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Adaptive Control Scheme for Plants with Time-varying Structure Using On-line Parameter Estimation

Angel González Raúl Ordóñez,

Abstract— **A Direct Adaptive Control Scheme to stabilize strict feedback plants with a time-varying structure is presented. Semi-global asymptotic stability is achieved with limited knowledge of the plant dynamics. Simulation of the scheme using a slender delta wing rock phenomenon model as practical example, confirming the results of the mathematical formulation of the control scheme. Various initial conditions are simulated showing the ability of the scheme to stabilize the plant over a wide range of operation.**

I. INTRODUCTION

The development of adaptive controllers for linear plants has been studied over a number of years [1]. A further challenge is posed when attempting to design the same controllers for nonlinear plants. Nonlinear plants often have complex dynamic structures that often render the control problem intractable. Early efforts to control nonlinear systems were on plants with analyzable nonlinearities for which control laws can be devised and stability analysis of the closed loop system can be performed [2], [3], [4]. Many advances in nonlinear control have been obtained by adapting ideas from linear control theory, such as dead zone, leakage, parameter projection, and dynamic normalization [5], providing important foundations for the development of nonlinear control methods. Adaptive control aims at stabilizing plants with dynamics that change through time. This change may depend on variables that may be exogenous to the plant itself. The control adapts itself to the plant changes, in order to obtain a stable response of the closed-loop system. To enable adaptation, some amount of knowledge of the plant characteristics is needed. This knowledge may range from a complete mathematical description of the plant dynamics including limits of operation, to a general mathematical description of the plant and some limits of operation. In this work, a scheme to control nonlinear plants with a particular time-varying structure [6] is presented. This scheme has the particularity that it does not require exact knowledge of the time-varying structure of the system in order to generate a stabilizing control law. This fact allows obtaining local asymptotic stability for strict feedback systems [7], with time-varying structure. This work assumes no knowledge of the exogenous variable dependence model and its limits of operation, obtaining asymptotic stability of the closed loop system. Simulation of this control scheme is presented using the slender delta wing rock phenomenon model developed by by Nayfeh, Elzbeda, and Mook [8], as practical example. Asymptotic stability is achieved using different initial conditions for the wing roll angle and rate.

II. THEORETICAL BACKGROUND

This work continues the works developed by Ordóñez and Passino [12], [13], [6] which presented an extension of the class of strict feedback systems considered by Polycarpou and Mears [10] and Zhang [11] with the additional concept of a dynamic structure that depends on time through a scheduling variable. In these works, Ordóñez and Passino developed both indirect and direct adaptive control approaches for a nonlinear systems with a time-varying structure. This class of systems is a generalization of the class of strict feedback systems traditionally considered in the literature and can be expressed as

$$\begin{aligned} \dot{x}_i &= \sum_{j=1}^{R} \rho_j(v) \left(\phi_{i,j}(X_i) + \psi_{i,j}(X_i) x_{i+1} \right), \\ i &= 1, .., n-1 \\ \dot{x}_n &= \sum_{j=1}^{R} \rho_j(v) \left(\phi_{n,j}(X_n) + \psi_{n,j}(X_n) u \right), \end{aligned} \tag{1}$$

where $X_i = [x_1, x_2, \dots, x_i]^\top$, $i = 1, \dots, n$, $X_n \in \mathbb{R}^n$ is the state vector, assumed available for measurement, and $u \in \mathbb{R}$ is the control input. The variable $v \in \mathbb{R}^q$ may be an additional input or an exogenous scheduling variable with $(n-1)$ bounded and measurable derivatives. The functions $\rho_j(v)$, $j = 1, \dots, R$, may be considered as interpolating functions that describe the time-varying structure of the system, since they combine the system (1) in strict feedback form, the combination depending on the variable v. In [12], [13], the dynamics of the system $(\phi_{i,j}, \psi_{i,j})$ are estimated on-line while the interpolation function $\rho_j(v)$ is assumed to be known (e.g., computed using previously recorded data). In the present work, that assumption of knowledge of $\rho_j(v)$ is relaxed, assuming only to know the bounds of the scheduling variable, and using such scheduling variable as an input of the on-line approximator. In order to do this, the system is conveniently expressed in the form

$$\begin{aligned} \phi_i^c(X_i, v) &= \sum_{j=1}^{R} \rho_j(v) \phi_{i,j}(X_i) \\ \psi_i^c(X_i, v) &= \sum_{j=1}^{R} \rho_j(v) \psi_{i,j}(X_i). \end{aligned} \tag{2}$$

Transforming the plant expression into

$$\begin{aligned} \dot{x}_i &= \phi_i^c(X_i, v) + \psi_i^c(X_i, v) x_{i+1}, \quad i = 1, .., n-1 \\ \dot{x}_n &= \phi_n^c(X_n, v) + \psi_n^c(X_n, v) u. \end{aligned} \tag{3}$$

This last equation includes the time-varying structure of the system in its dynamics, to facilitate the mathematical derivations.

III. Direct adaptive control theorem

This section presents the stability analysis of the direct adaptive control (DAC) scheme for plants with a time-varying structure dependent on a scheduling variable. **Theorem 1**:

Consider the system (1)*, and for convenience, define*

$$\phi_i^c(X_i,v)=\sum_{j=1}^{R}\rho_j(v)\phi_{i,j}(X_i,v)$$
$$\psi_i^c(X_i,v)=\sum_{j=1}^{R}\rho_j(v)\psi_{i,j}(X_i,v). \tag{4}$$

With these definitions, system (1) *becomes*

$$\dot{x}_i=\phi_i^c(X_i,v)+\psi_i^c(X_i,v)x_{i+1},\quad i=1,..,n-1$$
$$\dot{x}_n=\phi_n^c(X_n,v)+\psi_n^c(X_n,v)u, \tag{5}$$

with state vector X_n and scheduling vector v, both available for measurement and v bounded, and satisfying the assumptions

$$0\le\rho_j(v)\le 1$$
$$\sum_{j=1}^{R}\rho_j(v)<\infty, \tag{6}$$

for all $v\in\mathbb{R}^q$, and

$$\phi_i^c(0,v)=0$$
$$\psi_i^c(X_i,v)\neq 0, \tag{7}$$
$$\tag{8}$$

together with

$$0<\underline{\psi}_i^c\le\psi_i^c(X_i,v)\le\bar{\psi}_i^c<\infty$$
$$\left|\dot{\psi}_i^c\right|=\left|\sum_{j=1}^{R}\left(\frac{\partial\rho_j(v)}{\partial v}\dot{v}\psi_i^j(X_i)+\rho_j(v)\frac{\partial\psi_i^j(X_i)}{\partial X_i}\dot{X}_i\right)\right|\le\psi_{i_d}^c. \tag{9}$$

For all $X_i\in\mathbb{R}^i$, $v\in\mathbb{R}^q$. Assume also that $v(0)\in\mathcal{S}_v\subset\mathbb{R}^q, X_i(0)\in\mathcal{S}_{X_i}\subset\mathbb{R}^i, i=1,\ldots,n$, where $\mathcal{S}_v$ and $\mathcal{S}_{X_i}$ are compact sets of arbitrary size specified a priori. Consider the diffeomorphism

$$z_1=x_1$$
$$z_i=x_i-\hat{\alpha}_{i-1}-\alpha_{i-1}^s,\quad i=2,\ldots,n, \tag{10}$$

with $i=1,\ldots,n$,

$$\hat{\alpha}_i=\sum_{j=1}^{R}\hat{\theta}_{i,j}^{\top}\zeta_{i,j}(X_i,v)$$
$$\alpha_i^s=-k_iz_i-z_{i-1},\quad with\quad k_i>0\quad and\quad z_0=0. \tag{11}$$

Assume the functions $\zeta_{i,j}(X_i,v)$ to be at least $n-i$ times continuously differentiable, with those $n-i$ derivatives finite. Consider the adaptation laws for the parameter vectors $\hat{\theta}_{i,j}\in\mathbb{R}^{N_{i,j}}, N_{i,j}\in\mathbb{N}$,

$$\dot{\hat{\theta}}_{i,j}=-\gamma_{i,j}\zeta_{i,j}z_i-\sigma_{i,j}\hat{\theta}_{i,j} \tag{12}$$

with $\gamma_{\alpha_{i,j}}>0$, $\sigma_{\alpha_{i,j}}>0$, $i=1,\ldots,n$, $j=1,\ldots,R$. Then, the control law

$$u=\hat{\alpha}_n+\alpha_n^s \tag{13}$$

guarantees boundedness of all signals and convergence of the states to the residual set

$$\mathcal{D}_d=\left\{X_n\in\Re^n:\sum_{i=1}^{n}z_i^2\le\frac{2\underline{\psi}_mW_d}{\beta_d}\right\}. \tag{14}$$

where $\underline{\psi}_m=\min_{1\le i\le n}\bar{\psi}_i^c$, β_d is a constant, and W_d measures approximation errors and ideal parameter sizes, and its magnitude can be reduced through the choice of the design constants k_i, $\gamma_{i,j}$ and $\sigma_{i,j}$.

Proof: The proof is performed inductively and requires n steps. In the first step let $z_1=x_1$ and $z_2=x_2-\hat{\alpha}_1-\alpha_1^s$ where $\hat{\alpha}_1$ is the approximation to an ideal signal α_1^* that produces global asymptotic stability without the need of the stabilizing term α_1^s, defined below. Let $c_1>0$ be a constant such that $c_1>\frac{\psi_{1_d}^c}{2\underline{\psi}_1^c}$, and define

$$\alpha_1^*(x_1,v)=\frac{1}{\psi_1^c}\left(-\phi_1^c-c_1z_1\right). \tag{15}$$

It is assumed that the ideal control α_1^* is smooth and, hence, it can be approximated with arbitrary accuracy for v and x_1 within the compact sets $\mathcal{S}_v\subset\mathbb{R}^q$ and $\mathcal{S}_{x_1}\subset\mathbb{R}$, respectively, as long as the size of the approximator can be made arbitrarily large. For an approximator of finite size, let

$$\alpha_1^*(x_1,v)=\sum_{j=1}^{R}\theta_{1,j}^{*^{\top}}\zeta_{1,j}(x_1,v)+\delta_{\alpha_1}(x_1,v), \tag{16}$$

where the parameter vector $\theta_{1,j}^*\in\mathbb{R}^{N_{1,j}}, N_{1,j}\in\mathbb{N}$, is optimum in the sense that it minimizes the representation error δ_{α_1} over the set $\mathcal{S}_v\times\mathcal{S}_{x_1}$ and a suitable set parameter space $\Omega_{1,j}$, and $\zeta_{1,j}(x_1,v)$ is defined via the choice of the approximator (e.g., the elements of $\zeta_{1,j}(x_1,v)$ may be polynomials, or the output of a hidden layer of neurons in a feedforward neural network). As a result of the stability proof, the approximator parameters are bounded by the adaptation laws, so $\Omega_{1,j}$ does not have to be explicitly defined, and no means to keep the parameters bounded is needed. δ_{α_1} arises because $N_{1,j}$ is finite, but may be arbitrarily small by increasing the size of the estimator. This means there exists a constant bound $d_{\alpha_1}>0$, such that $|\delta_{\alpha_1}|\le d_{\alpha_1}<\infty$. Let $\Phi_{1,j}=\hat{\theta}_{1,j}-\theta_{1,j}^*$ denote the parameter error, and approximate (16) with

$$\hat{\alpha}_1(x_1,v,\hat{\theta}_{1,j})=\sum_{j=1}^{R}\hat{\theta}_{1,j}^{\top}\zeta_{1,j}(x_1,v). \tag{17}$$

Note that the time varying structure of the system is not explicitly reflected in the approximator ($\rho_j(v)$ is not included in the approximator definition), but it is included in the controller via the definition of $\zeta_{1,j}(x_1, v)$. Hence, $\zeta_{1,j}(x_1, v)$ must contain means to estimate the plant dynamics including their dependencies on the exogenous or scheduling variable v. That is, $\zeta_{1,j}(x_1, v)$ uses the variable v as one of its inputs to approximate the dynamics of the system directly. Consider the dynamics of the transformed state,

$$\begin{aligned}\dot{z}_1 &= \phi_1^c + \psi_1^c(z_2 + \hat{\alpha}_1 + \alpha_1^s) + \psi_1^c(\alpha_1^* - \alpha_1^*) \\ &= -c_1 z_1 + \psi_1^c z_2 + \psi_1^c \left(\sum_{j=1}^{R} \Phi_{1,j}\zeta_{1,j} - \delta_{\alpha_1} \right) + \psi_1^c \alpha_1^s.\end{aligned} \tag{18}$$

Let $V_1 = \frac{1}{2\psi_1^c} z_1^2 + \frac{1}{2}\sum_{j=1}^{R} \frac{\Phi_{1,j}^\top \Phi_{1,j}}{\gamma_{1,j}}$ and examine the derivative,

$$\dot{V}_1 = \frac{2\psi_1^c\,(2z_1\dot{z}_1) - 2z_1^2\dot{\psi}_1^c}{4{\psi_1^c}^2} + \sum_{j=1}^{R} \frac{\Phi_{1,j}^\top \dot{\Phi}_{1,j}}{\gamma_{1,j}}. \tag{19}$$

Using (18),

$$\begin{aligned}\dot{V}_1 = &-\frac{c_1 z_1^2}{\psi_1^c} + z_1 z_2 + z_1 \sum_{j=1}^{R} \Phi_{1,j}\zeta_{1,j} - z_1\delta_{\alpha_1} \\ &+ z_1\alpha_1^s - \frac{1}{2} z_1^2 \frac{\dot{\psi}_1^c}{\psi_1^c} + \sum_{j=1}^{R} \frac{\Phi_{1,j}^\top \dot{\Phi}_{1,j}}{\gamma_{1,j}}.\end{aligned} \tag{20}$$

Noting that $\dot{\Phi}_{1,j} = \dot{\hat{\theta}}_{1,j}$, choose the adaptation law,

$$\dot{\hat{\theta}}_{1,j} = -\gamma_{1,j}\zeta_{1,j} z_1 - \sigma_{1,j}\hat{\theta}_{1,j}, \tag{21}$$

with the design constants $\gamma_{1,j} > 0, \sigma_{1,j} > 0, j = 1, \ldots, R$. Notice that for any constant $k_1 > 0$

$$-z_1\delta_{\alpha_1} \le |z_1|\, d_{\alpha_1} \le k_1 z_1^2 + \frac{d_{\alpha_1}^2}{4k_1}. \tag{22}$$

Choose

$$\alpha_1^s = -k_1 z_1. \tag{23}$$

Notice that completing squares,

$$-\Phi_{1,j}^\top \hat{\theta}_{1,j} \le -\Phi_{1,j}^\top \left(\Phi_{1,j} + \theta_{1,j}^*\right) \le -\frac{|\Phi_{1,j}|^2}{2} + \frac{\left|\theta_{1,j}^*\right|^2}{2}. \tag{24}$$

And finally, observe that,

$$-\frac{z_1^2}{\psi_1^c}\left(c_1 + \frac{\dot{\psi}_1^c}{2\psi_1^c}\right) \le -\frac{z_1^2}{\psi_1^c}\left(c_1 - \frac{\psi_{1_d}^c}{2\underline{\psi}_1^c}\right) \le -\frac{\bar{c}_1 z_1^2}{\bar{\psi}_1^c}, \tag{25}$$

with $\bar{c}_1 = c_1 - \frac{\psi_{1_d}^c}{2\underline{\psi}_1^c}$. Then,

$$\begin{aligned}\dot{V}_1 \le &-\frac{\bar{c}_1 z_1^2}{\bar{\psi}_1^c} - \frac{1}{2}\sum_{j=1}^{R} \sigma_{1,j}\frac{|\Phi_{1,j}|^2}{\gamma_{1,j}} + z_1 z_2 \\ &+ \frac{d_{1,j}^2}{4k_1} + \frac{1}{2}\sum_{j=1}^{R} \sigma_{1,j}\frac{\theta_{1,j}^*}{\gamma_{1,j}}.\end{aligned} \tag{26}$$

completing the first step of the proof.

Continuing in the same manner up to the n^{th} step, we let $z_n = x_n - \hat{\alpha}_{n-1} - \alpha_{n-1}^s$, with $\hat{\alpha}_{n-1}$ and α_{n-1}^s defined as in (11). Let

$$\alpha_n^*(X_n, v) = \frac{1}{\psi_2^c}\left(-\phi_n^c - c_n z_n + \dot{\hat{\alpha}}_{n-1} + \dot{\alpha}_{n-1}^s\right), \tag{27}$$

with $c_n > \frac{\psi_{n_d}^c}{2\underline{\psi}_n^c}$, and its representation

$$\alpha_n^*(X_n, v_n) = \sum_{j=1}^{R} {\theta_{n,j}^*}^\top \zeta_{n,j}(X_n, v_n) + \delta_{\alpha_n}(X_n, v_n), \tag{28}$$

for $X_n \in \mathbb{R}^n, v \in \mathcal{S}_v^q$, and the parameter vector $\theta_{n,j}^* \in \mathbb{R}^{N_{n,j}}, N_{n,j} \in \mathbb{N}$ which minimizes the representation error δ_{α_n} over $\mathcal{S}_v \times \mathcal{S}_{X_n}$ and a compact parameter set $\Omega_{n,j}$ under some optimization criterion. Hence, there exists a constant $d_{\alpha_n} > 0$, such that $|\delta_{\alpha_n}| \le d_{\alpha_n} < \infty$. Let $\Phi_{n,j} = \hat{\theta}_{n,j} - \theta_{n,j}^*$, and consider

$$u = \hat{\alpha}_n + \alpha_n^s. \tag{29}$$

Then,

$$\begin{aligned}\dot{z_n} =&\phi_n^c + \psi_n^c(\hat{\alpha}_n + \alpha_n^s) - \dot{\hat{\alpha}}_{n-1} - \dot{\alpha}_{n-1}^s \qquad (30) \\ &+ \psi_n^c(\alpha_n^* - \alpha_n^*) \\ =& -c_n z_n + \psi_n^c \left(\sum_{j=1}^{R} \Phi_{n,j}\zeta_{n,j} - \delta_{\alpha_n}\right) + \psi_n^c \alpha_n^s.\end{aligned} \tag{31}$$

Choose the Lyapunov candidate

$$V = V_{n-1} + \frac{1}{2\psi_n^c} z_n^2 + \frac{1}{2}\sum_{j=1}^{R} \frac{\Phi_{n,j}^\top \Phi_{n,j}}{\gamma_{n,j}} \tag{32}$$

and examine its derivative

$$\begin{aligned}\dot{V}_n =&\dot{V}_{n-1} - \frac{c_n z_n^2}{\psi_n^c} + z_n \sum_{j=1}^{R} \Phi_{n,j}\zeta_{n,j} - z_n\delta_{\alpha_n} \\ &+ z_n\alpha_n^s - \frac{1}{2} z_n^2 \frac{\dot{\psi}_n^c}{\psi_n^c} + \sum_{j=1}^{R} \frac{\Phi_{n,j}^\top \dot{\Phi}_{n,j}}{\gamma_{n,j}}.\end{aligned} \tag{33}$$

It may be shown inductively that

$$\begin{aligned}\dot{V}_{n-1} \le &-\sum_{i=1}^{n-1} \frac{\bar{c}_i z_i^2}{\bar{\psi}_i^c} - \frac{1}{2}\sum_{i=1}^{n-1}\sum_{j=1}^{R} \sigma_{i,j}\frac{|\Phi_{i,j}|^2}{\gamma_{i,j}} + z_{n-1}z_n \\ &+ \sum_{i=1}^{n-1} \frac{d_{\alpha_i}^2}{4k_i} + \frac{1}{2}\sum_{i=1}^{n-1}\sum_{j=1}^{R} \sigma_{i,j}\frac{|\theta_{i,j}^*|^2}{\gamma_{i,j}}.\end{aligned} \tag{34}$$

with $\bar{c}_i = c_i - \frac{\psi_{i_d}^c}{2\underline{\psi}_i^c} > 0, i = 1, \ldots, n$. Choosing the adaptation laws for $\hat{\theta}_{n,j}$ in (12) and of α_n^s in (11), together with the observations that

$$-\Phi_{n,j}^\top \hat{\theta}_{n,j} \le -\frac{|\Phi_{n,j}|^2}{2} + \frac{\left|\theta_{n,j}^*\right|^2}{2}, \tag{35}$$

$$-z_n\delta_{\alpha_n} \le |z_n|\, d_{\alpha_n} \le k_n z_n^2 + \frac{d_{\alpha_n}^2}{4k_n}, \quad \text{with } k_n > 0, \tag{36}$$

$$-\frac{z_n^2}{\psi_n^c}\left(c_n+\frac{\dot{\psi}_n^c}{2\psi_n^c}\right)\leq-\frac{\bar{c}_n z_n^2}{\bar{\psi}_n^c}, \tag{37}$$

we obtain that

$$\dot{V}\leq-\sum_i^n\frac{\bar{c}_i z_i^2}{\bar{\psi}_i^c}-\frac{1}{2}\sum_{i=1}^n\sum_{j=1}^R\sigma_{i,j}\frac{|\Phi_{i,j}|^2}{\gamma_{i,j}}+W_d, \tag{38}$$

with

$$W_d=\sum_{i=1}^n\frac{d_{\alpha_i}^2}{4k_i}+\frac{1}{2}\sum_{i=1}^n\sum_{j=1}^R\sigma_{i,j}\frac{|\theta_{i,j}^*|^2}{\gamma_{i,j}}. \tag{39}$$

Then, if

$$\frac{\bar{c}_i z_i^2}{\bar{\psi}_i^c}\geq W_d$$

or

$$\frac{1}{2}\sum_{i=1}^n\sum_{j=1}^R\sigma_{i,j}\frac{|\Phi_{i,j}|^2}{\gamma_{i,j}}\geq W_d, \tag{40}$$

it follows that $\dot{V}\leq 0$. Furthermore, by letting $\underline{\psi}_m=\min_{1\leq i\leq n}(\underline{\psi}_i^c)$, $\bar{\psi}_m=\max_{1\leq i\leq n}(\bar{\psi}_i^c)$, and defining

$$\begin{aligned}\bar{c}_0&=\min_{1\leq i\leq n}(\bar{c}_i)\\ \psi_m&=\frac{\underline{\psi}_m}{\bar{\psi}_m}\\ \sigma_0&=\min_{1\leq i\leq n}(\sigma_{i,j}),\end{aligned} \tag{41}$$

we have

$$\begin{aligned}-\frac{\bar{c}_i z_i^2}{\bar{\psi}_i^c}\leq-\bar{c}_0\frac{z_i^2}{\bar{\psi}_i^c}\leq-\bar{c}_0\frac{z_i^2\psi_i^c}{\psi_i^c\bar{\psi}_i^c}\leq-\bar{c}_0\psi_m\frac{z_i^2}{\psi_i^c}\\ -\frac{1}{2}\sum_{i=1}^n\sum_{j=1}^R\sigma_{i,j}\frac{|\Phi_{i,j}|^2}{\gamma_{i,j}}\leq-\sigma_0\frac{1}{2}\sum_{i=1}^n\sum_{j=1}^R\frac{|\Phi_{i,j}|^2}{\gamma_{i,j}}.\end{aligned} \tag{42}$$

Then, letting $\beta_d=\min(2\bar{c}_0\psi_m,\sigma_0)$, we have that if

$$V=\frac{1}{2}\sum_{i=1}^n\frac{z_i^2}{\psi_i^c}+\frac{1}{2}\sum_{i=1}^n\sum_{j=1}^R\frac{|\Phi_{i,j}|^2}{\gamma_{i,j}}\geq V_0,\quad\text{with}\quad V_0=\frac{W_d}{\beta_d}, \tag{43}$$

then $\dot{V}\leq 0$ and all signals in the closed loop are bounded. Further

$$\dot{V}\leq-\beta_d V+W_d \tag{44}$$

implying that

$$0\leq V(t)\leq\frac{W_d}{\beta_d}+\left(V(0)-\frac{W_d}{\beta_d}\right)e^{-\beta_d t} \tag{45}$$

so the transformed states and the parameter error vector converge both to a bounded set. Finally, from (45), we conclude that the vector X_n converges to a residual set

$$\mathcal{D}_d=\left\{X_n\in\Re^n:\sum_{i=1}^n z_i^2\leq\frac{2\underline{\psi}_m W_d}{\beta_d}\right\}. \tag{46}$$

■

Remark 1. From 45 it is possible to obtain an estimate of the region of convergence of the adaptive controller, which can be used to *a priori* design the approximator's compact set of operation to ensure stability in the sense that the state does not exit this compact set during transient.

Remark 2. Notice that the assumption of $\psi_i^c>0,\quad i=1,\ldots,n$ is only to simplify the analysis and implies no loss of generality, given that the fundamental requirement is that ψ_i^c be bounded away from zero by a constant. The stability proof can accommodate negative cases.

Remark 3. Notice that explicit knowledge of the bounds established in assumption (9) is not needed but only knowledge of their existence is required.

Remark 4. Notice that the direct adaptive approach presented here relies on linearly parameterized function approximators. Nevertheless, nonlinearly parameterized approximators can be integrated into the analysis by following the usual approach of linearizing the approximators, and then lumping the higher order terms of the Taylor series expansions into the errors δ_{α_i} [14], [15], or by using the mean value theorem, as in [10].

Remark 5. This scheme relaxes the need of knowledge about the system dynamics and the exogenous variable v. Only the bounds of the exogenous variable are needed in order to have an adequate approximation of the stabilizing functions $\hat{\alpha}$. The inclusion of the estimation of the time-varying structure of the plant also potentially reduces the amount of calculations needed in the physical implementation of the controller, depending on the number of centers selected for the approximation structure. It also adds an amount of complexity to the approximator structure, dependent on the order of the scheduling vector. That is, the size of the approximation structure is dependent on the number of variables used by it, and the addition of new variables to the structure increases the size of the vectors involved in an exponential fashion.

Remark 6. The representation error bounds and the size of the ideal parameter vector must be known to compute the size of the residual set to which the states converge. The size of this residual set can be reduced by manipulating the design constants $\gamma_{i,j}$, $\sigma_{i,j}$, and $k_{i,j}$, but meeting some particular performance specification can only be done a posteriori if knowledge of the errors and parameters is lacking.

IV. Simulation

The adaptive control scheme formulated in section III is simulated using a model of the slender delta wing rock phenomenon as practical example. Wing rock is a highly nonlinear phenomenon in which the aircraft experiences limit cycle roll oscillations at high angles of attack. The magnitude and frequency of the oscillations are generally strongly dependent on aircraft configuration and angle of attack, as well as other flow conditions. Figure 1 shows a schematic representation of the wing roll angle oscillations. The wing rock phenomenon has been extensively studied and various models have been developed to describe it. For the purpose of the simulations in this work, the wing rock phenomenon was modelled using the equations developed by Nayfeh, Elzbeda, and Mook [8]. This model is a nonlinear dynamic description for slender delta wing roll angle, obtained from a wind tunnel

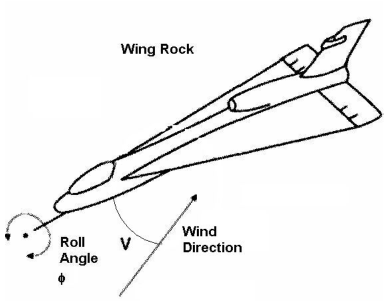

Fig. 1. Wing Rock Phenomenon

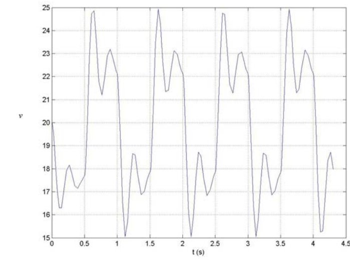

Fig. 2. Angle of attack variation

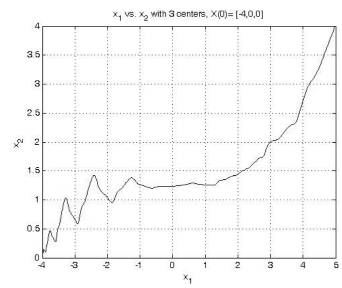

Fig. 3. Phase plane without control applied.

experiment, and is given by

$$\dot{\phi} = \omega^2\phi + \mu_1\dot{\phi}^3 + b_1\dot{\phi}^2 + \mu_2\phi^2\dot{\phi} + b_2\phi\dot{\phi}^2 + g\delta_a, \quad (47)$$

where ϕ is the wing roll angle, while the coefficients ω^2, μ_1, b_1, μ_2, and b_2 are dependent on the angle of attack and the characteristics of the wing, g is a gain and δ_a is the control signal. In wind tunnel experiments using 80° sweep delta wings [16], oscillations were recorded for angle of attack values of up to 60°, the maximum amplitude of the oscillation being about 44° (0.7679 rad) for an angle of attack of about 33°. The maximum value of the roll rate for a wing similar to the one used for this work is about 19.23 rad$/s$ or $1101.8°/s$ for an angle of attack of about 30°. This particular value is the oscillation frequency once the phenomenon is steadily established. In this work a more conservative value of the roll rate $(45\frac{\circ}{s}$, on the assumption that the controller is in operation before the phenomenon starts.

Equation (47) can be expressed in state variable form by letting $x_1 = \phi$, $x_2 = \dot{\phi}$, as

$$\begin{aligned}
\dot{x_1} &= x_2 \\
\dot{x_2} &= \sum_{j=1}^{R} \rho_j(v)(\omega_j^2 x_1 + \mu_{1,j}x_2^3 + b_{1,j}x_2^2 \\
&\quad + \mu_{2,j}x_1^2 x_2 + b_{2,j}x_1 x_2^2) + gx_3 \\
\dot{x_3} &= -\frac{1}{\tau}x_3 + \frac{1}{\tau}u, \qquad (48)
\end{aligned}$$

where x_3 is the output of a linear actuator with constant τ. The dependence of the coefficients in the second state equation on angle of attack v (used instead of the standard α of the aircraft literature to avoid confusion) are taken into account using the model developed by Ordóñez and Passino [13], [12], by inserting the interpolating functions $\rho_j(v)$.

A. Simulation Conditions

The basic simulation conditions are $\phi(0) = -4°$, angle of attack varying between 15° and 25°, according to the affine system

$$\begin{bmatrix} \dot{v_1} \\ \dot{v_2} \end{bmatrix} = \begin{bmatrix} 0 & 25 \\ -25 & 10 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} + \begin{bmatrix} 0 \\ 500 \end{bmatrix} + \begin{bmatrix} 0 \\ 62.5 \end{bmatrix} r, \quad (49)$$

where v_1 is the angle of attack, v_2 is its derivative, and r is a command signal taking values between minus one and one. System (49) has poles $-5 \pm j24.5$ and equilibrium at $v_1 = 20$, $v_2 = 0$. Notice that this system is used to generate an angle of attack signal simulating very rough control conditions. The plot of such a signal vs. time may be seen in Figure 2. According to [8] the angle of attack has a stable focus at the origin for angles about 19.5° or less. For higher angles, the origin becomes unstable and a limit cycle appears around it. However, in both cases the system is unstable and may diverge to infinity (the wing rotating faster and faster). By setting r to a square function of frequency 1 Hz in system (49) the angle of attack becomes smaller and larger than 19.5° once a second. For convenience we will refer to the roll angle, the roll rate, and the actuator output as x_1, x_2, x_3 respectively, or as $X = [x_1, x_2, x_3]^\top$ collectively. In Figure 3 the behavior of the roll angle and the roll rate for initial conditions $X(0) = [-4, 0, 0]^\top$, $v(0) = [20, 0]^\top$ is shown. A second set of values for the initial conditions of the states is used in order to test the operation of the control scheme at high initial conditions. For this particular simulation, $X(0) = [45, 45, 0]^\top$, according to the maximum value of the roll angle observed in similar wind tunnel experiments, and a reasonably high roll rate, to ensure the controller is able to achieve stability even when the wing is already rocking.

B. Simulation Results

The control scheme presented in the adaptive control Theorem 1 is used to stabilize the wing rock problem using the model described in 48, with the first initial conditions set. The design constant settings for this initial simulation are $\gamma = [1, 1, 1]^\top$, $\sigma = [1, 1, 1]^\top$, $k = [1, 1, 1]^\top$. Figure

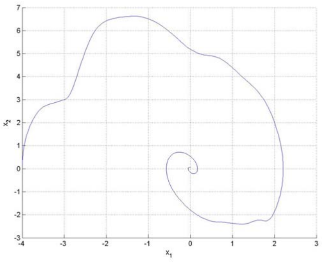

Fig. 4. Phase Plane at $X(0) = [-4, 0, 0]^\top$.

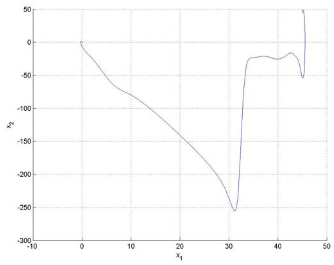

Fig. 5. Phase Plane for $X(0) = [45, 45, 0]^\top$.

4 shows the phase plane for that simulation, showing the convergence of the roll angle and the roll rate to zero. No intent to improve the performance of the controller is made in this simulation by setting the constants γ and σ to one. The second simulation is performed using the initial conditions $X(0) = [45, 45, 0]^\top, v = [20, 0]^\top$, as mentioned. Convergence is obtained for such initial conditions using $\sigma = [.5, .5, .5]^\top$, $k = [10, 10, 10]^\top$, and $\gamma = [0.5, 0.5, 0.5]^\top$. Figure 5 and 6 show the phase plane and the states vs time respectively for this simulation. Notice that the value of the roll rate reaches a maximum absolute value of $193.89255.31°/s$ $(4.456$ rad/s) for these particular initial conditions. Also the actuator output reaches values in the order of the thousands which is unreal for a physical actuator.

V. Conclusions

A Direct Adaptive Control Scheme was mathematically formulated showing that it is feasible to control a strict-feedback system with time-varying structure with a very limited knowledge of that structure. The simulations performed using the slender delta wing rock phenomenon example show that this scheme is able to stabilize this class of systems, with only information about the bounds of the scheduling or exogenous variable. The proof of the DAC theorem shows that it is possible to determine the size of a residual set where all the states will converge to, given that enough information about the system is provided. The assumptions made for the development of the DAC theorem that ψ_i^c be bounded away from zero by a constant, that the states are bounded and available for measurement, and that the exogenous variable is also bounded and available for measurement are not always easy to comply with when designing a real system, because the states are not always available. However, in most cases it is possible to obtain reasonable estimation of the value of the states using standard techniques.

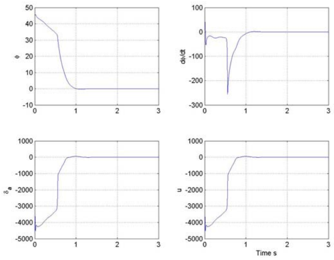

Fig. 6. States vs. time for $X(0) = [45, 45, 0]^\top$.

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