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(μ-Mu)-Iteration Technique : Application to attitude control of satellite with large flexible appendages

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***Abstract*— This paper addresses the problem of increasing robustness, with respect to real parameter variations, for a satellite with large flexible appendages. A first control law is designed using, for example, a classical frequency-domain method. Subsequently, robustness is improved iteratively, using the (μ-Mu)-iteration technique, which is based on alternating between real μ-analysis and modal Multi-model design.**

I. INTRODUCTION

Future space missions are likely to involve large and lightweight structures for antennas, sunshields, solar sails and solar arrays. The ESA Aurora program already includes the vision of large vehicles, including cruise transfer vehicles for manned exploration and large deployable structures, like the 40-metre deployable antennas of the Mars Express radar altimeter Figure 1. Examples in telecommunications are the large new for instance on INMARSAT-4 Figure 1 or large thin-film solar arrays for high power telecommunication platforms.

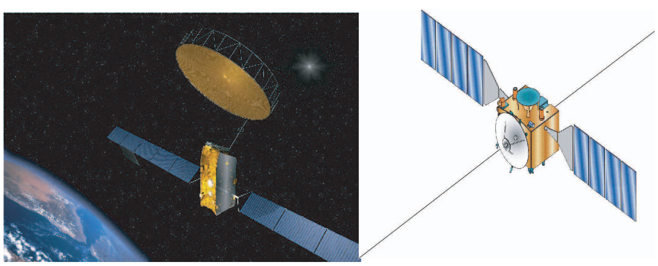

Fig. 1. Flexible Structures Example: Telecom satellite and Mars Express

Satellites with large flexible structures lead to challenging issues in terms of structure design, implementation on the satellite main body, strains transmitted to the structure, impact on the sensors and actuators and control law design. These large structures features low stiffness to avoid prohibitive mass and the corresponding missions faces the new challenge of robust control with uncertain, large and flexible structures. One of the main characteristics of large flexible space structures is the significant uncertainty in the knowledge of the spacecraft dynamics. Indeed, models of flexible appendages are always only partly representative of the true spacecraft dynamics. Furthermore, the other characteristics of flexible modes (e.g. damping, amplification factor specific to space applications Figure 2, and also couplings between axes, number of flexible modes) also greatly impact the design. The real challenge of large flexible appendages control lies in the combination of all these factors associated to the high level of uncertainties in their knowledge. It is the combination of small damping, uncertainty on the resonance frequency of a large number of modes and potential couplings Figure 3 that motivates the evaluation of sophisticated and dedicated control strategies.

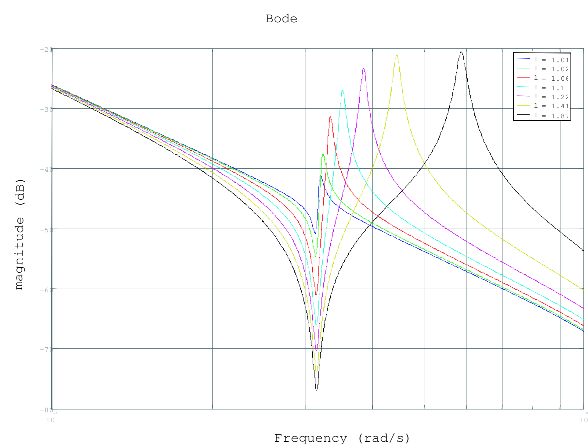

Fig. 2. Examples of flexible modes impact: amplification factor

One of the main issues in the design of the Attitude and Orbit Control System (AOCS) is to derive a robust controller that satisfies performance specifications both in the time and frequency domains and in the presence of large uncertainties (plant changes and/or external disturbances) due to the size and the poor characterization of the deployed structures. As these control techniques are critical to future missions, ESA has initiated several studies to evaluate, benchmark and demonstrate the benefits of robust control techniques when applied to the design of AOCS for missions including large flexible structures. In a previous publication [2], EADS Astrium has presented the application of two promising techniques, $H\infty$ and QFT, and demonstrated their interest in the design of robust controllers dedicated to the AOCS of satellites with large flimsy appendages. In the frame of this study, the (μ-Mu)-iteration technique has been retained as an alternate technique, because it allows to take into account the flexible dynamics in the design phase without inverting the dynamics and thus ensuring good robustness stability with respect to the flexible dynamics uncertainty variations.

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below, analysis is made by considering a *continuum* of models under the "$(M-\Delta)$" form, so, even if synthesis is made considering a finite set of models, the resulting design is valid over the continuum of models considered for analysis. We shall call "(μ-Mu)-iteration" this design cycle, which alternates μ-analysis and *Mu*lti-model design. As a matter of fact, lower bound of μ permit to obtain Δ_i, which destabilized the closed loop system. The open loop system closed by Δ_i will be named "worst case model" and will be taken into account during the Multi-model synthesis.

This paper is structured as follows. After a presentation about Multi-model eigenstructure assignment by dynamic feedback and tools used for computing upper and lower bounds of μ without frequency gridding, the design procedure (μ-Mu)-iteration is described. The design cycle is applied to the attitude control of a satellite with large flexible appendages. Starting with the problem definition, the initialization step will be outlined. Then the robustness improvement obtained by (μ-Mu)-iteration will be discussed in detail.

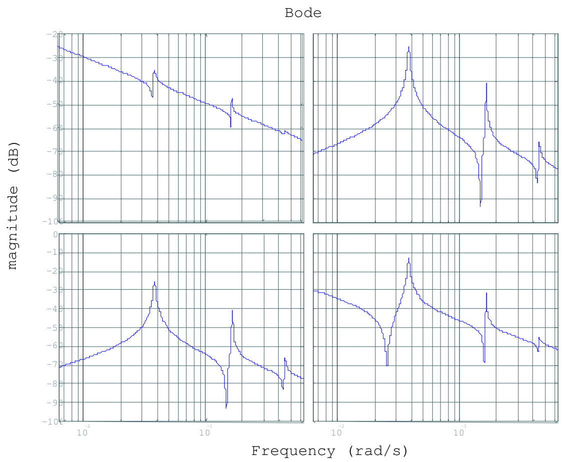

Fig. 3. Examples of flexible modes impact: couplings

Recent developments in modal control

Up to now, modal control is used for dealing with time domain specifications (settling time, damping, cross coupling). Despite its effectiveness for this kind of specifications, only minor improvements have been performed to treat robustness aspects by solving non convex optimizations. In the same time, some techniques have been developed to deal with robustness (μ-synthesis, quadratic stability ...). They have shown to be very conservative as they attempt to treat, in the same manner, all the parametric configurations (use of an unique Lyapunov function for example). Recent improvements of the aforementioned techniques [12] & [1] require high level of computation and lead to very high controller orders. Recently, modal approaches have been revisited using Multi-model setting, with the purpose of taking into account real parameter variations. This alternative technique has been successfully applied on aircraft autopilot design [6]. In this paper, an iterative technique based on real μ-analysis and modal Multi-model design is proposed. This technique, named (μ-Mu)-iteration will permit to proceed to Multi-model synthesis of complex systems and to validate the closed loop behaviors on a continuum of models [4]. This design cycle has no theoretical guarantee of convergence but will be shown to be, from a practical point of view, very efficient.

Proposed design cycle :

- Compute the lower bounds validated by upper bound of μ,
- identify the worst case model,
- use Multi-model technique to treat the "set of worst cases".

The computation of lower bounds validated by upper bound of μ allows us to identify the worst case model. Multi-model design permits the designer to treat a "set of worst cases". It is worth noting that, in the proposed design cycle described

II. Eigenstructure assignment by dynamic feedback

A. Notations

We shall consider the following linear system with n states, m inputs, p outputs:

$$\begin{aligned} \dot{x} &= Ax + Bu \\ y &= Cx + Du \end{aligned} \tag{1}$$

where x is the vector of states, y the vector of measurements and u the vector of inputs, $A \in \mathbb{R}^{n\times n}$, $B \in \mathbb{R}^{n\times m}$, $C \in \mathbb{R}^{p\times n}$ and $D \in \mathbb{R}^{p\times m}$.

The feedback that will be used in this paper is a dynamic feedback $K(s)$. We shall denote the state space form of $K_{dyn}(s)$ as follows:

$$\begin{array}{rcccl} \dot{x}_c &=& A_c x_c &+& B_c u_c \\ y_c &=& C_c x_c &+& D_c u_c \end{array} \tag{2}$$

$A_c \in \mathbb{R}^{n_c\times n_c}$, $B_c \in \mathbb{R}^{n_c\times p}$, $C_c \in \mathbb{R}^{m\times n_c}$ and $D_c \in \mathbb{R}^{m\times p}$.

$$K_{dyn}(s) = C_c(sI - A_c)^{-1}B_c + D_c \tag{3}$$

The connection of (1) and (2) with $u = y_c$ and $u_c = y$ is equivalent to the following augmented system $(\overline{A}, \overline{B}, \overline{C}, \overline{D})$

$$\begin{array}{ll} \overline{A} = \begin{bmatrix} A & 0 \\ 0 & 0 \end{bmatrix} ; & \overline{B} = \begin{bmatrix} B & 0 \\ 0 & I_{n_c} \end{bmatrix} \\ \overline{C} = \begin{bmatrix} C & 0 \\ 0 & I_{n_c} \end{bmatrix} ; & \overline{D} = \begin{bmatrix} D & 0 \\ 0 & 0 \end{bmatrix} \end{array} \tag{4}$$

controlled by the proportional feedback

$$\overline{K} = \begin{bmatrix} D_c & C_c \\ B_c & A_c \end{bmatrix} \tag{5}$$

B. Single-model modal control

Proposition 2.1 from [6] generalizes the traditional eigenstructure assignment of [11] for the use of dynamic controllers.

Proposition 2.1: The triple $\mathcal{T}_i = (\lambda_i, v_i, w_i)$ satisfying

$$\begin{bmatrix} A - \lambda_i I & B \end{bmatrix} \begin{pmatrix} v_i \\ w_i \end{pmatrix} = 0 \quad (6)$$

is assigned by the dynamic gain $K_{dyn}(s)$ if and only if

$$K_{dyn}(\lambda_i) \underbrace{\left[C\, v_i + D\, w_i\right]}_{E} = w_i \quad (7)$$

The input directions w_i and right eigenvectors v_i associated to the closed loop eigenvalue λ_i also depend on the perturbation matrix Δ, however this dependence is omitted in the formulae in order to simplify notation. They can be fixed by various methods. The method respecting mostly the system's natural behaviour is the *orthogonal projection* of the eigenvector $v_{i,ol}$ associated to the open loop eigenvalue $\lambda_{i,ol}$ on a vector v_i belonging to λ_i

$$v_i = V(\lambda_i) \left[V(\lambda_i)^T V(\lambda_i)\right]^{-1} V(\lambda_i)^T v_{i,ol} \quad (8)$$

where $V(\lambda_i)$ and $W(\lambda_i)$ are two matrices with $\mathrm{rank}\left[V(\lambda_i)\right] = m$ such that

$$\begin{bmatrix} A - \lambda_i I & B \end{bmatrix} \begin{pmatrix} V(\lambda_i) \\ W(\lambda_i) \end{pmatrix} = 0$$

See [10] for the theoretical background and further details. For aeronautical applications refer to [6] & [7]. This projection is used in the following.

C. Multi-model modal control

Multi-model eigenstructure assignment [10] is done by simultaneously assigning triples $\mathcal{T}_i$ for several models, which reduces to solve a set of equality constraints of type (7) for the transfer matrix $K_{dyn}(s)$ where its entries are

$$K_{ij,dyn}(s) = \frac{\mathcal{N}_{ij}(s)}{\mathcal{D}_{ij}(s)} = \frac{b_{ij,k}\, s^k + \cdots + b_{ij,1}\, s + b_{ij,0}}{a_{ij,k}\, s^k + \cdots + a_{ij,1}\, s + a_{ij,0}} \quad (9)$$

The choice of the models to treat with and the triples to assign is determined by an analysis of the stability and/or performance robustness (see III). Common or different denominators $\mathcal{D}_{ij}(s)$ of the matrix $K_{dyn}(s)$ are fixed *a priori*. This asumption is not very restrictive because if we take into account the bandwidth in which control effect is expected and roll-off requirements there is not so much freedom. The free design parameters are the numerator coefficients denoted $b_{ij,k}$. Generally, this choice offers too many degrees of freedom for the resolution of the equality constraints so that the problem is solved by minimizing a criterion of the type $||K_{dyn}(j\omega_i) - K_0(j\omega_i)||_F$ over a certain frequency interval ω_i where $K_0(s)$ is a reference controller synthesized for an initial model M_0. This reduces to minimize a quadratic criteria under linear constraints.

Remark: Although the following benchmark application is a SISO problem, the Robust Modal Control theory has been presented in a general MIMO framework.

III. μ ANALYSIS FOR FLEXIBLE SYSTEM

As it is necessary to identify worst cases to do multi-model control, an efficient technique for computing a lower bound of μ is necessary. Standard μ-analysis is not efficient for flexible structure. The main raisons are, first, if frequency gridding is used, peak values of μ are generally missed, second, uncertainties are usually real and in this case the standard numerical tools cannot compute the lower bound of μ. To sort out these difficulties, a specific algorithm has been developed in [8], [3] & [9]. The idea behind this algorithm consists of shifting the eigenvalues toward the imaginary axis with a minimum perturbation. This technique permits to detect worst case models, which are necessary in the (μ-Mu)-iteration design. Afterward, lower bound is validated using upper-bound especially adapted to flexible structures [9] & [5]. The proposed technique checks whether a given test value, for example the maximum value of the lower bound augmented by a given percentage, is larger than the upper bound of μ over the whole frequency range. At each step of the proposed algorithm, scaling are computed at a given frequency. The frequency intervals, for which this scaling permits to conclude that the best value is larger than μ, are eliminated. For more details about this algorithm see [9].

IV. (μ-MU)-ITERATION, DESIGN PROCEDURE

The design procedure called (μ-Mu)-iteration used alternatively multi-model modal control and μ-analysis. Three steps compose the procedure, initialization, μ-analysis and multi-model modal control.

Initialization : Step 0
Design an initial feedback $K_0(s)$ on a nominal model M_0 with Δ_0. All kinds of synthesis methods can be applied at this step ($\mathcal{H}_\infty$ control, LQG optimal control, μ–synthesis, *etc*...). In the case of initial non–modal approaches, it is necessary to find an equivalent modal dynamic controller. To obtain this equivalent controller, a modal analysis of the closed loop system (M_0,$K_0(s)$) is performed in order to identify the dominant modes and associated eigenvectors. This eigenstructure is assigned using Proposition 2.1 to design $K_{equ}(s)$.

μ-analysis : Step 1
Perform μ-analysis. If the design is satisfactory for all the relevant values of the matrix Δ: stop. Otherwise identify the worst case model M_{wc} using the lower bound.

Multi-model design : Step 2
Improve the behavior of the worst case model keeping the good properties of all the models treated at the previous iteration(s) M_0, ..., M_{wc-1} (in particular, the behavior of the initial feedback relative to the nominal model is preserved). Go to step 1.

V. APPLICATION: ROBUST CONTROL OF A SATELLITE

A. Presentation of the model

The model used to test the (μ-Mu)-iteration technique corresponds to the attitude control on one axis for a Telecom satellite type. Up to 4 flexible modes are taken into account in the flexible appendage model. The considered dynamics can be divided as follows:

- inertia of the rigid body dynamics :
$$\frac{1}{Is^2}$$
- actuation loop delay $(0.5s)$ modelled with a second order Pade filter.
- 4 flexible mode dynamics (with cantilever pulsation ω_j , amplification factor : ratio between free-free pulsation/cantilever pulsation λ_j and damping of the flexible ξ_j) with transfer functions of the form (the indice j corresponds to the j^{th} flexible mode):

$$H_j(s) = \frac{(\lambda_j^2 - 1)/I}{s^2 + 2\lambda_j^2 \xi_j \omega_j s + \lambda_j^2 \omega_j} \tag{10}$$

The considered uncertain parameters are the pulsations , the damping and the satellite inertia I, with a variation of 15% for each uncertain parameter. Nominal inertia of the satellite is set to $6450 kg.m2$, nominal pulsations of the four flexible modes are set to $[0.3248, 0.3563, 0.9613, 1.0681] rad.s{-}1$ and nominal damping is set to 1.7%. The challenge resides in the very low values of the modes frequencies and damping, and on the large uncertainties. The LFT form of the linear system has then been built. In our case, the uncertain parameters only appear at the level of the flexible modes dynamics. The transfer function associated to those dynamics can be written as follows:

$$H_j(s) = \frac{y_j}{u_j}(s) = \frac{(\lambda_j^2 - 1)/I}{s^2 + 2\lambda_j^2 \xi_j \omega_j s + \lambda_j^2 \omega_j} \tag{11}$$

Or after factorisation :

$$\ddot{y}_j = -\lambda_j^2 \omega_j (2\xi_j \dot{y}_j + \omega_j y_j) + \frac{\lambda_j^2 - 1}{I} u_j \tag{12}$$

This allows to obtain a scheme were the pulsation parameter ω_j is only repeated twice.

B. Initial law

The (μ-Mu)-iteration technique requires initial controllers before starting the improvement, i.e. increase their robustness stability, while keeping equivalent the other performances. The type of controllers that have been retained corresponds to very simple controllers: phase-lead controller to ensure sufficient robustness stability margins for the nominal dynamics associated to a low-pass filtering to reduce the impact of high frequency noises on the control loop:

- controller 1 : $50deg$ maximum phase brought at $0.01Hz$, and cut-off frequency of 2^{nd} order filter : $0.1Hz$ with a damping of 0.3
- controller 2 : $50deg$ maximum phase brought at $0.1Hz$, and cut-off frequency of 2^{nd} order filter : $1Hz$ with a damping of 0.3

Both controllers have 1 zero and 3 poles : 1 zero and 1 pole for the phase lead and 2 poles for the low pass filter.

To illustrate the (μ-Mu)-iteration technique, the design procedure is applied and detailed on the first controller.

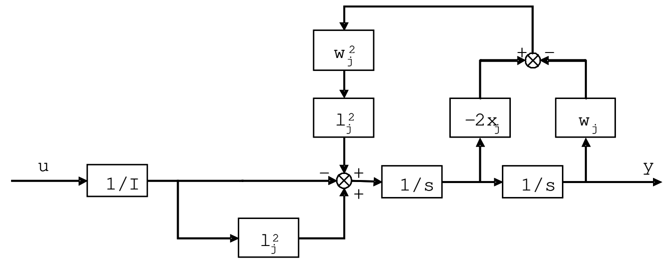

Fig. 4. Scheme with two occurrences of pulsation parameters

C. Design procedure

Initialization : Step 0

The modal analysis of the closed loop, with initial feedback, is performed in order to identify the dominant eigenstructure (eigenvalues and eigenvectors). After analysis, only the 2 lowest frequency modes are taken in consideration and will be kept during all the following iterations.A dynamic feedback is computed, which permits to assign this eigenstructure. During the calculation, a criterion is required to get a unique solution among the infinity of solutions. Here is chosen, the minimisation of the norm of the difference between the initial controller and the resulting dynamic feedback determined at given frequencies. The 3 poles of the denominator of the transfer function of the dynamic feedback are the same as the ones of the initial controller. Thus, the new controller will be similar to the initial one. The structural constraints are limited to the choice of the numerator degree. In order to restrict the effect of the flexible modes, the degree difference between the numerator and the denominator is fixed to 1. This will lead to $20dB$ attenuation at high frequencies.

μ analysis : Step 1

The ("$M - \Delta$") formulation used for the real μ-analysis is shown in Figure 5. After having closed the feedback loop, μ analysis is performed to detect the destabilizing Δ.

The lower bound is computed using algorithm [8]. The results of the algorithm are illustrated in Figure 6. The horizontal line represents the upper limit of the upper bound (and the curves represent mappings defined in [9] which are used for the validation of this upper limit).

A peak of $\mu \approx 0.7$ occurs at about $0.32 \, rad/s$. In order to reduce this peak the worst case model is identified. The algorithm developed in [8] delivers the worst case perturbation matrix Δ_{wc}.

Multi-model modal control : Step 2

This time the nominal system is closed by the perturbation matrix Δ_{wc}, thus we get the worst case model M_{wc}.

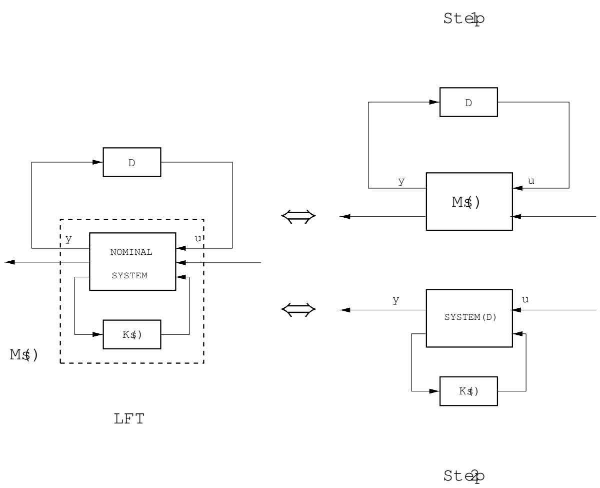

Fig. 5. LFT of the closed loop system

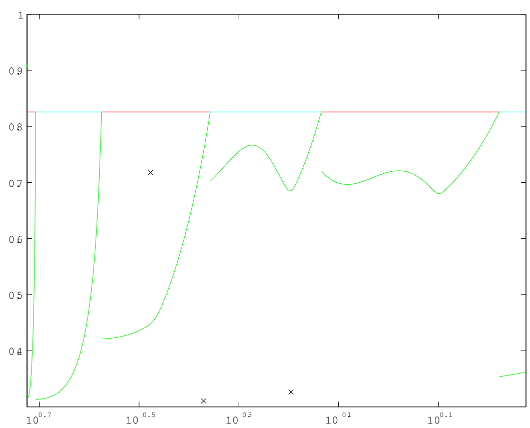

Fig. 6. Upper (solid line) and lower (*) bounds with the initial and equivalent controller

A new dynamic feedback conserving the assignment of the initial eigenstructure and respecting a new constraint relative to the critical model is calculated. The last one consists of assigning the destabilizing mode (a simple plot of the eigenvalues, see Figure 7, answers that question) on the limit of stability.

During the calculation a criterion is required to get one unique solution among the infinity of solutions. Here, the minimization of the norm of the difference between the initial feedback and the resulting dynamic controller determined at some frequencies is chosen.

Go to Step 1: Once again, the system is closed by the new feedback. And so on. One iteration has been enough for a reduction of the peak value and thus an improvement of the robustness characteristics.

In Figure 8 the results of μ-analysis after one iteration are presented. In fact, the peak value of 1 is reduced to 0.6. Furthermore, one can remark that the other peaks do not change at all. This is important to know that acting on one peak do not have unknown impacts on other peaks. The real-μ lower bound has to be verified using real-μ upper bound. This algorithm permits us to conclude that the peak values of μ are less than 0.36 and show a gain of 44% in terms of robustness.

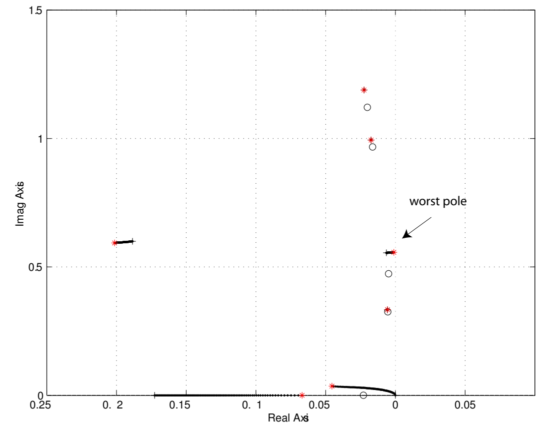

Fig. 7. Root locus with equivalent controller and worst cas model. "+" denotes open loop poles and "*" denotes the closed loop

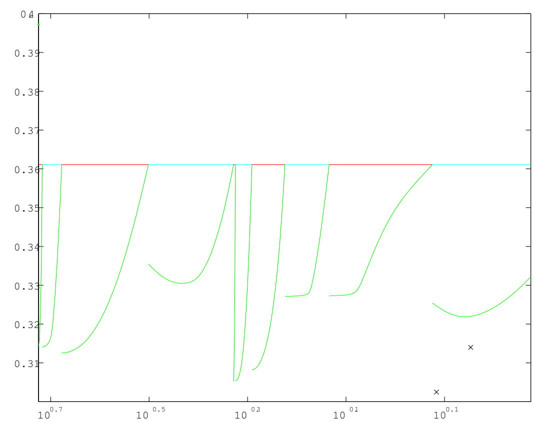

Fig. 8. Upper (solid line) and lower (*) bounds with the robust controller.

Figure 9 shows the time responses of the system closed by the robust and the initial controller. Figure 10 shows the nichols diagram with initial and robust controller.

D. Results Discussion

One iteration has been enough for a reduction of the peak value and thus an improvement of the robustness characteristics.

VI. Conclusions

Starting from a model of the dynamics uncertainties, represented using the $(M - \Delta)$ form of the dynamics model, the (μ-Mu)-iteration technique allows to directly take into account the parametric dependency into the design phase. The flexible dynamics uncertainty can be automatically analysed and resulting worst-cases directly integrated into the design phase to improve controller robustness.

In the frame of this study, a purely analytical model has been developed to represent the large flexible modes of a spacecraft. The dynamical model is based on fixed-free beam that can be added independently and fully parameterised in order to represent most of the large flexible structures (antenna, sunshield booms, solar arrays). The fundamental

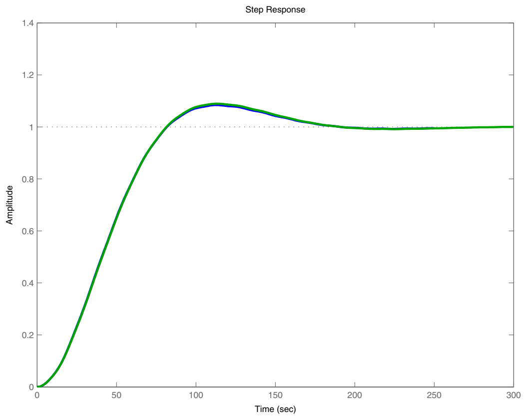

Fig. 9. Step responses with initial and robust controller

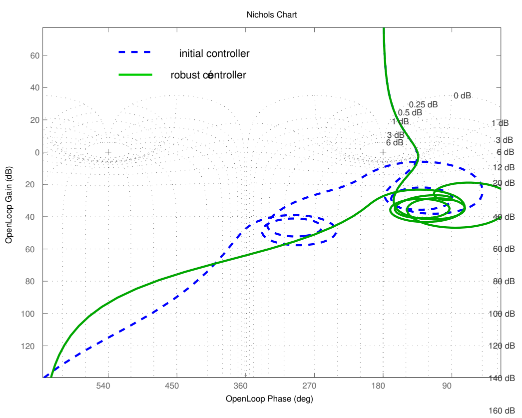

Fig. 10. Nichols diagram with initial and robust controller.

equation of the flexible structures allows to derive the 6-DoF dynamical equation for the complete system (rigid plus flexible dynamics).

The design procedure has been also applied on this analytical dynamical model of Inmarsat4-type Telecom Satellite. The controller that has been tested corresponds to the second controller of the previous analysis, i.e. the controller with a large bandwidth. The μ-analysis of the final controller shows a real improvement of the initial controller robustness : the lower bound of the initial controller was about 2 and the lower bound of the final robust controller is 0.64.
The resulting compensator, in the two cases, has the same order as the initial one. This kind of design procedure, despite his lack of theoretical convergence, has shown to be, from a practical point of view, very efficient. An other application on large aircraft autopilot design can be found in [4].
In this paper, the (μ-Mu)-iteration has been applied to improve the robustness stability. It would be possible to apply the same procedure by using a performance criteria given by an area on the pole map, as show Figure 11.

The application presented was a SISO problem. It is important to note that the method can be extended for a MIMO problem without any adaptation.

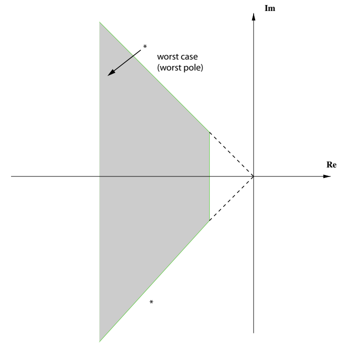

Fig. 11. Area defined by the performance requirements and worst case analysis.

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REFERENCES

[1] E. Apkarian, P. Feron and P. Gahinet. Parameter-dependent lyapunov functions for robust control of systems with real parametric uncertainty. *IEEE Transactions on Automatic Control*, pages 1241–1264, 1997.
[2] G. Bodineau, S. Boulade, B. Frappard, W. Chen, S. Salehi, and F. Ankersen. Robust control of large flexible appendages for future space missions. *In Proc. 14th IFAC World Congress, Beijing, China*, pages 1034–1037, July 2004.
[3] G. Looye C. Döll, J.F. Magni and S. Bennani. Robustness analysis applied to autopilot design part 2: Evaluation of a new tool for μ-analysis. *ICAS Congress, Melbourne, Australia*, 1998.
[4] C. Chiappa, J.F. Magni, C. Döll, and Y. Le Gorrec. Improvement of the robustness of an aircraft autopilot designed by an H_∞ technique. *In Proc. CESA'98 Conference, Nabeul-Hammamet, Tunisia*, 1:1016–1020, April 1998.
[5] B. Frapard, B. Girouart, C. Döll, J.F. Magni, and C. Chiappa. Mixed μ-analysis for flexible structures. Part 2:Application to the SPOT family of earth observation satellites. *In Proc. 14th IFAC World Congress, Beijing, China*, P:361–366, July 1999.
[6] Y. Le Gorrec, J.F. Magni, C. Döll, and C. Chiappa. A modal multi-model control design approach applied to aircraft autopilot design. *AIAA Journal of Guidance, Control, and Dynamics*, 21(1):77–83, 1998.
[7] J.F. Magni. Multimodel eigenstructure assignment in flight–control design. *Aerospace Science and Technology*, 3(3):141–151, May 1999.
[8] J.F. Magni and C. Döll. A new simple lower bound of the mixed structured singular value. *Asian Control Conference, Seoul, Korea*, pages I:847–850, 1997.
[9] J.F. Magni, C. Döll, C. Chiappa, B. Frappard, and B. Girouart. Mixed μ-analysis for flexible structures. Part 1:Theory. *In Proc. 14th IFAC World Congress, Beijing, China*, P:325–360, July 1999.
[10] J.F. Magni, Y. Le Gorrec, and C. Chiappa. A multimodel-based approach to robust and self-scheduled control design. *In Proc. 37th I.E.E.E. Conf. Decision Contr., Tampa, Florida*, pages 3009–3014, 1998.
[11] B.C. Moore. On flexibility offered by state feedback in multivariable system beyond closed loop eigenvalue assignment. volume AC-21, pages 659–692, 1976.
[12] P.M. Young. Controller design with parametric uncertainty. *Int. J. Robust Nonlinear Control*, vol.3:469–509, 1996.