \end{array} \right] < 0$$

Moreover, the above inequality can be strengthened into

$$\left[\begin{array}{ccc} \Gamma_{11} & * & * \\ \Sigma_{21} & \Sigma_{22} & * \\ \Sigma_{31} & \Sigma_{32} & -\tilde{Q} \end{array} \right] + \epsilon_1 \left[\begin{array}{c} \Sigma_{61}^T \\ \Sigma_{62}^T \\ 0 \end{array} \right] \left[\begin{array}{ccc} \Sigma_{61} & \Sigma_{62} & 0 \end{array} \right]$$

$$+\epsilon_1^{-1} \left[\begin{array}{c} \Sigma_{41}^T \\ 0 \\ \tilde{P}^T \bar{E}_\tau^T \end{array} \right] \left[\begin{array}{ccc} \Sigma_{41} & 0 & \bar{E}_\tau \tilde{P} \end{array} \right] + \epsilon_2 \left[\begin{array}{c} \Sigma_{71}^T \\ \Sigma_{72}^T \\ 0 \end{array} \right] \left[\begin{array}{c} \Sigma_{71} \end{array} \right.$$

$$\left. \begin{array}{cc} \Sigma_{72} & 0 \end{array} \right] + \epsilon_2^{-1} \left[\begin{array}{c} \Sigma_{51}^T \\ 0 \\ \tilde{P}^T \bar{A}_\tau^T \end{array} \right] \left[\begin{array}{ccc} \Sigma_{51} & 0 & \bar{A}_\tau \tilde{P} \end{array} \right] < 0 \quad (40)$$

here, $\Gamma_{11} = \Sigma_{11} + \tilde{P}^T N^T S N \tilde{P} + \tilde{W}^T R \tilde{W}$.Obviously (40) is equivalent to (37a) from a Schur Complement argument.

Now, we can obtain $\bar{P} = \left[\begin{array}{cc} (\tilde{P}_{11}^{-1})^T & (\tilde{P}_{11}^{-1})^T U \\ 0 & (\tilde{P}_{22}^{-1})^T \end{array} \right], \bar{Q} = (\tilde{P}^{-1})^T \tilde{Q} \tilde{P}^{-1}, \bar{Z} = (\tilde{P}^{-1})^T \tilde{Z} \tilde{P}^{-1}$ and Substitute them into (33). Then J^* in (39) can be obtained. It completes the proof.

To this end, we can get the guaranteed cost controller of system (1): $u(t) = Kx(t), K = \tilde{W}\tilde{P}^{-1}N^{-1}$ and the guaranteed cost: $J^* = \phi^T(0)\hat{P}E\phi(0) + \int_{-\tau}^{0} \phi^T(s)\hat{Q}\phi(s)ds + \int_{-\tau}^{0}\int_{\theta}^{0} \dot{\phi}^T(\alpha)E^T\hat{Z}E\dot{\phi}(\alpha)d\alpha d\theta$. Here $\hat{P} = (N^{-1})^T\bar{P}M,\ \hat{Q} = (N^{-1})^T\bar{Q}N^{-1},\ \hat{Z} = M^T\bar{Z}M$ are all solvable.

IV. EXAMPLES

Example 1. Consider the uncertain singular time-delay system (1) and the cost functional (3). The uncertainty satisfies (2) and the constant matrices are given below

$$E = \left[\begin{array}{cc} 1 & 0 \\ 0 & 0 \end{array} \right], A = \left[\begin{array}{cc} 1 & 1 \\ -1 & -2 \end{array} \right], A_\tau = \left[\begin{array}{cc} 0.5 & 0 \\ -0.05 & -0.05 \end{array} \right],$$

$$D = \left[\begin{array}{ccc} 0.1 & 0 & 0.1 \\ 0 & 1 & 0 \end{array} \right], E_1 = \left[\begin{array}{cc} 1 & 1 \\ 0 & 0 \\ 0 & 0 \end{array} \right], E_\tau = \left[\begin{array}{cc} 0 & 0 \\ 0 & 0 \\ 1 & 1 \end{array} \right],$$

$$B = \left[\begin{array}{c} 2 \\ 1 \end{array} \right], E_2 = \left[\begin{array}{c} 0 \\ 1 \\ 0 \end{array} \right], R = 1,\ S = I_2, F = diag\{r, q, s\},$$

$$|\ r\ | \leq 1,\ |\ s\ | \leq 1,\ |\ q\ | \leq 1, \phi(t) = \left[\begin{array}{c} -10 \\ 11.5 \end{array} \right], t \in [-1.5, 0].$$

Using the LMI Toolbox of MATLAB, the delay-independent results given in [10] is not feasible. While if we let $M = \left[\begin{array}{cc} -1 & -0.5 \\ 0 & 0.5 \end{array} \right], N = \left[\begin{array}{cc} -1 & 0 \\ 0.5 & -1 \end{array} \right]$ and transform it into equivalent system, (38a), (38b), (38c) are feasible. When $\varepsilon_1 = 0.5, \varepsilon_2 = 0.7$, the solutions of (38a), (38b), (38c) are

$$\tilde{X}_{11} = \left[\begin{array}{cc} 0.018 & 0.004 \\ 0.004 & 0.009 \end{array} \right], \tilde{X}_{12} = \left[\begin{array}{cc} 0.009 & 0 \\ 0.002 & 0 \end{array} \right], U = 0.219$$

$$\tilde{X}_{22} = \left[\begin{array}{cc} 0.031 & 0 \\ 0 & 69.83 \end{array} \right], \tilde{Y}_1 = \left[\begin{array}{cc} -0.0002 & 0 \\ 0.0029 & 0 \end{array} \right],$$

$$\tilde{Y}_2 = \left[\begin{array}{cc} 0.017 & 0 \\ 0 & 0 \end{array} \right], \tilde{Z} = \left[\begin{array}{cc} 0.027 & 0 \\ 0 & 69.825 \end{array} \right],$$

$$\tilde{P} = \left[\begin{array}{cc} 0.141 & 0 \\ 0 & -0.132 \end{array} \right], \tilde{Q} = \left[\begin{array}{cc} 0.070 & -0.010 \\ -0.010 & 0.048 \end{array} \right],$$

$$\tilde{W} = \left[\begin{array}{cc} 0.115 & -0.035 \end{array} \right].$$

Then we get the guaranteed cost controller of system (1): $u(t) = Kx(t),\quad K = \tilde{W}\tilde{P}^{-1}N^{-1} = [-0.9429 - 0.2622]$ and the guaranteed cost: $J^* = 26.4436$. It illustrates the validity of the arithmetic provided in this paper.

Fig 1 illustrates that the states asymptotically converge to zero.

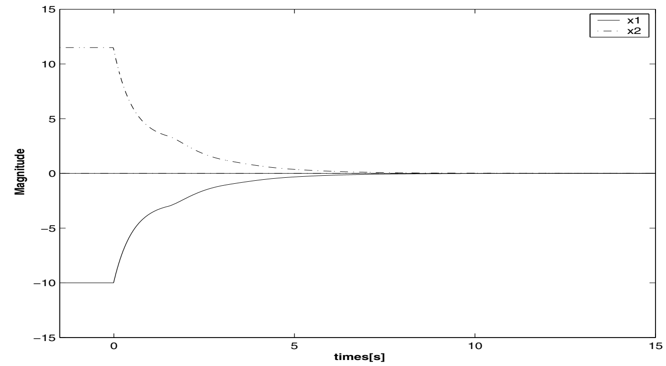

Fig. 1. The simulation results of the equivalent closed-loop system

V. CONCLUSIONS

This paper discusses the problem of state feedback guaranteed cost controller design for singular time-delay systems with norm-bounded parameter uncertainty. A delay-dependent stability criterion for the normal singular time-delay system is firstly established, which guarantees the system to be regular, impulse free and asymptotically stable. The novelty of this paper is the delay-dependent stability criterion for singular time-delay systems, which improves the results in [8] and [9] to a certain extent. Applying the delay-dependent stability criterion proposed here, we obtain a delay-dependent sufficient condition for the existence of the state feedback guaranteed cost controller in terms of linear matrix inequalities.

REFERENCES

[1] Chang,S.S.L.and Peng,T.K.C.(1972).Adaptive guaranteed cost control of systems with uncertain parameters.IEEE Transactions on Automatic Control,17,474-483.

[2] Moheimani,S.O.R.and Petersen,I.R.(1997).Optimal quadratic guaranteed cost controlof a class of uncertain time-delay systems. IEE Proceedings-Control,Theory and Applications,144,183-188.

[3] Esfahani,S.H.and Moheimani,S.O.R.(1998).LMI approach to suboptimal guaranteed cost control for uncertain time-delay systems.IEE Proceedings-Control,Theory and Applications,145,491-498.

[4] Yu,L.and Chu,J.(1999).An LMI approach to guaranteed cost control of a linear uncertain time-delay systems.Automatica,35,1155-1159.

[5] Lee,Y.S.,Moon,Y.S.,Kowon,W.H.(2001).Delay-dependent guaranteed cost control for uncertain systems state-delayed systems.Proceedings of the American control conference,Arlington,USA(pp.3376-3381).

[6] Chen,W.H.,Xu,J.X.,(2003b).Guaranteed cost control for uncertain Markovina jump systems mode-dependent delays.IEEE Trans. Automat. Contr.48,2270-2277.

[7] Chen,W.H.,Guan,Z.H.and Lu,X.(2003a).Delay-dependent guaranteed cost control for uncertain discrete-time systems with delay.IEE Proceedings-Control,Theory and Applications,150,412-416.

[8] S. Xu, P. V. Dooren, R. Stefan and J. Lam, "Robust stability and stabilization for singular systems with state delay and paramete uncertainty," *IEEE Trans. Automat. Contr.*, vol. 47, pp. 1122-1128, 2002.

[9] J. Feng, S. Zhu and Z. Cheng, "Guaranteed cost control of linear uncertain singular time-delay systems," *in Proc. 41st IEEE Conf. Decision Control*, pp. 1802-1807, Las Vegas, Nenasa USA, 2002.

[10] L. Dai, *Singular Control Systems*, Berlin: Springer-Verlag, 1989.

[11] Moon,Y.S.,Park,P.G.,Kwon,W.H.and Lee,Y.S.Delay-dependent robust stabilization of uncertain state-delayed systems.International Journal of Control,74(2001), 1447-1455.

[12] Ghaoui,L.E.,Oustry,F.and AitRami,M.(1997).A cone complementarity linearization algorithm for static output-feedback and related problems.IEEE Transactions on Automatic Control,42,1171-1176.

[13] Wu-Hua Chen,Zhi-Hong Guan,Xiaomei Lu.Delay-dependent output feedback guaranteed cost control for uncertain time-delay systems.Automatic 40(2004),1263-1268.