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Observability recovering by additional sensor implementation in linear structured systems

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Abstract— **In this paper, we consider linear structured systems which represent a large class of parameter dependent linear systems. We focus our attention on observability analysis. It turns out that the system is structurally observable if and only if the system is output connected and contains no contraction. We analyze in a structural way the possible causes for unobservability and tackle the problem of observability recovering by additional sensor implementation. Using classical tools of graph theory, we give the minimal number and the location of additional sensors to be implemented in order to avoid contractions. We give also the minimal number of states which have to be measured for ensuring output connectivity.**
We think that the results presented here provide a deeper understanding of structural unobservability and can be useful for solving problems as fault detection without a priori assuming the system observability.

I. INTRODUCTION

In this paper, we consider linear structured systems which represent a large class of parameter dependent linear systems. To such systems one can associate graphs which allow to study in a simple way generic or structural properties or to give solvability conditions for standard control problems as decoupling or disturbance rejection [1]. The structural controllability problem has been studied by several authors in the seventies [2], [3], [4]. These results can be easily dualized to get conditions for structural observability.
When the system is not observable, it is of interest whenever possible, to add new sensors at some cost to reach the observability conditions. This problem of additional sensor implementation to get useful information on the system is of practical interest for example in the Fault Detection and Isolation problem. Many references can be found in the literature reporting applications in chemical engineering, see for example [5], [6]. In these application oriented papers, qualitative models are used which do not include the accurate dynamical behavior of the system. The observability in this case reduces to the fact that each variable is connected to a sensor. Here we restrict our attention to classes of linear models but with full dynamics description. In this paper, we analyze in a structural way the possible causes for unobservability and tackle the problem of observability recovering by additional sensor implementation.
It turns out that the system is structurally observable if and only if the system is output connected and contains no contraction. Using classical tools of graph theory, we analyse first the contractions and prove that the Dulmage-Mendelsohn decomposition allows to characterize these contractions. We give then the minimal number and the location of additional sensors to be implemented in order to avoid contractions. Using strongly connected components of the system associated graph, we give also the minimal number of states which have to be measured for ensuring output connectivity.
We think that the results presented here provide a deeper understanding of structural unobservability and can be useful for solving problems as fault detection without a priori assuming the system observability.
The outline of this paper is as follows. The linear structured systems are presented in section 2. We revisit structural observability conditions in section 3. In section 4 we consider the observability recovering problem with additional sensors. The study is decomposed in two steps, first we study output connection property and then contraction avoidance. Our results are illustrated on simple examples in section 5. Some concluding remarks end the paper.

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II. LINEAR STRUCTURED SYSTEMS

In this part, we recall some definitions and results on linear structured systems. More details can be found in [1].
We consider linear systems with parameterized entries and denoted by Σ_Λ. In this paper, we will only be concerned with observability and we will not take into account input variables.

$$\Sigma_\Lambda \begin{cases} \dot{x}(t) = Ax(t) \\ y(t) = Cx(t) \end{cases} , \tag{1}$$

where $x(t) \in \mathbb{R}^n$ is the state vector and $y(t) \in \mathbb{R}^p$ the measured output vector. A and C are matrices of appropriate dimensions.
This system is called a linear structured system if the entries of the composite matrix $J = \begin{bmatrix} A \\ C \end{bmatrix}$ are either fixed zeros or independent parameters (not related by algebraic equations). $\Lambda = \{\lambda_1, \lambda_2, \ldots, \lambda_k\}$ denotes the set of independent parameters of the composite matrix J. For the sake of simplicity the dependence of the system matrices on Λ will not be made explicit in the notation. A structured system represents a large class of parameter dependent linear systems. The structure is given by the location of the fixed zero entries of J.
For such systems, one can study generic properties i.e. properties which are true for almost all values of the parameters collected in Λ [7], [8]. More precisely a property

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is said to be generic (or structural) if it is true for all values of the parameters (i.e. any $\Lambda \in \mathbb{R}^k$) outside a proper algebraic variety of the parameter space. A directed graph $G(\Sigma_\Lambda) = (Z, W)$ can be easily associated to the structured system Σ_Λ of type (1) where the matrix $\begin{bmatrix} A \\ C \end{bmatrix}$ is structured:

- the vertex set is $Z = X \cup Y$ where X and Y are the state and output sets given by $\{x_1, x_2, \ldots, x_n\}$ and $\{y_1, y_2, \ldots, y_p\}$ respectively,
- the edge set is $W = \{(x_i, x_j) | A_{ji} \neq 0\} \cup \{(x_i, y_j) | C_{ji} \neq 0\}$, where A_{ji} (resp. C_{ji}) denotes the entry (j, i) of the matrix A (resp. C).

Moreover, recall that a path in $G(\Sigma_\Lambda)$ from a vertex $i_{\mu 0}$ to a vertex $i_{\mu q}$ is a sequence of edges $(i_{\mu 0}, i_{\mu 1}), (i_{\mu 1}, i_{\mu 2}), \ldots, (i_{\mu q-1}, i_{\mu q})$ such that $i_{\mu t} \in Z$ for $t = 0, 1, \ldots, q$ and $(i_{\mu t-1}, i_{\mu t}) \in W$ for $t = 1, 2, \ldots, q$. Moreover, if $i_{\mu 0} \in X$ and, $i_{\mu q} \in Y$, the path is called a state-output path. The system Σ_Λ is said to be output-connected if for any state vertex x_i there exists a state-output path with initial vertex x_i.
A path which is such that $i_{\mu 0} = i_{\mu q}$ is called a circuit. A set of paths with no common vertex is said to be vertex disjoint.

Example 1: Let Σ_Λ be the linear structured system defined by its structured matrices.

$$A = \begin{bmatrix} 0 & 0 & 0 \\ \lambda_1 & \lambda_2 & \lambda_3 \\ 0 & 0 & 0 \end{bmatrix}, C = \begin{bmatrix} \lambda_4 & \lambda_5 & \lambda_6 \end{bmatrix}, \quad (2)$$

Its associated graph $G(\Sigma_\Lambda)$ is given in Figure 1.

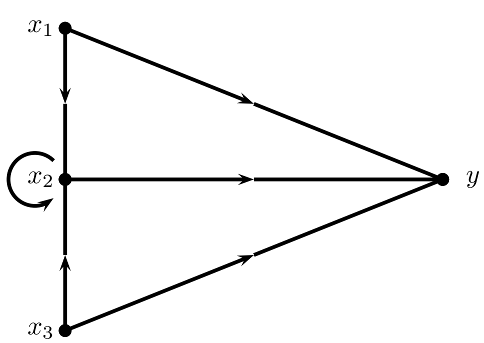

Fig. 1. $G(\Sigma_\Lambda)$ for Example 1

III. Observability of linear structured systems

The structural controllability or its dual notion of observability has been studied in several papers [2], [3], [4]. Recall the graph characterization of the structural observability, which will be useful later [7].

Proposition 1: Let Σ_Λ be the linear structured system defined by (1) with its associated graph $G(\Sigma_\Lambda)$. The system (in fact the pair(C, A)) is structurally observable if and only if:

1) The system Σ_Λ is output-connected,
2) There exists a set of vertex disjoint circuits and state-output paths which cover all state vertices.

Define now for observability the concept of contraction which is the dual notion of the dilation defined by Lin in the study of controllability [2].

Definition 1: Let Σ_Λ be the linear structured system defined by (1) with its associated graph $G(\Sigma_\Lambda)$ with vertex set Z and edge set W. Consider a set S made of k_S state vertices. Denote $E(S)$ the set of vertices w_i for $i = 1, \ldots, l_S$ of Z, such that there exists an edge (x_j, w_i) of W where $x_j \in S$.
S is said to be a contraction if

$$k_S - l_S = d_S > 0 \quad (3)$$

d_S is called the contraction defect of S. Define $\hat{d}$ the contraction defect of Σ_Λ as the maximal value of d_S for any contraction S.
Proposition 1 was stated initially by Lin, in the single-input single-output case, using the above defined contraction concept, Proposition 1 can be reformulated as follows.

Proposition 2: Let Σ_Λ be the linear structured system defined by (1) with its associated graph $G(\Sigma_\Lambda)$. The system (in fact the pair(C, A)) is structurally observable if and only if:

1) The system Σ_Λ is output-connected,
2) $G(\Sigma_\Lambda)$ contains no contraction.

Consider again Example 1, the set $S = \{x_1, x_2, x_3\}$ is clearly a contraction since $E(S) = \{x_2, y\}$ and $k_S - l_S = 1$. Then Σ_Λ is not structurally observable.

IV. Observability recovering with additional sensors

In this section, we will focus our interest on non structurally observable systems. We assume that some additional sensors can be implemented on the system at some cost. We look for the minimal number of such additional sensors which are required for ensuring structural observability. We will tackle this problem in two stages. In a first step, we will study the output connection property (Propositions 1 and 2 part 1) and give the minimal number of states which have to be measured by the additional sensors to meet this property. In a second step, we will study the contraction suppression by adding sensors and give the minimal number of such additional sensors.
Define the new output vector z which collects the new measurements:

$$z(t) = Hx(t), \quad (4)$$

$z(t) \in \mathbb{R}^q$, where $z_i(t)$ is the measurement obtained from the i-th additional sensor.
Define now the composite system denoted by Σ_Λ^c.

$$\Sigma_\Lambda^c \begin{cases} \dot{x}(t) = Ax(t) \\ y(t) = Cx(t) \\ z(t) = Hx(t) \end{cases} \quad (5)$$

To this composite system we can associate the graph $G(\Sigma_\Lambda^c)$ as before.

A. Output connection property

Let Σ_Λ be the linear structured system defined by (1) with its associated graph $G(\Sigma_\Lambda)$. Two vertices v_i and v_j of $G(\Sigma_\Lambda)$ are said to be equivalent if there exists a path from v_i to v_j and a path from v_j to v_i. In this context v_i is assumed to be equivalent to itself. The equivalent classes corresponding to this equivalence relation are called the strongly connected components of $G(\Sigma_\Lambda)$. Standard combinatorial optimization algorithms exist to get the canonical decomposition of the graph into strongly connected components. The output vertices y_i are strongly connected components composed of unique vertex. The strongly connected components can be endowed with a natural partial order. The strongly connected components $\mathcal{C}_i$ and $\mathcal{C}_j$ are such that $\mathcal{C}_i \preceq \mathcal{C}_j$ if there exists an edge (v_j, v_i) where $v_i \in \mathcal{C}_i$ and $v_j \in \mathcal{C}_j$. The infimal elements with this order are the strongly connected components with no outgoing edges. Notice that the output vertices are such infimal elements.

Definition 2: Let Σ_Λ be the linear structured system defined by (1) with its associated graph $G(\Sigma_\Lambda)$. Decompose $G(\Sigma_\Lambda)$ in strongly connected components. We call infimal unobservable components the infimal strongly connected components which are not output vertices. Denote $\bar{d}$ the number of such infimal unobservable components. $\bar{d}$ is called the output connectivity defect.

Proposition 3: Let Σ_Λ be the linear structured system defined by (1) with its associated graph $G(\Sigma_\Lambda)$ and additional sensors z. The composite system is Σ_Λ^c and its associated graph is $G(\Sigma_\Lambda^c)$. Σ_Λ^c is output connected if and only if in any infimal unobservable component of $G(\Sigma_\Lambda)$ there exists a vertex x_i and there exists an edge (x_i, z_j) where z_j is an output vertex corresponding to an additional sensor. This implies that the number of states which have to be measured by the additional sensors is at least $\bar{d}$, the output connectivity defect.

Sketchy proof: Sufficiency

Any state vertex of Σ_Λ is connected to one or more infimal components of $G(\Sigma_\Lambda)$. When the condition is satisfied the infimal components of Σ_Λ are output connected, therefore all state vertices are output connected.

Necessity

Assume that there exists an infimal unobservable component of $G(\Sigma_\Lambda)$ which has no vertex connected to an additional sensor. It follows that there is no path from vertices of this component to $Y \cup Z$ and then Σ_Λ^c is not output connected.

Remark 1: The above proposition states that the number of states which have to be measured by the additional sensors is at least $\bar{d}$. Since several states can influence a single sensor, the minimal number of additional sensors may be less than $\bar{d}$ as shown by the following:

Example 2: Let Σ_Λ be the linear structured system defined by its structured matrices.

$$A = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}, C = \begin{bmatrix} 0 & 0 \end{bmatrix}, \tag{6}$$

In this case $\bar{d} = 2$ and a unique sensor measuring both states, $H = [\lambda_1 \ \lambda_2]$, is sufficient for ensuring output connectivity (see Figure 2).

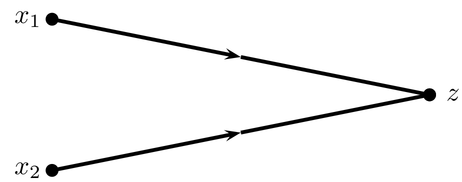

Fig. 2. $G(\Sigma_\Lambda)$ for Example 2

B. Contraction characterization

1) The bipartite graph: In this subsection, we will analyse in a finer way contractions. This will be performed using a bipartite graph associated with the system Σ_Λ. We consider a linear structured system Σ_Λ of type (1) as previously. Let us now introduce the bipartite graph $B(\Sigma_\Lambda)$ as follows.

The bipartite graph of this system is $B(\Sigma_\Lambda) = (V^+, V^-; W')$ where the sets V^+ and V^- are two disjoint vertex sets and W' is the edge set. The vertex set V^+ is given by X^+, the vertex set V^- is given by $X^- \cup Y$, with $X^+ = \{x_1^+, \ldots, x_n^+\}$ the first set of state vertices, $X^- = \{x_1^-, \ldots, x_n^-\}$ the second set of state vertices and $Y = \{y_1, \ldots, y_p\}$ the set of output vertices. Notice that here we have split each state vertex x_i of $G(\Sigma_\Lambda)$ into two vertices x_i^+ and x_i^-. The edge set W' is described by $W_A \cup W_C$ with $W_A = \{(x_j^+, x_i^-) | A_{ij} \neq 0\}$ and $W_C = \{(x_j^+, y_i) | C_{ij} \neq 0\}$. In the latter, for instance $A_{ij} \neq 0$ means that the (i,j)-th entry of the matrix A is a parameter (structurally nonzero). In the same way, to the composite system Σ_Λ^c defined in (5) we can associate the bipartite graph $B(\Sigma_\Lambda^c)$ adding the vertices $\{z_1, \ldots, z_q\}$ and the corresponding incident edges.

A *matching* in a bipartite graph $B = (V^+, V^-; W')$ is an edge set $M \subseteq W'$ such that the edges in M have no common vertex. We denote by $\partial^+ M$(resp. $\partial^- M$) the set of vertices in V^+(resp. V^-) incident to the edges in M. A matching M is called maximal if it has a maximum cardinality. The cardinality of a matching, i.e. the number of edges it consists of, is also called its size. The maximal matching problem is the problem of finding a matching of maximal cardinality. This problem can be solved using very efficient algorithms based on alternate augmenting chains or ideas of maximum flow theory [9]. This notion allows a simple characterization of the generic rank of a structured matrix in terms of its bipartite graph [10] which in our problem can be stated as follows.

Proposition 4: Let Σ_Λ be the linear structured system defined by (1) with its associated bipartite graph $B(\Sigma_\Lambda)$. The generic rank of $\begin{bmatrix} A \\ C \end{bmatrix}$ is the size of a maximal matching in $B(\Sigma_\Lambda)$.

In particular, it can be shown that there is no contraction in Σ_Λ if and only if there exists a size n matching in $B(\Sigma_\Lambda)$. With a matching M we associate a graph $B_M =$

$(V^+, V^-; \overline{W'})$ so that: $(v,w) \in \overline{W'} \Leftrightarrow (v,w) \in W'$ or $(w,v) \in M$. Denote $S^+ = V^+ \backslash \partial^+ M$ and $S^- = V^- \backslash \partial^- M$. Denote $v \rightsquigarrow w$ if there is a path from v to w in B_M.

2) The Dulmage-Mendelsohn (DM) decomposition: The DM-Decomposition allows to decompose a bipartite graph B into unique partially ordered irreducible bipartite subgraphs $B_i = (V_i^+, V_i^-; W_i')$ called the DM-components:

Algorithm 1: DM-Decomposition [7]

1. Find a maximum matching M on $B = (V^+, V^-; W')$.
2. Let $V_0 = \{V_0^+ \cup V_0^-\} = \{v \in V^+ \cup V^- | w \rightsquigarrow v$ on B_M for some w in $S^+\}$ where $V_0^+ = \{V^+ \cap V_0\}$ and $V_0^- = \{V^- \cap V_0\}$. V_0 is called the minimal inconsistent part or horizontal tail of B.
3. Let $V_\infty = \{V_\infty^+ \cup V_\infty^-\} = \{v \in V^+ \cup V^- | v \rightsquigarrow w$ on B_M for some w in $S^-\}$ where $V_\infty^+ = \{V^+ \cap V_\infty\}$ and $V_\infty^- = \{V^- \cap V_\infty\}$.
4. Let V_i (i=1,...r) be the strong components of the graph obtained from B_M by deleting the vertices of $V_0 \cup V_\infty$ and the edges incident thereto.
5. Define the partial order $\prec$ on $\{V_i | i = 0, 1, ...r, \infty\}$ as follows: $V_i \prec V_j \Leftrightarrow v_j \rightsquigarrow v_i$ on B_M for some $v_i \in V_i$ and $v_j \in V_j$.

Example 3: Consider the system Σ_Λ which associated graph $G(\Sigma_\Lambda)$ is depicted in Figure 3 and consider the associated bipartite graph $B(\Sigma_\Lambda)$. We will find the DM-Decomposition of $B(\Sigma_\Lambda)$. First of all, a maximum matching is defined by $\{x_1^+ \rightarrow x_4^-; x_4^+ \rightarrow x_5^-; x_5^+ \rightarrow x_6^-; x_6^+ \rightarrow x_7^-; x_7^+ \rightarrow x_8^-; x_8^+ \rightarrow y_2\}$, so that we have $S^+ = \{x_2^+; x_3^+\}$ and $S^- = \{x_1^-; x_2^-; x_3^-; y_1\}$. Then $V_0 = \{x_2^+; x_5^-; x_4^+; x_3^+; x_6^-; x_5^+\}$ and $V_\infty = \{x_1^-; x_2^-; x_3^-; y_1; x_1^+; x_4^-\}$; Finally, $V_1 = \{x_6^+; x_7^-\}$; $V_2 = \{x_7^+; x_8^-\}$ and $V_3 = \{x_8^+; y_2^-\}$ as shown in the Figure 4.

3) Application to the contraction: Notice that the DM-Decomposition characterizes all the maximal matchings. In particular, any maximal matching is incident to all vertices of V_0^-. From Proposition 4 it follows that $card(V_0^+) - card(V_0^-)$ represents the rank defect of $\begin{bmatrix} A \\ C \end{bmatrix}$.

Proposition 5: Let Σ_Λ be the linear structured system defined by (1) with associated graph $G(\Sigma_\Lambda)$ and associated bipartite graph $B(\Sigma_\Lambda)$. Among all the contractions of $G(\Sigma_\Lambda)$ having the maximal defect $\hat{d}$, the contraction $\{x_i^+ | x_i^+ \in V_0^+\}$ is the contraction with minimal cardinality, where $V_0^+ = \{V^+ \cap V_0\}$ and V_0 is the minimal inconsistent part of the DM-Decomposition of $B(\Sigma_\Lambda)$.

Proof: follows directly from the DM-Decomposition properties and from the definition of a contraction.

Proposition 6: Consider Σ_Λ the linear structured system defined by (1) with associated graph $G(\Sigma_\Lambda)$ and associated bipartite graph $B(\Sigma_\Lambda)$. Let V_0 be the minimal inconsistent part of the DM-Decomposition of $B(\Sigma_\Lambda)$.

1) The minimal number of additional sensors such that the composite system Σ_Λ^c has no contraction is $\hat{d}$ the contraction defect of Σ_Λ. Furthermore, these additional sensors must measure $\hat{d}$ states of V_0^+.
2) The set of solutions is parameterized by the set of maximal matchings of V_0 in the following way. For a maximal matching M_0 of V_0, the states of V_0 which have to be measured such that Σ_Λ^c has no contraction are $S_0^+ = V_0^+ \backslash \partial^+ M_0$ and $\partial^+ M_0$ is the set of vertices of V_0^+ incident to the edges in M_0.

Remark 2: These $\hat{d}$ additional sensors must be independent in the sense that there exists a size $card(V_0^+)$ matching between the states of V_0^+ and the states of $V_0^- \cup \hat{d}$ additional output sensors.

Sketchy proof: Sufficiency

If we add to a maximal matching M of $B(\Sigma_\Lambda)$ the $\hat{d}$ edges between S_0^+ and the additional sensors which form a matching of size $\hat{d}$, we get a size n matching in $B(\Sigma_\Lambda^c)$. This proves that the rank of $\begin{bmatrix} A \\ C \\ H \end{bmatrix}$ is n, therefore the composite system has no contraction.

Necessity

The minimal number of additional sensors is clearly $\hat{d}$ since each one can increase the rank of $\begin{bmatrix} A \\ C \end{bmatrix}$ by at most one unit.

Consider a solution of the problem which provides us with a matching M_c of size n of $B(\Sigma_\Lambda^c)$. M_c is incident to all vertices of V_0^+ then in M_c there are at least $\hat{d}$ edges from vertices of V_0^+ to additional sensors output vertices.

C. Observability recovering

To be observable, the composite system with additional sensors must satisfy output connectivity property and absence of contraction. Proposition 3 and Proposition 6 give the minimal number of states to be measured to satisfy separately these conditions. It is clear that $\bar{d} + \hat{d}$ well chosen additional sensors are enough to recover observability. Since both conditions are not independent, it may happen that observability of the composite system can be obtained with less than $\bar{d} + \hat{d}$ additional sensors.

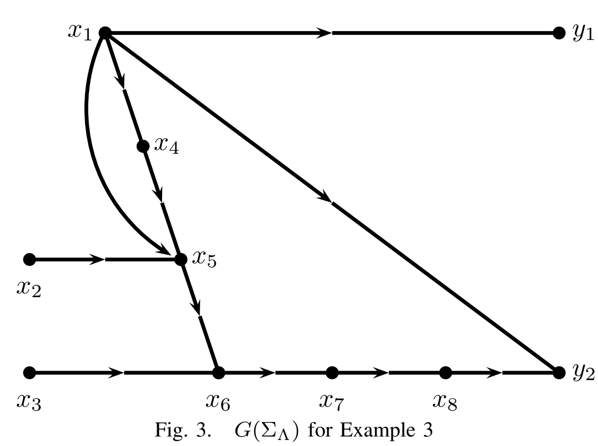

Fig. 3. $G(\Sigma_\Lambda)$ for Example 3

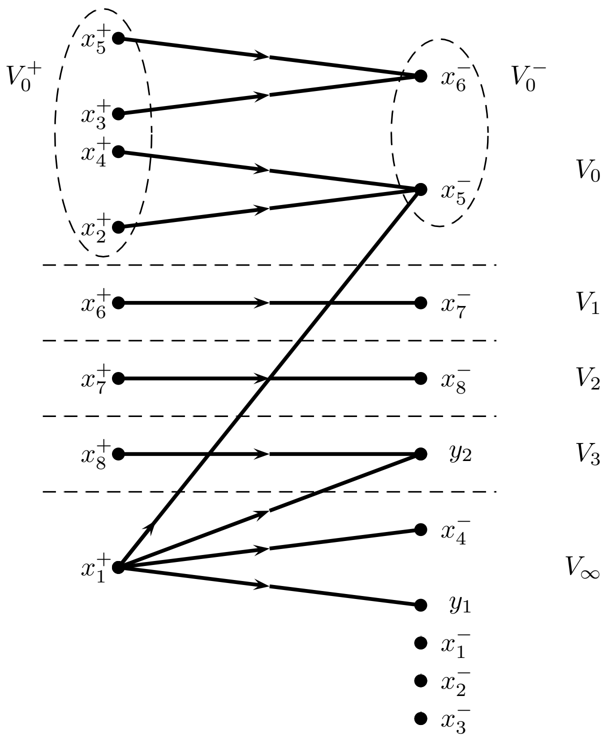

Fig. 4. $B(\Sigma_\Lambda)$ for Example 3

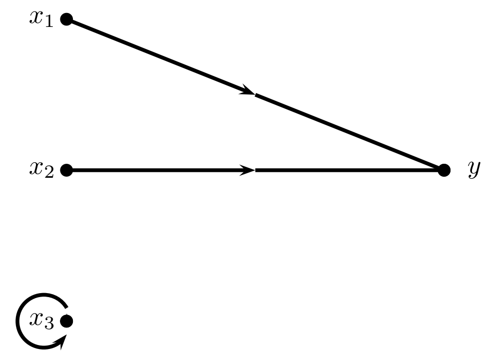

Fig. 5. $G(\Sigma_\Lambda)$ for Example 4

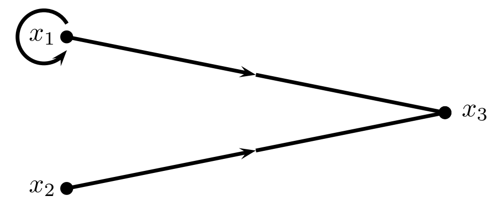

Fig. 6. $G(\Sigma_\Lambda)$ for Example 5

V. Illustrative examples

Consider firstly Example 3, which is clearly output connected (from Figure 3). Using Proposition 2 and Proposition 6, we need $\hat{d} = Card(V_0^+) - Card(V_0^-) = 2$ additional sensors for ensuring structural observability. The two states to be measured belong to $V_0^+ = \{x_2; x_3; x_4; x_5\}$. In $B(\Sigma_\Lambda)$ the parameterization of the four matchings on V_0 determines all the possible solutions i.e. we have to measure one state of $\{x_3; x_5\}$ together with one state of $\{x_2; x_4\}$.
Consider now Example 4 of Figure 5. The output connectivity defect $\bar{d}$ is one, the contraction defect $\hat{d}$ is one. In this case, the minimal number of additional sensors for ensuring structural observability is equal to $\bar{d} + \hat{d} = 2$. It shows that this bound of $\bar{d} + \hat{d}$ is sometimes reached. This is not always the case as shown in Example 5 of Figure 6. The infimal unobservable component is $\{x_3\}$ then $\bar{d} = 1$, and $S = \{x_3\}$ is a contraction with $\hat{d} = 1$. In this case $\bar{d} + \hat{d} = 2$ while measuring $\{x_3\}$ is enough to ensure structural observability.

VI. Concluding remarks

In this paper we have considered the observability recovering for structured systems by means of additional sensors. We gave bounds on the minimal number of states to be measured by these new sensors. Although the results are complete and easy to check for each of the conditions to be satisfied i.e. output connectivity and contraction avoidance, some work remains to be done to get the minimal number of additional sensors to fulfill both conditions simultaneously.

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