

# Flexible Satellite Attitude Control Via Sliding Mode Technique

P. Guan, X.J. Liu, and J.Z. Liu

**Abstract**—Input-output (I/O) linearization is a typical nonlinear control method in the multi-input multi-output (MIMO) system. In this paper, adaptive fuzzy sliding mode control incorporating input-output linearization is proposed to constitute the hybrid controller. The proposed method is then applied to the attitude maneuver control of the flexible satellite. The basic control structure is presented. The adaptive fuzzy control is utilized to approach the nonlinear control part of sliding mode control and the adaptive law is derived. The parameter of the adaptive fuzzy control is adjusted on-line to deal with the satellite uncertainty, thus the robustness is obtained. Simulation results show that precise attitude control is accomplished in spite of the uncertainty in the system.

## I. INTRODUCTION

SATELLITE dynamics exhibit in general highly nonlinear behavior with environmental disturbances and poorly known parameters. Robustness against modeling uncertainties and unknown disturbances for the satellite attitude control system is widely treated in the literature. One of the techniques is input-output (I/O) linearizing control that is often employed in the nonlinear multi-input multi-output (MIMO) system. Although the technique of I/O linearization results in input-output decoupling, it has limitations. The technique relies on the exact cancellation of nonlinear terms and the resulting controller's robustness cannot be guaranteed for the nonlinear system with uncertainties. In recent years, the enhancing robustness of I/O linearization controllers has been widely discussed. Reference [1] obtained robustness of linearization control system by the integral error feedback. For pitch axis maneuver, attitude control is accomplished in spite of the parameter uncertainty in the system. References [2]-[4] combine the I/O linearization with sliding mode control to form a hybrid controller. The resulting hybrid controllers increase the robustness of the I/O linearization controllers. However, it requires that the uncertainty is matched and the upper bound of uncertainty is known in advance. In practice, the bound of uncertainty is very difficult to be known exactly. If the upper bound is estimated too large, it may excite high-frequency dynamics, and result in the heavy chattering phenomena.

Manuscript received February 28, 2005. This work was supported in part by the National Nature Science Foundation of China under Grant 69804003

P. Guan is with the Department of Automation, Beijing Institute of Machinery, Beijing, 100085, P.R. China (e-mail: guanpingliu@tom.com).

X.J. Liu is with the Department of Automation, North China Electric Power University, Beijing, 102206, P.R. China (corresponding author. Tel: 86-10-51963882; e-mail: liuxj@ncepubj.edu.cn).

J.Z. Liu is with the Department of Automation, North China Electric Power University, Beijing, 102206, P.R. China (e-mail: nepuljz@263.net)

Recently fuzzy control is introduced into the sliding mode control for attenuating the chattering phenomena. In [5], the authors developed the control method based on the principle of sliding mode control to handle the chattering problem effectively by the application of fuzzy set theory. Paper [6] proposed fuzzy controller with the fuzzy sliding surface, and proved the stability of the fuzzy control system and the boundedness of the tracking error. The design procedures of these methods usually focus on giving the fixed fuzzy control rules, which are usually inappropriate and possess weak adaptability to the uncertainty. Therefore, various tuning methods are usually employed to improve the performance of fuzzy sliding mode controllers.

In this paper, adaptive fuzzy sliding mode control (AFSMC) is incorporated with I/O linearization to control the attitude maneuver of the flexible satellite. Based on I/O linearization, the motion equations of the flexible satellite are decomposed into three decoupled subsystems. Three adaptive fuzzy sliding mode control laws are derived similarly to control respectively the three subsystems. According to the principle of sliding mode control, the control law consists of two parts: the equivalent control term and the nonlinear control term. The adaptive fuzzy control is utilized to approximate the nonlinear control term for attenuating the chattering phenomena. The on-line parameter adaptation law is derived to make the system output track the desired attitude asymptotically. The expression of the proposed AFSMC is simple and independent of the boundary of the uncertainty. The AFSMC can effectively cope with the uncertainty of flexible satellite through on-line learning, thus possesses the good robustness. Simulation results show that precise attitude control is accomplished in the presence of the uncertainty.

The organization of this paper is as follows. Section II describes the attitude control problem. The theory of feedback linearization is applied to the uncertain nonlinear system in section III. The AFSMC law is derived in section IV. Section V presents simulation results. The final section summarizes the research and conclusions.

## II. PROCEDURE FOR PAPER SUBMISSION

The flexible satellite with a solar panel moving in a circular orbit, an inverse square gravitational field, equipped with reaction jets for orientation control is considered. The equations of motion about the center of mass of the satellite is as follows [7]:

$$J\dot{\omega} + \omega^*J\omega + C\ddot{\eta} = T_a + u \quad (1)$$

$$\ddot{\eta} + 2\xi\Lambda\dot{\eta} + \Lambda^2\eta + C^T\dot{\omega} = 0 \quad (2)$$

$$\dot{q} = \frac{1}{2}(q^\times + q_0 I)\omega \quad \dot{q}_0 = -\frac{1}{2}q^T \omega \quad (3)$$

where  $J \in R^{3 \times 3}$  denotes the inertia matrix of satellite,  $\omega \in R^3$  denotes the angular velocity of the satellite with respect to an inertial frame,  $u \in R^3$  denotes the vector of control torques,  $T_d \in R^3$  denotes the vector of external disturbances,  $C$  is the coupling matrix between rigid body and appendage,  $\eta$  is the vector of five-order modal displacements,  $\xi$  is the modal damping matrix,  $\Lambda$  is modal frequency matrix.  $q \in R^3$  and  $q_0 \in R$  denote the quaternion of the satellite with respect to an inertial frame and satisfy the constraint  $q^T q + q_0^2 = 1$ , and  $I$  denotes a  $3 \times 3$  identity matrix.

For any vector  $r = [r_1 \ r_2 \ r_3]^T$ , the notation  $r^\times$  stands for cross product matrix

$$r^\times = \begin{bmatrix} 0 & -r_3 & r_2 \\ r_3 & 0 & -r_1 \\ -r_2 & r_1 & 0 \end{bmatrix} \quad (4)$$

Satellite commonly operates in the presence of various disturbances. The inertia matrix of satellite is usually not known exactly. In addition, oscillation of the flexible appendage also influence satellite attitude. Therefore, satellite control design needs take these uncertainties into account. In this paper, it is assumed that  $J = J^* + \Delta J$ , where  $J^*$  is a known matrix,  $\Delta J$  denotes the uncertain parts of the matrices  $J$ . In order to apply the proposed design method, we define state variable  $x = (q^T, \omega^T)^T$  and the controlled output  $y = q$ . The satellite motion (1)~(3) can be put in the form of MIMO uncertain nonlinear system

$$\dot{x} = f(x) + \Delta f(x) + [g(x) + \Delta g(x)]u \quad (5)$$

$$y = h(x) = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \end{bmatrix} x \quad (6)$$

$$\text{where } f(x) = \begin{bmatrix} \frac{1}{2}(q^\times + q_0 I)\omega \\ (J^*)^{-1}(-\omega^\times J^* \omega) \end{bmatrix}, \quad \Delta f(x) = \begin{bmatrix} 0 \\ \Delta f_2 \end{bmatrix},$$

$$g(x) = \begin{bmatrix} 0 \\ (J^*)^{-1} \end{bmatrix}, \quad \Delta g(x) = \begin{bmatrix} 0 \\ \Delta g_2 \end{bmatrix},$$

$$u = [u_1, u_2, \dots, u_m]^T, \quad y = [y_1, y_2, \dots, y_m]^T, \quad m=3.$$

$\Delta f(x)$  and  $\Delta g(x)$  denote the terms in (5) which arise due to satellite uncertainties.  $\Delta f_2$  is a function of  $\Delta J, T_d, \ddot{\eta}$ .  $\Delta g_2$  is a function of  $\Delta J$ . The control objective is to design a control law  $u(t)$  which make the output  $y$  tracks a desired trajectory  $y_d = q_d(t) = (q_{d1}, q_{d2}, q_{d3})^T$  in the presence of bounded disturbance  $\Delta f$  and  $\Delta g$ .

In the following section, the nonlinear dynamics of (5) and (6) is first I/O linearised, and is converted into three decoupled subsystems. Then the adaptive fuzzy sliding mode

control law is designed for each subsystem. The fuzzy rules parameters are on-line modified to effectively adaptive to satellite uncertainties and avoid the chattering phenomenon.

### III. I/O LINEARIZATION OF SATELLITE ATTITUDE SYSTEM WITH UNCERTAINTIES

If  $\Delta f$  and  $\Delta g = 0$ , the nonlinear system (5) corresponds to the nominal system. The I/O linearization process is performed on the nominal system by assuming that the nonlinear system (5) has a vector relative degree  $[r_1, \dots, r_m]$

$$\begin{bmatrix} y_1^{(r_1)} \\ y_2^{(r_2)} \\ \vdots \\ y_m^{(r_m)} \end{bmatrix} = B(x(t)) + A(x(t)) \begin{bmatrix} u_1(t) \\ u_2(t) \\ \vdots \\ u_m(t) \end{bmatrix} \quad (7)$$

$$\text{where } A(x) = \begin{bmatrix} L_{g_1} L_f^{(r_1-1)} h_1(x) & \dots & L_{g_m} L_f^{(r_1-1)} h_1(x) \\ \dots & \dots & \dots \\ L_{g_1} L_f^{(r_m-1)} h_m(x) & \dots & L_{g_m} L_f^{(r_m-1)} h_m(x) \end{bmatrix},$$

$$B(x) = \begin{bmatrix} L_f^{(r_1)} h_1(x) \\ \vdots \\ L_f^{(r_m)} h_m(x) \end{bmatrix} \quad (8)$$

In (8),  $L^i$  ( $k=r_1-1, \dots, r_m-1$ ) denotes the  $k$ th successive Lie derivative. A major drawback of I/O linearization is that it relies on the exact cancellation of nonlinear terms in order to achieve linear input-output relation. Thus the presence of uncertainties causes loss of I/O decoupling, steady-state tracking errors and deteriorated transient responses. However, for a certain class of uncertainties which obey the so called 'matching condition', I/O linearization is guaranteed [3], [4].

Matching condition: If the system has a vector relative degree  $[r_1, \dots, r_m]$ , then the addition of perturbations  $\Delta f$  and  $\Delta g$  do not change the relative degree of the system.  $\Delta f(x, t)$  and  $\Delta g_i(x, t) \in \ker [dh_i, dL_t h_i, dL_t^2 h_i, \dots, dL_t^{r_i-2} h_i]$ ,  $i=1, 2, \dots, m$

where  $\ker(\dots)$  denotes the kernel of a matrix. This 'matching condition' guarantees that the perturbations  $\Delta f$  and  $\Delta g$  do not appear in derivatives of  $y_i$  of order less than  $r_i$ . Therefore if the uncertainties  $\Delta f$  and  $\Delta g$  satisfy 'matching condition', there exists diffeomorphic coordinate transformation  $T(x) = (\xi, \eta)$  which transforms system (5) and (6) into the nominal form [3], [4]:

$$\begin{cases} \dot{\xi}^{(r)} = B + \Delta B + (A + \Delta A)u \\ \dot{\eta} = W \end{cases},$$

$$y^{(r)} = \xi^{(r)} = B + \Delta B + (A + \Delta A)u \quad (9)$$

$$\text{where } \xi^{(r)} = [(\xi_1^{(1)})^{(r_1)}, (\xi_1^{(2)})^{(r_2)}, \dots, (\xi_1^{(m)})^{(r_m)}]^T,$$

$y^{(r)} = [y_1^{(r_1)}, y_2^{(r_2)}, \dots, y_m^{(r_m)}]^T$ .  $\Delta A$  and  $\Delta B$  arise due to uncertainty  $\Delta f$  and  $\Delta g$  in linearization process. The matrix A and B are defined similarly as (8) with the argument  $x$  replaced with  $T^{-1}(\xi, \eta)$ . If the matrix A is nonsingular, the control law is:

$$u = A^{-1}(v - B) \quad (10)$$

where  $v$  is a new synthetic input. Substituting (10) into (9), (5) and (6) are converted into the form of the decoupled subsystems as follows:

$$\begin{aligned} \dot{\xi}_1^i &= \xi_2^i \\ &\vdots \\ \dot{\xi}_{r_i-1}^i &= \xi_{r_i}^i \\ \dot{\xi}_{r_i}^i &= v_i + d_i(\xi, \eta) \\ \eta &= W \\ y_i &= \xi_1^i, \quad i=1, 2, \dots, m \end{aligned} \quad (11)$$

Performing I/O linearisation on the satellite dynamic (5), it can easily be verified that it has a relative degree  $[2, 2, 2]$ . In the satellite maneuver control process, the matrix A is nonsingular for all quaternion  $(q_0, q)$ . Therefore, selecting the new state variable

$$\begin{aligned} z &= T(x) = [z_{11}, z_{12}, z_{21}, z_{22}, z_{31}, z_{32}]^T \\ &= [h_1(x), L_f h_1(x), h_2(x), L_f h_2(x), h_3(x), L_f h_3(x))]^T \end{aligned}$$

the satellite dynamic (5) and (6) can be converted into three decoupled pseud-linearity subsystems as follows:

$$\begin{aligned} \dot{z}_{i1} &= z_{i2} \\ \dot{z}_{i2} &= v_i + d_i(z) \\ y_{i1} &= z_{i1}, \quad i=1, 2, 3 \end{aligned} \quad (12)$$

$d_i(z)$  denote the term in (12) which arise due to satellite uncertainties  $\Delta f$  and  $\Delta g$ . In these subsystems,  $d_i(z)$  may be regarded as the bounded disturbance,  $|d_i(z)| \leq D_i$ ,  $D_i > 0$ ,  $i=1, 2, 3$ .

#### IV. ADAPTIVE FUZZY SLIDING MODE CONTROLLER

##### A. Sliding mode control

In this section, the sliding mode controller is designed for each subsystem of (12). The control object is to make satellite attitude  $z_i = (z_{i1}, z_{i2})^T$  track the desired attitude  $z_{di} = (z_{di}, \dot{z}_{di})^T$ ,  $i=1, 2, 3$ .

Let the tracking error be  $e_i = z_{di} - z_i = (e_i, \dot{e}_i)^T$

Then a sliding surface in the space of the error state can be defined as:

$$s_i = -(k_i e_i + \dot{e}_i) = -k_i^T e_i, \quad i=1, 2, 3 \quad (13)$$

where the coefficients  $k_i = [k_i, 1]^T$  are the coefficient of the Hurwitzian polynomial  $\lambda_i + k_i$ . If the initial error  $e_i(0) = 0$ , then the tracking problem can be considered as

the state error vector  $e_i$  remaining on the sliding surface  $s_i(e_i, t) = 0$  for all  $t \geq 0$ .

The process of sliding mode control can be divided into two phases: the approaching phase ( $s_i \neq 0$ ) and the sliding phase ( $s_i = 0$ ). A sufficient condition to guarantee that the trajectory of the error vector  $e_i$  will translate from the approaching phase to the sliding phase is to select the control strategy such that:

$$s_i \dot{s}_i \leq -\eta |s_i|, \quad \eta > 0 \quad (14)$$

Corresponding to two phases, two types of control law can be derived separately. In the sliding phase, we have  $s_i = 0$  and  $\dot{s}_i = 0$ , then the equivalent control  $u_{eqi}$  that will force the system dynamics to stay on the sliding surface can be obtained as follows

$$\dot{s}_i = -k_i \dot{e}_i - \ddot{e}_i = -k_i \dot{e}_i + v_i + d_i - \ddot{z}_{di} = 0 \quad (15)$$

$$u_{eqi} = k_i \dot{e}_i - d_i + \ddot{z}_{di} \quad (16)$$

In the approaching phase, where  $s_i \neq 0$ , in order to satisfy the sliding condition (14), a nonlinear control term  $u_{Ni}$  must be added and the resulting sliding mode control law will be:

$$v_i = u_{eqi} - u_{Ni}, \quad u_{Ni} = (D_i + \eta_s) \text{sgn}(s_i) \quad (17)$$

$$\text{where } D_i + \eta_s \geq \eta > 0, \quad \eta_s = \eta_{s1} + \eta_{s2}, \quad \eta_{s1} > \eta_{s2} > 0$$

It is obvious that the sign of nonlinear control term  $u_{Ni}$  is changed while the state trajectories cross the sliding surface, incurring heavily chattering phenomenon. Furthermore nonlinear control term  $u_{Ni}$  often cause high-frequency unmodelled dynamics. To avoid the chattering phenomenon, the adaptive fuzzy control is utilized to approximate the nonlinear control law  $u_{Ni}$ . In the next section, the adaptive fuzzy controller is constructed, with adaptive adjusting law of the rules parameters.

##### B. Adaptive fuzzy sliding mode control

The adaptive fuzzy controller is implemented by the Takagi-Sugeno(T-S) model with constant consequent. The  $l$ th fuzzy rule is:

$$R_l: \text{if } s_i \text{ is } A_l^i \text{ then } \hat{u}_{Ni} \text{ is } C_l.$$

Where  $A_l^i$  represent the fuzzy set of input variables ( $s_i$ ).  $C_l$  denotes constant value,  $l=1, 2, \dots, L$  ( $L$  is number of rules). On the premise of precise attitude control, the only rules parameters  $C_l$  ( $l=1, 2, \dots, L$ ) are on-line adjusted, so as to reduce the computational burden in the control process. The output of the adaptive fuzzy controller is:

$$\hat{u}_{Ni} = C_{fi}^T \Psi(s_i) \quad (18)$$

$$\text{Where } \Psi(s_i) = [\psi_1(s_i), \psi_2(s_i), \dots, \psi_L(s_i)]^T,$$

$$\psi_l(s_i) = \frac{\mu_{A_l^i}(s_i)}{\sum_{l=1}^L \mu_{A_l^i}(s_i)}, \quad C_{fi} = [C_1, C_2, \dots, C_L]^T$$

$C_{\bar{f}_i}$  is the adjustable parameter vector.  $\mu_{A_i}(s_i)$  is Gaussian membership function.

In this paper, the adaptive fuzzy control  $\hat{u}_{Ni}$  is used to approximate the nonlinear control  $u_{Ni}$ . The resulting control law is as follows:

$$v_i = k_i \dot{e}_i + \ddot{z}_{di} - \hat{u}_{Ni} \quad (19)$$

In the following, the adaptive law for updating rule parameter  $C_{\bar{f}_i}$  is derived. Let  $C_{\bar{f}_i}^*$  denotes the optimal  $C_{\bar{f}_i}$ , defined as:

$$C_{\bar{f}_i}^* = \arg \min_{|C_{\bar{f}_i}| \leq M} \left[ \sup_{s_i \in \mathbb{R}} |\hat{u}_{Ni}(s_i | C_{\bar{f}_i}) - u_{Ni}| \right] \quad (20)$$

Then,

$$\begin{aligned} \dot{s}_i &= -k_i \dot{e}_i + v_i + d_i - \ddot{z}_{di} \\ &= -k_i \dot{e}_i + d_i - \ddot{z}_{di} + k_i \dot{e}_i + \ddot{z}_{di} - \hat{u}_{Ni}(s_i | C_{\bar{f}_i}) \\ &= -\hat{u}_{Ni}(s_i | C_{\bar{f}_i}) + d_i \\ &= -\hat{u}_{Ni}(s_i | C_{\bar{f}_i}) + \hat{u}_{Ni}(s_i | C_{\bar{f}_i}^*) + d_i - \hat{u}_{Ni}(s_i | C_{\bar{f}_i}^*) \\ &= \phi_i^T \psi(s_i) + d_i - \hat{u}_{Ni}(s_i | C_{\bar{f}_i}^*) \end{aligned} \quad (21)$$

where  $\phi_i = C_{\bar{f}_i}^* - C_{\bar{f}_i}$ .

Now consider the Lyapunov candidate:

$$V_i = \frac{1}{2} (s_i^2 + \frac{1}{r_i} \phi_i^T \phi_i) \quad (22)$$

where  $r_i$  is positive constants. The time derivative of  $V_i$  is

$$\begin{aligned} \dot{V}_i &= s_i \dot{s}_i + \frac{1}{r_i} \phi_i^T \dot{\phi}_i \\ &= s_i \phi_i^T \psi(s_i) + \frac{1}{r_i} \phi_i^T \dot{\phi}_i - s_i \hat{u}_{Ni}(s_i | C_{\bar{f}_i}^*) + s_i d_i \\ &\leq \frac{1}{r_i} \phi_i^T (r_i s_i \psi(s_i) + \dot{\phi}_i) - s_i (D_i + \eta_{s1}) \operatorname{sgn}(s_i) + s_i d_i \\ &< \frac{1}{r_i} \phi_i^T (r_i s_i \psi(s_i) + \dot{\phi}_i) - |s_i| \eta_{s1} \end{aligned} \quad (23)$$

Choosing the adaptive law (recalling that  $\dot{\phi}_i = -\dot{C}_{\bar{f}_i}$ )

$$\begin{aligned} \dot{C}_{\bar{f}_i} &= r_i s_i \psi(s_i), \\ \text{then} \\ \dot{V}_i &< -|s_i| \eta_{s1} < 0 \end{aligned} \quad (24)$$

To avoid that  $C_{\bar{f}_i}$  take arbitrarily large values, the estimate of  $C_{\bar{f}_i}$  is restricted to a compact set  $B(M)$  (where  $\|C_{\bar{f}_i}\| \leq M$  denotes a ball of radius  $M$ ). Using a Lyapunov function, an adaptive law with projection, can be defined as

$$\dot{C}_{\bar{f}_i} = r_i s_i \Psi(s_i) + \alpha_0 r_i s_i \frac{C_{\bar{f}_i} C_{\bar{f}_i}^T \Psi(s_i)}{|C_{\bar{f}_i}|^2} \quad (25)$$

$$\text{where } \alpha_0 = \begin{cases} 1 & \text{if } |C_{\bar{f}_i}| = M \text{ and } s_i C_{\bar{f}_i}^T \Psi(s_i) > 0 \\ 0 & \text{if } |C_{\bar{f}_i}| \leq M \text{ and } s_i C_{\bar{f}_i}^T \Psi(s_i) \leq 0 \end{cases}$$

Based on the above discussion, the proposed control design is outlined as follows. In satellite attitude control system, the overall control law is:

$$u = A^{-1}(x)[v - B(x)] \quad (26)$$

where

$$v = [v_1, v_2, v_3]^T, \quad v_i = k_i \dot{e}_i + \ddot{z}_{di} - C_{\bar{f}_i}^T \psi(s_i), \quad i=1, 2, 3.$$

In control law,  $\ddot{z}_{di}$  denotes the derivatives of the desired quaternion  $q_{di}$  of second order, error  $e_i = q_i - q_{di}$ ,  $q_i$  denotes the actual quaternion,  $i=1, 2, 3$ . In the simulation, the fuzzy sets of input variables  $s_i$  in the  $i$ th T-S model are defined as negative big (NB), negative middle (NM), negative small (NS), zero (E), positive small (PS), positive middle (PM), and positive big (PB). The initial mean of NB, NM, NS, E, PS, PM, and PB is respectively -0.36, -0.24, -0.12, 0, 0.12, 0.24, and 0.36. The corresponding initial variance is 0.056. The initial fuzzy rules are given in table 1, where  $i=1, 2, 3$ . The desired trajectory  $q_{di}(t)$  of satellite attitude is selected as:

$$\ddot{q}_{di} + 2\xi \omega_n \dot{q}_{di} + \omega_n^2 q_{di} = \omega_n^2 R_i \quad (27)$$

where  $\xi = 0.707$ ,  $\omega_n = 0.08$ , external input  $R_i = q_i^* = 0$ ,  $i=1, 2, 3$ .

TABLE 1  
THE INITIAL FUZZY RULES

| $s_i$          | NB     | NM     | NS     | E | PS    | PM    | PB    |
|----------------|--------|--------|--------|---|-------|-------|-------|
| $\hat{u}_{Ni}$ | -0.006 | -0.004 | -0.002 | 0 | 0.002 | 0.004 | 0.006 |

## V. SIMULATION

In this section the proposed AFSMC is applied to the attitude control of flexible satellite. The first elastic mode is considered for the simplification. The parameter values used for the flexible satellite are as follows: inertia matrix

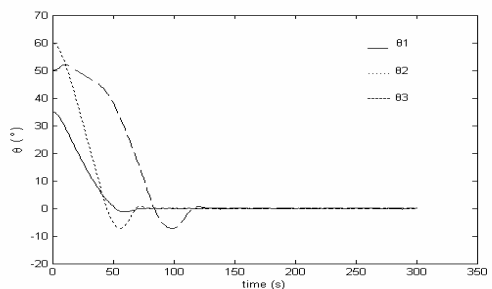
$$I = \begin{bmatrix} 6100 & -90 & 20 \\ -90 & 5070 & -1100 \\ 20 & -1100 & 8400 \end{bmatrix} \text{kg} \cdot \text{m}^2, \quad \text{coupling matrix}$$

$C = [0.3 \ 18 \ -21]^T$  ( $\text{kg}^{1/2} \cdot \text{m}$ ), modal frequency  $\Lambda = 1.02 \text{ rad/s}$ , modal damping  $\xi = 0.001$ , control torque range  $[-10, 10]$  Nm, orbital rate  $\omega_0 = 1.078 \times 10^{-3} \text{ rad/s}$ .

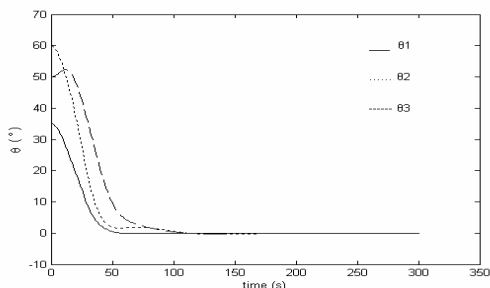
Further, the follow initial conditions are chosen as: the actual attitude  $q_0(0) = 0.6850$ ,  $q(0) = [0.0345 \ 0.5422 \ 0.4853]$  (namely roll angle  $\theta_1 = 35^\circ$ , pitch angle  $\theta_2 = 60^\circ$ , yaw angle  $\theta_3 = 50^\circ$ ),  $\omega(0) = [0.04 \ 0.04 \ 0.04]^\circ/\text{s}$ ,  $\eta(0) = 0$ ,  $\dot{\eta}(0) = 0$ , the desired attitude  $q_0 = 1$ ,  $q_d = [0 \ 0 \ 0]$ . The parameters of the adaptive law are selected as  $r_1 = 0.001$ ,  $r_2 = 0.0016$ ,

$$r_3 = 0.0012.$$

The conventional sliding mode control (CSMC) is first constituted to make comparison with AFSMC for the flexible satellite. The CSMC law is  $v_i = k_i \dot{e}_i + \ddot{z}_{di} - k_i \text{sgn}(s_i)$ ,  $k_i = \eta_s + D_i = 0.003$ ,  $i=1,2,3$ . Fig.1 shows the three attitude angles under the AFSMC and the CSMC in the case of nominal inertia matrix. Under the AFSMC, attitude angles converge to zero in 93s and the overshoot is  $-0.3432^\circ$ . Under the CSMC, the response time is around 116s and the overshoot is  $-7.28^\circ$ . Fig.2 shows the results in the case of inertia matrix increment of 20%. Under the CSMC, the response time is extended to around 142s and the overshoot arrived to  $-10.61^\circ$ . Under the AFSMC, the response time and the overshoot are almost unmodified. It is seen that the AFSMC is insensitive to the parameter change of the flexible satellite, thus has the strong robustness.

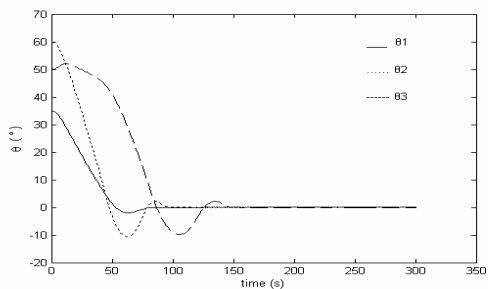


a: CSMC

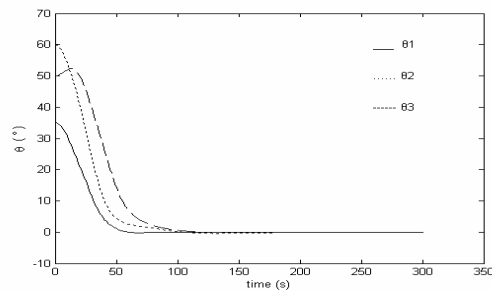


b: AFSMC

Fig.1. Attitude angles in the case of nominal parameter



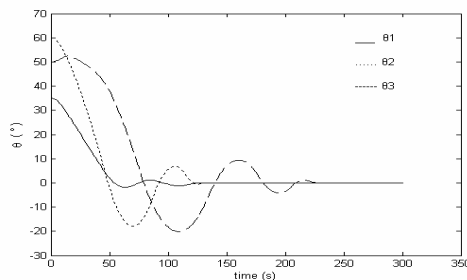
a: CSMC



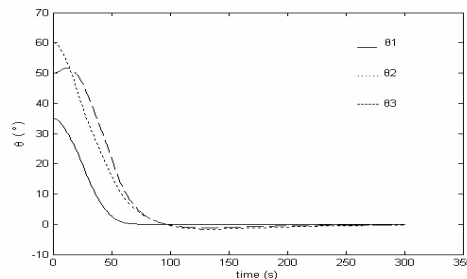
b: AFSMC

Fig.2 Attitude angles in the case of inertia matrix increment of 20%

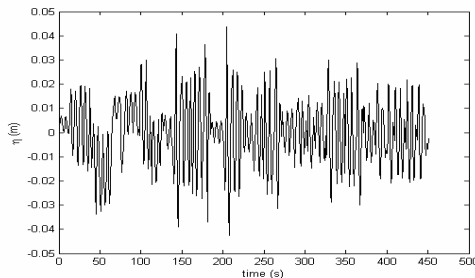
To further demonstrate the effectiveness of the AFSMC to larger variations of inertia matrix, the inertia matrix is increased to 150%J. Fig.3 shows the response of the satellite using the AFSMC and CSMC control. Still the modal displacement and control torque  $u_1$  of the roll subsystem are depicted in this case. Under the CSMC, the response time is extended to around 220s and the overshoot arrived to  $-20.11^\circ$ . The modal displacement of panel is between  $-0.01\text{m}$  and  $0.01\text{m}$  at 450s. The heavily chattering of control torque is observed. Under the AFSMC, the response time is around 150s and the overshoot is  $-1.64^\circ$ , the oscillation of panel attenuates to zero in about 350s, the control torque is smooth and possesses no chattering.



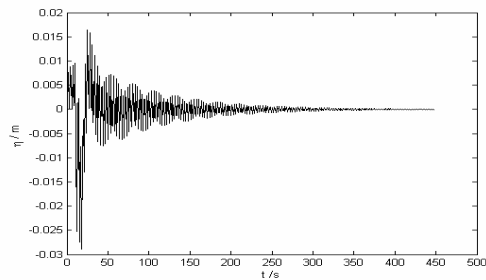
a: attitude angles under CSMC



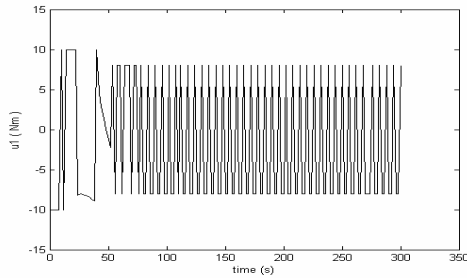
b: attitude angles under AFSMC



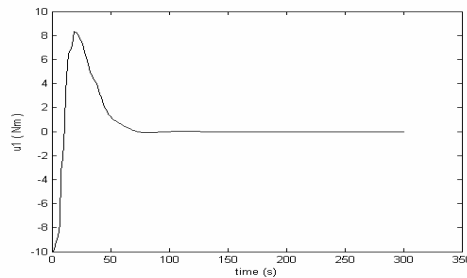
c: modal displacement under CSMC



d: modal displacement under AFSMC



e: control torque  $u_1$  under CSMC



f: control torque  $u_1$  under AFSMC

Fig. 3 Satellite attitude in the case of inertia matrix increment of 50%

Comparing with the CSMC, the proposed AFSMC can effectively compensate for the effect of parametric uncertainty by updating the rule parameter vector  $C_f$ . The AFSMC offers quicker response and is robust to the uncertainties of the flexible satellite. It could effectively damp out the oscillation of the solar panel that is yielded in the attitude maneuver process, and attenuate the chattering phenomenon, so that the precise attitude control of flexible satellite is obtained.

## VI. CONCLUSION

Adaptive fuzzy sliding mode control incorporating with I/O linearization is performed on flexible satellite. Based on I/O linearization, the fuzzy control rules employed in MIMO system are decomposed to single input single output fuzzy rules. This makes it easy to incorporate the experts' experience into the control system. In sliding mode control law, the adaptive fuzzy control is used to approximate the nonlinear control term; thus the proposed method doesn't depend on the bound of the system uncertainty. The proposed AFSMC can effectively cope with the uncertainty by adjusting the rule parameter vector in the control process, so as to avoid the large nonlinear control term. The chattering phenomenon inherent to the conventional sliding mode

control is simultaneously attenuated. Simulation results show the derived controller achieves robust tracking performance for large parameter uncertainty.

## REFERENCES

- [1] N. S. Sahjendra and Y. Woosoon, "Feedback linearization and solar pressure satellite attitude control," *IEEE Trans. on aerospace and electronic systems*, vol.32, no.2, pp.732-741, 1996.
- [2] N. S. Sahjendra and I. Ashok, "Nonlinear decoupling sliding mode control and attitude control of spacecraft," *IEEE Trans. on aerospace and electronic systems*, vol.25, no.5, pp.621-633, 1989.
- [3] H. Elmali and N. Olgac, "Robust output tracking of MIMO nonlinear systems via sliding mode technique," *Automatic*, vol.28, no.1, pp.145-151, 1992.
- [4] H. Elmali and N. Olgac, "Satellite attitude control via sliding mode with perturbation estimation," *IEE Proc.-control theory application*, vol.143, no.3, pp.276-282, 1996.
- [5] G. C. Hwang, and S. Chang, "A stability approach to fuzzy control design for nonlinear system," *Fuzzy Sets and Systems*, vol.48, pp.279-287, 1992.
- [6] S. W. Kim, and J. J. Lee, "Design of a fuzzy controller with fuzzy sliding surface," *Fuzzy Sets and Systems*, vol.71, no.3, pp.359-367, 1995.
- [7] R. Zhang, *Satellite orbital dynamics and attitude control*. Beijing university of Aeronautics and Astronautics Press. 1998. (in Chinese)