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A Non-Dexterous Dual Arm Robot's Feasible Orientations Along Desired Trajectories: Analysis & Synthesis

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***Abstract*— A dual-arm robotic manipulator needs to maintain its end-effectors' orientations with reference to each other as defined by the task constraints at various locations. To give one example, the task of operating a pair of pliers by rigidly grasping its handles will require that the orientation of the two end-effectors with respect to each other should vary as a function of their respective locations. Due to the wrist joint range limitations, dexterity is very limited and it may not be possible to achieve a desired maneuver along a desired trajectory. We present here a systematic approach to judge the feasibility of such demands and a method to find other reasonable target trajectories or orientations. The approach here is based on simple geometry with little computation load.**

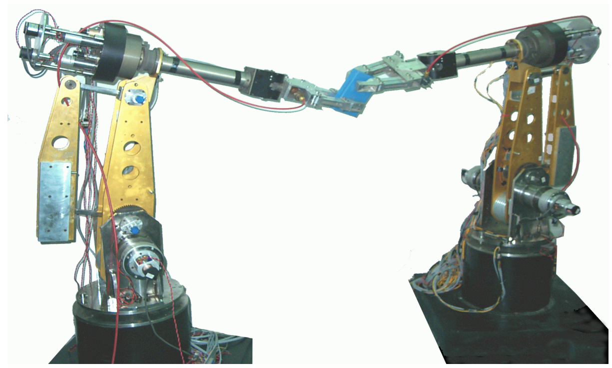

Fig. 1. A cooperative manipulator developed at Division of Remote Handling & Robotics, BARC, India.

I. Motivation

Manipulation of an object using a dual-arm manipulator imposes certain constraints that are not apparent in the case of single arm manipulation. In the case of dual-arm handling of an object, the end-effectors need to maintain (either a constant or a well-defined one) their orientation with reference to each other during the execution of the task, (see figure(1)). The mechanical joint limits of the two robotic arms ensure that only certain set of orientation is available in any given position. In other words, the wrist is not dexterous enough to execute the tasks. In such situation, it becomes very difficult to specify the maneuvers that needs to be done for executing a task successfully. Even the simplest possible task, like picking a object from one location and placing it in another, may become a frustrating experience because one of the arm would not able to generate required orientation at certain position. We felt that there is a requirement of an approach that will systematically eliminate the unfeasible zones in the workspace for a demanded orientation so that task is planned in only certain feasible sectors. We also wish to analyze how we should specify and modify the task orientations and positions, so that the task is executed without encountering the joint constraint.

To do this we focus on the end-effector, consider it's tip and wrist as the points of interest. The manipulator joints are treated as the variables that can be freely (subject to their limits) used to relocate the wrist and tip point.

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II. Related work

The dexterous workspace has been defined by Kumar and Waldron [1] as the subspace within which a vector on the end-effector may assume all orientations. A general discussion regarding workspace shapes and the effects of hand sizes are examined in detail in [2]. Lai and Menq [3] have presented analytical expressions for dexterous workspace. They have pointed out that only if the wrist is dexterous, i.e. the wrist has no joint limitations, the dexterous workspace exists. The service angle and the concept of service regions are discussed in [4]. An overview of research in dexterous manipulation is presented in [5]. A comparison of various methods for determining the boundaries to manipulator workspaces are made in [6]. The maps illustrating all possible orientations of a manipulator, called atlases of orientability, are presented in [7]. The limited feasibility of a fixed orientation trajectory has been discussed in [8].

We use exponential coordinates, a good description of which can be seen in [9], to generate rotational matrices. In exponential notation, any rotation of a rigid body can be represented using two variables, namely, the axis of rotation (ω) and the magnitude of the rotation (θ):

$$R(\omega,\theta) = e^{\hat{\omega}\theta}. \tag{1}$$

Here, R is a 3×3 rotation matrix and $\hat{\omega}$ is a skew-symmetric matrix. It is an operator that will yield a cross product if operated on a vector i.e. $\hat{\omega}\eta = \omega \times \eta$ where $\eta \in \Re^3$. Here it is just used as a matrix multiplied with a scalar.

In case the axis of rotation does not pass through origin, but through a point $q \in \Re^3$, the transformation of a point $p(0) \in \Re^3$ with a rotation θ about an axis ω is represented as

$$p(\theta) = e^{\hat{\xi}\theta} p(0), \tag{2}$$

where

$$\hat{\xi} = \begin{bmatrix} \hat{\omega} & v \\ 0 & 0 \end{bmatrix}, \quad v = -\omega \times q. \tag{3}$$

The use of these coordinates help in visualization of the placement of the wrist while generating the orientations with the tip of the end-effector fixed at a specified location.

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III. THE ORIENTATION OF AN END-EFFECTOR

The robot considered here is a six axis revolute joint robot, figure(2). The figure shows the second and third link, labeled as l_2 and l_3. The first three joints are labeled as q_1, q_2 and q_3. Last three joints intersect at the wrist.

The joint angles are denoted by q_i and their respective axes are denoted by ω_i, where $i = 1 \ldots 6$. The joint angles q_i with upper and lower limit values given as U_i and L_i are computed using a function ϕ, so that

$$q_i = \phi_i(\lambda) = (U_i + L_i)/2 + ((U_i - L_i)/2)\sin\lambda, \tag{4}$$

where $-\pi/2 < \lambda < \pi/2$. The center position is assumed to be zero and movement towards clockwise or counterclockwise is treated as positive and negative reading respectively. Thus, we can use $\phi_i(\lambda)$ for arriving at a feasible q_i for any value of $\|\lambda\| \leq \pi/2$ in kinematics expressions. In this article, we use q_i directly for our analysis, where q_i is considered within its limit. The purpose of its introduction here is to demonstrate the applicability of the present analysis, to a real robot with different limits for each joint, in an algorithm.

The orientation of an end-effector can be described by the orientation of the body frame with reference to the spatial frame. Thus, the orientation R is given by three vectors:

$$R = [a \quad o \quad n], \tag{5}$$

where, a is the approach vector and is directed from the wrist to the end-effector tip. We notice that ω_2 and ω_3 are parallel thus we can write,

$$R = e^{\hat{\omega}_1 q_1} e^{\hat{\omega}_2 (q_2+q_3)} e^{\hat{\omega}_4 q_4} e^{\hat{\omega}_5 q_5} e^{\hat{\omega}_6 q_6}, \tag{6}$$

where q_i is corresponding to the location of the end-effector tip. Here, the ω_is are represented with reference to the base frame, *not* with referance to the previous link, and thus, can be readily eavaluated by inspection.

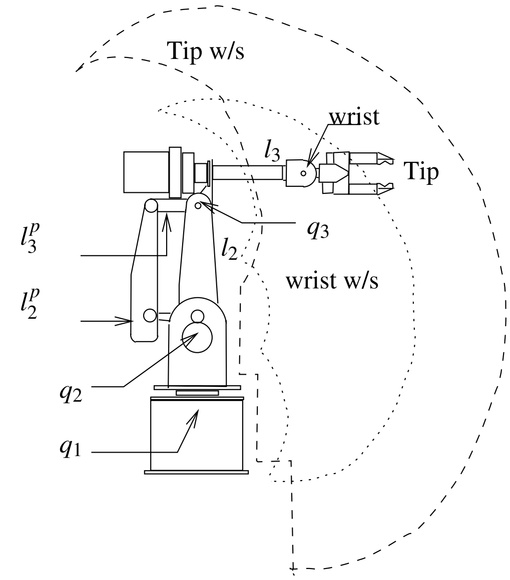

Fig. 2. A six d-o-f manipulator

IV. THE MEAN ORIENTATION

It is proposed that we can define a *mean orientation* associated with each point in the manipulator workspace. This is the orientation of the end-effector with it's last three joints, q_4, q_5, q_6, (for a 6-d.o.f. robot) at their center (mean) position ($q_i(\lambda_c) = 0$). Thus they constitute an I (identity) rotation. Further, we can associate a vector $q \in \Re^3$ with each workspace location, specifying the wrist location in terms of the first three joint angles. If we fix the last three joints at zero degrees, then each end-effector tip position p is uniquely associated with an unique $q(p)$ as there is no redundancy. The *mean orientation* of a point p is given as

$$\begin{aligned} R_p^M &= e^{\hat{\omega}_1 q_1(p)} e^{\hat{\omega}_2 (q_2(p)+q_3(p))} & (7) \\ &= [a_p^M \quad o_p^M \quad n_p^M] & (8) \end{aligned}$$

As can be interpreted from the above equation, there will be a mean orientation that is uniquely associated with the point p. There may be some other p that also has same mean orientation as seen from the above equation.

V. THE SET OF ORIENTATIONS AT THE POINT p

A dual-arm manipulation task will require to place the second arm's end-effector tip at a desired location p_d with a specific orientation R_d, where R_d and p_d depend on the orientation and the location of the first arm as well as the size of the object. If inverse kinematics solution yields the feasible joint angles then the task can be executed. However, it may not be possible to arbitrarily demand the position, orientation and hope to get the solution every time. It is, therefore, necessary that the set of orientations available at a location or the set of locations where a certain orientation is available is ascertained. This will ensure that the task is planned successfully and the error margins are

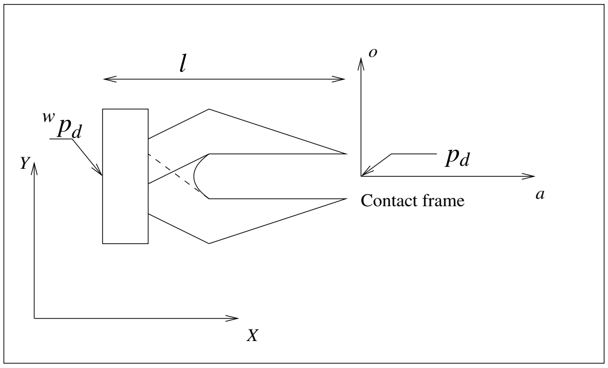

Fig. 3. End-effector orientation

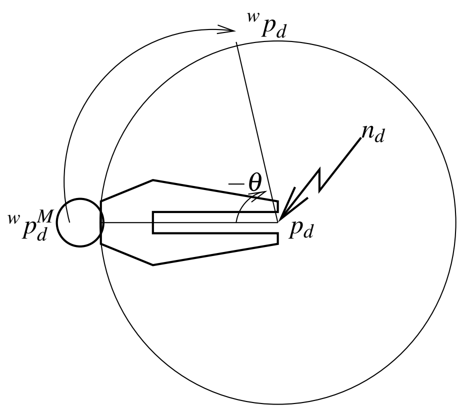

Fig. 4. Rotation of the vector connecting the wrist with the tip

reasonable. This is due to the fact that online adjustment of computed position and orientation may be needed on the basis of actual feedback. Thus, if location p_d and orientation $R_d = [a_d \quad o_d \quad n_d]$ is obtained with the joints already at their end-limits, the minor compliance can not be provided.

The set of orientations available at any location can be generated by freezing the tip at that point and then rotating the wrist using the full range of the last 3 joints. We maintain the axes of these rotations parallel to the *mean* approach vector a_d^M (4th joint's axis while in mean orientation) and the normal vector n_d (5th joint's axis under transformation due to the rotation of q_4) respectively to cover the entire subspace systematically. That is, rotate q_4 by an angle ϕ_1, this will make the axis ω_5 parallel to, say, n_{d1} (a new axis direction of q_5). Now the wrist can be moved on a circular trajectory with n_{d1} as an axis using the joints q_1, q_2 and q_3. In similar way, moving q_4 by different ϕ, we can generate a series of circles that make a sphere, figure (6). All the axes are assumed to be passing through the end-effector tip. The axis of joint q_4 and q_5 are orthogonal by construction and subject to the joint limits, it is feasible to generate n_d that is orthogonal to a_d.

We call it the service sphere corresponding to the service point p_d.

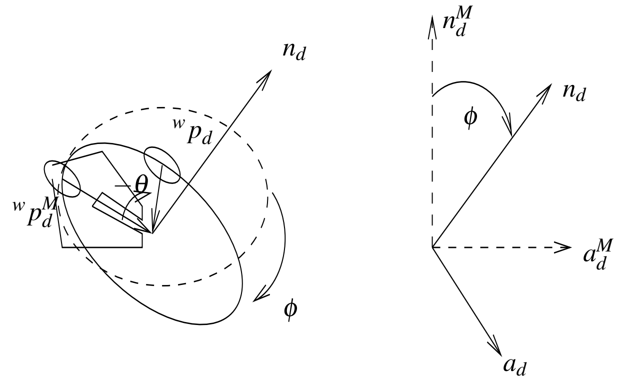

Fig. 5. Rotation of the wrist

VI. Problem Description

To begin with, we assume that the object frame and end-effector frame have identical orientation. Hence the orientation requirement of the object and end-effector is the same. In so far as the translation position (the tip location) is concerned, we assume that the task is defined inside the wrist workspace and end-effector remains inside the tip workspace. For a given $g_d = (p_d, R_d)$ we can anticipate following three situations, assuming that the task is to handle a flask half-filled with water:

1) It is solvable with all the six joints within their limit.
2) It is solvable only if the orientation is modified. *For example, we may allow little tilting of the flask, ensuring that the water does not spill.*
3) It is solvable only if the tip location is modified. *We can hold it at some other location to keep it vertical.*

Now, the modified orientations and locations need not be unique and many choices may be there. But we can impose a few more constraints to narrow down the choices.

VII. The Modifications in the Orientation

Assuming that the desired orientation of the end-effector R_d may not be available at p_d, we proceed to investigate if we can modify the end-effector orientation requirement and still keep the object in its place with the desired orientation.

The mean orientation at p_d is

$$R_d^M = [a_d^M \quad o_d^M \quad n_d^M]. \tag{9}$$

The wrist at this orientation is located at

$$^{w}p_d^M = p_d - l a_d^M \tag{10}$$

where l is the length of the end-effector. Whereas the desired position of the wrist at the desired orientation is

$$^{w}p_d = p_d - l a_d. \tag{11}$$

The orientation of the end-effector can be changed if we move the wrist in all possible directions while maintaining the tip at the desired point. The locus of the wrist will lie on a sphere if movements in all the directions are possible.

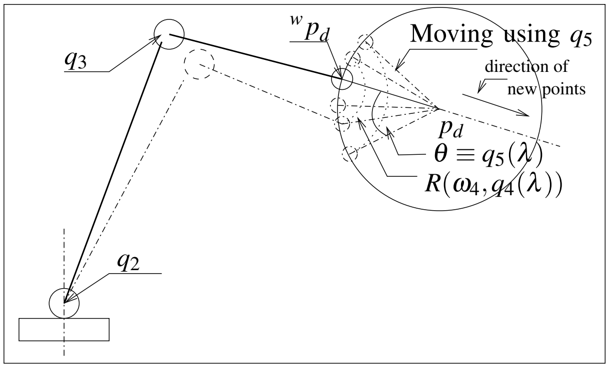

Fig. 6. The service sphere

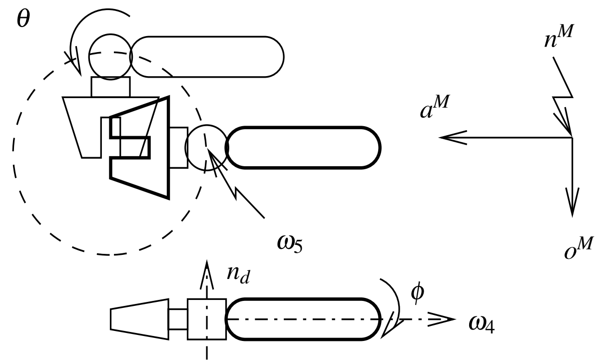

Fig. 7. Rotation of joint 4 by ϕ and 5 by θ

We propose that we attempt to rotate the point $^w p_d^M$ about an axis n_d that is passing through p_d, to reach the point $^w p_d$, figure (4) and figure (5). The axis n_d is perpendicular to the plane of vector $p_d - {}^w p_d$ and $p_d - {}^w p_d^M$. Therefore, it is given as

$$n_d = (p_d - {}^w p_d^M) \times (p_d - {}^w p_d) \tag{12}$$

The angle between these two vector is

$$\cos\theta = \frac{(p_d - {}^w p_d^M)^T (p_d - {}^w p_d)}{\|(p_d - {}^w p_d^M)\| \|(p_d - {}^w p_d)\|} \tag{13}$$

Since n_d passes through p_d, we find the twist ξ as in (3)

$$\xi = \begin{bmatrix} -(n_d \times p_d) \\ n_d \end{bmatrix} \tag{14}$$

Thus we can write the wrist point location as

$${}^w p_d = \left(e^{\hat{\xi}_d \theta}\right) ({}^w p_d^M). \tag{15}$$

A. The questions about the wrist point

The desired orientation leaves us no choice but to keep the wrist point at the location given by $^w p_d$. There may not be any problem in placing the wrist at the desired location using q_1, q_2 and q_3. However, in order to keep the end-effector tip at the desired position, we need to use q_4 and q_5. Thus, ω_5 must be made parallel to n_d by rotating the q_4 by ϕ and then q_5 can be turned by an angle θ to get the approach vector a_d, figure(7). We need to investigate following points:

- Is $^w p_d$ inside the workspace?
- Is θ within the limits q_5?[1].
- Can we actually rotate the axis of q_5, namely ω_5, so that it becomes parallel to n_d using $R(\omega_4, \phi)$.(note that $\omega_4 \equiv a_d^M$)

[1]axis n_d and ω_5 would be made parallel using $R(a_d^M, \phi)$

B. Shifting the wrist towards $^w p_d$

We attempt the last question first. It can be seen that n_d can be created by rotating the mean normal vector about an axis parallel to a_d^M using the joint 4:

$$n_d = (e^{\hat{a}_d^M \phi}) n_d^M \tag{16}$$

As before, we must check whether the angle ϕ violates the limits of joint 4. If it is valid then we can get n_d. Use n_d to rotate the end-effector and check if θ is also suitable for achieving the desired $^w p_d$.

Finally, we can further alter the normal vector , n_d, using the last joint q_6, without affecting the approach vector a_d. Figure (7) shows the joint 4 and joint 5 that are used to move the wrist points to different locations.

C. The required modifications

The motivation for starting this work is due to the fact that it is difficult to predict or anticipate the service positions where the desired orientation will be available. If we could really move q_5 by θ and q_4 by ϕ as described in the previous subsection, we would not have bothered to find them in the first place because *the inverse kinematic solution would have been already available*. Since the constraints do not always allow us to achieve the desired orientation, we must select some alternative orientation.

An alternative orientation R could be decided on the basis of task requirement. In some tasks, only one of the column of the matrix R may be critical. For example, in a situation where liquid is to be handled, the normal vector (assuming the object and end-effector orientation is same) must be the z axis. In some other tasks like assembly of cylindrical objects, only the approach vector may be important. But in the case of rigid object handling, the desired orientation requirement may have to be changed by changing the initial orientation between the grippers of the two manipulators by gripping the object at a different angle at the starting point. This is the situation that will be interesting and also more frequent in case of dual arm manipulation. It implies that we should develop some algorithm to ascertain the range of orientations available at certain points as well as the trajectory which connects these points.

D. A new approach for selecting orientations

The orientation is described by an orthogonal matrix and it is not simple to specify some range or boundary of available orientations in conventional sense. Though, we can specify the range of yaw, pitch and roll available at a point, it is not, however, very clear as to what way we can call some orientation closest to the desired one compared to another. We propose to use R_d^M as the reference orientation for the service position and the attempt to move the point ${}^w p_d^M$ to the target location ${}^w p_d$ using q_4 and q_5. The closest location, where the wrist point can reach, may be called a nearest possible (feasible) orientation, $R(n_p, \theta_p)$.

As explained in the previous section, availability of the range of joints q_5 and q_4 to the extent of θ and ϕ may not be present. We may fall short of desired target by $\delta\phi$ and $\delta\theta$. To maintain the object in specified orientation, we must, therefore, introduce an *additional* orientation between the object and the end-effector. This orientation can be computed as: (we use n_p and θ_p as the possible (feasible) orientation at the service point.)

$$R_d = R_{(n_p,\theta_p)} R_{grip} \quad \theta_p \neq 0 \tag{17}$$

$$R^{-1}_{(n_p,\theta_p)} R_d = R_{grip} \tag{18}$$

We can find n_p from the following equation

$$n_p = R_{(a_d^M,\phi_p)} n_d^M, \tag{19}$$

where ϕ_p is the maximum possible rotation available about axis a_d^M.

If we choose ϕ_p and θ_p as zero, it is possible to use mean orientation and grip orientation alone to generate desired orientation for the gripped object:

$$R_{grip} = (R_d^M)^T R_d, \tag{20}$$

so that last three joints need not be moved. This way we compensate the error between the desired and available $\delta\theta$ and $\delta\phi$ by introducing R_{grip} between the end-effector and the contact frame. Given this choice, it is easy to see that θ_p and ϕ_p need not be the extreme limits but we can have a reasonable value so that the constraints at other points are also satisfied.

Now, we settle for a (modified) desired orientation and still denote it by R_d to save notational complexity. We can write (modified)

$$R_d = R_d^M R_{T_d}, \tag{21}$$

where R_{T_d} is the orientation generated by last three joints. The mean orientation is nothing but the orientation achieved using first three joint. (Note that it will be now within the limits due to the algebraic manipulation done to calculate an acceptable R_d earlier.)

Now, we need to pick object from a point p_{pick}, in such a way, so that the orientation between the gripper and the object is R_{grip}. The object is assumed to be in reference orientation. Thus

$$R_{grip} = R_{pick}^M R_{T_{pick}}. \tag{22}$$

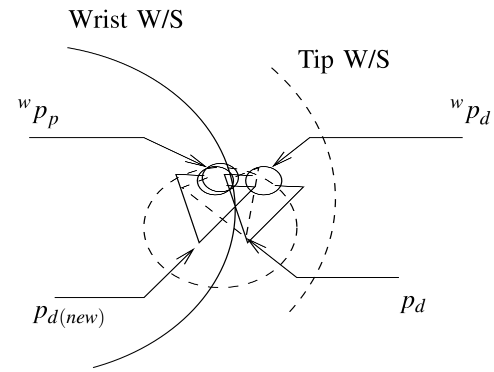

Fig. 8. Possible locations where the wrist can be shifted

Is $R_{T_{pick}}$ available ? If it is not available we have to look for a better R_{grip}. We can choose one R_{grip} that is calculated by selecting the smallest of θ_p and ϕ_p from all the points it has to service. It is assumed that the object orientation and service point (end-effector tip) can not be modified due to the nature of the task (rigid object manipulation with dual arms). Same exercise is to be repeated for all the service points where we need certain orientation and arrive at an optimum R_{grip} that satisfies all the points. The introduction of this additional orientation may push some otherwise feasible orientation out of the range. In that case some of the service points will have to be discarded to manage maximum service points. The gripper can be designed to suit the object geometry and preferred grip orientation. It is not unreasonable to expect that the contact surface and shape between the object and the end-effector can be designed suitably to generate a range of orientation.

VIII. Modification in the desired location

We may choose some other suitable point where the task can be done without changing the orientation requirement. In many situations the location may not be very crucial. For example, in a task of pouring liquid from one container to another using dual arm, it does not matter where in the workspace it is done, as long as the liquid is transferred successfully.

A. Altering the wrist locations

The wrist point may be outside the workspace of first three joints for a particular orientation. If the location of the service point is not critical, we can go to the nearest location (in some sense like minimum joint movements) and do the task. To examine a wrist position, with end-effector of length l and approach vector a_d,

$${}^w p_d = p_d - l a_d, \tag{23}$$

we can find if it is inside the first three joint limits. Otherwise we can find another ${}^w P_d'$ as

$${}^w P_d' = p_d - \alpha(l a_d), \tag{24}$$

where α is chosen to satisfy the requirement of placing the wrist point inside the workspace. This choice is meant for preserving the orientation at the cost of relocating the service point. But it is not necessary that the wrist workspace will be nearest or available in that direction(a_d).

Another approach proposed is to relocate the wrist on the intersection of the wrist workspace and service sphere (${}^{w}p_p$), figure (8). Then we change the tip position to $p_{d(new)}$ to generate desired orientation. This solution is applicable if q_4 and q_5 continue to remain within the joint limits during re-positioning of the wrist point on the service sphere, but the desired wrist point ${}^{w}p_d$ falls outside the wrist workspace i.e. one of the joints q_1, q_2 or q_3 reaches its limit. We can have a modified service position is

$$p_{d(new)} = {}^{w}p_p + l a_d \tag{25}$$

This change of service point is not often objectionable even in the case of fixed point assembly jobs too. One can always grip the object from a neighboring point (on the object) so that the object itself (or the point of interest on the object) remains on the desired trajectory.

B. Selective relocation of the service points

As discussed in the previous section, it may occur in some cases that it is impossible to maintain (p_d, R_d) for all the service points. In these situation, we can think of performing the relocation of some of the points, where change in location does not make much difference in the task to be performed at those points. In two arms manipulation, humans do these type of relocation almost without noticing.

IX. Conclusions and future work

It is very difficult to fabricate a robot end-effector with a dexterous wrist and almost all robots have limited joint movements. A robotic task, however, may require a series of manipulations. The end-effector will be expected to smoothly achieve certain orientations at certain locations. A user may not be comfortable in calculating inverse kinematics for each and every alternative points to discover some, if any, feasible zones. Closed form analytical solutions to this problem do not seem feasible. We have presented here an approach that will simplify the task planning with very little computation load and can be converted easily to an algorithm. A search algorithm that uses iterations to find a trajectory in case of a rigid rod can be seen in [10], to get an idea of the computation load.

There may be some unintentional shift in the object orientation while grasping it. This will require re-calculation of the service locations and orientations. But the object's altered orientation may be difficult to quantify in a dynamic environment. It may, therefore, be a good idea to introduce an impedance control in the neighborhood of the desired orientation so that $\delta\phi$ and $\delta\theta$ is self-corrected by interaction forces. Our current research effort is to write an algorithm based on the ideas presented in this paper.

X. Acknowledgment

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