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Robust control of nonlinear uncertain systems with sandwiched backlash.

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***Abstract*— This paper proposes a novel approach for the stabilization problem of a sandwich nonlinear system with a backlash nonlinearity between two dynamic blocks. To this purpose a robust sliding mode controller has been designed ensuring the convergence of the system trajectories in a neighborhood of the origin of arbitrary width (independently of the backlash size). Theoretical results have been validated by simulation.**

***Index Terms*— Backlash, Robust Control, Sliding Mode Control, sandwich systems.**

I. Introduction

The development of control techniques able to compensate for the presence of non-differentiable nonlinearities is, at present, a challenging issue in control theory, mostly in view of the practical relevance of the problem. A wide range of physical systems and devices (i.e. motor servo systems, mechanical actuators, electronic relay circuits) show non-smooth behaviors, memory effects and/or time delays. In other words, non-smooth nonlinearities such as hysteresis, backlash, dead-zone, are always present in real control plants, due to both physical imperfections and inherent characteristics of the controlled system.

Backlash affects almost each mechanical system, and ignoring its presence during control design causes a severe deterioration of system performance which can even lead to instability. As pointed out in [4], controlled systems containing backlash show steady-state errors and/or limit cycles, and plant variables show oscillations wider than the backlash gap.

Various approaches have been developed since 1940 to face this intriguing problem. Two main research thrusts can be found in the literature, the former using PID controllers and the latter based on the describing function methods. However, as discussed in the recent survey [4], only in the last few years significant new developments arose, though a lot of research remains to be done. It is worth mentioning that the so called sandwich systems have been addressed in [12], [2], [1] where time optimal control laws are sought to traverse the backlash gap, and in [10], [9] where the solution proposed is cancelling the actuator dynamics by state feedback and then applying the inverse nonlinearity; both approaches have been widely discussed in [11].

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Anti-backlash mechanical devices have also been developed in the framework of precision position control systems. Unfortunately, this solution adds a resonance and limits the achievable closed-loop bandwidth [4]. Moreover, mechanical anti-gear devices have been often criticized [8], [7] since they could make more expensive, cumbersome and heavy the overall system, can be brittle and usually need maintenance and care.

It follows that some compensation of the detrimental effects of backlash, in terms of closed loop system performance, can be only achieved if control laws are synthesized taking into account the backlash characteristics during control design, though the non-differentiable nature of the nonlinearity.

A key point in pursuing this approach is the definition of a suitable model, to be used for control design, describing the nonlinear behavior of the backlash. An important issue to be considered is that, in real plants, the actuator dynamics is not negligible, and hence it is rather impossible to cross the backlash gap instantaneously. This aspect reveals that the inverse compensation techniques are difficult to be applied in practice and can have evident drawbacks. This issue has been confirmed by experimental findings [6] showing that the search for instantaneous jumps of the backlash gap can produce performance degradation, while measurement noise induces chattering in the control law.

It should be noticed that the avoiding sudden crossings of the backlash gap has been further strongly suggested by the experimental results reported in [4]. In an effort of privileging good closed loop performances, avoiding such instantaneous jumps, this paper proposes a controller ensuring the convergence of the system trajectories in a neighborhood of the origin of arbitrary width (independent of the backlash size) for a sandwich system with backlash. The present note extends recently presented results [3].

II. System model and problem statement

Consider a SISO uncertain nonlinear dynamical system described by:

$$\dot{\mathbf{x}} \quad = \quad \mathbf{f}(\mathbf{x}) + \mathbf{\Delta f}(\mathbf{x}) + \mathbf{g}(\mathbf{x})u + \mathbf{d}(\mathbf{x}) \qquad (1)$$

where $\mathbf{x}(t) \in \mathbb{R}^n$ is the *system state vector* at time t, $u(t) \in \mathbb{R}$ is the *system input*, $\mathbf{g}(\mathbf{x}) : \mathbb{R}^n \rightarrow \mathbb{R}^n$ is the smooth state-input map, $\mathbf{f}(\mathbf{x}) : \mathbb{R}^n \rightarrow \mathbb{R}^n$ is a smooth function describing

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the known plant dynamics, and finally $\mathbf{\Delta f}(\mathbf{x}) : \mathbb{R}^n \to \mathbb{R}^n$ and $\mathbf{d}(\mathbf{x}) : \mathbb{R}^n \to \mathbb{R}^n$ account for parameter variations and exogenous disturbances respectively. The nonlinear system is assumed to be preceded by an actuating device (see Fig.1) whose dynamics are described by

$$\dot{\mathbf{x}}_A = \mathbf{f_A}(\mathbf{x_A}) + \mathbf{\Delta f_A}(\mathbf{x_A}) + \mathbf{g_A}(\mathbf{x_A})v + \mathbf{d_A}(\mathbf{x}_A) \quad (2)$$

$$\mathbf{w} = h_A(\mathbf{x}_A) \quad (3)$$

where $\mathbf{x}_A(t) \in \mathbb{R}^{n_A}$ is the *actuator state vector* at time t, $v(t) \in \mathbb{R}$ is the *actual input*, $\mathbf{g_A}(\mathbf{x_A}) : \mathbb{R}^{n_A} \to \mathbb{R}^{n_A}$ is the smooth actuator state-input map, $\mathbf{f_A}(\mathbf{x_A}) : \mathbb{R}^{n_A} \to \mathbb{R}^{n_A}$ is a smooth function describing the known actuator dynamics, and finally $\mathbf{\Delta f_A}(\mathbf{x_A})$ and $\mathbf{d_A}(\mathbf{x}_A) : \mathbb{R}^{n_A} \to \mathbb{R}^{n_A}$ describe eventual uncertain terms in the actuators.

The actuator output $w(t) \in \mathbb{R}$ is connected to the system input $u(t)$ by a backlash-like nonsmooth block $u = \text{bk}[w]$ (see Fig.2). A compact analytical description of the backlash is [8]:

$$\dot{u} = \begin{cases} m\dot{w} & \text{if } \dot{w} > 0 \text{ and } u = m(w - c_r) \text{ or} \\ & \text{if } \dot{w} < 0 \text{ and } u = m(w + c_l) \\ 0 & \text{otherwise} \end{cases} \quad (4)$$

It is likely to consider coefficients c_r and c_l uncertain, meaning that the width of the backlash gap is uncertain, too. It is assumed that such coefficients have bounded uncertainties:

$$m|c_r| \leq \rho_{cr} \quad m|c_l| \leq \rho_{cl} \quad \Delta = \max(\rho_{cr}, \rho_{cl})$$

so that the following holds:

Assumption 2.1: The coefficients c_l, c_r describing backlash are uncertain but bounded by a known constant, hence

$$|\text{bk}[w]| \leq |mw| + \Delta \quad (5)$$

The above assumptions concern the knowledge of the backlash nonlinearity. The following extra assumptions on the system and actuator dynamics are introduced; the first one is not restrictive as it states the existence of a sliding mode surface for the system (1), the second one is equivalent to the assumption that the actuator dynamics have to be relative degree constant and equal to one:

Assumption 2.2: There exists a smooth function

$$s(\mathbf{x}) : \mathbb{R}^n \to \mathbb{R} \quad (6)$$

such that:

- the system dynamics on the surface $s(\mathbf{x}) = 0$ are asymptotically stable.
- the function $s(\mathbf{x})$ is such that the following relation holds:
$$\frac{\partial s(\mathbf{x})}{\partial \mathbf{x}}\mathbf{g}(\mathbf{x}) = \nabla s\, \mathbf{g}(\mathbf{x}) \neq 0 \quad \forall \mathbf{x} \in \mathbb{R}^n$$

Assumption 2.3: The actuator dynamics satisfies

$$\nabla\, h_A(\mathbf{x}_A)\mathbf{g_A}(\mathbf{x_A}) \neq 0 \quad \forall \mathbf{x}_A \in \mathbb{R}^{n_A}$$

Remark 2.1: Note that, by the above assumptions and the smoothness of $s(\mathbf{x})$, $\mathbf{g}(\mathbf{x})$, $\mathbf{g_A}(\mathbf{x_A})$ and $h_A(\mathbf{x}_A)$ it can be assumed without loss of generality that:

$$\begin{cases} \nabla s\, \mathbf{g}(\mathbf{x}) > 0 & \forall \mathbf{x} \in \mathbb{R}^n \\ \nabla\, h_A(\mathbf{x}_A)\mathbf{g_A}(\mathbf{x_A}) > 0 & \forall \mathbf{x}_A \in \mathbb{R}^{n_A} \end{cases}$$

The next assumptions basically state that bounded uncertainties are considered in this paper.

Assumption 2.4: There exist smooth functions $(\rho_f)_i : \mathbb{R}^n \to \mathbb{R}$, $(\rho_d)_i : \mathbb{R}^n \to \mathbb{R}$, $i = 1, \ldots, n$, $\rho_A : \mathbb{R}^{n_A} \to \mathbb{R}$ such that:

$$|[\mathbf{\Delta f}(\mathbf{x})]_i| \leq (\rho_f(\mathbf{x}))_i \;\forall \mathbf{x} \in \mathbb{R}^n \; i = 1, \ldots, n$$

$$|[\mathbf{d}(\mathbf{x})]_i| \leq (\rho_d(\mathbf{x}))_i \;\forall \mathbf{x} \in \mathbb{R}^n \; i = 1, \ldots, n$$

$$|\nabla\, h_A(\mathbf{x}_A)[\mathbf{\Delta f_A}(\mathbf{x_A}) + \mathbf{d_A}(\mathbf{x}_A)]| \leq \rho_A(\mathbf{x}_A) \;\forall \mathbf{x}_A \in \mathbb{R}^{n_A}$$

It follows from the above assumptions that

$$|\nabla s\, [\mathbf{\Delta f}(\mathbf{x}) + \mathbf{d}(\mathbf{x})]| \leq \rho(\mathbf{x}) \;\forall \mathbf{x} \in \mathbb{R}^n$$

for a known positive scalar $\rho(\mathbf{x})$. Note that the assumptions above stated do hold for a wide class of nonlinear plants, especially mechanical systems.

Problem 2.1: The addressed problem, provided that Assumptions 2.1 to 2.4 are satisfied, is finding a feedback controller guaranteeing the robust stabilization of system (1) in the presence of the actuating device described by the uncertain dynamics (2), (3), (4).

III. Preliminaries and notation

In order to concisely state the main result, some definitions will be given in the following. They formalize some relationships between the sliding surface, system dynamics and actuator characteristics. It is useful to define the functions below, built considering both system and actuator uncertainties:

$$\begin{aligned} \omega(\mathbf{x}) &\overset{\text{def}}{=} \nabla s\, \mathbf{f}(\mathbf{x}); \\ r(\mathbf{x}) &\overset{\text{def}}{=} \nabla s\, \mathbf{g}(\mathbf{x}) \\ \delta(\mathbf{x}) &\overset{\text{def}}{=} \nabla s\, [\mathbf{\Delta f}(\mathbf{x}) + \mathbf{d}(\mathbf{x})] \end{aligned}$$

$$\begin{aligned} \omega_A(\mathbf{x}_A) &\overset{\text{def}}{=} \nabla\, h_A(\mathbf{x}_A)\mathbf{f_A}(\mathbf{x_A}) \\ r_A(\mathbf{x}_A) &\overset{\text{def}}{=} \nabla\, h_A(\mathbf{x}_A)\mathbf{g_A}(\mathbf{x_A}) \\ \delta_A(\mathbf{x}_A) &\overset{\text{def}}{=} \nabla\, h_A(\mathbf{x}_A)[\mathbf{\Delta f_A}(\mathbf{x_A}) + \mathbf{d_A}(\mathbf{x}_A)] \end{aligned}$$

The following functions will be used hereafter, ς being a real scalar variable:

$$\text{sgn}(\varsigma) \stackrel{\text{def}}{=} \begin{cases} 1 & \text{if } \varsigma \geq 0 \\ -1 & \text{otherwise} \end{cases}$$

$$\text{sat}(\varsigma) \stackrel{\text{def}}{=} \begin{cases} 1 & \text{if } \varsigma \geq 1 \\ \varsigma & \text{if } |\varsigma| < 1 \\ -1 & \text{otherwise} \end{cases}$$

In the following, the $sign$ function will be approximated by means of the so called the 'bell' functions ϕ_ε, given below, which belong to the $\mathcal{C}^\infty$ class over the whole real axis but are nonzero in the interval $(-\varepsilon, \varepsilon)$:

$$\phi_\varepsilon(\varsigma) \stackrel{\text{def}}{=} \begin{cases} C_\varepsilon e^{\left(\frac{1}{\left(\frac{\varsigma}{\varepsilon}\right)^2 - 1}\right)} & \text{if } |\varsigma| < \varepsilon, \\ 0 & \text{otherwise,} \end{cases}$$

C_ε being a positive real number such that

$$C_\varepsilon^{-1} = \frac{1}{2} \int_{-\infty}^{+\infty} \phi_\varepsilon(x) \mathrm{d}x = \int_0^\varepsilon e^{\frac{1}{\left(\frac{x}{\varepsilon}\right)^2 - 1}} \mathrm{d}x.$$

The following function will be used hereafter to approximated the $sign$ function:

$$\psi_\varepsilon(\varsigma) \stackrel{\text{def}}{=} \int_{-\infty}^{\varsigma} \phi_\varepsilon(\xi) d\xi - 1.$$

since it owns the desirable properties of belonging to the $\mathcal{C}^\infty$ class over the whole real axis (being an integral function of a $\mathcal{C}^\infty$ integrand), and satisfies $\psi_\varepsilon(\alpha) = -1 \ \ \forall \alpha < -\varepsilon$ and $\psi_\varepsilon(\alpha) = 1 \ \ \forall \alpha > \varepsilon$. It follows that this function coincides with the $sign$ function outside the interval $(-\varepsilon, \varepsilon)$.

IV. MAIN RESULT

Define the following sliding surface

$$\begin{aligned} \sigma(\mathbf{x_A}, \mathbf{x}) \ &\stackrel{\text{def}}{=}\ h_A(\mathbf{x}_A) + m^{-1}[r(\mathbf{x})]^{-1} \cdot \\ &\cdot \{\omega(\mathbf{x}) + \bar{\kappa}(\mathbf{x}) \ \ \psi_\varepsilon(s(\mathbf{x}))\} \end{aligned} \tag{7}$$

where $\bar{\kappa}(\mathbf{x}) \triangleq \Delta + \eta + \rho(\mathbf{x})$, η and ϵ being positive design constants. The following result holds.

Theorem 4.1: It is given the system (1) containing uncertain actuator nonlinearities described by (2), (3), (4). Under Assumptions 2.1-2.4, there exist computable functions $v_e(t)$, $v_n(t)$ such that the following control law

$$v = v_e + \theta v_n \text{sgn}(\sigma(\mathbf{x_A}, \mathbf{x})) \tag{8}$$

with arbitrary $\theta > 1$, ensures the achievement of a sliding motion on the surface $\sigma(\mathbf{x_A}, \mathbf{x}) = 0$, and this condition ensures the convergence of system (1) trajectories to an ϵ-neighborhood of the stabilizing surface $s(\mathbf{x})$ in finite time.

Proof. According to Assumption 2.2, the achievement of a sliding motion on the surface $s(\mathbf{x}) = 0$ guarantees plant asymptotic stabilization. This condition can be achieved if the following sliding mode existence condition $s(\mathbf{x})\dot{s}(\mathbf{x}) < -\eta|s(\mathbf{x})|$ is globally satisfied at each time instant, which ensures also a finite maximum reaching time equal to $T_0 = \dfrac{|s(\mathbf{x}(0))|}{\eta}$. The previous condition gives:

$$\begin{aligned} s(\mathbf{x})\dot{s}(\mathbf{x}) \ &= \ s(\mathbf{x}) \left[\nabla s \ \ (\mathbf{f}(\mathbf{x}) + \mathbf{g}(\mathbf{x})u + \mathbf{d}(\mathbf{x}))\right] \\ &= \ s(\mathbf{x}) \left[\omega(\mathbf{x}) + r(\mathbf{x})u + \delta(\mathbf{x})\right] < -\eta|s(\mathbf{x})|. \end{aligned}$$

It is easy to verify that the last inequality is fulfilled by the following input u:

$$\begin{cases} u = -[r(\mathbf{x})]^{-1} \{\omega(\mathbf{x}) + \kappa(\mathbf{x}) \text{sgn}[s(\mathbf{x})]\} \\ \kappa(\mathbf{x}) = \rho(\mathbf{x}) + \eta \end{cases} \tag{9}$$

and, in view of (4), by the following actuator output w:

$$w = -m^{-1}[r(\mathbf{x})]^{-1} \{\omega(\mathbf{x}) + (\kappa(\mathbf{x}) + \Delta) \text{sgn}[s(\mathbf{x})]\}.$$

In order to ensure convergence to an ϵ-neighborhood of the stabilizing surface $s(\mathbf{x})$, the previous discontinuous control law can be replaced by the following continuous controller [5]:

$$w = -m^{-1}[r(\mathbf{x})]^{-1} \left\{\omega(\mathbf{x}) + \bar{\kappa}(\mathbf{x}) \text{sat} \left[\frac{s(\mathbf{x})}{\epsilon}\right]\right\} \tag{10}$$

where $\bar{\kappa}(\mathbf{x}) = \kappa(\mathbf{x}) + \Delta$.

In this work the above saturation function is replaced by the ψ_ε function; this choice is done in order to deal with a $\mathcal{C}^\infty$ class function that coincides the discontinuous $sign$ function outside the interval $(-\varepsilon, \varepsilon)$.

Finally, resorting to the actuator output map (3), the expression (10) gives:

$$h_A(\mathbf{x}_A) + m^{-1}[r(\mathbf{x})]^{-1} \{\omega(\mathbf{x}) + \bar{\kappa}(\mathbf{x}) \ \ \psi_\varepsilon(s(\mathbf{x}))\} = 0. \tag{11}$$

Comparing (11) with (7), it is immediate to verify that imposing the achievement of a sliding motion on (7) means constraining the trajectories of the cascaded 'actuator-plant' system on the surface $\sigma(\mathbf{x}_A, \mathbf{x}) = 0$. The latter condition ensures that the variable $w(t)$ satisfies (10) and hence this implies confining the original plant trajectories within an ϵ-neighborhood of the surface $s(\mathbf{x}) = 0$. This, in turn, implies plant stabilization.

The next step is to design a control law v allowing the achievement of a sliding mode on (7) with finite time reaching, in order to ensure that the system input u is perfectly the desired one from a (known) time on. To this purpose, the computation of $\dfrac{d\sigma(\mathbf{x_A}, \mathbf{x})}{dt}$ gives:

$$\begin{aligned} \frac{d\sigma(\mathbf{x_A}, \mathbf{x})}{dt} \ &= \ \nabla \ h_A(\mathbf{x_A})\dot{\mathbf{x}}_\mathbf{A} + m^{-1} \frac{d}{dt} \left\{[r(\mathbf{x})]^{-1} \omega(\mathbf{x})\right\} \\ &+ \ m^{-1} \frac{d}{dt} \left\{[r(\mathbf{x})]^{-1} \bar{\kappa}(\mathbf{x}) \ \ \psi_\varepsilon(s(\mathbf{x}))\right\} \end{aligned}$$

Performing successive derivations one has:

$$\frac{d\sigma(\mathbf{x_A},\mathbf{x})}{dt} = \nabla\ h_A(\mathbf{x}_A)\dot{\mathbf{x}}_\mathbf{A} + m^{-1}\Big\{-[r(\mathbf{x})]^{-2}\omega(\mathbf{x})\frac{\partial r(\mathbf{x})}{\partial \mathbf{x}}$$
$$+[r(\mathbf{x})]^{-1}\frac{\partial \omega(\mathbf{x})}{\partial \mathbf{x}} + \ \psi_\varepsilon(s(\mathbf{x}))\cdot\Big(-[r(\mathbf{x})]^{-2}(\mathbf{x})\rho(\mathbf{x})\frac{\partial r(\mathbf{x})}{\partial \mathbf{x}}$$
$$+[r(\mathbf{x})]^{-1}\frac{\partial \rho(\mathbf{x})}{\partial \mathbf{x}}\Big)+\epsilon^{-1}[r(\mathbf{x})]^{-1}\bar{\kappa}(\mathbf{x})\ \ \phi_\varepsilon(s(\mathbf{x}))\,\nabla s\ \Big\}\dot{\mathbf{x}}$$

and, defining:

$$\varphi(\mathbf{x})\overset{\mathrm{def}}{=}m^{-1}\Big\{-[r(\mathbf{x})]^{-2}\omega(\mathbf{x})\frac{\partial r(\mathbf{x})}{\partial \mathbf{x}} + [r(\mathbf{x})]^{-1}\frac{\partial \omega(\mathbf{x})}{\partial \mathbf{x}}$$
$$+\ \ \psi_\varepsilon(s(\mathbf{x}))\left(-[r(\mathbf{x})]^{-2}(\mathbf{x})\rho(\mathbf{x})\frac{\partial r(\mathbf{x})}{\partial \mathbf{x}} + [r(\mathbf{x})]^{-1}\frac{\partial \rho(\mathbf{x})}{\partial \mathbf{x}}\right)$$
$$+\ \epsilon^{-1}[r(\mathbf{x})]^{-1}\bar{\kappa}(\mathbf{x})\ \ \phi_\varepsilon(s(\mathbf{x}))\,\nabla s\ \Big\}$$

one finally has:

$$\frac{d\sigma(\mathbf{x_A},\mathbf{x})}{dt} \quad = \quad \nabla\ h_A(\mathbf{x}_A)\dot{\mathbf{x}}_\mathbf{A} + \varphi(\mathbf{x})\dot{\mathbf{x}}$$

By resorting to equations (1), (2), the latter expression becomes:

$$\begin{aligned}\frac{d\sigma(\mathbf{x_A},\mathbf{x})}{dt} &= \nabla\ h_A(\mathbf{x_A})[\mathbf{f_A}(\mathbf{x_A}) + \mathbf{\Delta f_A}(\mathbf{x_A})\\ &+ \mathbf{g_A}(\mathbf{x_A})v + \mathbf{d_A}(\mathbf{x}_A)] + \varphi(\mathbf{x})\cdot\\ &\quad \big[\mathbf{f}(\mathbf{x}) + \mathbf{\Delta f}(\mathbf{x}) + \mathbf{g}(\mathbf{x})u + \mathbf{d}(\mathbf{x})\big]\end{aligned}$$

and, using (3) and (4), yields:

$$\begin{aligned}\frac{d\sigma(\mathbf{x_A},\mathbf{x})}{dt} &= \omega_A(\mathbf{x}_A) + r_A(\mathbf{x_A})v + \delta_A(\mathbf{x}_A)\\ &+ \varphi(\mathbf{x})\big\{\mathbf{f}(\mathbf{x}) + \mathbf{\Delta f}(\mathbf{x}) + \mathbf{g}(\mathbf{x})\cdot\\ &\quad \mathrm{bk}[\mathbf{h_A}(\mathbf{x_A})] + \mathbf{d}(\mathbf{x})\big\}.\end{aligned}$$

The final step of the design process solving Problem 2.1 is to determine the input v imposing the condition

$$\dot{\sigma}(\mathbf{x_A},\mathbf{x})\sigma(\mathbf{x_A},\mathbf{x}) < -\mu|\sigma(\mathbf{x_A},\mathbf{x})|. \qquad (12)$$

Assume $\sigma(\mathbf{x_A},\mathbf{x}) > 0$. In this case (12) corresponds to $\dot{\sigma} < -\mu$. It can be easily verified that, if one sets

$$\begin{aligned} v_e \quad &= \quad -[r_A(\mathbf{x_A})]^{-1}\left[\omega_A(\mathbf{x_A}) + \varphi(\mathbf{x})\left(\mathbf{f}(\mathbf{x}) + \right.\right. \qquad (13)\\ &+ \quad \left.\left.\mathbf{g}(\mathbf{x})mh_A(\mathbf{x}_A)\right)\right] \qquad (14)\end{aligned}$$

and

$$v_n = -\left(\mu + \alpha(\mathbf{x}) + |\varphi(\mathbf{x})\mathbf{g}(\mathbf{x})|\Delta + \rho_A(\mathbf{x}_A)\right) \qquad (15)$$

with

$$\alpha(\mathbf{x}) = \sum_{n=1}^{n}|\varphi_i(\mathbf{x})|\left((\rho_f(\mathbf{x}))_i + (\rho_d(\mathbf{x}))_i\right)$$

then (12) is fulfilled in the worst case.

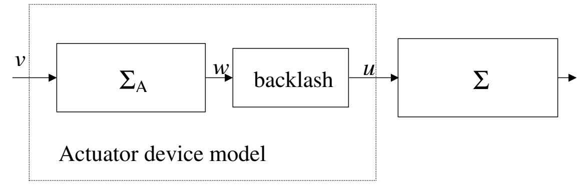

Fig.1 - Block scheme of the sandwich system.

Analogously, if $\sigma(\mathbf{x_A},\mathbf{x}) < 0$, the sliding mode condition becomes $\dot{\sigma} > \mu$. Again, (8),(13),(15) satisfy (12) in the worst case.

■

Remark 4.1: Theorem 4.1 provides a feedback controller ensuring the convergence to an ϵ-neighborhood of the stabilizing sliding surface $s(\mathbf{x})$. Though this control law cannot guarantee the asymptotical stabilization of system (1) in the presence of the actuating device described by the uncertain dynamics (2), (3), (4), it ensures the boundedness of the plant state variables within arbitrary bounds.

V. Simulation Results

In order to validate previous theoretical results, the proposed control approach has been applied by simulation, in this preliminary version of the paper, on a simple linear system described by the following model:

$$\left\{\begin{array}{lcl}\dot{x}_1 &=& x_2\\ \dot{x}_2 &=& c_1x_1 + c_2x_2 + u\end{array}\right. \qquad (16)$$

where the coefficients c_1 and c_2 belong to the interval $[-1.2, -0.8]$. Note that simulations on more complex and meaningful systems are being currently carried out. The considered plant is preceded by an actuator device whose behavior is described by:

$$\left\{\begin{array}{lcl}\dot{x}_\mathbf{A} &=& c_3x_\mathbf{A} + v\\ w &=& x_\mathbf{A}\end{array}\right.$$

with the coefficients c_3 belonging to the interval $[-10, -5]$, and where the variable v is the actual control variable. A backlash nonlinearity is considered present between the system input $u(t)$ and the actuator output. The backlash characteristics $u(t) = \mathrm{bk}[w]$ have unitary slope:

$$\dot{u} = \left\{\begin{array}{ll}\dot{w} & \text{if } \dot{w} > 0 \text{ and } u = (w - c_r) \text{ or}\\ & \text{if } \dot{w} < 0 \text{ and } u = (w + c_l)\\ 0 & \text{otherwise}\end{array}\right. \qquad (17)$$

Coefficients c_r and c_l are uncertain, meaning that the width of the backlash gap is uncertain, too. It is assumed that such coefficients have the following bounds:

$$|c_r| \le \rho_{cr} = 1.5 \quad |c_l| \le \rho_{cl} = 1.5$$

so that the backlash width is bounded by:

$$2\Delta = 2\max\left(\rho_{cr}, \rho_{cl}\right) = 3.$$

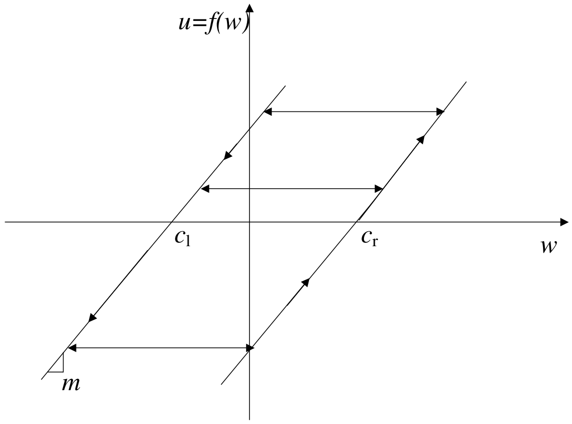

Fig.2 - Characteristics of the backlash nonlinearity.

The following standard sliding surface (6) for the system (1) as been chosen, satisfying the conditions of Assumption (2.2):

$$s(\mathbf{x}) = x_1 + x_2 = 0.$$

The parameter η has been set equal to 1 and the the sliding surface (7) considered for controlling the system (1), (2) and (4) is:

$$\sigma(\mathbf{x_A}, \mathbf{x}) = \\ = x_A + c_1 x_1 + (c_2+1)x_2 + [3.5 + \rho(\mathbf{x})]\psi_\varepsilon(x_1+x_2)$$

The obtained simulation results are shown in Fig.3, whose panels $a), b)$ show the time evolution of the two state variables.

In the simulation initial conditions have been set $\mathbf{x}_0 = [10 \;\; 10 \;\; 10]^T$, but further simulations (not reported for sake of brevity) have shown no particular dependence of controller performances on the initial conditions, as theoretically expected. With this initial conditions the sliding surface is reached within a time less than $T_0 = 20$[sec].

It can be immediately verified that control goals stated in Problem 2.1 have been achieved. Panel 3 $c)$ shows the time evolution of the sliding surface, which behaves as theoretically expected. System trajectories are in fact driven onto the stabilizing sliding surface $s(\mathbf{x})$ within a finite time. Finally, panel 3 $d)$ shows the actuator output before the backlash block, panel 3 $e)$ the time evolution of the system input (i.e. the signal w after the backlash block) and panel 3 $f)$ displays the points (marked) of the backlash characteristics used by the controller.

It is meaningful to see that, though the system input $u(t)$ vanishes as time increases, actuator output $w(t)$ does not. This is due to the presence of the steady state value $w(t) = 1.25$ (of which the controller is of course unaware) needed for achieving the desired zero value of the control input. In figure 3 $f)$ this corresponds to the left hand side of the backlash characteristics.

VI. Conclusion

In this work the problem of stabilizing a system with sandwiched backlash has been addressed. In particular a backlash of uncertain width between two nonlinear uncertain SISO systems has been considered.

With the aim of ensuring good closed loop performances, also avoiding sudden crossings of the backlash gap (i.e. avoiding compensation techniques based on the search of inverses), this note has proposed a controller ensuring the convergence of the system trajectories in a neighborhood of the origin of arbitrary width, independent of the backlash size, for a sandwich system with backlash.

Simulation results show effectiveness of the proposed controller.

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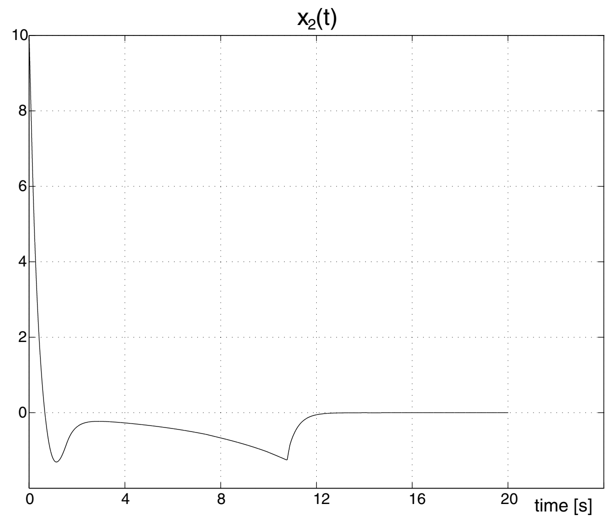

Fig.3a) - Simulation results: system state variable $x_2(t)$.

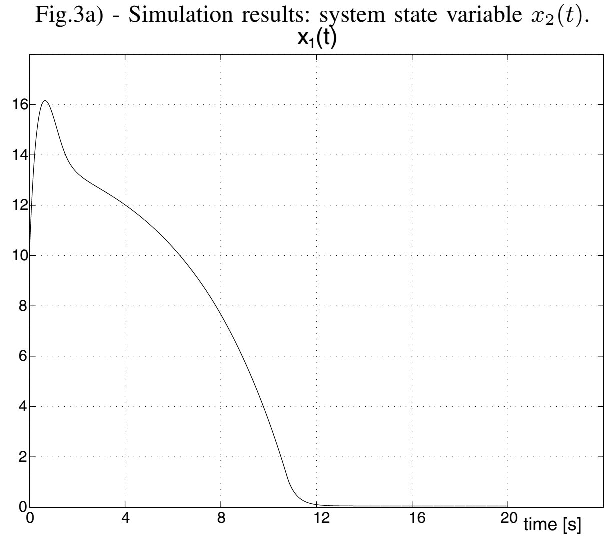

Fig.3b) - Simulation results: system state variable $x_1(t)$.

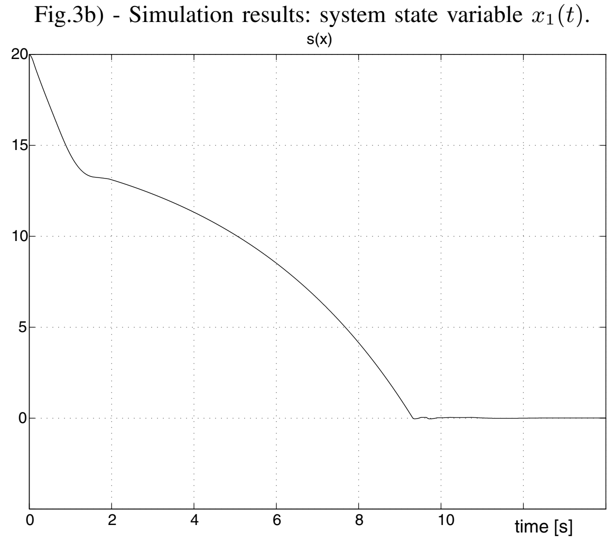

Fig.3 c) - Simulation results: time evolution of $s(\mathbf{x}(t))$

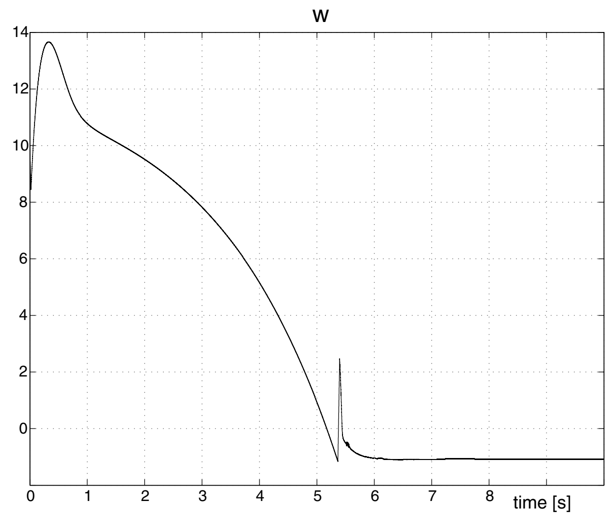

Fig.3 d) - Simulation results: actuator output variable

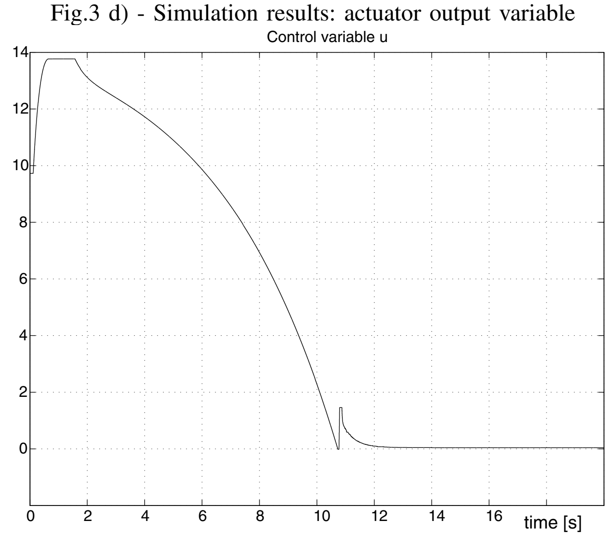

Fig.3 e) - Simulation results: system input

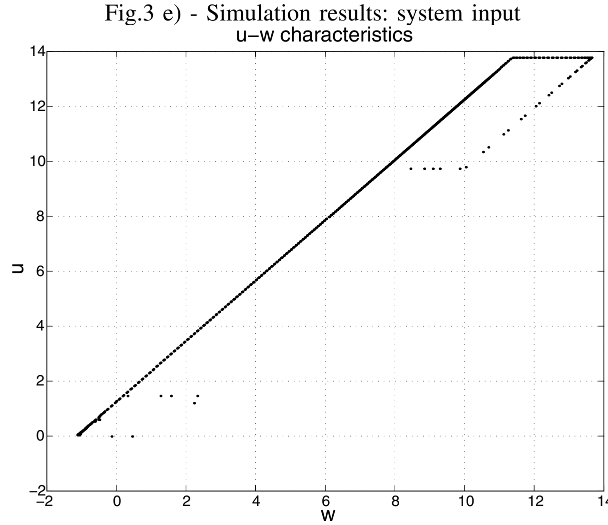

Fig.3 f) - Simulation results: backlash characteristic between system input and actuator output