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Nonlinear Observer Design for an On-line Estimation of the Cerebrospinal Fluid Outflow Resistance

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Abstract. This paper suggests an on-line identification algorithm for estimating a cerebrospinal fluid outflow resistance - one important lumped parameter in the well-known finite-dimensional nonlinear model describing a behaviour of intracranial pressure of humans. The results of on-line tests of the presented algorithm on recorded experimental data are given.

I. INTRODUCTION

This note is devoted to the design of an on-line estimator for important lumped parameter – the so-called cerebrospinal fluid outflow resistance – of the well-known and well accepted nonlinear model describing the quantitative behaviour of intracranial pressure, see [1], [2], [3], [4] and others. Following this model the changes in the intracranial pressure P_{ic} are governed by the differential equation

$$\frac{d}{dt}P_{ic}(t) = -\frac{k}{R}\left[P_{ic}(t)\right]^2 + \left(kI_a(t) + \frac{kP_r}{R}\right)P_{ic}(t), \quad (1)$$

where k is a constant parameter - a compliance of the cerebrospinal fluid system - that could be regarded as fixed and known for each patient; P_r is a constant parameter - a resting pressure of the patient - regarded here as known; R is a constant parameter - an outflow resistance of absorption pathways regarded here as unknown; $I_a(t)$ is an additional inflow representing an active infusion of the cerebrospinal liquid (or its substitute) into the system.

One can always assume that the initial condition $P_{ic}(0)$ in (1) is positive and that[1]

$$P_{ic}(0) \geq P_r$$

Constant parameters k and P_r in (1) are not, in fact, known for any new patient, but their estimates are considered as available.

One of important issues in the current medical research related to *idiopathic normal-pressure hydrocephalus* disease [4], [5] is finding a reliable and fast method/algorithm for estimating a missing value of the parameter R - an outflow resistance in (1). Abnormal value of R of a patient is commonly followed by a shunt surgery that alters the hydrodynamical system and improves patients' condition [6].

Currently-in-use estimation schemes rely on (1) but have more or less off-line and static nature. A number of steady state conditions are achieved by a constant or controlled infusion of artificial cerebrospinal fluid for shaping constant pressure levels. Mean levels of the intracranial pressure P_{ic} and the additional inflow I_a on these levels are used for determination of R-estimate from the algebraic equation - the right hand side of (1) provided that another parameter P_r is already precomputed somehow.

Although outflow resistance is an accepted parameter for predicting clinical outcome after shunting, all current methods and systems for infusion tests have been regarded as technically difficult to operate and interpret. As a result they have not reached widespread use [7]. All available methods and algorithms lack in the important feedback to the investigator regarding the accuracy of the estimated parameters, both during investigation and in the subsequent analysis. It is believed that an adaptive regulation and optimization of the treatment time with respect to accuracy in the process of estimating outflow resistance, would greatly improve infusion tests.

The main contribution of this note is a new on-line identification algorithm that enables to up-date an estimate for the constant parameter R in real time.

The important feature of the proposed on-line procedure is that it is independent on particular form of inflow testing signal I_a used today for generating step-like intracranial pressure values[2], see for instance Fig. 3.

The result is presented in the form of a nonlinear observer developed for (1) and a nonlinear up-dating law for R-value parameter estimate. The local convergence of the estimate to the true value of R is proven. The theoretical result is further illustrated and verified by series of tests implemented on recorded experimental data and performed on a especially built testing equipment.

The paper is organized as follows. Section 2 contains the main result of the note, results of on-line estimation

[1] For values of P_{ic} less than the value of resting pressure P_r the model (1) is not valid.

[2] that is nontrivial problem in itself taking into account (a) constraints imposed on possible pattern for inflow-outflow signal I_a; (b) a measurement noise due to other physiological processes in the body like heart-beat, breathing etc.

for outflow resistance implemented on the experimental data are discussed in Section 3. Some conclusions are drawn in Section 4.

II. Main Result

Lemma 1: Consider a nonlinear system (1) and an observer

$$\begin{aligned}\frac{d}{dt}\hat{P} &= -\frac{k}{\hat{R}}\hat{P}^2 + \left(kI_a + \frac{kP_r}{\hat{R}}\frac{\hat{P}}{P_{ic}}\right)\hat{P} - \\ &\qquad -c\left(1-\frac{\hat{P}}{P_{ic}}\right)\hat{P}\end{aligned} \tag{2}$$

$$\frac{d}{dt}\hat{R} = \gamma\hat{R}^2\left(k\frac{P_r}{P_{ic}}-1\right)\left(\frac{1}{\hat{P}}-\frac{1}{P_{ic}}\right) \tag{3}$$

with γ, c being some constants. Suppose that the inflow signal $I_a(t)$ is a bounded continuous function

$$-K_1 \le I_a(t) \le K_2, \quad \forall\, t \ge 0$$

and such that there exist positive constants ε and $\triangle$ such that

$$\varepsilon \le \int_t^{t+\triangle} (P_{ic}(\tau)-P_r)^2 d\tau, \quad \forall\, t \ge 0 \tag{4}$$

If $\gamma > 0$ and c is chosen so that

$$c < kI_a(t) - \delta \quad \forall\, t \tag{5}$$

with some $\delta > 0$, then
1) The observer state $[\hat{P}(t),\, \hat{R}(t)]$ asymptotically converges to true values of the intracranial pressure $P_{ic}(t)$ and the outflow resistance R of the system (1) provided that the initial conditions $\hat{P}(0)$, $\hat{R}(0)$ for (2), (3) are chosen sufficiently close to $P_{ic}(0)$ and R;
2) For any initial conditions $[P_{ic}(0),\, \hat{P}(0),\, \hat{R}(0)]$, except those that belong to a particular 2-dimensional manifold in the phase space of the augmented system (1), (2), (3), along the corresponding solution of (1), (2), (3) the values of the observer state $[\hat{P}(t),\, \hat{R}(t)]$ asymptotically converges to true values of the intracranial pressure $P_{ic}(t)$ and the outflow resistance R of the system (1). This means that for almost any vector of initial conditions the solution of the system (1), (2), (3) gives asymptotically unbased estimate for the parameter R value. ■

Proof: Let us first observe that the equation (1) is the famous Bernoulli equation and can be rewritten as a linear one after the change of coordinate. Indeed, introducing formally new variable

$$x = 1/P_{ic},$$

one can rewrite (1) as follows

$$\begin{aligned}\frac{d}{dt}x &= -\frac{1}{P_{ic}^2}\frac{d}{dt}P_{ic} = \frac{k}{R} - \left(kI_a + \frac{kP_r}{R}\right)\frac{1}{P_{ic}} \\ &= \frac{k}{R} - \left(kI_a + \frac{kP_r}{R}\right)x\end{aligned} \tag{6}$$

Denote

$$\theta = \frac{1}{R}$$

and consider a standard observer for x and θ

$$\dot{\hat{x}} = -kI_a\hat{x} + \hat{\theta}\,(k - kP_r x) + c(\hat{x}-x) \tag{7}$$

$$\dot{\hat{\theta}} = v \tag{8}$$

where a constant c and a function v are variables to be chosen so that the error dynamics

$$\dot{e} = [c - kI_a]\, e + (k - kP_r x)\,\tilde{\theta} \tag{9}$$

$$\dot{\tilde{\theta}} = v \tag{10}$$

becomes asymptotically stable. Here

$$e = \hat{x} - x, \quad \tilde{\theta} = \hat{\theta} - \theta.$$

The time-derivative of a Lyapunov function candidate

$$V(e,\tilde{\theta}) = \frac{1}{2}e^2 + \frac{1}{2\gamma}\tilde{\theta}^2 \tag{11}$$

along a solution of the error system (9), (10) takes the form

$$\begin{aligned}\frac{d}{dt}V &= [c - kI_a]\,e^2 + \left[\frac{1}{\gamma}v + (k - kP_r x)e\right]\tilde{\theta} \\ &= [c - kI_a]\,e^2\end{aligned} \tag{12}$$

The last line is valid provided that the updating law for $\tilde{\theta}$ is chosen as follows

$$v = -\gamma(k - kP_r x)e = -\gamma(k - kP_r x)(\hat{x} - x). \tag{13}$$

If the value of the constant c chosen as in the relation (5), then the standard analysis (based on the Barbalat lemma) shows that

$$e(t) = \hat{x}(t) - x(t) \to 0 \text{ as } t \to +\infty.$$

To state the convergence of $\hat{\theta}$ to the true value θ, we need to check the *persistence of excitation* condition for the signal

$$1 - P_r x(t) = 1 - \frac{P_r}{P_{ic}(t)}$$

Its validity is equivalent to the assumption (4) implicitly imposed on inflow function $I_a(t)$.

Coming back to original coordinates $\hat{P}$, $\hat{R}$ and P_{ic} from $\hat{x}$, $\hat{\theta}$ and x respectively, one can observe that a singularity in $\hat{P}$ and $\hat{R}$ might happen when $\hat{x}$ and $\hat{\theta}$ get zero values. The same singularities in x and θ are not feasible because the values of pressure and the resistance are always positive and strictly separated from zero.

To avoid the singularity $\hat{P}$ and $\hat{R}$ from consideration, one can invoke the differential relation (12). It will basically say that if the initial conditions $\hat{P}(0)$, $\hat{R}(0)$ of the observer are chosen closed enough to minimum value of the Lyapunov function V, i.e. they are close enough to the true values $P_{ic}(0)$, R, then the sub-level set of the function V will never contain points with $\hat{x} = 0$ and $\hat{\theta} = 0$. The observer (7), (8) rewritten in $\hat{P}$ and $\hat{R}$ coordinates is given in (2), (3). This finishes the proof of the part 1 of Lemma.

To verify the statement given in the second part of Lemma, one needs to observe that the singularity in a solution of (6), (7), (8) and (13) rewritten in original coordinates P_{ic}, $\hat{P}$ and $\hat{R}$ can only occur when $\hat{x}$ and/or $\hat{\theta}$ get zero values. While in the coordinates x, $\hat{x}$, $\hat{\theta}$ this singularity in solutions does not exist and solutions are well-defined for all time instants. The solutions of (6), (7), (8) and (13), for which the component $\hat{x}(t)$ gets zero values for some time instant $t = T$, form a 2-dimensional submanifold in the 3-dimensional phase space of the system. The same is true for those solutions, for which the component $\hat{x}(t)$ gets zero values for some time instant $t = T$. The union of these two 2-dimensional submanifold describes to the set exceptional initial conditions, for which the observer state $[\hat{P}, \hat{R}]$ does not converge to the true $[P_{ic}(t), R]$. This finishes the proof of the second part of Lemma. ■

Remark 1: The first part of Lemma states local asymptotic stability of the error dynamics (9), (10), (13) for the suggested observer (2), (3) any for any realistic infusion policy I_a, provided that the persistent excitation condition (4) holds. The second part of Lemma complements the first one saying that the asymptotic stability of the error dynamics is almost global. The set of exceptional initial conditions, for which convergence of estimates to the true value is not guaranteed, is a 2-dimensional manifold in the 3-dimensional phase space of overall system. ■

III. Verification of an Estimation Method on Experimental Data

Here the description of experimental set-up and some results of estimation of the resistance on the recorded experimental data are given.

The developed apparatus for performing infusion tests included a PC for control and recording, two pressure transducers, and a peristaltic pump for infusion of artificial CSF. Data collection and communication between software and hardware were performed using two data acquisition cards, PCI-MIO16X50 and PCI-6503 (National Instruments, Inc., Austin, TX).

The software was developed in LabVIEW. The apparatus is at daily clinical use at the Department of Neurology, Umeå University Hospital. In this study a constant pressure level protocol for performing investigations were utilized (Fig. 3).

The experimental results presented in this note were performed on a physically built hydrodynamic model of the CSF system consisting of a cavity formed in plexiglass. The sketchy drawing of test-bed is shown on Fig. 1

The shape of the cavity corresponded to the compliance of the CSF system with a pressure volume index chosen to be 25.9 ml, that is

$$k = 1/(0.4343 \cdot 25.9) \approx 0.0889.$$

The resting pressure P_r was simulated by a continuous overflow container placed at a level of 1.5 kPa, i.e.

$$P_r \approx 1.5 \ (kPa)$$

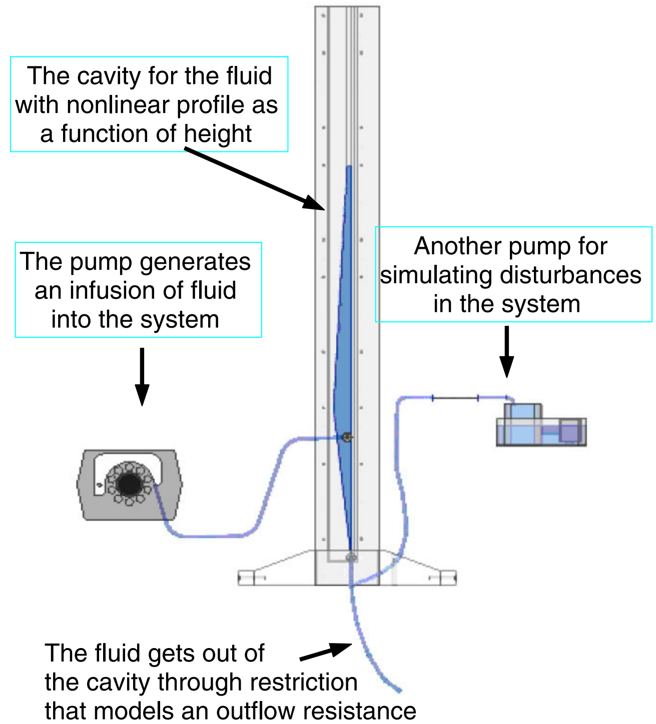

Fig. 1. The test-bed apparatus for performing infusion tests (built at the Department of Biomedical Engineering and Informatics, Umeå University Hospital) includes: the tower modeling the pressure increase as a nonlinear function of fluid height in the tower; two pumps for infusion of artificial CSF and for modeling variety of physiological disturbances in the process; the outflow pathway through restriction to model outflow resistance.

Through a thin steel pipe the cavity (CSF space) and the overflow container were connected, and the resistance of the steel pipe simulated R of the CSF system. Another computer controlled peristaltic pump was connected to the cavity and was used to produce disturbing physiological pressure waves in the model.

The recorded in experiment values for an additional inflow $I_a(t)$ and observed pressure levels $P_{ic}(t)$ are shown on Figures 2 and 3 respectively.

The observer parameters c and γ have been chosen as

$$c = -1, \quad \gamma = 10,$$

while the initial conditions $\hat{P}(0)$, $\hat{R}(0)$ are

$$\hat{P}(0) = 1.5, \quad \hat{R}(0) = 0.02$$

Figure 4 the on-line estimate for R value is depicted, while the true value of R is not exactly known, but the estimate $R = 0.037$ from off-line computation could be considered as the 'true' one.

Figure 5 shows the error signal - the difference between $P_{ic}(t)$ and its estimate $\hat{P}(t)$. As seen the mismatch does not exceed ± 0.5 (kPa) after transition.

IV. Conclusions

This note has suggested a new on-line estimation procedure for a cerebrospinal fluid outflow resistance - one

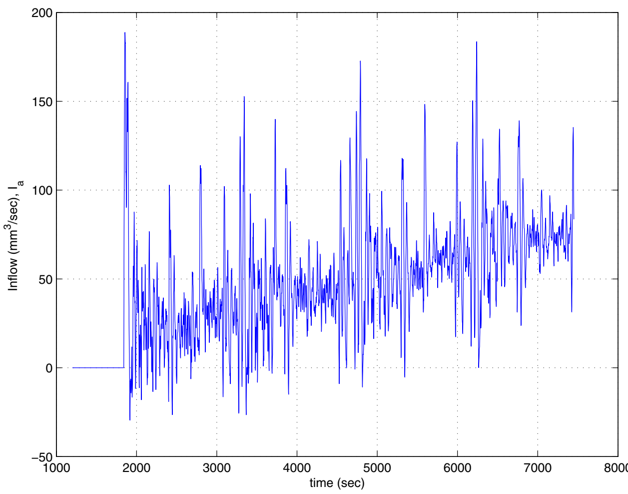

Fig. 2. The inflow $I_a(t)$ of the artificial cerebrospinal liquid into the system.

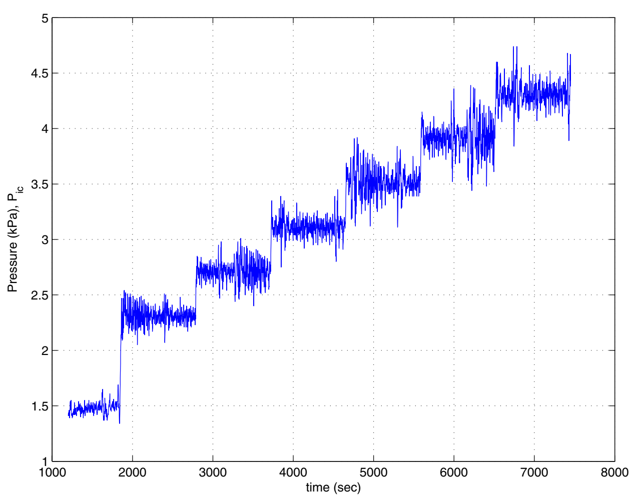

Fig. 3. The corresponding values of the pressure $P_{ic}(t)$ in response to the active inflow depicted on the Fig. 2. Here one can observe the step like pressure response usually generated in experiments for further offline estimation of the resistance value R.

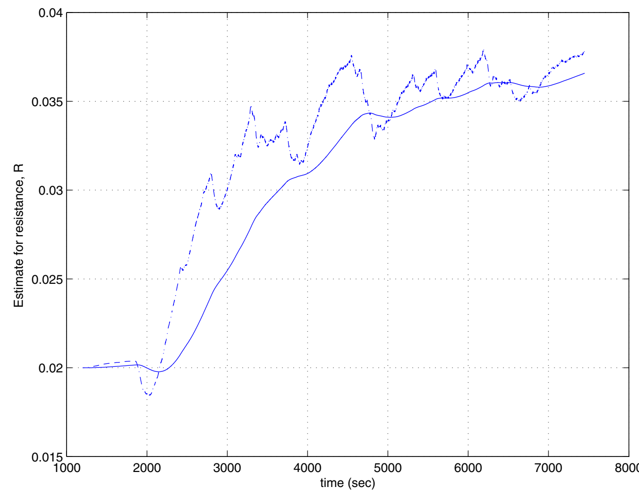

Fig. 4. The behaviour of the resistance estimate computed on-line as solution of the observer (2), (3), while the true value of resistance is around 0.037. Here the dotted curve represents the unfiltered estimate value $\hat{R}$, while the solid line is the result of low-pass filter applied for $\hat{R}$.

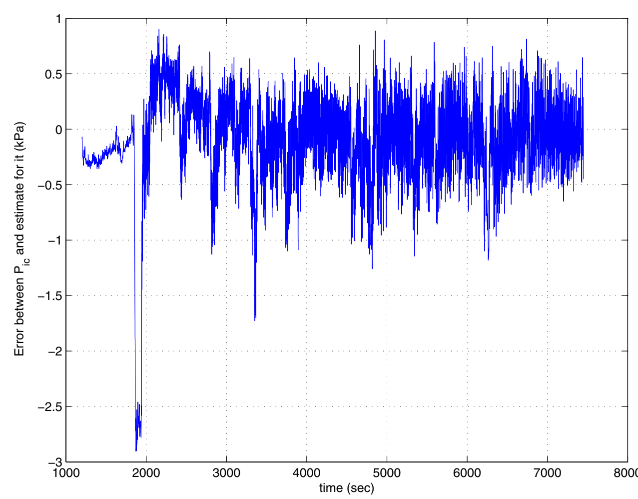

Fig. 5. The error signal $e(t) = P_{ic}(t) - \hat{P}(t)$ is depicted. The error signal remains within ± 0.5 (kPa). The remaining is mainly due to high frequency components that are not covered by the model (1).

of important lumped parameter in the well-known finite-dimensional nonlinear model describing a behaviour of an intracranial pressure. The suggested estimation method relies on the fact that the dynamical model of the intracranial pressure is well described by the Bernoulli equation. Being nonlinear this equation can be transform into an equivalent linear form, where the parameter for identification comes in affine form. Applying the standard estimation procedure for this equivalent model, it can be further transformed into the nonlinear observer for the original system. These results are further examined on real experimental data. Small part of these tests is included and illustrates the method.

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