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LPV Modeling of Atmospheric Re-entry Demonstrator for Guidance Re-entry Problem

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Abstract— **This paper describes the status of an on-going research work and investigates the problem of entry guidance of the Atmospheric Re-entry Demonstrator (ARD). We propose an entry guidance scheme that uses the flatness property. First, it is shown that the longitudinal dynamics of the ARD is flat and by using flatness property a feasible desired output and the associated nominal control input trajectory are easily generated. Next, a methodology to construct a nonlinear error model, which describes the dynamics of the trajectory tracking error, is derived using coordinate transformations of the flat outputs. A first order linearization of the nonlinear error model is performed along the nominal trajectory in order to formulate the nonlinear ARD longitudinal dynamics as Linear Parameter Varying (LPV) model. LPV control design techniques can then be used to construct the global control law guaranteeing stability and performance.**

I. INTRODUCTION

Atmospheric re-entry presents challenges in several domains of engineering and science, being one of principal researches in space technology. In order to validate Europe's knowledge in re-entry flight, European Space Agency (ESA) developed a flight demonstrator named Atmospheric Re-entry Demonstrator (ARD). The ARD is an Apollo type capsule, with a sphero-conical shape as shown in figure.1, it's maximum diameter is 2.80 [m] for a 2.04 [m] length and the overall mass is 2800 [Kg] (see for instance [1]).

Fig. 1. Atmospheric re-entry demonstrator.

The ARD first flight was successfully completed on 1998 aboard Ariane 5 flight 503 and consisted of launch phase, suborbital ballistic phase and re-entry phase. Its main objectives were to validate in flight key technical expertise and technologies, required for mastery of orbital return: Accuracy of the aerodynamics models, performance of the thermal protection system, performance and robustness of Navigation, Guidance and control (NGC) flight software developed by European Aeronautic Defence and Space Company (EADS) and behavior of the parachute chains and mastery of splash-down.

The entry phase in earth atmosphere corresponds to an altitude range from 120 [Km] to nearly 0 [Km]. This phase is characterized by large variations of environment because of large variations of atmospheric conditions (e.g. density and temperature) and aerodynamics coefficients. The re-entry phase is a difficult operation which requires in particular the ARD NGC system to be robust enough to operate in the flight environment. The ARD trajectory is controlled by the guidance loop, the primary task of entry guidance is: given the ARD model, determine the controls that steer the vehicle on a feasible trajectory, a trajectory within the entry corridor defined by heating, acceleration, dynamic pressure, and controllability limits that achieve the specified target conditions within the specified error margin.

Several guidance schemes have been proposed for re-entry, probably the guidance law most referred to is the Shuttle re-entry guidance. Since its inception in the early seventies, numerous papers have been published [2, 3].

In the present study, a re-entry guidance scheme employing flatness concept (introduced by Fliess et al [4]) and Linear Parameter Varying (LPV) techniques has been proposed for the ARD mission. The flatness theory is used to generate by a reference model the reference profile of the states and the commands satisfying equilibrium, heat rate, dynamic pressure and load factor constraints, while the LPV techniques can be used to design the control law to follow the predicted reference profile.

The main contribution of this paper is considered to be the flatness demonstration of the longitudinal model of the ARD and the modeling of the ARD longitudinal nonlinear flat dynamics as LPV systems. The strategy adopted here consists to reduce the complex entry guidance problem into a simple trajectory generation and trajectory tracking problem. This approach is similar to the two degrees of freedom scheme proposed in [5]. Firstly, reference trajectory profile of the outputs and the nominal commands are generated by a reference model using flatness theory. Secondly, the

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dynamics of the trajectory tracking error is determined by using coordinate transformation of the flat outputs, and then a first order linearization is performed around the reference trajectory. Finally, once the LPV model is formed, there exist several design techniques [6, 7, 8] for designing LPV controller guaranteeing stability and desired performance of the system.

The remainder of this paper is organized as follows. The ARD dynamical equations are presented in section 2. In section 3, flatness theory and its application to trajectory generation is reviewed. The flatness of the simplified model which represents the ARD longitudinal dynamic is demonstrated in section 4. Section 5 consists on the modeling of the ARD non linear (simplified) flat model as LPV model. Finally, Section 6 presents some concluding remarks.

II. The ARD model

The dynamical equations in this section are borrowed from Vinh [9] and Betts [10]. The flight dynamics of the re-entry vehicle are approximated by the rotational and translational motion of rigid body with respect to a stationary, spherical planet, the equations of motion are given by

$$\begin{aligned}
\dot{R} &= V\sin\gamma \\
\dot{\theta} &= \frac{V}{R}\frac{\cos\gamma\sin\psi}{\cos\theta} \\
\dot{\phi} &= \frac{V}{R}\cos\gamma\cos\psi \\
\dot{V} &= -\frac{D(\alpha,M)}{m} - g\sin\gamma \\
\dot{\gamma} &= \frac{L(\alpha,M)}{mV}\cos\mu + \left(\frac{V}{R}-\frac{g}{V}\right)\cos\gamma \\
\dot{\psi} &= \frac{L(\alpha,M)}{mV\cos\gamma}\sin\mu + \frac{V}{R}\cos\gamma\sin\psi\tan\theta
\end{aligned} \tag{1}$$

where the position coordinates are the radial distance from the center of the earth to the vehicle R, the longitude θ, and the latitude ϕ and the velocity coordinates are the velocity magnitude V, the flight path angle γ and the azimuth angle ψ measured from the North in a clockwise direction. Also, $g=\mu_T/R^2$ with μ_T is a gravitational constant. $L(\alpha,M)$ and $D(\alpha,M)$ represent the lift and drag accelerations and are given by

$$\begin{aligned}
L(\alpha,M) &= \frac{1}{2}\frac{\rho V^2 S\, C_L(\alpha,M)}{m} \\
D(\alpha,M) &= \frac{1}{2}\frac{\rho V^2 S\, C_D(\alpha,M)}{m}
\end{aligned} \tag{2}$$

where α is the angle of attack and $M=V/V_s$ is the Mach number with V_s is the sound velocity. S is the vehicle reference area, m is the mass of the vehicle, $C_L(\alpha, M)$ and $C_D(\alpha, M)$ are the lift and the drag coefficients, $\rho(R)$ is the R-dependant density and it is given as follow

$$\rho = \rho_0 \exp(-(R-R_e)/h_{ref}) \tag{3}$$

where ρ_0 is the density at sea level (where the altitude h is equal to zero), h_{ref} is the associated altitude to the model and R_e is the radius of the earth.

The lift-to-drag (L/D) ratio is given by the following expression

$$L/D = C_L(M)/C_D(M) \tag{4}$$

Throughout entry, m is assumed constant. Besides α is equal to the natural trim angle-of-attack of the capsule, which, for a given Mach number, is fixed by its shape and its layout (location of the center of gravity). Therefore the lift and drag coefficients, as well as the lift-to-drag-ratio, can be assumed to be functions of M only. They are depicted in figure 2.

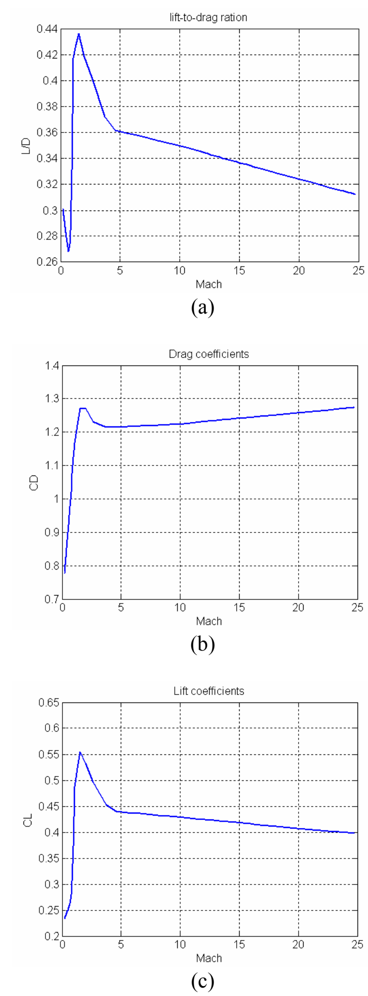


Fig. 2. (a) Lift-to-drag-ratio profile, (b) Lift coefficient profile, (c) Drag coefficient profile.

The data are borrowed form [11]. Since the ARD is essentially a scaled version of the Apollo Command Module, Apollo aerodynamics were used for this study.

The two controls that will be considered in this study are $\cos\mu$ the bank angle and $C_D(M)$ the drag coefficient.

III. Review of differential flatness

In this section a very brief review of the differential flatness is given. It is shown how we can apply these tools to the problem of trajectory generation.

A. *Differential flatness*

Differential flatness was first introduced by Fliess et *al* [4] in a differential algebraic context. The important property of flat systems is that we can find a set of variables (equal in number to the number of inputs) such that all states and inputs can be expressed in terms of those outputs and a finite number of their time derivatives without any integration procedure. More precisely, we consider the dynamical system of the general form

$$\begin{aligned} \dot{x}(t) &= f\big(x(t), u(t)\big) \\ y(t) &= h\big(x(t), u(t)\big) \end{aligned} \tag{5}$$

where t is the time variable, x is the n-dimensional state vector, u is the m-dimensional input vector, y is the m-dimensional tracking output vector, f (.) and h (.) are a nonlinear functions. The system is differentially flat if we can find a set of variables $z(t)\in\Re^m$ which are differentially independent, called flat outputs, of the form

$$z(t) = \Phi\big(x(t), u(t), \dot{u}(t), \cdots, u^{(a)}(t)\big) \tag{6}$$

such that

$$\begin{aligned} x(t) &= \Psi_x\big(z(t), \dot{z}(t), \cdots, z^{(b)}(t)\big) \\ u(t) &= \Psi_u\big(z(t), \dot{z}(t), \cdots, z_1^{(b+1)}(t)\big) \end{aligned} \tag{7}$$

where Φ and Ψ are smooth functions, $z^{(a)}(t)$ and $z^{(b)}(t)$ are respectively the a and b order time derivative of $z(t)$.

B. *Trajectory generation*

The trajectory generation problem for general systems described in (5) consists in finding the control $[0,T]\ni t\rightarrow u(t)$ steering the system from the state $x=p$ at $t=0$ to the state $x=q$ at $t=T$ (see for instance [12]).

The problem involves the solution of two point boundary value problems which are hard to solve numerically but when the system is flat, the problem is equivalent to finding a flat output $[0,T]\ni t\rightarrow z(t)$ such that

$$x(0) = p = \Psi_x\big(z(0), \dot{z}(0), \cdots, z^{(b)}(0)\big) \tag{8}$$

and

$$x(T) = q = \Psi_x\big(z(T), \dot{z}(T), \cdots, z^{(b)}(T)\big) \tag{9}$$

In general, the problem consists in finding a smooth function $t\rightarrow z(t)$ with prescribed values for some of derivatives at time 0 and at time T, such that

$$[0,T]\ni t \rightarrow x(t) = \Psi_x\big(z(t), \dot{z}(t), \cdots, z^{(b)}(t)\big) \tag{10}$$

and

$$[0,T]\ni t \rightarrow u(t) = \Psi_u\big(z(t), \dot{z}(t), \cdots, z^{(b+1)}(t)\big) \tag{11}$$

IV. Flatness of the ARD model

This section is devoted to flatness checking of the longitudinal ARD model. The global model given in the section 2 is not flat. In (T. Neckel [13]) it has shown by using the ruled manifold criterion introduced by P. Rouchon [14], which is a necessary condition for a system to be flat, that the global model is not flat. Therefore, to circumvent this problem, it is useful to treat the dynamic model (1) as a longitudinal model and lateral model. So, it is very easy to check flatness of the two models separately.

In this note we consider only the longitudinal motion i.e., we neglect the equation of motion $\dot{\psi}$ describing the lateral motion and, under spherical planet hypothesis; we introduce a new variable X which describes the downrange position (the curvilinear X-coordinate along the projection of the trajectory on the ground).

By using these assumptions, we consider the following set of simplified differential equations governing the longitudinal motion of the ARD

$$\begin{aligned} \dot{R} &= V\sin\gamma \\ \dot{X} &= V\cos\gamma \\ \dot{V} &= -\frac{1}{2}\frac{\rho S V^2}{m}C_D(M) - g\sin\gamma \\ \dot{\gamma} &= \frac{1}{2}\frac{\rho S V^2}{mV}C_L(M)\cos\mu + \left(\frac{V}{R} - \frac{g}{V}\right)\cos\gamma \end{aligned} \tag{12}$$

The system is a nonlinear model with four states (R, X, V, γ) and two control inputs $C_D(M)$ and $\cos\mu$. The "flat" outputs of the system are given by the radial distance R and the downrange position X, which we denote by z_1 and z_2 respectively. So, the system inputs $C_D(M)$ and $\cos\mu$ and the system states V and γ are expressible in terms of the proposed flat outputs z_1 and z_2 and its first and second order time derivatives. Their expressions are given as follows

$$
\begin{aligned}
R &= z_1 \\
X &= z_2 \\
V &= \sqrt{\dot{z}_1^{\,2} + \dot{z}_2^{\,2}} \\
\gamma &= \arctan \frac{\dot{z}_1}{\dot{z}_2}
\end{aligned}
\tag{13}
$$

It is convenient in the sequel, as shown in [13], to solve for the derivatives $\dot{v}$ and $\dot{\gamma}$.

$$
\begin{aligned}
\dot{V} &= \frac{\ddot{z}_1 \dot{z}_1 + \ddot{z}_2 \dot{z}_2}{\sqrt{\dot{z}_1^{\,2} + \dot{z}_2^{\,2}}} \\
\dot{\gamma} &= \frac{\ddot{z}_1 \dot{z}_1 + \ddot{z}_2 \dot{z}_2}{\dot{z}_2^{\,2} \left(1 + \left(\frac{\dot{z}_2}{\dot{z}_1} \right)^2 \right)}
\end{aligned}
\tag{14}
$$

The drag coefficient C_D and the bank angle $\cos \mu$ can be written as follows

$$
\begin{aligned}
C_D &= \frac{2\,m\,(-\dot{V} - g \sin \gamma)}{\rho V^2 S} \\
\cos \mu &= \frac{2m}{\rho S V (L/D) C_D} \left(\dot{\gamma} - \left(\frac{V}{R} - \frac{g}{V} \right) \cos \gamma \right)
\end{aligned}
\tag{15}
$$

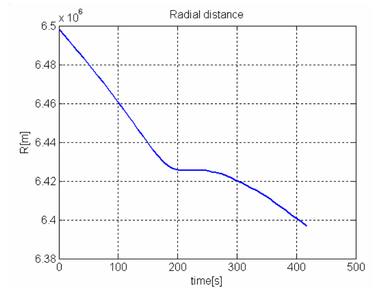
with

$$
\begin{aligned}
\rho &= \rho_0 \exp(-(z_1 - R_e)/h_{ref}) \\
g &= \mu_T / z_1^{\,2}
\end{aligned}
\tag{16}
$$

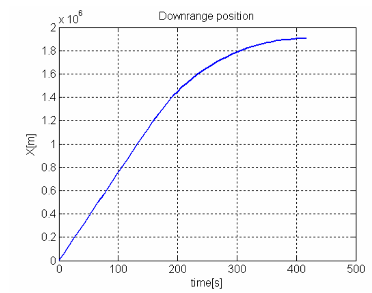
Remark: Note that in order to ensure lateral control authority, several bank reversals must be planned along the trajectory i.e., the sign of the bank angle is reversed. Currently one of our research activities on flatness property of the global ARD dynamics consists in an integration of hidden control input included in the control loop as yaw reference input. This approach increases the number of input and reduces the constraint on flatness property. However this approach does not respect the standard decoupling between guidance loop and control loop.

V. Trajectory Generation

This section is devoted to use the admissible reference trajectory of the flat outputs $z_{1d}(t)$ and $z_{2d}(t)$, depicted in figure 3, in order to generate the desired ARD states and controls. Note that the reference trajectory $z_{1d}(t)$ and $z_{2d}(t)$ are assumed to a priori known. Here, they are obtained using a simulator of ARD developed in a MATLAB®/SIMULINK environment.



(a)



(b)

Fig. 3. (a) Radial distance reference trajectory profile, (b) Downrange position reference trajectory profile.

In order to generate the reference states (V_d, γ_d) and the nominal controls (C_{Dd}, $\cos \mu_d$) trajectory we use only the desired trajectory of the flat coordinates and then, by using the equations (13) and (15), we obtain the desired trajectories shown in figure 4 and figure 5 and written as follows

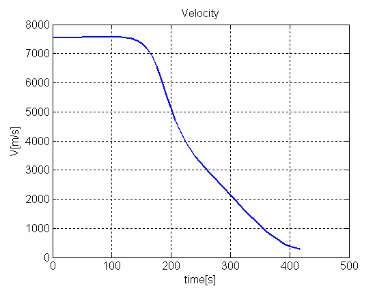
$$
\begin{aligned}
R_d &= z_{1d} \\
X_d &= z_{2d} \\
V_d &= \sqrt{\dot{z}_{1d}^{\,2} + \dot{z}_{2d}^{\,2}} \\
\gamma_d &= \arctan \frac{\dot{z}_{1d}}{\dot{z}_{2d}}
\end{aligned}
\tag{17}
$$

and

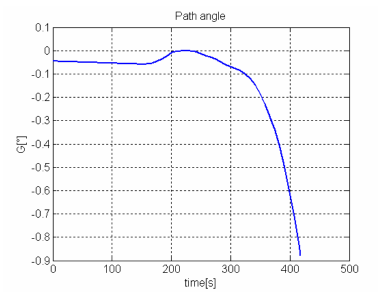
$$
\begin{aligned}
C_{Dd} &= \frac{2m(-\dot{V}_d - g \sin \gamma_d)}{\rho_d V_d^{\,2} S} \\
\cos \mu_d &= \frac{2m}{\rho_d S V_d (L/D) C_{D_d}} \left(\dot{\gamma}_d - \left(\frac{V_d}{R_d} - \frac{g_d}{V_d} \right) \cos \gamma_d \right)
\end{aligned}
\tag{18}
$$

such that

$$
\begin{aligned}
\dot{v}_d &= \frac{\ddot{z}_{1d} \dot{z}_{1d} + \ddot{z}_{2d} \dot{z}_{2d}}{\sqrt{\dot{z}_{1d}^{\,2} + \dot{z}_{2d}^{\,2}}}, \qquad
\dot{\gamma}_d = \frac{\ddot{z}_{1d} \dot{z}_{1d} + \ddot{z}_{2d} \dot{z}_{2d}}{\dot{z}_{2d}^{\,2} \left(1 + \left(\frac{\dot{z}_{2d}}{\dot{z}_{1d}} \right)^2 \right)}, \\
\rho_d &= \rho_0 \exp(-(z_{1_d} - R_e)/h_{ref}), \qquad g_d = \mu_T / z_{1_d}^{\,2}
\end{aligned}
\tag{19}
$$

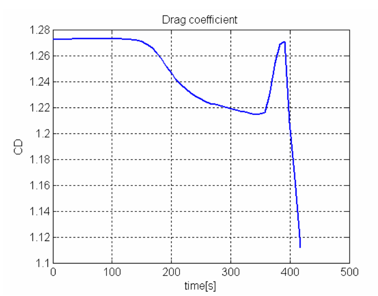


(a)

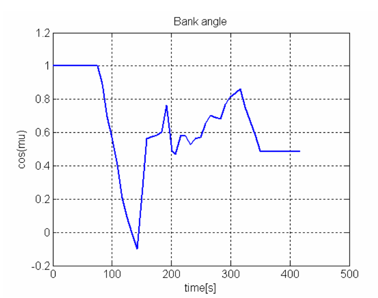


(b)

Fig. 4. (a) Velocity desired trajectory profile, (b) Path angle desired trajectory profile.



(a)



(b)

Fig. 5. (a) Drag desired trajectory profile, (b) Bank angle desired trajectory profile.

VI. LPV MODELING OF THE ARD LOGITUDINAL MODEL

The goal of this section is to describe the parameterization of the ARD flat model as an LPV model.

Let us consider the following coordinate transformation of the flat outputs

$$\xi(t)=\left[z_1(t), \dot{z}_1(t), z_2(t), \dot{z}_2(t)\right]^T=\left[\xi_{11}(t), \xi_{12}(t), \xi_{21}(t), \xi_{22}(t)\right]^T \quad (20)$$

which permits the rewriting of the nonlinear flat system (12) under the following canonical form (see, e. g. [15] and reference therein for more details)

$$\begin{aligned}
&\dot{\xi}_{11}(t)=\xi_{12}(t)\\
&\dot{\xi}_{12}(t)=\left[-\frac{1}{2m}\rho S V^2 C_D-g\sin\gamma\right]\sin\gamma+\\
&\qquad V\left[\frac{1}{2m}\rho S V^2 (L/D) C_D\cos\mu+\left(\frac{V}{R}-\frac{g}{V}\right)\cos\gamma\right]\cos\gamma\\
&\dot{\xi}_{21}(t)=\xi_{22}(t)\\
&\dot{\xi}_{22}(t)=\left[-\frac{1}{2m}\rho S V^2 C_D-g\sin\gamma\right]\cos\gamma-\\
&\qquad V\left[\frac{1}{2m}\rho S V^2 (L/D) C_D\cos\mu+\left(\frac{V}{R}-\frac{g}{V}\right)\cos\gamma\right]\sin\gamma
\end{aligned} \quad (21)$$

Let $\xi_{11d}(t)$, $\xi_{21d}(t)$, $\xi_{21d}(t)$, $\xi_{22d}(t)$ are the desired trajectory of the new states $\xi_{11}(t)$, $\xi_{21}(t)$, $\xi_{21}(t)$, $\xi_{22}(t)$.

The dynamics of tracking errors ($e_{11}(t)$, $e_{12}(t)$, $e_{21}(t)$, $e_{22}(t)$) is defined as the difference between the actual and the desired trajectories.

$$\begin{aligned}
e_{11}(t)&=\xi_{11}(t)-\xi_{11d}(t)\\
e_{12}(t)&=\xi_{12}(t)-\xi_{12d}(t)\\
e_{21}(t)&=\xi_{21}(t)-\xi_{22d}(t)\\
e_{22}(t)&=\xi_{22}(t)-\xi_{22d}(t)
\end{aligned} \quad (22)$$

The velocity of the error vector $(\dot{e}_{11}(t), \dot{e}_{12}(t), \dot{e}_{21}(t), \dot{e}_{22}(t))$ is described as:

$$\begin{aligned}
\dot{e}_{11}(t)&=e_{12}(t)\\
\dot{e}_{12}(t)&=\dot{\xi}_{12}(t)-\dot{\xi}_{12d}(t)\\
\dot{e}_{21}(t)&=e_{22}(t)\\
\dot{e}_{22}(t)&=\dot{\xi}_{22}(t)-\dot{\xi}_{22d}(t)
\end{aligned} \quad (23)$$

A first order linearization of (23) around the desired trajectories $\xi_{11d}(t)$, $\xi_{21d}(t)$, $\xi_{21d}(t)$, $\xi_{22d}(t)$, $C_{Dd}(t)$ and cos $\mu_d(t)$ is performed. It is useful to use software that supports mathematical symbolical operations (for instance MAPLE). We obtain the LPV system which can be described in state-space form as

$$\begin{pmatrix}\delta\dot{e}_{11}(t)\\ \delta\dot{e}_{12}(t)\\ \delta\dot{e}_{21}(t)\\ \delta\dot{e}_{22}(t)\\ \hline \delta\Delta y_1(t)\\ \delta\Delta y_2(t)\end{pmatrix}=\left(\begin{array}{cccc|cc}0&1&0&0&0&0\\ A_{21}(t)&A_{22}(t)&0&A_{24}(t)&B_{41}(t)&B_{42}(t)\\ 0&0&0&1&0&0\\ A_{41}(t)&A_{42}(t)&0&A_{44}(t)&B_{41}(t)&B_{42}(t)\\ \hline 1&0&0&0&0&0\\ 0&0&1&0&0&0\end{array}\right)\begin{pmatrix}\delta e_{11}(t)\\ \delta e_{12}(t)\\ \delta e_{21}(t)\\ \delta e_{22}(t)\\ \hline \delta\Delta u_1(t)\\ \delta\Delta u_2(t)\end{pmatrix} \quad (24)$$

where

$$A_{12}(t)=\frac{\partial\delta\ddot{e}_{12}(t)}{\partial\delta e_{11}(t)},\ A_{22}(t)=\frac{\partial\delta\ddot{e}_{12}(t)}{\partial\delta e_{12}(t)},\ A_{23}(t)=\frac{\partial\delta\ddot{e}_{12}(t)}{\partial\delta e_{22}(t)},$$
$$A_{41}(t)=\frac{\partial\delta\ddot{e}_{22}(t)}{\partial\delta e_{11}(t)},\ A_{42}(t)=\frac{\partial\delta\ddot{e}_{22}(t)}{\partial\delta e_{12}(t)},\ A_{43}(t)=\frac{\partial\delta\ddot{e}_{22}(t)}{\partial\delta e_{22}(t)}, \tag{25}$$
$$B_{21}(t)=\frac{\partial\delta\ddot{e}_{12}(t)}{\partial\delta\Delta u_1(t)},\ B_{22}(t)=\frac{\partial\delta\ddot{e}_{12}(t)}{\partial\delta\Delta u_2(t)},\ B_{41}(t)=\frac{\partial\delta\ddot{e}_{22}(t)}{\partial\delta\Delta u_1(t)},$$
$$B_{42}(t)=\frac{\partial\delta\ddot{e}_{22}(t)}{\partial\delta\Delta u_2(t)}$$

with ($A_{21}(t)$, $A_{22}(t)$, $A_{24}(t)$, $A_{41}(t)$, $A_{42}(t)$, $A_{44}(t)$, $B_{21}(t)$, $B_{22}(t)$, $B_{41}(t)$, $B_{42}(t)$) are measured in real-time and consists the following vector of real-time varying parameters

$$\theta(t)=(\theta_1(t),\ \theta_2(t),\ \theta_3(t),\ \theta_4(t),\ \theta_5(t),\\ \theta_6(t),\ \theta_7(t),\ \theta_8(t),\ \theta_9(t),\ \theta_{10}(t)) \tag{26}$$

For instance $\theta(t)$ can be represented as

$$\theta_i(t)=\theta_{i0}+\delta\theta(t)\theta_{i1},\quad -1\le\delta\theta(t)\le+1,\quad i=1,\ldots,10 \tag{27}$$

where

$$\theta_{i0}=\frac{\theta_{i\max}+\theta_{i\min}}{2},\quad \theta_{i1}=\frac{\theta_{i\max}-\theta_{i\min}}{2} \tag{28}$$

Once the LPV model is formed, there is more than one synthesis technique for designing an LPV controller. Currently there are two predominant synthesis techniques, Linear Quadratic Gaussian (LQG) control design [8] and H_∞ control design [7]. The design of the corresponding LPV controller is a topic of our current work.

VII. Conclusion

This paper has developed a general framework of ARD entry guidance problem using flatness property. The results can be summarized as follows. Essentially, it has been shown how to construct an error system which describes the dynamics of the tracking error by using coordinate transformation. Then, it was shown that the longitudinal ARD model which has a differentially flatness properties can be extended, by linearization of the error model along nominal trajectory, to a LPV model for which LPV synthesis technique may be used to design the LPV controller that tracks the admissible input trajectories of the longitudinal ARD model.

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