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Power Control Algorithm for Providing Packet Error Rate Guarantees in Ad-Hoc Networks

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Abstract— **We investigate the issue of power control in mobile ad-hoc networks under distributed resource allocations, where the interference at the receivers is not known in advance. Based on a model that captures the physical layer and power control issues more accurately, we develop a new power control algorithm that can provide a physical/link layer quality-of-service in the form of packet error rate. We then formulate the problem of minimizing the average aggregate transmission power as an optimization problem, and show that our proposed power control algorithm converges to a solution of the optimization problem.**

I. Introduction

Unlike wireline or even cellular networks with a fixed infrastructure, multi-hop wireless networks can be deployed without any centralized agents and be self organized through neighbor discovery and link establishment. One of the fundamental differences between a cabled network and a (multi-hop) wireless network is the characteristics of the communication medium. In a wireline network, links are dedicated to point-to-point communication between two end nodes and the topology of a network does not change frequently. On the other hand, in a wireless network links are fictitious as connectivity (*i.e.,* ability to communicate) is determined by achievable signal to interference and noise ratio (SINR). Hence, the connectivity of the nodes (*i.e.,* topology of the network) is determined not only by the distance between nodes, but also by the density of the communicating nodes [2] as well as the performance of the underlying physical layer algorithms and availability of resources such as energy.

The lack of a fixed infrastructure and/or a time-varying topology due to mobility in a multi-hop wireless network renders a centralized packet scheduling inefficient, if not impossible, because of prohibitive required communication overhead and delays. Similarly, coordinated scheduling based on a *pre-determined* sequence of packet scheduling vectors agreed to by *all* nodes, is difficult to realize in practice when topology varies with time. Therefore, nodes must rely on *distributed* packet scheduling and necessary physical layer resource allocations (*e.g.,* power control) to support the packet scheduling, possibly with some local coordination.

Distributed packet scheduling and power control results in *random* interference at the receivers as the interference cannot be predicted accurately without the *full knowledge* of the set of transmitters and their transmission powers during packet reception. This is because the interference experienced at a receiver depends on the set of links that are accessing the channel simultaneously and their transmission powers. This potentially widely varying, *unknown* interference at a receiver causes uncertainty in achieved SINR value during packet reception even when accurate channel gains are available. For the same reason the traditional approach to power control that aims to achieve certain target SINR may not be feasible in a distributed environment.

Most of previously proposed physical layer models adopted for performance evaluation and algorithm design, however, do not sufficiently capture the effects of stochastic nature of interference at the receivers. Thus, the simulation results obtained using these inaccurate physical layer models can be misleading and give misguiding intuition. Consequently, algorithms designed based on such premises will not perform satisfactorily in practice. Here we only focus on the aspect of a physical layer model that decides whether a packet transmission is successful or not.

In this paper we investigate the issue of power control in multi-hop wireless networks where packet scheduling is carried out by the nodes in a distributed manner *without* centralized coordination. We explicitly model the events of successful transmission of packets using link curves based on the achieved SINRs. Using this more detailed physical/link layer model, we propose a new power control algorithm that can provide a physical/link layer quality-of-service (QoS) in the form of guaranteed (average) packet error rate (PER). For algorithm design and performance evaluation, PER is the suitable physical/link layer parameter to consider, for the performance of higher layer protocols depends on the achieved PERs at the physical/link layer and the SINR affects their performance only *indirectly* through achieved PER.

We show that our novel and yet *simple* approach leads to a new paradigm for *robust* algorithm design that does not require unrealistic assumptions on the interference estimation or physical layer behavior, with *minimal* communication overhead. We then formulate the problem of minimizing the average aggregate transmission power of the nodes as an optimization problem. We demonstrate that our proposed power control algorithm converges to a solution of the optimization problem under both synchronous and asynchronous update rules.

This paper is organized as follows: Section II explains the implications of random interference at the receivers and motivates our approach. Our proposed power control algorithm is described in Section III. We study the problem of minimizing the average total transmission power of the nodes as an optimization problem and establish the convergence of our proposed algorithm to a solution of the optimization problem in Section IV.

II. Stochastic Nature of Interference & its Implications on Network Performance

In a multi-hop wireless network it is unlikely that there will be a centralized controller that carries out packet scheduling, power control, and other physical layer resource allocation. If no such centralized agent is available, the nodes must rely on distributed packet scheduling and power control. When packet scheduling is carried out in a distributed manner, since the set of transmitter-receiver pairs is time-varying and is not known in advance, neither the transmitters nor receivers can accurately predict the interference during the packet reception.

In practice the probability of successful transmission is given by a link curve (corresponding to a selected modulation and coding scheme (MCS), packet size, etc.). A link curve gives the PER as a function of the achieved SINR, and is typically a continuous function of the SINR. Hence, in the presence of random interference the achieved PER depends on the distribution of interference and the sensitivity of the link curve to the SINR. A set of link curves for a TDMA system is displayed in Fig. 1 [1], [5]. The measured data points are shown as '*', and the solid lines are fitting curves, which will be discussed in more details in subsection III-A.

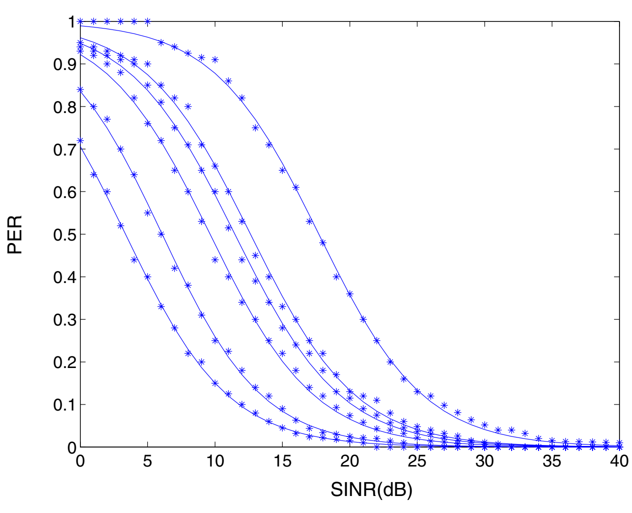


Fig. 1. Link curves of a TDMA system [5].

In this paper we select the PER as the right physical/link layer QoS parameter. As argued earlier, adoption of PER as a meaningful physical/link layer QoS parameter, instead of achieved SINR, is more natural as the performance of higher layer protocols does not depend directly on the achieved SINR, but *indirectly* through the achieved PER. Moreover, unlike in the case where the interference can be accurately estimated/predicted, achieving certain target SINR as a physical layer QoS parameter is not possible or even desirable in multi-hop wireless networks in the absence of centralized resource allocation.

III. Proposed Power Control Algorithm

In this section we first outline how one can approximate the PER from a link curve, and then, using the derived approximation, describe the proposed power control algorithm that can handle the issue of random interference and provide PER guarantees. Using a numerical example, we demonstrate that the proposed algorithm does achieve the target PERs.

A. Approximation of Packet Error Rate

Our proposed approach to power control does not make any assumptions regarding the nature/distribution of the interference, and is simple and robust. It is based on the observation that the link curves for most MCS schemes (or at least the portion of interest) [5], [6] can be well approximated by the following family of functions

$$PER(SINR) = \frac{1}{1 + e^{k(SINR_{dB} - z)}} \quad (1)$$

where $SINR_{dB}$ is SINR in dB, *i.e.*, $SINR_{dB} = 10\log_{10}(SINR)$, and k and z are two fitting parameters that determine the slope and the position of a link curve, respectively. These parameters can be determined from the given link curves *off-line*.

The fitting curves for link curves of a TDMA system are shown as solid curves in Fig. 1 for different MCS schemes. One can clearly see that these fitting curves match the measured link data very closely. Link curves for systems other than the TDMA system are similar in shape but with different parameters. Several example link curves for a CDMA system are given in [6].

For a reasonably small target PER, we can approximate (1) as follows:

$$PER(SINR) \approx e^{-k(SINR_{dB} - z)} = e^{kz} SINR^{-\alpha} \quad (2)$$

where $\alpha = 10k/\ln 10$ and determines the sensitivity of PER to SINR. Since the realized SINR is a random variable (rv) due to random interference, assuming necessary ergodicity, the realized average PER is given by

$$PER_{avg} \approx e^{kz} \mathbf{E}\left[SINR^{-\alpha}\right] \ . \quad (3)$$

In the rest we replace the approximation in (2) and (3) with an equality.

B. Proposed Power Control Algorithm

Suppose that an average interference value is used as an estimate of interference for computing the transmission power that will satisfy a target SINR, $SINR_{target}$, corresponding to certain target PER according to a given link curve. Under this assumption, one can show that $\mathbf{E}\left[SINR^{-1}\right] = SINR_{target}^{-1}$ [7]. From this equality it is plain to see that the target PER of the algorithm can be expressed as

$$PER_{target} = e^{kz} SINR_{target}^{-\alpha} = e^{kz} \mathbf{E}\left[SINR^{-1}\right]^{\alpha} \ ,(4)$$

where the first equality follows from (2). Note that if $\alpha \approx 1$, then

$$PER_{avg} = e^{kz}\mathbf{E}\left[SINR^{-\alpha}\right] \approx e^{kz}\mathbf{E}\left[SINR^{-1}\right]^{\alpha} = PER_{target} \ .$$

Thus, if $\alpha \approx 1$ the realized PER_{avg} will be close to PER_{target}. However, when α deviates considerably from one, the target PER may not be achieved by simply using the average interference as an estimate. Note that for α larger than one, from Jensen's inequality [3] we have

$$e^{kz}\mathbf{E}\left[SINR^{-\alpha}\right] \geq e^{kz}\mathbf{E}\left[SINR^{-1}\right]^{\alpha} \ .$$

This usually results in the realized PERs larger than the target PERs, and the physical/link layer QoS is violated. For the link curves for a CDMA system in [6] the values of α are much larger than one.

From (3) and (4) we observe that the discrepancy between a target PER and a realized PER is due to the fact that $\mathbf{E}\left[SINR^{-1}\right]^{\alpha}$ and $\mathbf{E}\left[SINR^{-\alpha}\right]$ differ. Therefore, in order to achieve the average PER close to the target PER, the transmitter of link l must select the transmission power so that

$$e^{kz}\mathbf{E}\left[SINR_l^{-\alpha}\right] = e^{kz}\frac{\mathbf{E}\left[(\text{Interference}_l)^{\alpha}\right]}{(P_l \cdot G_l)^{\alpha}} = PER_{target} \ . \qquad (5)$$

Here Interference_l includes both the interference and noise at the receiver of link l, and $\mathbf{E}\left[(\text{Interference}_l)^{\alpha}\right]$ is the mean of $(\text{Interference}_l)^{\alpha}$ at the receiver of link l *during packet receptions*. From (5) it is clear that the transmission power should be set to

$$P_l = \left(e^{kz}\frac{\mathbf{E}\left[(\text{Interference}_l)^{\alpha}\right]}{PER_{target} \cdot G_l^{\alpha}}\right)^{1/\alpha} \ . \qquad (6)$$

Note from (6) that when accurate channel gains are available, the transmitter requires only one parameter $\mathbf{E}\left[\text{Interference}_l^{\alpha}\right]$ to compute the transmission power. This parameter can be estimated using exponential averaging. In other words, the estimate for link l is updated after each packet transmission over link l according to

$$\mathbf{E}\left[(\text{Interference}_l)^{\alpha}\right]_{new} = (1-\omega) \cdot \mathbf{E}\left[(\text{Interference}_l)^{\alpha}\right]_{old} + \omega \cdot (\text{Interference}_{l,cur})^{\alpha} \qquad (7)$$

where $\text{Interference}_{l,cur}$ is the new experienced interference. This estimate can be either fed to the transmitter by the receiver when it experiences a significant change in its value or piggybacked in the acknowledgment after each transmission.

In practice, in order for exponential averaging in (7) to be effective, the averaging constant w must be selected large enough so that the estimate can be updated in a timely manner with time-varying channel conditions due to mobility and (slow) fading. However, if a link is not used often enough, the receiver may not be able to update the estimates often enough and these estimates may not be accurate.

In order to solve this problem we can maintain only one estimate at each receiver rather than keeping one estimate per link. Hence, after every packet reception the node updates the estimate according to (7). This reduces the number of parameters each node needs to maintain to one, leading to a more scalable algorithm regardless of the density of the network, and faster convergence of the estimates. In the numerical example in the following subsection, we adopt this simpler version of the algorithm and show that it achieves realized PERs close to the target PERs.

C. Numerical Example

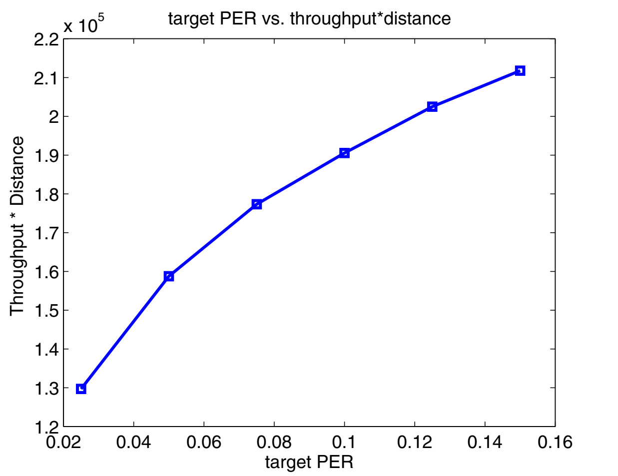
We simulated our power control algorithm with various target PERs. In our example 100 nodes are randomly placed in a 1 km $\times$ 1 km rectangular region. For the simplicity of demonstration we assume a discrete-time system throughout and the time is slotted into contiguous timeslots.

The scheduling algorithm we use for simulation is simple. In each timeslot we find a set of links to be transmitted in a sequential manner as follows. In each iteration we randomly select a candidate transmitter from a set of eligible transmitters. Then, we check if there exists a valid receiver to which the candidate transmitter can communicate while satisfying the target PER requirement subject to a maximum power budget of 10. If so, a packet transmission is scheduled to a receiver randomly selected among such nodes. We repeat this until no more transmitter-receiver pair can be scheduled without violating the physical/link layer constraint. The noise power at the receivers is set to 10^{-14}.

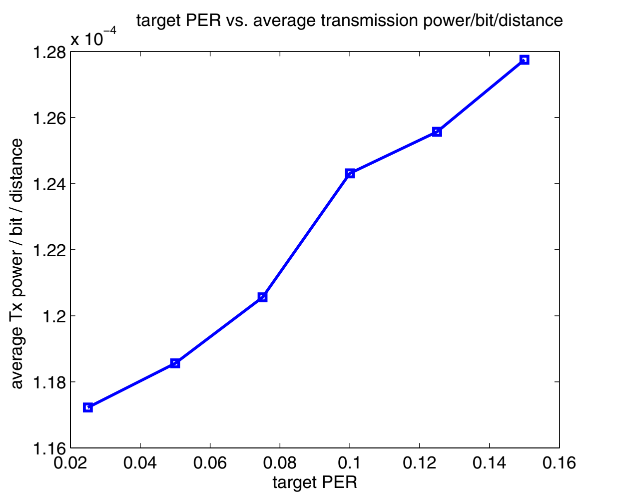
Clearly, this scheduling policy is not designed to support any flow rates between source and destination pairs. Instead this scheduling policy typically selects very different scheduling vectors (a set of scheduled links) from one timeslot to next, hence ensuring sufficient randomness in interference at the receivers. However, we suspect that this is a reasonable approximation to the network behavior when the network is congested and many queues are not empty. When the network (or a neighborhood) is congested, queues begin to build up and nodes will choose different links to transmit on in consecutive timeslots (if possible) rather than transmitting to the same neighbor for many consecutive timeslots. This will prevent other queues from overflowing and experiencing high packet drop probabilities, produce more smooth flow of packets throughout the network, and reduce the delay jitter of packets. Therefore, the set of scheduled links will change dynamically from one timeslot to next as done in our simulation. This will also result in *weak* temporal correlation in the interference experienced at the receivers and make it difficult to accurately predict the interference to be experienced during a packet reception from the current estimate.

The same issue exists in an asynchronous system as well, whether it is a TDMA, CDMA, or OFDM system. In these asynchronous systems, including a CDMA system where pseudo-noise sequences are assigned to different links, the experienced interference during a packet reception depends on the set of other simultaneous packet transmissions and the amounts of overlap in time.

For simulation we selected one of the link curves of the TDMA systems shown in Fig. 1 and modified the parameters so that the value of α is 2.



(a)



(b)

Fig. 2. Plot of (a) network transport throughput vs. target PER and (b) average transmission power per successfully delivered bit per unit distance.

Numerical results are shown in Fig. 2. Here we only show the results with lognormal fading. We define the network transport throughput to be the product of the throughput (in bits) and the distance (in meters) packets travel between nodes [4]. We adopt the network transport throughput as the performance metric because in a multi-hop wireless network the *total* end-to-end throughput (in bits) of the source-destination pairs depends on the distances between the source nodes and the destination nodes. Hence, in order for the metric to be invariant of the locations of the source and destination nodes, the end-to-end throughput of a source-destination pair needs to be scaled by the distance between the nodes. One important thing to note from Figs. 2(a) and 2(b) is that both achieved network transport throughput (bits $\times$ distance) and average transmission power per successfully delivered bit per unit distance increase with the target PER over the region of interest (PER $\leq$ 15 percent). In fact, the gain in throughput is more than 61 percent (from 1.3×10^5 to 2.1×10^5) when the target PER is increased from 2 percent to 15 percent. The average transmission power per successfully delivered bit per unit distance also increases by almost 10 percent at the same time (from 1.172×10^{-4} to 1.279×10^{-4} in Fig. 2(b)). This suggests that there exists an intrinsic *trade-off* between the network transport throughput and energy consumption with the target PER as the control parameter. To the best of authors' knowledge, this trade-off has not been studied in the literature.

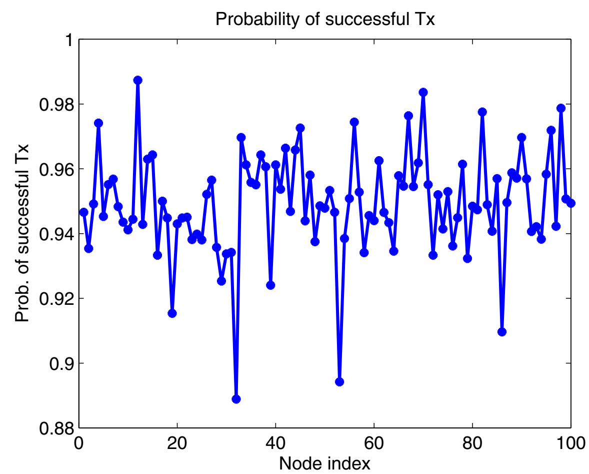


Fig. 3. Probability of successful transmission (target PER = 0.05).

We also plot the probability of successful transmission at the nodes (as receivers) in Fig. 3. As one can see the achieved PERs at the nodes remain close to the target PER of 0.05 for most nodes. Thus, this demonstrates that our proposed power control algorithm can achieve the target PERs even when the interference at the receivers cannot be predicted accurately. Some nodes experience PERs larger than the target PER because each node maintains only one estimate of interference as mentioned earlier and/or they do not receive a sufficient number of packets and do not update their estimates frequently enough.

Our simulation results also reveal that the transmission power among the nodes varies widely, which depends on the characteristics of the interference experienced at the receivers. Large variance of interference implies larger $\mathbf{E}\left[(\text{Interference}_l)^\alpha\right]$, resulting in more restrictive physical/link layer constraints, and higher transmission power. Consequently, fewer number of simultaneous transmissions are possible in a neighborhood.

IV. Optimal Power Control & Convergence

The previous section tells us that one can provide physical/link layer QoS in the form of PER under distributed scheduling even when the exact value of interference is not known at a receiver during packet reception. In a wireless network many nodes are expected to operate on batteries and, hence, are energy constrained. Therefore, the power control algorithm should not only satisfy the physical/link layer QoS, but should also minimize the energy consumption of the nodes at the same time. This problem can be studied in an optimization framework.

A. Optimization Formulation

Let $\mathcal{I} = \{1, \ldots, I\}$ denote the set of nodes and $\mathcal{L} = \{1, \ldots, L\}$ the set of unidirectional links. Here a link is a pair of nodes (i, j) such that node i can communicate to node j. We are given a set of source-destination pairs $\mathcal{K} = \{1, \ldots, K\}$. Each source-destination pair has certain flow rate demand/requirement, and the demand (in bits per timeslot) of the k-th pair is denoted by x_k. The routes of the source-destination pairs are fixed, and the routing matrix is given by a $K \times L$ matrix A, *i.e.,* $A_{kl} = 1$ if the route of the k-th source-destination pair traverses link l, and $A_{kl} = 0$ otherwise. Let $\tilde{\mathcal{L}} = \{l \in \mathcal{L} | A_{kl} = 1 \text{ for some } k \in \mathcal{K}\ \}$, and $\tilde{L} = |\tilde{\mathcal{L}}|$. In the rest of this section we focus on the links in $\tilde{\mathcal{L}}$ as other links are not being used. For simplicity assume that the transmission rates (in the unit of bits per timeslot) of the links are constant, which are given by a diagonal transmission rate matrix R. In other words, $R_{ll}, l \in \tilde{\mathcal{L}}$, is the transmission rate of link l.

Let $\underline{s}$ be an $\tilde{L} \times 1$ scheduling vector where $s_l = 1$ if link $l \in \tilde{\mathcal{L}}$ is on, *i.e.,* transmitter of link l sends a packet to the receiver of link l. We only consider scheduling vectors that satisfy the following: no node (i) receives and transmits simultaneously, (2) receives from more than one node, or (3) transmits to more than one receiver. Obviously, some, if not all, assumptions can be relaxed, depending on the capabilities of devices. We denote the set of scheduling vectors satisfying these conditions by $\mathcal{S}$.

The state of the system is modeled by an ergodic discrete-time Markov chain. Here the state can, for example, comprise the queue sizes of the links. The medium access control (MAC) scheme that yields a feasible scheduling vector $\underline{s} \in \mathcal{S}$ in each timeslot, is assumed to be stationary, *i.e.,* channel access by the nodes depends on the state of the system, but not on time. For example, under a random access scheme, each node will attempt to schedule a link with a non-empty queue with certain probability that may depend on the queue size. A pair of links that have the same destination node will collide and a subset of the attempted links that do not collide with others will be scheduled successfully. Under this assumption, given the system state, one can compute the probability a particular scheduling vector will result, which clearly depends on the adopted MAC scheme. In addition, we assume that the channel access is carried out using small control packets (*e.g.,* request-to-send (RTS) and clear-to-send (CTS) packets in IEEE 802.11) and the energy consumption for transmitting a control packet is assumed to be much smaller than that of data packet transmission.

We assume that the system is at steady state, and the stationary distribution is given by π. We denote the resulting steady-state distribution over $\mathcal{S}$ by $\mathbf{d}$. In other words, $\mathbf{d}_{\underline{s}}, \underline{s} \in \mathcal{S}$, is the probability that scheduling policy selects scheduling vector $\underline{s}$ at steady state. This probability is given by

$$\mathbf{d}_{\underline{s}} = \sum_{\theta \in \Theta} \pi_\theta \cdot \mathbf{p}_{\underline{s}|\theta} \ ,$$

where Θ is the state space of the system, and $\mathbf{p}_{\underline{s}|\theta}$ denotes the conditional probability that scheduling vector $\underline{s}$ will be selected given that the system is at state θ.

We assume that the resulting distribution $\mathbf{d}$ satisfies

$$(I_{\tilde{L} \times \tilde{L}} - \Gamma) \cdot R \sum_{\underline{s} \in \mathcal{S}} \mathbf{d}_{\underline{s}} \cdot \underline{s} \geq \tilde{A}^T \underline{x} \ , \tag{8}$$

where $\Gamma = \text{diag}(\gamma_l; l \in \tilde{\mathcal{L}})$, γ_l is the target PER of link l, $I_{\tilde{L} \times \tilde{L}}$ is an $\tilde{L} \times \tilde{L}$ identity matrix, and $\tilde{A}$ is a submatrix of the routing matrix A only with the columns corresponding to the links in $\tilde{\mathcal{L}}$. The left hand side of (8) is the vector of the average goodput over the links, and the right hand side is the link demands determined by the rate demand vector $\underline{x}$ and the routing matrix $\tilde{A}$. If the transmission power levels are fixed and we ignore channel fading, the distribution of the interference experienced at the receivers is completely determined by the distribution $\mathbf{d}$ and transmission power vector $\mathbf{p}$.

We formulate the problem of power control as the following optimization problem:

$$\begin{aligned} \underset{\mathbf{p} \in \mathcal{P}}{\text{minimize}} \quad & \mathbf{p}^T \Big(\sum_{\underline{s} \in \mathcal{S}} \mathbf{d}_{\underline{s}} \cdot \underline{s} \Big) \\ \text{subject to} \quad & PER_l(\mathbf{p}, \mathbf{d}) \leq \gamma_l \ , \end{aligned} \tag{9}$$

where $\mathcal{P} = \prod_{l \in \tilde{\mathcal{L}}} [\mathbf{p}_{l,\min}, \ \mathbf{p}_{l,\max}]$, and $\mathbf{p}_{l,\min}$ and $\mathbf{p}_{l,\max}$ are the minimum and maximum power constraints of link l, respectively. The minimum power constraint exists because the transmission power of a radio device cannot be arbitrarily small. We assume that the solution of (9) is an interior point of $\mathcal{P}$, *i.e.,* the constraints are not active at the solution. Note that the PERs depend both on the transmission powers and the distribution $\mathbf{d}$ because the interference experienced at the receivers depends on both.

We assume that the channel gains are fixed, and they are denoted by $G_l, l \in \tilde{\mathcal{L}}$. We define $\mathbf{G} = \text{diag}(G_l, l \in \tilde{\mathcal{L}})$ and, for each $l \in \tilde{\mathcal{L}}$,

$$\mathbf{d}^l_{\underline{s}} = \mathbf{P} \left[\text{scheduling vector } \underline{s} \text{ selected} \mid \text{link } l \text{ scheduled} \right] \ .$$

Note that $\mathbf{d}^l_{\underline{s}}$ is the conditional probability that the scheduling vector $\underline{s}$ is selected given that link l is scheduled. Let $\mathbf{G}^l = \text{diag}(G_{Tx(l')Rx(l)}; l' \in \tilde{\mathcal{L}})$, where $Tx(l)$ and $Rx(l)$ are the transmitter and receiver of link l, respectively. Under this assumption, it is plain to see that

$$\begin{aligned} \mathbf{E}\left[SINR_l^{-\alpha}\right] &= \sum_{\underline{s} \in \mathcal{S}: \underline{s}_l = 1} \mathbf{d}^l_{\underline{s}} \left(\frac{\mathbf{p}^T \mathbf{G}^l \underline{s} - G_l \cdot \mathbf{p}_l + n_l}{G_l \cdot \mathbf{p}_l} \right)^\alpha \\ &= \frac{\mathbf{E}\left[(\text{Interference}_l)^\alpha\right]}{(G_l \cdot \mathbf{p}_l)^\alpha} \end{aligned} \tag{10}$$

where $n_l > 0$ is the noise power at the receiver of link l.

One can easily see from (10) that $\mathbf{E}\left[SINR_l^{-\alpha}\right]$ is convex in each $\mathbf{p}_{l'}, l' \neq l$ if $\alpha \geq 1$ and is strictly convex if there exists $\underline{s}'$ such that $\mathbf{d}^l_{\underline{s}'} > 0$, $\underline{s}'_l = \underline{s}'_{l'} = 1$ and $\alpha > 1$. As most of the link curves, if not all, that we have seen have α larger than one, we assume that $\alpha > 1$ [5], [6].

B. Uniqueness of Solution

Define a multi-dimensional mapping $F(\mathbf{p})$, where

$$F_l(\mathbf{p}) = \min\Big\{\mathbf{p}_{l,\max}, \max\Big\{\mathbf{p}_{l,\min}, \left(e^{kz}\frac{\mathbf{E}\left[(\text{Interference}_l(\mathbf{p}))^\alpha\right]}{\gamma_l \cdot G_l^\alpha}\right)^{1/\alpha}\Big\}\Big\} \ . \quad (11)$$

It is easy to see that a solution to the optimization problem in (9) must be a fixed point of the mapping F. This is because at any solution the transmission power of each link must be the smallest transmission power that satisfies the PER constraint given the transmission powers of other links, which is obtained from the mapping F.

The following lemma tells us that there exists a unique fixed point of the mapping F.

Lemma 1: There exists a unique fixed point of the mapping $F(\cdot)$.

Combined with the previous observation that the solution to (9) is a fixed point of the mapping F, Lemma 1 tells us that the unique fixed point of the mapping is the solution to (9).

We now investigate the problem of convergence of the distributed power control algorithm to the solution.

C. Synchronous Update

In this subsection we first consider the simpler case where the updates of the transmission powers are synchronized and are based on the latest values. Consider the following updating rule. We model the updates with a discrete-time model. For each $n = 0, 1, 2, \ldots$, let $\mathbf{p}(n) = (\mathbf{p}_l(n); l \in \tilde{\mathcal{L}})$, and each link updates its transmission power according to

$$\mathbf{p}_l(n+1) = F_l(\mathbf{p}(n)) \ . \quad (12)$$

Once all links update their transmission powers, they wait long enough so that they can estimate $\mathbf{E}\left[(\text{Interference}_l(\mathbf{p}))^\alpha\right]$. Once this estimate is available at all links, they repeat the above update procedure, based on the new estimates. This is called *Jacobi update scheme*.

We assume that $\mathbf{p}(0) \in \mathcal{P}$. The following lemma tells us that the link transmission powers $\mathbf{p}(n)$ converge to the solution.

Lemma 2: Under the update rule (12) we have $\lim_{n\to\infty} \mathbf{p}(n) = \mathbf{p}^\star$, where $\mathbf{p}^\star$ is the unique fixed point of the mapping F.

One can establish similar convergence results when transmission powers are updated according to

$$\mathbf{p}_l(n+1) = (1-\omega_l)\cdot \mathbf{p}_l(n) + \omega_l \cdot F_l(\mathbf{p}(n)) \ , \quad (13)$$

where $0 < \omega_l \le 1$. Note that our proposed algorithm in (6) and (7) is a variant of the update rule in (13).

D. Asynchronous update

The convergence results in the previous subsection assume that users are synchronized and the latest information is available for every link. However, in practice it is unlikely that such updates will take place simultaneously or even at the same update frequency, and in many cases only delayed information may be available depending on the update frequency and so on. Hence, it is important to show the convergence of the update algorithm under an asynchronous update scheme with possibly delayed information.

Let T_l be the set of periods at which the transmission power of link l is updated, and

$$\mathbf{p}_l(n+1) = F_l(\mathbf{p}(\tau_l(n))) \qquad \text{for all } n \in T_l \ , \quad (14)$$

where $0 \le \tau_l(n) \le n$. We assume that the sets $T_l, l \in \tilde{\mathcal{L}}$, are infinite and if $\{n_k\}$ is a sequence of elements in T_l that tends to infinity, then

$$\lim_{k\to\infty} \tau_l(n_k) = \infty \ .$$

This update scheme is called a *totally asynchronous update scheme*.

The following lemma tells us that the link transmission powers converge to the solution under totally asynchronous updates, starting from any initial vector $\mathbf{p}(0) \in \mathcal{P}$.

Lemma 3: Under the update rule (14) we have $\lim_{n\to\infty} \mathbf{p}(n) = \mathbf{p}^\star$ for all $\mathbf{p}(0) \in \mathcal{P}$.

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