

Integrated Structure and Control Design of Modular Tensegrities

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Abstract—This paper demonstrates a method for prestress optimization of tensegrity structures resulting in their optimal mixed dynamic and control performance. Prestress of a structure is parameterized using force-density variables that appear linearly in its linearized dynamic model. A feasible region for these parameters is defined in terms of the extreme directions of the prestress cone. A numerical method is proposed for computing this particular basis for the structure prestress cone. The problem is solved using a gradient method based on the sensitivity analysis. A numerical example of a cantilevered planar tensegrity beam is shown.

Keywords: Tensegrity Structures, Control, Optimization, Prestress

I. INTRODUCTION

Tensegrity structures are prestressable, stable, truss-like systems involving string elements. They were originally introduced and used as artistic forms. The renewed interest in their engineering applications has been generated by their favorable properties that include reconfigurability and structural efficiency. Because of the presence of the controllable string elements, tensegrities are suitable for applications in shape morphing structures capable of undergoing large shape changes. At the same time, utilization of the materials specialized for tensile loads, that have superior mechanical properties over materials available for compressive elements, yields lightweight structures. Related engineering problems have been topics of recent studies.

A majority of the published tensegrity research concerns form-finding and rigidity problems, [1], [2], [3], [4], [5]. The methods for optimal structural design of tensegrity structures have been proposed recently, [6], [7]. Several different dynamic models of tensegrities have been derived, e.g., [8], [9], and a number of control strategies are available, e.g., [10], [11], [12], [13], [14], [15], [16].

Traditionally, the structure and control designs have been two independent engineering problems. Suitability of structures for latter control implementation is usually ignored during their design phase which yields structures with severe performance limitations. Although designing structures and their control jointly has a clear advantage, mathematical tools for solving this problem are of a limited scope. There has been a number of results available on this subject, e.g. [17], [18], [19], [20], [21], [22]. Because of the non-convex nature of the problem the available methods for joint structure and control optimization provide at most a convergence to a local minimum.

In this paper, we seek the distribution of the structure prestress that yields the optimal LQR performance of the system for a given nodal configuration, support conditions and load scenario. We use the fact that the prestress parameters appear linearly in the linearized dynamic model to analytically define problem gradients using sensitivity analysis.

The paper is outlined as follows. We define tensegrity equilibrium conditions and other preliminary notions in Section 2. The linearized dynamic model is presented in Section 3 along with the definition of the feasible set of the design parameters. In Section 4, the optimization problem is defined first. This section is concluded with a numerical example. Conclusions are given in Section 5.

II. PRELIMINARY NOTIONS—TENSEGRITY EQUILIBRIUM CONDITIONS AND CONSTITUTIVE EQUATIONS

Let the set \mathbb{N} of n_n nodes of a tensegrity structure be given, and let the node $\nu_j \in \mathbb{N}$ be located at the position defined by the nodal vector $\mathbf{p}_j \in \mathbb{R}^3$. Denote the set of all nodal vectors \mathbb{P} .

Let the set \mathbb{E} of n_e elements of the structure be given, and let \mathbb{E}_s and \mathbb{E}_b denote its partitions into the sets of n_s strings and n_b bars respectively. The scalar z_i in the definition of the element $e_i = \{[\nu_j, \nu_k], z_i\} \in \mathbb{E}$ that connects the nodes ν_j and ν_k of the tensegrity, identifies the type of the element so that,

$$z_i = \begin{cases} 1, & e_i \in \mathbb{E}_s, \\ -1, & e_i \in \mathbb{E}_b. \end{cases} \quad (1)$$

We define an element vector, $\mathbf{g}_i \in \mathbb{R}^3$, as a vector along the length of an element $e_i = \{[\nu_k, \nu_j], z_i\}$. It emanates from the first node ν_k and terminates at the second node ν_j of the element, i.e.,

$$\mathbf{g}_i = \mathbf{p}_j - \mathbf{p}_k.$$

It is obvious that magnitude of an element vector \mathbf{g}_i is equal to its length $\|\mathbf{g}_i\|$, which is denoted by l_i .

We define now the element force vector $\mathbf{f}_{ji} \in \mathbb{R}^3$ that represents the contribution of the internal force of the element e_i , to the total nodal force at the node ν_j as,

$$\mathbf{f}_{ji} = c_{ji}(\lambda_i + \lambda_i^d)\mathbf{g}_i, \quad f_i = \lambda_i\|\mathbf{g}_i\| = \lambda_i l_i, \quad (2)$$

where the element elastic force density λ_i and the damping force density λ_i^d are scalars. Obviously scalars c_{ji} in (2) that are defined as typical elements of the matrix $C(\mathbb{E}) \in \mathbb{R}^{n_n \times n_e}$, have one of the three possible values, $c_{ji} = \pm 1$ or $c_{ji} = 0$.

Let the vector space of vectors $\mathbf{x} \in \mathbb{R}_m^n$ be defined as follows,

$$\mathbf{x}^T = [\mathbf{x}_1^T \ \dots \ \mathbf{x}_n^T], \quad \mathbf{x}_i \in \mathbb{R}^m, \quad \mathbb{R}^m = \mathbb{R}_1^m. \quad (3)$$

Vector of nodal vectors $\mathbf{p} \in \mathbb{R}_3^{n_s}$, vector of element vectors $\mathbf{g}(\mathbb{E}, \mathbb{P}) \in \mathbb{R}_3^{n_e}$, vector of force densities $\boldsymbol{\lambda} \in \mathbb{R}^{n_e}$ and vector $\mathbf{z} \in \mathbb{R}^{n_e}$ are formed by collecting all node vectors \mathbf{p}_i , element vectors \mathbf{g}_i , force densities λ_i and all individual element type identifiers.

Let the member-node incidence matrix of the oriented graph associated with \mathbb{E} be denoted $M(\mathbb{E}) \in \mathbb{R}^{n_e \times n_n}$ and let $\mathbf{M} \in \mathbb{R}^{3n_e \times 3n_n}$ be defined as $\mathbf{M} = M \otimes I_3$. The typical element m_{ij} of the matrix M is $m_{ij} = 1$ or $m_{ij} = -1$ if the element e_i terminates at or emanates from the node ν_j , otherwise $m_{ij} = 0$. Let the n_s string elements in \mathbb{E}_s be numbered first. Then, vector \mathbf{g} and matrix \mathbf{M} can be partitioned as follows,

$$\mathbf{g} = \begin{bmatrix} \mathbf{g}_s \\ \mathbf{g}_b \end{bmatrix} = \mathbf{M}\mathbf{p}, \quad \mathbf{M} = \begin{bmatrix} \mathbf{S}^T \\ \mathbf{B}^T \end{bmatrix}, \quad \mathbf{S} \in \mathbb{R}^{3n_n \times 3n_s}.$$

One can show that equilibrium conditions for the prestressed structure with properly loaded strings, and matrix $\mathbf{C} = \mathbf{C} \otimes I_3$ can be defined and written as,

$$\mathbf{C}\tilde{\mathbf{g}}\boldsymbol{\lambda} = 0, \quad \|\boldsymbol{\lambda}\| > 0, \quad \underline{\lambda}_i \geq 0, \quad e_i \in \mathbb{E}_s, \quad (4)$$

$$\mathbf{C} = \begin{bmatrix} -\mathbf{S} & \mathbf{B} \end{bmatrix}, \quad \mathbf{g} = \mathbf{M}\mathbf{p} = \begin{bmatrix} \mathbf{S}^T \\ \mathbf{B}^T \end{bmatrix} \mathbf{p}, \quad (5)$$

if the linear operator $\tilde{\cdot}$ acting on the vector $\mathbf{x} \in \mathbb{R}_m^n$ is defined as follows,

$$\tilde{\mathbf{x}} := \text{blockdiag}\{\mathbf{x}_1 \dots, \mathbf{x}_i, \dots, \mathbf{x}_n\} \in \mathbb{R}^{mn \times n}, \quad \mathbf{x}_i \in \mathbb{R}^m.$$

The relationship between the elastic force density variables and the structural parameters depends on the particular constitutive model. Let the elements of the structure be built from a linear elastic material, and let their mass, cross-sectional area, Young's modulus, rest lengths, and viscous damping coefficients be given in the following vectors, $\mathbf{m}, \mathbf{a}, \mathbf{y}, \mathbf{l}_0, \boldsymbol{\mu} \in \mathbb{R}^{n_e}$. Elastic force densities can then be computed from the Hooke's law,

$$\lambda_i(\mathbf{p}) = \begin{cases} z_i \frac{y_i a_i}{l_i(\mathbf{p}) l_{0_i}} (l_i(\mathbf{p}) - l_{0_i}), \\ 0, & \text{if } e_i \in \mathbb{E}_s \text{ and } l_i(\mathbf{p}) \leq l_{0_i}. \end{cases} \quad (6)$$

Similarly, damping force densities can be computed as,

$$\lambda_i^d(\mathbf{p}, \dot{\mathbf{p}}) = \begin{cases} z_i \frac{1}{l_i(\mathbf{p})} \mu_i \dot{l}_i(\mathbf{p}, \dot{\mathbf{p}}), \\ 0, & e_i \in \mathbb{E}_s \text{ and } l_i(\mathbf{p}) \leq l_{0_i}. \end{cases} \quad (7)$$

The non-smooth nonlinear relationships in (6) and (7) reflect the fact that the string elements cannot transmit a load when they are slack.

A more detailed explanation of tensegrity equilibrium equations (4-5) is given in [5].

III. A LUMPED MASS DYNAMIC MODEL OF A TENSEGRITY STRUCTURE

Assume that all string elements of a stationary tensegrity structure are taut when the structure is subjected to the prestress $\boldsymbol{\lambda}$ in the equilibrium configuration \mathbf{p} . The finite variations of the equilibrium variables \mathbf{p} and $\boldsymbol{\lambda}$ are denoted \mathbf{u} and $\Delta\boldsymbol{\lambda}$ respectively. Assume further that the set of independent nodal constraints are defined in the following form,

$$C_u \mathbf{u} = 0, \quad (8)$$

and that the SVD of the matrix C_u is,

$$C_u = U\Sigma V^T = U \begin{bmatrix} \Sigma_1 & 0 \end{bmatrix} \begin{bmatrix} V_1^T \\ V_2^T \end{bmatrix} = U\Sigma_1 V_1^T, \\ U U^T = U^T U = I, \quad V V^T = V^T V = I.$$

Hence, $\mathbf{u} = V_2 \underline{\mathbf{u}}$, and the linearized lumped mass model of the structure in the presence of the external nodal forces, given in the vector $\mathbf{w} \in \mathbb{R}_3^{n_s}$, can be defined as,

$$V_2^T \mathcal{M} V_2 \ddot{\underline{\mathbf{u}}} + V_2^T \mathcal{D} V_2 \dot{\underline{\mathbf{u}}} + V_2^T \mathcal{K}(\boldsymbol{\lambda}, \mathbf{p}, \mathbf{y}, \mathbf{a}) V_2 \underline{\mathbf{u}} = \\ = V_2^T \mathbf{C} \tilde{\mathbf{g}} \Delta\boldsymbol{\lambda} + V_2^T \mathbf{w}.$$

For more details on the derivation of this model of constrained structure dynamics consult [23]. Here, we give the final compact form of the mass, damping and stiffness matrices, \mathcal{M}, \mathcal{D} and \mathcal{K} ,

$$\mathcal{K}(\boldsymbol{\lambda}, \mathbf{p}, \mathbf{y}, \mathbf{a}) = \mathbf{C} \tilde{\mathbf{g}} \tilde{\mathbf{y}} \tilde{\mathbf{a}} \tilde{\mathbf{l}}^{-3} \tilde{\mathbf{g}}^T \mathbf{C}^T - \mathbf{C} \hat{\boldsymbol{\lambda}} \mathbf{M} \\ \mathcal{D} = \mathbf{C} \tilde{\mathbf{g}} \tilde{\mathbf{l}}^{-2} \tilde{\boldsymbol{\mu}} \tilde{\mathbf{g}}^T \mathbf{C}^T, \\ \mathcal{M} = \hat{\mathbf{m}}^n, \quad \mathbf{m}^n = 0.5 \text{abs}(\mathbf{C}) \mathbf{m},$$

A. State space model of the system

A parameterized state space form of the linear dynamic model of the structure is obtained after identifying, nodal displacements $\underline{\mathbf{u}}$ and nodal velocities $\dot{\underline{\mathbf{u}}}$ as the state variables, $\Delta\boldsymbol{\lambda}$ as the control input variables, and \mathbf{w} as the exogenous inputs,

$$\dot{\mathbf{x}} = A(\boldsymbol{\lambda}) \mathbf{x} + B \Delta\boldsymbol{\lambda} + B_w \mathbf{w}, \\ A = \begin{bmatrix} 0 & I \\ -\underline{\mathcal{M}}^{-1} \underline{\mathcal{K}}(\boldsymbol{\lambda}, \mathbf{p}, \mathbf{y}, \mathbf{a}) & -\underline{\mathcal{M}}^{-1} \underline{\mathcal{D}} \end{bmatrix}, \quad (9) \\ B = \begin{bmatrix} 0 \\ \underline{\mathcal{M}}^{-1} V_2^T \mathbf{C} \tilde{\mathbf{g}} \end{bmatrix}, \quad B_w = \begin{bmatrix} 0 \\ \underline{\mathcal{M}}^{-1} V_2^T \end{bmatrix}.$$

1) *Identifying the structure design freedom:* Recall that the prestress parameters, $\boldsymbol{\lambda}$, of the structure model in (9) represent equilibrium force densities. Any freedom in the model must comply with this constraint. Consider an equilibrium tensegrity structure that in the configuration \mathbf{p} has a multidimensional prestress cone with the basis $\Lambda(\mathbf{p})$, and let,

$$\boldsymbol{\lambda} = \Lambda(\mathbf{p}) \underline{\boldsymbol{\lambda}}. \quad (10)$$

Let the columns of the prestress cone basis, $\Lambda(\mathbf{p})$, represent the extreme directions of the tensegrity cone, so that every feasible force density vector, $\boldsymbol{\lambda} \in \Lambda(\mathbf{p})$, can be written as, $\boldsymbol{\lambda} = \Lambda(\mathbf{p}) \underline{\boldsymbol{\lambda}}$, for some choice of $\underline{\boldsymbol{\lambda}}$ with $\underline{\lambda}_i \geq 0$. The basis

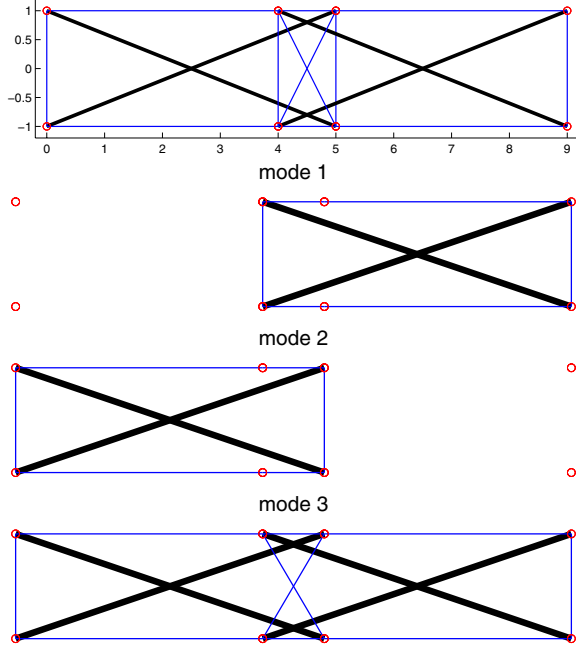


Fig. 1. Tensegrity structure with three prestress modes (top), and the three extreme direction of its prestress cone

for the prestress cone with this property can be computed applying an algorithm that is derived from an implementation of the simplex method for linear programming. This algorithm seeks and identifies all vertices of the linear constraint $Nx \geq 0$. Fig. 1 depicts an application of the algorithm on identifying the extreme directions of the prestress cone of the structure that has three prestress modes.

The prestress parameters, λ , must be additionally constrained to ensure that the actual elements of the structure can be built after defining the prestress of the structure. If the cross-sectional area of the elements, denoted \mathbf{a} , is given, and the material of the structure is fixed, by fixing its Young's modulus \mathbf{y} , this constraint must guarantee that the resulting element rest-lengths are feasible, that is $l_{0i} > 0$. Hence, from (6), force density variables must satisfy,

$$\lambda_i l_i < e_i a_i, \quad e_i \in \mathbb{E}_b. \quad (11)$$

Imposing a similar but more restrictive constraint,

$$\lambda_i < \epsilon \frac{e_i a_i}{l_i}, \quad 0 < \epsilon < 1, \quad (12)$$

on all elements of the structure ensures that the prestress of the elements does not exceed their yield stress, and constrains the structure from this mode of failure.

IV. THE DESIGN OF THE STRUCTURE FOR THE OPTIMAL LQR PERFORMANCE – OPTIMIZATION OVER THE PRESTRESS CONE

A. Formulation of the problem

We assume that only the bar elements are used for controlling a structure resulting in the following reduction

of the number of control variables,

$$\Delta \lambda = \begin{bmatrix} 0_{n_s \times n_b} \\ I_{n_b} \end{bmatrix} \Delta \lambda_b. \quad (13)$$

This and the fact that the bars can sustain both compression and tension eliminate the need for imposing additional saturation constraints on the control variables.

The assumption is made that the covariance matrix of external disturbance W , and the control and output variance penalty matrices $R \succ 0$ and $Q \succeq 0$ are given. The problem of the optimal selection of structure prestress for the optimal LQR performance can be cast in the following form:

$$\min_{\underline{\lambda}, G} Tr[(G^T R G + Q)X] \quad (14)$$

subject to

$$(A(\underline{\lambda}) + B G)X + X(A(\underline{\lambda}) + B G)^T + B_w W B_w^T = 0,$$

$$\underline{\lambda} \geq 0,$$

$$\lambda_i \leq \epsilon \frac{a_i y_i}{l_i},$$

$$\text{where } \lambda = \Lambda \underline{\lambda}$$

B. An algorithm for designing the structure for optimal LQR performance

The following algorithm provides a monotonic decrease of the cost function of the problem (14) if an initial guess for $\underline{\lambda}$ yields a stable matrix $A(\underline{\lambda})$.

- Step 1. Pick feasible $\underline{\lambda}$, $A(\underline{\lambda})$.
- Step 2. Solve the LQR problem for X and G ignoring the constraint on feasible $\underline{\lambda}$.
- Step 3. Compute the gradient \mathbf{grad} of $(G^T R G + Q)X$ with respect to $\underline{\lambda}$.
- Step 4. Compute the projected gradient $\mathbf{gradproj}$ with respect to constraint on feasible $\underline{\lambda}$.
- Step 5. Update $\underline{\lambda}$ and $A(\underline{\lambda})$ using the line search parameter α that yields a sufficient decrease of the objective, and satisfies that $\underline{\lambda} - \alpha \mathbf{gradproj}$ remains feasible.
- Step 6. Repeat Steps 2–5 until optimality conditions are satisfied.

1) Computing the gradient of the problem objective function:

Theorem 1: The sensitivity $\partial f(x^*(y))/\partial y$ of the following optimization problem

$$\begin{aligned} x^*(y) = \underset{x}{\text{arg min}} \quad & f(x) \\ \text{subject to} \quad & c(x, y) = 0 \end{aligned}$$

is

$$\frac{\partial f(x^*(y))}{\partial y} = \frac{\partial L}{\partial y} \Big|_{x=x^*, t=t^*} = t^{*T} \frac{\partial c(x, y)}{\partial y} \Big|_{x=x^*} \quad (15)$$

where t^{*T} is the vector of lagrange multipliers at the optimal point x^* .

Proof: Lagrangian L ,

$$L(x, y, t) = f(x) + t^T c(x, y),$$

of the problem must satisfy,

$$L(x^*(y), y, t^*(y)) = f(x^*(y)) + t^{*T}(y)c(x^*(y), y) = f(x^*(y)).$$

at any optimal point x^*, t^* . Differentiating the Lagrangian at the optimal point, x^*, t^* , yields,

$$\begin{aligned} \frac{\partial f(x^*(y))}{\partial y} &= \frac{\partial L(x^*(y), y, t^*(y))}{\partial y} = \\ &= \frac{\partial L}{\partial x} \Big|_{x=x^*, t=t^*} \frac{\partial x^*(y)}{\partial y} + \frac{\partial L}{\partial t} \Big|_{x=x^*, t=t^*} \frac{\partial t^*(y)}{\partial y} + \\ &\quad \frac{\partial L}{\partial y} \Big|_{x=x^*, t=t^*}, \end{aligned}$$

and finally, using the first order optimality conditions, $\frac{\partial L}{\partial x} \Big|_{x=x^*, t=t^*} = 0$, $\frac{\partial L}{\partial t} \Big|_{x=x^*, t=t^*} = 0$,

$$\frac{\partial f(x^*(y))}{\partial y} = \frac{\partial L}{\partial y} \Big|_{x=x^*, t=t^*} = t^{*T} \frac{\partial c(x, y)}{\partial y} \Big|_{x=x^*}.$$

Theorem 1 suggests a method for computing the gradient of the objective function. From the equation in (15), computation of the gradient involves the dual variables t^* . The dual problem associated with the LQR problem in the Step 2 of the solution algorithm is,

$$\min_{\underline{\lambda}, G} Tr[B_w W B_w^T T]$$

subject to

$$(A(\underline{\lambda}) + BG)^T T + T(A(\underline{\lambda}) + BG) + G^T R G + Q = 0,$$

so that, after finishing the execution of the Step 2, the dual variables in $T = T^T$ can be recovered by solving the Lyapunov equation,

$$(A(\underline{\lambda}) + BG)^T T + T(A(\underline{\lambda}) + BG) + G^T R G + Q = 0.$$

An equivalent form of the Theorem 1 for the problems involving matrices results in the following,

$$\frac{\partial f(G^*(\underline{\lambda}))}{\partial \underline{\lambda}} = \frac{\partial L^*}{\partial \underline{\lambda}},$$

where,

$$L^* = Tr[(G^T R G + Q)X] + Tr[T((A(\underline{\lambda}) + BG)X + X(A(\underline{\lambda}) + BG)^T + B_w W B_w^T)].$$

Using several properties of the trace operator, the gradient of the objective function can be defined as,

$$\mathbf{grad}_i = \frac{\partial L^*}{\partial \underline{\lambda}_i} = 2Tr[\mathbf{M}V_2[XT]_{1,2}\mathbf{M}^{-1}V_2^T \mathbf{C}\hat{\lambda}_i].$$

C. Example

D. Example

The planar tensegrity structure depicted in Figure 2 is optimized. The structure is supported at the two leftmost nodes. A vertical zero mean white noise disturbance of a unit variance acts at the top rightmost node. The initial distribution of the prestress is uniform, $\underline{\lambda}^T = [0.1 \ 0.1 \ 0.1]^T$, as depicted in upper structure of Fig. 2.

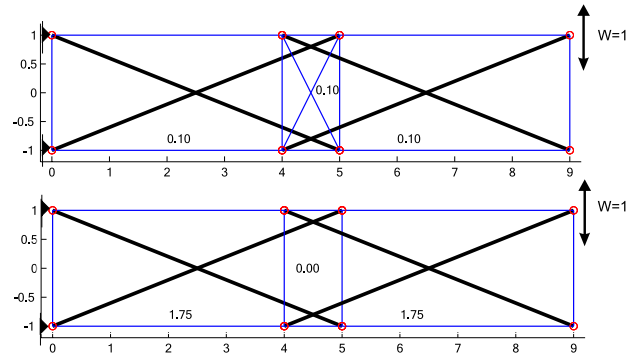


Fig. 2. Initial (up) vs. optimal (down) distribution of prestress in the tensegrity structure

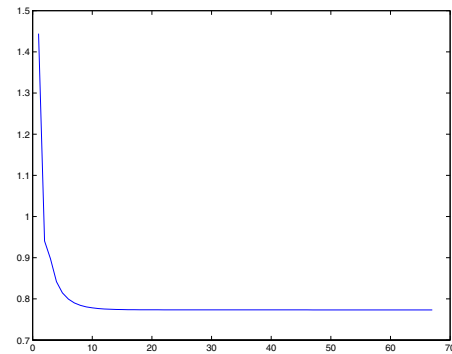


Fig. 3. Convergence of the algorithm-objective vs. iteration number

The outputs of interest are the nodal positions of all free nodes. The optimal distribution of the prestress is not uniform as indicated in the lower structure of Fig. 2. Only the elements that are prestressed are shown. Note that the two diagonal strings in the middle of the structure are slack since the corresponding prestress parameter is zero. The optimal prestress of all four bars is on its upper bound.

The effect of the optimal prestress on the open-loop plant is depicted in Fig. 4. This figure shows the vertical displacement of the top right node, and it is obvious that the optimal distribution of the prestress stiffens the plant. The closed loop behavior of the system is shown in Fig. 5 depicting the vertical displacement of the top right node. It is clear that the L_2 norm of the plant output is reduced compared to the initial plant for this particular choice of the penalty matrix $R = I$, and the matrix $Q = \text{blockdiag}\{I, 0\}$ that penalizes displacements of the free nodes but not their velocities. The same holds true for the control energy, although this is not a general rule independent from the choice of the matrices Q and R . Fig. 6 shows the control action of the controlled bar that is attached to the top right node. Clearly, the optimal LQR controller for the optimized plant requires less control energy than the initial optimal controller for the suboptimal plant.

V. CONCLUSIONS

This paper demonstrates a method for integrated plant and control design of tensegrity structures. A gradient based algorithm is proposed for solving the prestress selection problem. The objective gradient was computed analytically using the problem sensitivity analysis. There is no guarantee that the solution shown is a global optimum because of the non-convex nature of the problem. Nonetheless, the algorithm proposed guarantees a monotonic improvement of the performance of the system, and therefore can be considered a legitimate tensegrity design tool. One of the novelties introduced in this paper is the demonstration of the decomposition of the prestress of the structure into the extreme directions of the prestress cone.

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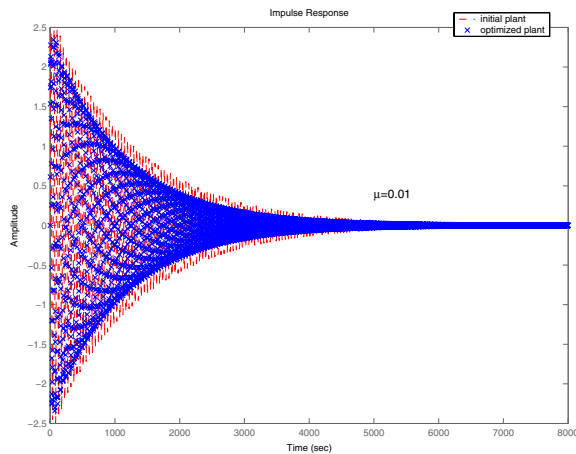


Fig. 4. Impulse response of the initial plant Vs. impulse response of the optimized plant – effect of the structure optimization on its dynamic performance

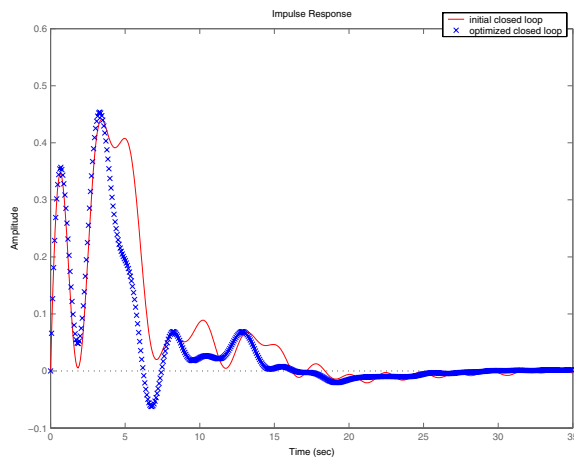


Fig. 5. Closed-loop impulse response of the initial plant Vs. the same response for the optimized plant

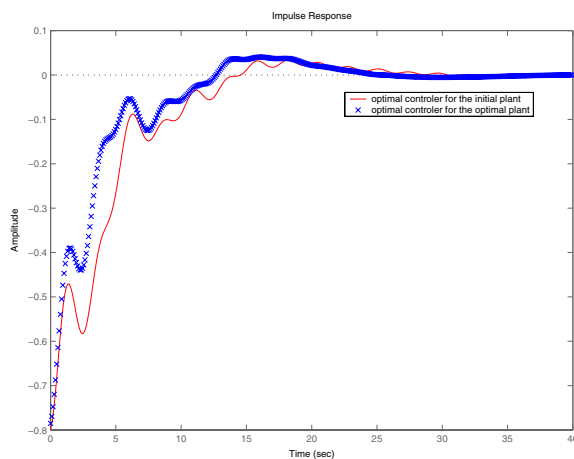


Fig. 6. Output of the controller for the initial and the optimized plant

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