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A Generalized Stochastic Petri Net Model for Management of Distributed Manufacturing Systems

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Abstract—**A Distributed Manufacturing System (DMS) is a collection of independent companies possessing complementary skills and integrated with transportation and storage systems. This paper deals with the issues of modeling and managing a DMS. The system is modeled as a timed discrete event dynamical system by generalized stochastic Petri nets. Moreover, two well known broad policies are considered to manage the DMS: make-to-stock and make-to-order. In order to compare the two management techniques and to show the effectiveness of each method, a case study is presented.**

I. Introduction

A Distributed Manufacturing System (DMS) is a collection of independent companies possessing complementary skills and integrated with transportation and storage systems, information and financial flows, with all entities collaborating to meet the market demand. In the related literature, distributed manufacturing systems are named Supply Chains (SC). Here, we prefer to generalize the structure considering a distributed manufacturing system that may not be arranged in a *chain*. Appropriate modeling and analysis of these highly complex systems are crucial for performance evaluation and comparison of competing DMS.

This paper deals with the problem of modeling and managing the DMS behavior. The analytical models of DMS or of SC can be classified in two types: models for network design and models for performance analysis [7]. A large amount of literature deals with the network design models, that are basically optimization models used to decide the structure of the system [5, 8]. On the other hand, few contributions face the problem of modeling the DMS to determine the performance analysis versus different management strategies. In [7] the system is modeled as a Discrete Event Dynamical System (DEDS), in which the evolution depends on the interaction of discrete events such as the arrival of components at the facilities (manufacturers or retailers), the departure of the transporters from the suppliers or the manufacturers, the start of assembly operations at the manufacturers, etc.

Generalized Stochastic Petri Nets (GSPN) model a particular example of SC and determine the decoupling

point location, i.e., the facility from which all finished goods are assembled after customer order confirmation. Moreover, in [4] a two product SC is modeled by complex-valued token Petri nets and the performance measures are determined by simulation.

The paper proposes a GSPN model describing in a modular way a generic DMS and to evaluate the system behavior under different management strategies. Moreover, we consider two types of broad policies to manage the DMS [6, 7]: *Make-To-Stock* (MTS) and *Make-To-Order* (MTO). More precisely, the MTS strategy manages the system by a push strategy, so that the end customers are satisfied from stocks of inventory of finished goods. On the other hand, in the MTO technique the customer orders trigger the flow of materials and the requirements at the different stages of the DMS. In this paper, we realize the MTO control strategy by using the just in time philosophy employing Kanbans. In order to compare the two management techniques, a case study is considered and some performance indices are computed, such as inventory costs and delay costs.

This paper is organized as follows. Section II describes the structure and the model of a DMS and Section III defines the performance measures obtained by the analysis of the GSPN. Moreover, Section IV introduces the case study and its performance measures are analyzed in Section V. Finally, Section VI summarizes the conclusions.

II. The System Model

A. The System Description

A DMS may be described as a set of facilities with materials that flow from the sources of raw materials to subassembly producers and onwards to manufacturers and consumers of finished products. The DMS facilities are connected by transporters of materials, semi-finished goods and finished products. More precisely, the entities of a DMS can be summarized as follows:

1- *Suppliers:* a supplier is a facility that provides raw materials, components, semi-finished products to manufacturers that make use of them.
2- *Manufacturers*: a manufacturer is a facility that transforms input raw materials/components into desired output products. It is the place where transformation processes occur.
3- *Logistics*: distributors and transporters play a critical role in distributed manufacturing. The attributes of logistics

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facilities are storage capacities, handling capacities, transportation times, operation and inventory costs.

4- *Customers*: (groups of) customers are sink nodes of material flows.

In the following we use the term *resources* only to refer to logistics entities such as transporters, buffers and warehouses. Let J be the set of final products that the DMS has to produce.

The dynamics of the production system is traced by the flow of products among resources. However, from a general point of view, we can model a DMS as a timed DEDS. In particular, we assume that an event of the DEDS occurs whenever a product acquires and/or releases a resource, starts or finishes an operation. Therefore, we consider the following events:

a) arrival of components at a supplier, a manufacturer and a customer;
b) departure of a truck from a supplier or a manufacturer;
c) start of assembling or transport operation;
d) start of a request.

Example 1: We consider a system, inspired by a case study presented in the related literature [8], where the product set $J=\{E, F\}$ is produced by the four stages depicted in Fig. 1. The first stage includes a number of component suppliers, the second stage is composed by subassembly manufacturers, the third stage is composed by brand manufacturers and the last stage is constituted by a set of retailers. Moreover, logistics service providers connect the DMS entities, that are located in different geographical sites. Here, we consider an example where two buyers order two brand of products (E and F). Such products are obtained by two manufacturers (M_3 and M_4) that are located in different sites. The two manufacturers assemble two types of products (C and D) obtained from two subassembly suppliers. The subassemblies C and D are in turn produced by the manufacturers of the second stage (M_1 and M_2), which receives the components of type A and B by the first supplier stage. Transporters connect the different stages and transfer material or product lots from a stage to another.
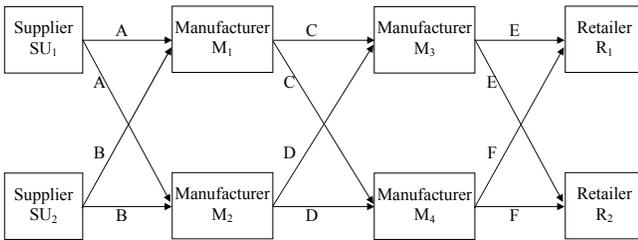

Fig. 1: The distributed manufacturing system configuration.

B. Introduction to Generalized Stochastic Petri Nets

A Generalized Stochastic (marked) Petri Net (GSPN) [1] is a bipartite digraph described by the seven-tuple $GSPN=(P, T, \textbf{Pre}, \textbf{Post}, F, S, \textbf{M})$, where P is a set of places, T is a set of transitions partitioned in the set T_I of immediate transitions and the set T_E of exponential transitions, $\textbf{Pre}$ and $\textbf{Post}$ are the pre-incidence and the post-incidence matrices, respectively, of dimension $|P| \times |T|$. Note that we use symbol $|A|$ to denote the cardinality of the generic set A. Moreover, $\textbf{F}$ is a firing time vector. The firing time of transition $t_j \in T_E$ is an exponentially distributed random variable with mean F_j (i.e., the j-th element of vector $\textbf{F}$). Each $t_j \in T_I$ has zero firing time, i.e., $F_j=0$. In addition, S is a set of elements called random switches, which associate probability distributions to subsets of conflicting immediate transitions. The state of a GSPN is given by its current marking, that is a mapping $\textbf{M}$: $P \to N$, where N is the set of non-negative integers. $\textbf{M}$ is described by a $|P|$-vector and the i-th component of $\textbf{M}$, indicated with $M(p_i)$, represents the number of tokens in the i-th place $p_i \in P$.

Given a GSPN and a transition $t \in T$, the following sets of places may be defined: $\bullet t=\{p \in P: Pre(p,t)>0\}$, named pre-set of t; $t \bullet=\{p \in P: Post(p,t)>0\}$, named post-set of t. A transition $t_j \in T$ is enabled at a marking $\textbf{M}$ if and only if for each $p_i \in \bullet t_j$, $M(p_i)>0$. When fired, t_j produces a new marking $\textbf{M'}$, denoted as $\textbf{M}[t_j>\textbf{M'}$, where for each $p_i \in P$ it holds $M'(p_i)=M(p_i)+Post(p_i,t_j)-Pre(p_i,t_j)$. A marking $\textbf{M'}$ is said reachable from $\textbf{M}$ if and only if there exists a sequence of transitions $t_1 t_2 \dots t_n$ such that $\textbf{M}[t_1>\textbf{M}_1[t_2>\textbf{M}_2 \dots \textbf{M}_{n-1}[t_n>\textbf{M'}$. The set of markings reachable from $\textbf{M}$ in the GSPN is denoted by $R(\textbf{M})$.

C. The DMS Model

We introduce the $GSPN=(P, T, \textbf{Pre}, \textbf{Post}, F, S, \textbf{M}_0)$ modeling the DMS as a GSPN, whose elements are defined as follows.

1) P is the set of places partitioned into three subsets: P_A (modeling activities such as transportation, storage in manufacturers or in warehouses), P_C (modeling the resource capacities) and P_K (modeling orders).

2) T is the set of transitions partitioned into two subsets: stochastic transitions T_E (modeling the activity durations, i.e., transportation and manufacturer operations) and immediate transitions T_I (modeling a resource acquisition or the start of an order).

In particular, each transportation stage is modeled by the subnet shown in Fig. 2 (solid lines): the immediate transition t models the transport beginning, the stochastic transition t' models the transport of the product, a token in $p \in P_A$ means that a lot is under transport.

Moreover, Fig. 3 (solid lines) shows the subnet modeling a manufacturer that is described by a stochastic transition (t), modeling the production of a lot of products, and two places ($p \in P_A$ and $p' \in P_A$): a token in p (p') represents a lot of material under operation (ready for transport), respectively.
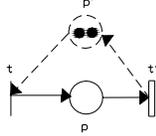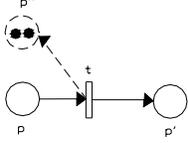
Fig. 2: The transportation stage model.

Fig. 3: The manufacturer model.　　Fig. 4: The customer model.

In addition, a customer is modeled by a place $p \in P_A$, where ready lots of products are stored, and an immediate transition t (see Fig. 4), modeling the start of a lot request. Note that a supplier can be simply modeled by the starting transition t of a transport.

After the identification of activity places and transitions, the net can be completed by adding capacity and order places and properly connecting them to the transitions. For example, when a transport is completed or a lot leaves a buffer, the corresponding resource is released and a token appears in the related transporters place. Figure 2 shows the transport subnet with two available transporters completed by the capacity place $p' \in P_A$ and two tokens (see dashed lines). Analogously, dashed lines in Fig. 3 show the capacity assigned to a manufacturer, i.e., an available inventory of two lots.

Obviously, the net structure determines the **Pre** and **Post** matrices. Moreover, elements of **F**, **S** and the initial marking **M₀** are defined in the case study (see Section IV).

III. ANALYSIS OF THE GSPN

The marking process of a $GSPN=(P, T, \textbf{Pre}, \textbf{Post}, \textbf{F}, S, \textbf{M}_0)$ can be shown to be a semi-Markov process with discrete state space, given by the reachability set $R(\textbf{M}_0)$ [9]. To study the Markov Chain (MC) embedded in the GSPN, we can examine the GSPN properties that are closely related to the existence of the steady-state probability distribution of the equivalent MC [3].

A. Computation of Performance Measures

From the performance evaluation point of view, it suffices to study the evolution of the MC tangible markings, i.e., markings in which only exponential transitions are enabled. Moreover, for a MC with K tangible states, the *limiting* or *steady-state* probabilities of the MC are given by π_k with $k=1 \ldots K$, which are independent of the initial state. The steady-state probabilities can be used to determine performance measures of the GSPN like throughput and mean waiting time as follows:

- the probability that a place p_i has exactly h tokens:

$$\mathrm{Prob}(p_i, h) = \sum_{k \in U_1} \pi_k , \qquad (1)$$

where $U_1 = \{k : k \in \{1,2,\ldots,K\} \text{ and } M_k(p_i)=h\}$;
- the expected number of tokens in a place p_i:

$$ET(p_i) = \sum_{h=1}^{H} h \, \mathrm{Prob}(p_i, h), \qquad (2)$$

where H is the maximum number of tokens that p_i may contain in any reachable marking;
- the throughput rate of an exponential transition t_j:

$$TR(t_j) = \sum_{k \in U_2} \pi_k F_j , \qquad (3)$$

where $U_2 = \{k : k \in \{1,2,\ldots,K\} \, t_j \text{ is enabled in } M_k\}$;
- the mean waiting time in a place p_i:

$$WAIT(p_i) = \frac{ET(p_i)}{\sum_{t_j \in p_i \bullet} TR(t_j)} . \qquad (4)$$

IV. THE CASE STUDY

In this section, we consider the DMS described in Example 1 and we manage the system using two types of broad strategies: MTO and MTS. The MTO technique can be considered as a pull strategy in which the customer orders trigger the flow of materials and the requirements at the different stages of the DMS. In particular, we realize this control strategy by using the just in time philosophy realized by Kanbans. However, in this context the transportation resources are very important and can significantly influence the system performance. Hence, we consider two different transport facilities: system S_1 exhibits only one logistics to manage transportation among the stages, while system S_2 considers different transport items devoted to transport each type of products. Figure 5 shows the GSPN modeling system S_1 and Fig. 6 shows the GSPN describing system S_2. Note that places p_3, p_4, p_{19}, p_{20}, p_{35} and p_{36} in Fig. 5 representing available transporters for the first, second and third stage respectively, are doubled by adding places p'_3, p'_4, p'_{19}, p'_{20}, p'_{35} and p'_{36} in Fig. 6, to model the available transporters for each type of products. The interpretations of places and of transitions are reported in Tables 1 and 2 respectively. In addition, Table 2 shows the mean firing times of the stochastic transitions (a zero firing time indicates that the transition is immediate). Both in Fig. 5 and 6 places p_{13}, p_{14}, p_{15} and p_{16}, p_{29}, p_{30}, p_{31} and p_{32}, p_{41}, p_{42}, p_{45} and p_{46} represent available Kanbans for the goods produced by the first, second and third stage, respectively. Furthermore, transitions t_{29}, t_{30}, t_{31} and t_{32} modeling order arrivals are immediate (see Table 2).

Fig. 5: GSPN model of system S_1.

Fig. 6: GSPN model of system S_2.

Fig. 7: GSPN model of system S_3.

TABLE 1
PLACE INTERPRETATION IN THE GSPN OF FIG. 5, 6 AND 7.

Name	Description
p_1 (p_5)	A lot of product A is transported to M_1 (M_2)
p_2 (p_6)	A lot of product B is transported to M_2 (M_1)
p_3 (p_3')	Pool of available transporters to transfer a lot of A to M_1 (M_2)
p_4 (p_4')	Pool of available transporters to transfer a lot of B to M_1 (M_2)
p_7 (p_9)	Lots of product A in M_1 (M_2) ready for processing
p_8 (p_{10})	Lots of product B in M_2 (M_1) ready for processing
p_{11}	Lots of product C in M_1 ready for transportation
p_{12}	Lots of product D in M_2 ready for transportation
p_{13} (p_{14})	Kanbans available for product A and supplier SU_1 (SU_2)
p_{15} (p_{16})	Kanbans available for product B and supplier SU_2 (SU_1)
p_{17}	Transportation of a lot of product C to M_3
p_{18}	Transportation of a lot of product D to M_4
p_{19} (p_{19}')	Pool of available transporter to transfer a lot of C to M_3 (M_4)
p_{20} (p_{20}')	Pool of available transporter to transfer a lot of C to M_3 (M_4)
p_{21}	Transportation of a lot of product C to M_4
p_{22}	Transportation of a lot of product D to M_3
p_{23} (p_{25})	Lots of product C in M_3 (M_4) ready for assembling
p_{24} (p_{26})	Lots of product D in M_4 (M_3) ready for assembling
p_{27}	Lots of product E in M_3 ready for transportation
p_{28}	Lots of product F in M_4 ready for transportation
p_{29} (p_{31})	Kanbans available for product C and manufacturer M_1 (M_2)
p_{30} (p_{32})	Kanbans available for product D and manufacturer M_2 (M_1)
p_{33}	Transportation of a lot of product E to R_1
p_{34}	Transportation of a lot of product F to R_2
p_{35} (p_{35}')	Pool of available transporters to transfer a lot of E to R_1 (R_2)
p_{36}	Pool of available transporters to transfer a lot of F to R_1 (R_2)
p_{37}	Transportation of a lot of product E to R_2
p_{38}	Transportation of a lot of product F to R_1
p_{39}	Lots of product E in R_1 ready for sale
p_{40}	Lots of product F in R_2 ready for sale
p_{41}	Kanbans available for manufacturer M_3
p_{42}	Kanbans available for manufacturer M_4
p_{43}	Lots of product E in R_2 ready for sale
p_{44}	Lots of product F in R_1 ready for sale
p_{45}	Kanbans available for manufacturer M_4
p_{46}	Kanbans available for manufacturer M_3

TABLE 2
TRANSITION INTERPRETATION IN THE GSPN OF FIG. 5, 6 AND 7.

Name	Description	Firing time [h]
t_1 (t_3)	Start of transport of a lot of product A to M_1 (M_2)	0
t_2 (t_4)	Start of transport of a lot of product B to M_2 (M_1)	0
t_5	Arrival of a lot of product A in M_1	10.00
t_6	Arrival of a lot of product B in M_2	11.70
t_7	Arrival of a lot of product A in M_2	8.30
t_8	Arrival of a lot of product B in M_1	6.67
t_9	End of production of a lot of product C	3.00
t_{10}	End of production of a lot of product D	4.00
t_{11} (t_{13})	Start of transport of a lot of product C to M_3 (M_4)	0
t_{12} (t_{14})	Start of transport of a lot of product D to M_4 (M_3)	0
t_{15}	Arrival of a lot of product C in M_3	16.70
t_{16}	Arrival of a lot of product D in M_4	1.90
t_{17}	Arrival of a lot of product C in M_4	2.20
t_{18}	Arrival of a lot of product D in M_3	14.20
t_{19}	End of production of a lot of product E	2.00
t_{20}	End of production of a lot of product F	1.50
t_{21} (t_{23})	Start of transport of a lot of product E to R_1 (R_2)	0
t_{22} (t_{24})	Start of transport of a lot of product F to R_2 (R_1)	0
t_{25}	Arrival of a lot of product E in R_1	1.70
t_{26}	Arrival of a lot of product F in R_2	1.33
t_{27}	Arrival of a lot of product E in R_2	2.50
t_{28}	Arrival of a lot of product F in R_1	5.80
t_{29} (t_{32})	Request of a lot of product E from R_1 (R_2)	0
t_{30} (t_{31})	Request of a lot of product F from R_2 (R_1)	0

TABLE 3
INTERPRETATION OF SOME PLACES IN THE GSPN OF FIG. 7.

Name	Description
p'_7 (p_{15})	Capacity of inventory in M_1 (M_2) for product A
p_{16} ($p_{8'}$)	Capacity of inventory in M_1 (M_2) for product B
p_{13}	Capacity of inventory in M_1 for product C
p_{14}	Capacity of inventory in M_2 for product D
p_{32} (p'_{24})	Capacity of inventory in M_3 (M_4) for product D
p_{31} (p'_{23})	Capacity of inventory in M_4 (M_3) for product C
p_{29}	Capacity of inventory in M_3 for product E
p_{30}	Capacity of inventory in M_4 for product F
p_{41} (p_{46})	Capacity of inventory in R_1 (R_2) for product E
p_{45} (p_{42})	Capacity of inventory in R_1 (R_2) for product F

The MTS strategy manages the system in a push perspective, so that end customers are satisfied from stocks of inventory of finished goods. In such a case, in order to limit the input of raw materials the capacities of the inventories are introduced in each facility. Figure 7 shows the GSPN describing Example 1 under the MTS strategy (i.e., system S_3) and the relative place and transition interpretations are respectively listed in Tables 1 and 2. In addition, Table 3 reports the interpretation of the resource capacity places of Fig. 7 that are not present in Table 1 or have a different meaning for system S_3. It is evident that the system under the MTS strategy exhibits limited buffer capacities for all the products in order to bound the production. Moreover, the elements of S for all systems S_1, S_2 and S_3 associate a uniform probability distribution to the subsets of conflicting immediate transitions. Finally, the initial marking M_0 of each system is defined as follows: $M_0(p_i)=0$ if $p_i \in P_A$, $M_0(p_i)=C_i$ if $p_i \in P_C$ and $M_0(p_i)=K_i$ if $p_i \in P_K$, where C_i and K_i denote the capacities and the number of available Kanbans, respectively. For the sake of simplicity, the initial marking of systems S_1, S_2 and S_3 is omitted in the corresponding Figs. 5, 6 and 7.

V. THE PERFORMANCE MEASURES

The introduced DMS models based on GSPN are solved as MC using the SPNP software package [2]. The selected performance measures are the total inventory cost C_I and the total delay cost C_D. The first performance index accounts for costs related to stocks, while the second collects costs due to delays in deliveries. Clearly, such indices are complementary to each other: in a MTS strategy the first index is prevalent, while a MTO strategy is characterized by the second measure being predominant. Obviously, the total cost associated to the DMS operation is: $C=C_I+C_D$. In addition, we call I_1 the unitary cost for inventories at the first stage of the DMS, while the cost per hour of delayed delivery at this stage is D_1. We consider two cases: $D_1=1.5I_1$ and $D_1=40I_1$ [7]. Moreover, product E is valued more than product F by 50%. In addition, the inventory in each stage is assumed to be 20% more expensive than stocks in the upstream stage. Furthermore, without loss of generality we assume $I_1=1$.

The SPNP package determines the average number of tokens in all the places of the considered GSPN and the

throughput rate of the system stochastic transitions. Hence, with the introduced data we determine the total inventory costs associated with the GSPN in systems S_1, S_2 and S_3 (Fig. 5, 6 and 7, respectively) as follows:

$$C_I = I_1 \sum_{i=7,8,9,10} ET(p_i) +$$
$$+1.2 \cdot I_1 \sum_{i=11,12,23,24,25,26} ET(p_i) + 1.44 \cdot I_1 \sum_{i=27,28} ET(p_i) \quad ,(5)$$

where the above expression accounts for costs of work in process during assembly and manufacturing operations (i.e., related to places p_7, p_8, p_9, p_{10}, p_{23}, p_{24}, p_{25} and p_{26}) and stocks in warehouses (i.e., related to places p_{11}, p_{12}, p_{27} and p_{28}). Similarly, under the above assumptions the total delay costs associated to the GSPN are as follows:

$$C_D = D_1 \cdot \left(WAIT(p_{38}) + WAIT(p_{34}) \right) +$$
$$+1.5 \cdot D_1 \cdot \left(WAIT(p_{33}) + WAIT(p_{37}) \right) \quad , \quad (6)$$

where the previous expression refers to the different costs of the waiting times of finished goods E and F (i.e., related to places p_{33} and p_{37} for product E and to p_{38} and p_{34} for product F).

First, we compare systems S_1 and S_2, managed by the MTO policy, considering a fixed number of transporter capacities (i.e., $M_0(p_i)=1$ for each $p_i \in P_C$) and varying in each system the number of available Kanbans. More precisely, the defined costs are obtained for systems S_1 and S_2 in the cases 1, 2, 3 and 4 shown in Table 4. Moreover, in case 5 system S_1 has the number of available Kanbans equal to case 1 but a double number of transporters. Table 5 reports the performance measures obtained in each case and Fig. 8 compares system S_1 and S_2 for similar values of delay and inventory costs (i.e., $D_1=1.5I_1$). More precisely, Fig. 8 shows that if the transportation capacity is minimum and the number of Kanbans assigned to the first and second DMS stages (i.e., to facilities SU_1, SU_2, M_1 and M_2 in Fig. 1) increases, then total costs increase too. Indeed, in such a case the stock costs of the two stages rise. On the contrary, if the number of Kanbans assigned to manufacturers M_3 and M_4 increases, then the total costs suffer a reduction (basically due to reduced delay costs). Moreover, doubling the pool of transporters for each stage in system S_1, the costs increase for the higher inventory costs (compare the results of cases 1 and 5 for S_1 in Table 4 and in Fig. 8).

Second, to compare the MTO and MTS strategy we consider systems S_2 and S_3. More precisely, we compare the performance measures evaluated for system S_2 in cases 1, 2, 3 and 4 with the performance measures obtained for system S_3 respectively in cases 6, 7, 8 and 9 (see Table 4). In each case systems S_2 and S_3 are characterized by the same transporter capacities. However, system S_2 has a finite number of Kanbans in each stage to realize the MTO policy, and system S_3 exhibits a finite capacity of each

manufacturer, so that MTS does not lead the system to infinite inventory. The corresponding results for system S_3 are reported in Table 5. Comparing the MTO and MTS results with $D_1=1.5I_1$, it can be inferred that the MTO policy leads to lower delay costs. However, MTS is characterized by an increase in the inventory costs, that are limited in the Kanban system. Hence, the MTO strategy is able to reduce stocks (and their costs) while restraining delays, so that overall costs are lower than with MTS (see Fig. 9).

TABLE 4
CAPACITIES AND KANBANS OF THE GSPN OF FIG. 5, 6 AND 7.

Case	System	$K_{13}=K_{14}$	$K_{29}=K_{30}$	$K_{41}=K_{42}$	$C_3=C_4$	$C_{19}=C_{20}$	$C_{35}=C_{36}$
1	S_1, S_2	1	1	1	1	1	1
2	S_1, S_2	2	1	1	1	1	1
3	S_1, S_2	1	2	1	1	1	1
4	S_1, S_2	1	1	2	1	1	1
5	S_1	1	1	1	2	2	2
Case	System	$C_{23}=C_{24}=C_{29}=C_{30}$	$C_7=C_8=C_{13}=C_{14}$	$C_{41}=C_{42}$	$C_3=C_4$	$C_{19}=C_{20}$	$C_{35}=C_{36}$
6	S_3	1	1	1	1	1	1
7	S_3	2	1	1	1	1	1
8	S_3	1	2	1	1	1	1
9	S_3	1	1	2	1	1	1

TABLE 5
PERFORMANCE MEASURES FOR THE GSPN OF FIG. 5, 6 AND 7.

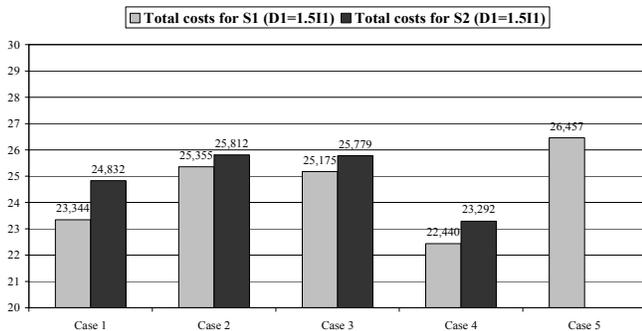| Case	System	Inventory costs	Delay costs	Total costs ($D_1=1.5I_1$)	Total costs ($D_1=40I_1$)
1	S_1	$3.166I_1$	$13.452D_1$	23.344	541.246
2	S_1	$4.895I_1$	$13.640D_1$	25.355	550.495
3	S_1	$4.647I_1$	$13.685D_1$	25.175	552.047
4	S_1	$3.166I_1$	$12.849D_1$	22.440	517.126
1	S_2	$4.636I_1$	$13.464D_1$	24.832	543.196
2	S_2	$5.613I_1$	$13.466D_1$	25.812	544.253
3	S_2	$5.575I_1$	$13.469D_1$	25.779	544.335
4	S_2	$3.090I_1$	$13.468D_1$	23.292	541.810
5	S_1	$6.282I_1$	$13.450D_1$	26.457	544.282
6	S_3	$4.712I_1$	$13.459D_1$	24.901	543.072
7	S_3	$7.809I_1$	$13.465D_1$	28.007	546.409
8	S_3	$6.868I_1$	$13.468D_1$	27.070	545.588
9	S_3	$3.907I_1$	$13.451D_1$	24.084	541.947

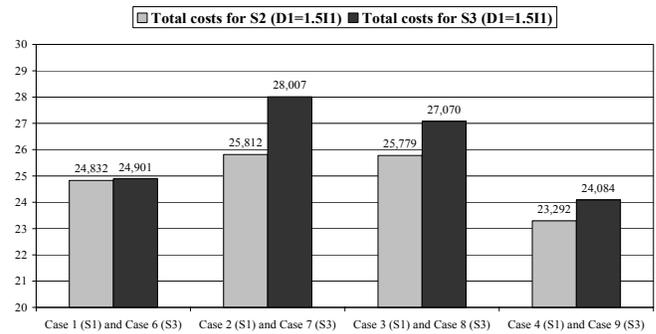Fig. 9: Total costs for systems S_2 and S_3 ($D_1=1.5I_1$).

Fig. 8: Total costs for systems S_1 and S_2 ($D_1=1.5I_1$).

VI. CONCLUSION

This paper presents a technique to model Distributed Manufacturing Systems (DMS) using Generalized Stochastic Petri Nets (GSPN). The modeling technique is illustrated via a case study, that is in turn controlled using a Make-To-Order (MTO) strategy employing the just in time philosophy based on Kanbans and the more traditional Make-To-Stock (MTS) technique. The considered DMS is solved using a GSPN software package and performance evaluation under the different management strategies is carried out. The obtained results show the ability of the MTO approach to reduce delay costs with respect to the MTS method.

The limit of the modeling technique is that discrete tokens can model lots of products. Consequently, transporters and manufacturers work on discrete amount of materials and products. To overcome this problem, future research could generalize the model using for instance continuous or hybrid Petri nets.

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