**Proceedings of the**
**44th IEEE Conference on Decision and Control, and**
**the European Control Conference 2005**
**Seville, Spain, December 12-15, 2005**



# Coordinated Decentralized MRAC of Delayed Plants with Actuator Failures

Boris M. Mirkin and Per-Olof Gutman

*Abstract*— **This paper develops adaptive state feedback coordinated decentralized control scheme for a class of dynamic systems with state delay in subsystems and in the interconnections and in the presence of unknown actuator failures in each subsystem. The main contributions of this paper are the development of a new controller parametrization which attempt to anticipate the future states and failures, the introduction of an appropriate Lyapunov-Krasovskii type functional to design the adaptation algorithms, and a stability proof.**

## I. INTRODUCTION

Control design for systems with actuator failures is an area of research that has been studied a lot in recent years. Many important results have been obtained. The designs have been based mainly on the following approaches: actuator fault detection and isolation methods [1], [2], [3], the robust fault accommodation approach [4], multiple model switching and tuning methods [5], [6], and the sliding mode method [7]. One of the main research directions is adaptive control, see e.g. [8], [9], [10], [11], [12], and references therein.

In in the recent series of papers, see, e.g. [10] and the books [11], [12] the model reference adaptive control (MRAC) technique was successful applied for the numerous problems with actuator failures in the centralized framework and in the delay-free case.

Yet, relatively few results using adaptive control for the important class of delayed systems with actuator failures are available in the literature. Time-delay is a natural component of dynamic processes in many engineering fields and its presence in the plants considerably complicates the design problem, see e.g. the recent papers [13] and [14] for centralized control cases. In [13] a fault detection and accommodation procedure is considered for stable nonlinear state delay plants with, based on an the iterative design of an observer which monitors the variations of the system dynamics. Within the framework of Linear Matrix Inequalities techniques, a robust state feedback linear controller $u = Kx(t)$ is designed for the stabilization of the linear plant with input delay, and actuator failures of stuck-type [14]. In [15] we proposed two adaptive state feedback control schemes for a class of linear systems with state delay in the presence of unknown actuator failures for the centralized control problem. To the best of the authors' knowledge, the decentralized MRAC design problem for plants with actuator failures and time delay has not been solved yet.

In this paper, using a Lyapunov approach and our adaptive decentralized control scheme with model coordination, see, e.g. [16], [17], we present a decentralized model reference adaptive controller (DMRAC) for a class of uncertain dynamic systems with state delays in subsystems and in the interconnections and in the presence of unknown actuator failures in each subsystem.

A special Lyapunov-Krasovskii functional is used to design the update mechanism for the controller parameters. For the updating of the controller parameters, we use a proportional, integral, time delayed (PITD) adaptation mechanism which possesses a better adaptation performance than the traditional I and PI schemes [15].

The main contributions of the paper are:

1) the enlargement of the class of systems with actuator failures that can be handled using model reference adaptive control;
2) a direct coordinated decentralized adaptive control law parametrization which attempt to anticipate the future states and actuator failures in subsystems;
3) the introduction of an appropriate Lyapunov-Krasovskii type functional to design the adaptation algorithms and to prove stability.

This paper is organized as follows: In section 2 the problem statement and preliminaries are presented. In section 3 the decentralized controller parametrization with reference model coordination is presented. The section includes the adaptive control scheme and the proof of stability. The simulation results are presented in Section 4.

## II. PLANT MODEL AND PROBLEM FORMULATION

We consider a class of uncertain systems, which are composed of $M$ multi-input multi-output subsystems with state delays in subsystems and in interconnections whose control components may fail at the time of operation described by

This work was supported by the Israel Science Foundation under Grant 38/03. The research of the first author was supported by the Center for Absorption in Science, Ministry of Immigrant Absorption, State of Israel.

B. Mirkin and Per-Olof Gutman are with Faculty of Civil and Environmental Engineering, The Division of Environmental, Water, and Agricultural Engineering Technion – Israel Institute of Technology, Haifa 32000, Israel (bmirkin),(peo)@tx.technion.ac.il

equations, suitably initialized, of the form

$$\begin{aligned}\dot{x}_i(t) =& A_i x_i(t) + A_{\tau i} x(t-\tau_i) + B_i u_{pi}(t) \\ &+ \sum_{l=1,\, l\neq i}^{M} B_i A_{il} x_l(t-\tau_{il}) \\ =& A_i x_i(t) + A_{\tau i} x(t-\tau_i) + \sum_{q=1}^{m_i} b_{iq} u_{piq}(t) \\ &+ \sum_{l=1,\, l\neq i}^{M} B_i A_{il} x_l(t-\tau_{il}) \\ u_{pi}(t) =& \Upsilon_i u_i(t) + (I-\Upsilon_i)\bar{u}_i,\ i=1,\ldots,M \end{aligned} \tag{1}$$

where, for the $i$-th subsystem $x_i \in \mathbb{R}^{n_i}$ is the state vector, $u_{pi}(t) = [u_{pi1}(t),\ldots,u_{piq}(t),\ldots,u_{pim_i}(t)]^T$ is the applied to the plant real control input vector, $b_{iq}$, $(q=1,2,\ldots,m_i)$ is the $q$th column of $B_i$, $\bar{u}_i = [\bar{u}_{i1},\ldots,\bar{u}_{iq}\ldots,\bar{u}_{im_i}]^T$ is some constant vector and $u_i(t) = [u_{i1}(t),\ldots,u_{iq}(t),\ldots,u_{im_i}(t)]^T$ is the control vector to be designed. The constant matrices $A_i, A_{\tau i} \in \mathbb{R}^{n_i\times n_i}$, $B_i \in \mathbb{R}^{n_i\times m_i}$, $A_{il} \in \mathbb{R}^{m_i\times n_l}$ have unknown elements. $\tau_{il} \in \mathbb{R}^{+}$, $i,l=1,\ldots,M$ are known time delays and $\sum_{i=1}^{M} n_i = n$.

The indicator matrix $\Upsilon_i = \mathrm{diag}[\upsilon_{i1},\ldots,\upsilon_{iq},\ldots,\upsilon_{im_i}] \in \mathbb{R}^{m_i\times m_i}$ describes the working condition of the actuators. $\upsilon_{iq}=1$ denotes that the $q$th actuator of the $i$th system is in normal mode, and the actually applied $q$th control input to the subsystem is $u_{iq}$. $\upsilon_{iq}=0$ denotes a failed actuator, e.g. stuck at certain position, and the actually applied $q$th control input to the subsystem equals $\bar{u}_{iq}$. If all actuators of the subsystem are in normal mode, the matrix $\Upsilon_i$ is an identity matrix $\Upsilon_i = I$. The constant value $\bar{u}_{iq}$ and the failure time instant $t_q$ are unknown, i.e. the type of actuator failures considered here is the same as in [9]:

$$u_{piq}(t) = \bar{u}_{iq}, \quad t \geq t_q, \quad q=1,2,\ldots,m_i \tag{2}$$

Note that it is postulated that a failed actuator never returns to normal operation. For the decentralized control problem considered in this paper, the first assumption is that *(A1)* the system (1) without actuator failures is decentralized state feedback stabilizable.

The problem is to design an adaptive feedback control, and tune, on-line, the controller parameters in order to achieve desired closed loop specifications when there are up to $m-1$ unknown actuator failures. The desired specification in this paper is that with a failure model (2), all signals of the closed loop system remain bounded, and that the each subsystems state $x_i(t)$ asymptotically exact follows the state $x_{ri}(t)$ of a stable reference model without delays

$$\dot{x}_{ri}(t) = A_{ri} x_{ri}(t) + b_{ri} r_i(t) \tag{3}$$

where $A_{ri} \in \mathbb{R}^{n_i\times n_i}$, $b_{ri} \in \mathbb{R}^{n_i}$ are known constant matrices, and $r_i(t) \in \mathbb{R}$ is a bounded reference input signal. I.e. we demand that $\lim_{t\to\infty}\|e_i(t)\| = \|x_i(t) - x_{ri}(t)\| = 0$, $i=1,\ldots,M$, i.e. also in the presence of up to $m-1$ actuator failures.

As in [10] for the centralized case, we assume that *(A2)* if the plant parameters and the actuator failures (up to $m-1$ failures) are known, the remaining subsystem actuators can still achieve the desired control objective.

## III. Proposed Error Equation Parametrization

Motivated by our previous works, see, e.g. [18], [16], [17], we will use the decentralized adaptive control scheme with reference model coordination to achieve the control objective. The control law for the $i$th local subsystem $u_{pi}(t)$ is chosen to be of the form

$$u_{pi}(t) = u_{pfi}(t) + u_{pci}(t) \tag{4}$$

where the part of the control law $u_{pfi}(t)$ is based only on the local signals of the $i$th subsystem, and the component $u_{cpi}(t)$ is the coordinated component which is based on the reference signals of the all other subsystems. Exchange of the reference signals between subsystems can be easily implemented in real-life control systems. Let us assume that all the parameters of (1) and the actuator failures are known, and let us define $u_{pi}(t)$ as

$$\begin{aligned} u^*_{pi}(t) = u^*_i(t) =& \Theta^{*T}_{ei} e_i(t) + \Theta^{*T}_{xi} x_{ri}(t) + \Theta^{*T}_{\tau i} x_{ri}(t-\tau_i) \\ &+ \Theta^*_{ri} r_i(t) + \Theta^*_i + u_{ci}(t) \end{aligned} \tag{5}$$

where the constant matrices $\Theta^*_{ei} = [\theta^*_{ei1}, \theta^*_{ei2},\ldots,\theta^*_{eim_i}] \in \mathbb{R}^{n_i\times m_i}$, $\Theta^*_{xi} = [\theta^*_{x1i}, \theta^*_{x2i},\ldots,\theta^*_{xm_i}] \in \mathbb{R}^{n_i\times m_i}$, $\Theta^*_{\tau i} = [\theta^*_{\tau 1i}, \theta^*_{\tau 2i},\ldots,\theta^*_{\tau m_i}] \in \mathbb{R}^{n_i\times m_i}$, $\Theta^*_{ri} = [\theta_{ri1}, \theta_{ri2},\ldots,\theta_{rim_i}]^T \in \mathbb{R}^{m_i}$ to be defined for perfect model-following, and the vector $\Theta^*_i = [\theta_{i1}, \theta_{i2},\ldots,\theta_{im_i}]^T \in \mathbb{R}^{m_i}$ we introduce, as in [10], [12], for cancel of the failed actuators effect. The coordinated component $u_{ci}$ is used to cancel the effect of the interconnections, as defined next.

The $q$th component of $u_{pi}$, $q=1,2,\ldots,m_i$, can be write

$$\begin{aligned} u^*_{piq}(t) =& u^*_{iq}(t) = u^*_{fiq}(t) + u_{ciq}(t) \\ u^*_{fiq}(t) =& \theta^{*T}_{eiq} e_i(t) + \theta^*_{xiq} x_{ri}(t) + \theta^{*T}_{\tau i} x_{riq}(t-\tau_i) + \theta^*_{riq} r_i(t) + \theta^*_{iq} \end{aligned} \tag{6}$$

Suppose there are $g$ failed actuators, that is

$$u_{pij}(t) = \bar{u}_{ij},\ \text{if } j = j_1, j_2, \ldots, j_g,\ 1 \leq g \leq m_i - 1$$

than with definition of $u^*_{fiq}(t)$ in (6) there exist [10], [15] constant parameters $\theta^*_{oxiq}$, $\theta^*_{o\tau iq}$ $\theta^*_{oriq}$ and $\theta^*_{oiq}$ such that $A_i - A_{ri} + b_{iq}\theta^{*T}_{oxiq} = 0$, $A_{\tau i} + b_{iq}\theta^{*T}_{o\tau iq} = 0$, $b_{iq}\theta^*_{oriq} - b_{ri} = 0$ and $b_{iq}\theta^*_{oiq} + \sum_{i=j_1,\ldots,j_g}^{m} b_{iq}\bar{u}_{iq} = 0$ and to hold

$$\begin{aligned} A_i - A_{ri} + \sum_{q\neq j_1,\ldots,j_g}^{m_i} b_{iq}\theta^{*T}_{xiq} =& 0 \\ A_{\tau i} + \sum_{q\neq j_1,\ldots,j_g}^{m} b_i \theta^{*T}_{\tau iq} =& 0 \\ \sum_{q\neq j_1,\ldots,j_g}^{m_i} b_{iq}\theta^*_{riq} - b_{ri} =& 0 \\ \sum_{q\neq j_1,\ldots,j_g}^{m_i} b_{iq}\theta^*_{iq} + \sum_{q=j_1,\ldots,j_g}^{m_i} b_{iq}\bar{u}_{iq} =& 0 \end{aligned} \tag{7}$$

Introducing the parameter errors $\tilde{\theta}_{eiq}(t)$, $\tilde{\theta}_{riq}(t)$, $\tilde{\theta}_{xiq}(t)$, $\tilde{\theta}_{\tau iq}(t)$ and $\tilde{\theta}_{\tau iq}(t)$

$$\begin{aligned} \tilde{\theta}_{eiq}(t) =& \theta_{eiq}(t) - \theta^*_{eiq} \qquad & \tilde{\theta}_{riq}(t) = \theta_{riq}(t) - \theta^*_{riq} \\ \tilde{\theta}_{iq}(t) =& \theta_{iq}(t) - \theta^*_{iq} \qquad & \tilde{\theta}_{xiq}(t) = \theta_{xiq}(t) - \theta^*_{xiq} \\ \tilde{\theta}_{\tau iq}(t) =& \theta_{\tau iq}(t) - \theta^*_{\tau iq} & \end{aligned} \tag{8}$$

where $\theta_{eiq}(t)$, $\theta_{\tau iq}(t)$, $\theta_{xiq}(t)$, $\theta_{riq}(t)$ and $\theta_{\tau iq}(t)$ are the adaptation gain matrices and applying (6) to the actual plant (1), then using (7) and (3) we obtain

$$\begin{aligned}\dot{e}_i(t) =& A_{ri}e_{ri}(t) + \sum_{l=1}^{M}\sum_{q=1}^{m_i} b_{iq}a_{ilq}^{*T}x_l(t-\tau_{il}) \\ & - \sum_{q\neq j_1,\dots,j_g}^{m_i} b_{iq}\theta_{\tau iq}^{*T}e_i(t-\tau_i) + \sum_{q\neq j_1,\dots,j_g}^{m_i} b_{iq}u_{ciq}(t) \\ & + \sum_{q\neq j_1,\dots,j_g}^{m_i} b_{iq}\big[\tilde{\theta}_{eiq}^T e_i(t) + \tilde{\theta}_{xiq}^T x_{ri}(t) + \tilde{\theta}_{\tau iq}^T x_{ri}(t-\tau_i) \\ & + \tilde{\theta}_{riq}r_i(t) + \tilde{\theta}_{iq}(t)\big] \end{aligned} \tag{9}$$

Let us define the coordinated component of control $u_{ciq}(t)$ as

$$u_{ciq}(t) = \sum_{l=1,\, l\neq i}^{M} \theta_{cilq}^T(t)x_{rl}(t-\tau_{il}) \tag{10}$$

Then with (10), using the value $b_{iq} = b_{ri}\theta_{oriq}^{*-1}$ from (7) and the error $e_l(t-\tau_{il}) = x_l(t-\tau_{il}) - x_{rl}(t-\tau_{il})$ we obtain

$$\begin{aligned}\dot{e}_i(t) =& A_{ri}e_i(t) + \sum_{l=1}^{M}\sum_{q=1}^{m_i} b_{ri}\theta_{oriq}^{*-1}a_{ilq}^{**T}e_l(t-\tau_{il}) \\ & + \sum_{q\neq j_1,\dots,j_g}^{m_i} b_{ri}\theta_{oriq}^{*-1}\big[\tilde{\theta}_{eiq}^T e_i(t) + \tilde{\theta}_{xriq}^T x_{ri}(t) \\ & + \tilde{\theta}_{riq}r_i(t) + \tilde{\theta}_{iq}(t)\big] \\ & + \sum_{l=1}^{M}\sum_{q\neq j_1,\dots,j_g}^{m_i} b_{ri}\theta_{oriq}^{*-1}\theta_{cilq}^T(t)x_{rl}(t-\tau_{il}) \\ & + \sum_{l=1}^{M}\sum_{q=1}^{m_i} b_{ri}\theta_{oriq}^{*-1}a_{ilq}^{*T}x_{rl}(t-\tau_{il}) \end{aligned} \tag{11}$$

where $\tau_{ii} = \tau_i$ and

$$a_{ilq}^{**} = \begin{cases} a_{ilq}^*, & l\neq i; \\ \bar{a}_{ilq}, & l = i. \end{cases} \qquad \bar{a}_{ilq} = \begin{cases} -\theta_{\tau iq}^*, & q\neq j_1,\dots,j_g; \\ 0, & q = j_1,\dots,j_g. \end{cases}$$

Because we can write

$$\begin{aligned} & \sum_{l=1}^{M}\sum_{q=1}^{m_i} b_{ri}\theta_{oriq}^{*-1}a_{ilq}^{*T}x_{rl}(t-\tau_{il}) \\ =& \sum_{l=1}^{M}\sum_{q\neq j_1,\dots,j_g}^{m_i} b_{ri}\theta_{oriq}^{*-1}\theta_{cilq}^{*T}x_{rl}(t-\tau_{il}) \end{aligned} \tag{12}$$

where $\theta_{ilq}^*$ is a some constant matrix we have

$$\begin{aligned}\dot{e}_i(t) =& A_{ri}e_i(t) + \sum_{l=1}^{M}\sum_{q=1}^{m_i} b_{ri}\theta_{oriq}^{*-1}a_{ilq}^{**T}e_l(t-\tau_{il}) \\ & + \sum_{q\neq j_1,\dots,j_g}^{m_i} b_{ri}\theta_{oriq}^{*-1}\big[\tilde{\theta}_{eiq}^T e_i(t) + \tilde{\theta}_{xriq}^T x_{ri}(t) \\ & + \tilde{\theta}_{riq}r_i(t) + \tilde{\theta}_{iq}(t)\big] \\ & + \sum_{l=1}^{M}\sum_{q\neq j_1,\dots,j_g}^{m_i} b_{ri}\theta_{oriq}^{*-1}\tilde{\theta}_{cilq}^T(t)x_{rl}(t-\tau_{il}) \end{aligned} \tag{13}$$

with $\theta_{cilq}(t) + \tilde{\theta}_{cilq}(t) = \theta_{ilq}^*$.

For the convenience of the analysis we introduce the following auxiliary variables and signals

$$\begin{aligned} \tilde{\theta}_{fiq} =& [\tilde{\theta}_{eiq}^T(t)(t),\ \tilde{\theta}_{xiq}^T(t),\ \tilde{\theta}_{\tau iq}^T(t),\ \tilde{\theta}_{riq}(t),\ \tilde{\theta}_{iq}(t)] \\ \omega_{fi}(t) =& [e_i^T(t),\ x_{ri}^T(t),\ x_{ri}^T(t-\tau_i),\ r_i(t),\ 1] \end{aligned} \tag{14}$$

and write the $q$th component of $u_i$ from (6) as

$$u_{iq}(t) = \theta_{fiq}^T(t)\omega_{fi}(t) + \sum_{l=1,\, l\neq i}^{M}\theta_{ciql}^T(t)x_{rl}(t-\tau_{il}) \tag{15}$$

then we obtain the basic tracking error equation for stability analysis and adaptation algorithms design

$$\begin{aligned}\dot{e}_i(t) =& A_{ri}e_i(t) + \sum_{l=1}^{M}\sum_{q=1}^{m_i} b_{ri}\theta_{oriq}^{*-1}a_{ilq}^{*T}e_l(t-\tau_{il}) \\ & + \sum_{q\neq j_1,\dots,j_g}^{m_i} b_{ri}\theta_{oriq}^{*-1}\tilde{\theta}_{fiq}^T(t)\omega_i(t) \\ & + \sum_{l=1}^{M}\sum_{q\neq j_1,\dots,j_g}^{m_i} b_{ri}\theta_{oriq}^{*-1}\tilde{\theta}_{cilq}^T(t)x_{rl}(t-\tau_{il}) \end{aligned} \tag{16}$$

## IV. ADAPTATION ALGORITHMS AND STABILITY ANALYSIS

To design the update laws for the control parameter matrices $\theta_{fiq}(t)$ and $\theta_{cilq}(t)$ in the adaptive control (15) we use the following Lyapunov-Krasovskii type functional

$$\begin{aligned} V(\star) =& \sum_{i=1}^{M}V_i, \quad V_i = V_{ei} + V_{\eta i} + \sum_{l=1}^{M}\Big(V_{\eta il} + V_{eil}\Big) \\ V_{ei} =& e_i^T(t)P_ie_i(t) \qquad V_{eil} = \tfrac{1}{M}\int_{t-\tau_{il}}^{t} e_l^T(s)Q_ee_l(s)ds \\ V_{\eta i} =& \sum_{q\neq j_1,\dots,j_g}^{m_i}\left|\theta_{oriq}^*\right|^{-1}\Big(\tilde{\eta}_{iq}^T\Gamma_{i1q}^{-1}\tilde{\eta}_{iq} \\ & + \int_{t-h_i}^{t}\eta_{iq}^T(s)\Gamma_{i1q}^{-1}\eta_{iq}(s)ds\Big) \\ V_{\eta il} =& \sum_{l=1}^{M}\sum_{q\neq j_1,\dots,j_g}^{m_i}\left|\theta_{oriq}^*\right|^{-1}\Big(\tilde{\eta}_{ilq}^T\Gamma_{i2q}^{-1}\tilde{\eta}_{ilq} \\ & + \int_{t-h_{il}}^{t}\eta_{ilq}^T(s)\Gamma_{i2q}^{-1}\eta_{ilq}(s)ds\Big) \end{aligned} \tag{17}$$

where

$$\begin{aligned} \tilde{\eta}_{iq} =& \tilde{\theta}_{fiq}(t) + \eta_{iq}^* + \eta_{iq}(t) + \eta_{iq}(t-h_i) \\ \tilde{\eta}_{ilq} =& \tilde{\theta}_{cilq}(t) + \eta_{ilq}^* + \eta_{ilq}(t) + \eta_{ilq}(t-h_{il}) \end{aligned} \tag{18}$$

and $Q_e = Q_e^T > 0$, $\Gamma_{i1q}^T = \Gamma_{i1q} > 0$ and $\Gamma_{i2q}^T = \Gamma_{i2q} > 0$ are matrices of corresponding dimensions. The matrix $P_i = P_i^T > 0$ is from the Lyapunov equation

$$A_{ri}^TP_i + P_iA_{ri} + Q_i + Q_e = 0, \quad Q_i^T = Q_i > 0 \tag{19}$$

and

$$\eta_{iq}^* = \frac{1}{2}r_0\theta_{oriq}^*[b_{ri}^TP_i\ \ 0\ \ 0\ \ 0\ \ 0] \quad r_0 > 0 \tag{20}$$

where $r_0 > 0$ is some scalar constant. This constant $r_0$ and the time-varying vectors $\eta_{iq}(t)$ and $\eta_{ilq}(t)$ are “artificial” whose values will be defined later. These parameters are used only in the process of the stability proof. The parameters $h_i$ and $h_{il}$ are design parameters in the delayed components of adaptation algorithms, as will be seen below.

We now choose the adaptation algorithms as

$$\theta_{fiq}(t) = -\eta_{iq}(0) - \eta_{iq}(t) - \eta_{iq}(t-h_i) - \int_0^t \eta_{iq}(s)ds,$$
$$\eta_{fiq}(t) = \text{sign}[\theta^*_{oriq}]\Gamma_{i1q}\omega_i(t)e_i^T(t)P_ib_{ri} \quad (21)$$
$$\theta_{ilq}(t) = -\eta_{ilq}(0) - \eta_{ilq}(t) - \eta_{ilq}(t-h_{ij}) - \int_0^t \eta_{ilq}(s)ds,$$
$$\eta_{ilq}(t) = \text{sign}[\theta^*_{oriq}]\Gamma_{i2q}x_{rl}(t-\tau_{il})e_i^T(t)P_ib_{ri} \quad (22)$$

or in differential form

$$\dot{\theta}_{fiq}(t) = -\eta_{iq}(t) - \dot{\eta}_{iq}(t) - \dot{\eta}_{iq}(t-h_i),$$
$$\eta_{iq}(t) = \text{sign}[\theta^*_{oriq}]\Gamma_{i1q}\omega_i(t)e_i^T(t)P_ib_{ri} \quad (23)$$
$$\dot{\theta}_{ilq}(t) = -\eta_{ilq}(t) - \dot{\eta}_{iql}(t) - \dot{\eta}_{ilq}(t-h_{il}),$$
$$\eta_{ilq}(t) = \text{sign}[\theta^*_{oriq}]\Gamma_{i2q}x_{rl}(t-\tau_{il})e_i^T(t)P_ib_{ri} \quad (24)$$

where $h_i$ and $h_{il}$ are some design parameters.

*Remark 1:* Although only the integral component $\eta_{iq}(t)$ ($\dot{\eta}_{iq}(t) = 0$ and $\dot{\eta}_{iq}(t-h_i) = 0$ in (23)) of the adaptation algorithm is needed for stability and exact asymptotic tracking, the use of the proportional and the proportional delayed terms in the adaptation algorithm (23) makes it possible to achieve better adaptation performance than the traditional integral (I) and proportional integral (PI) schemes see, e.g. [15] in which a proportional integral time delay (PITD) adaptation algorithm is used for the centralized adaptive control. This adaptation algorithm includes the traditional I and PI schemes [19] as a special case. The design parameters $h_i$ and $h_{ij}$ are chosen in the same way as the traditional gains $\Gamma_{i1q}$ and $\Gamma_{i2q}$ in (21) - (23).

Using (19), the time derivatives of the components of (17) along (16) can be written as

$$\begin{aligned}\dot{V}_{ei}|_{(16)} =& e_i^T(t)[A_{ri}^TP_i + P_iA_{ri}]e_i(t) \\ &+ 2e_i^T(t)P_i\sum_{l=1}^{M}\sum_{q=1}^{m_i} b_{ri}\theta^{*-1}_{oriq}a^{*T}_{ilq}e_l(t-\tau_{il}) \\ &+ 2e_i^T(t)P_i\sum_{q\neq j_1,\ldots,j_g}^{m_i} b_{ri}\theta^{*-1}_{oriq}\tilde{\theta}^T_{fiq}(t)\omega_i(t) \\ &+ 2e_i^T(t)P_i\sum_{q\neq j_1,\ldots,j_g}^{m_i}\sum_{l=1}^{M} b_{ri}\theta^{*-1}_{oriq}\tilde{\theta}^T_{cilq}(t)x_{rl}(t-\tau_{il})\end{aligned} \quad (25)$$

$$\dot{V}_{eil}|_{(16)} = \tfrac{1}{M}\Big(e_l^T(t)Q_ee_l(t) - e_l^T(t-\tau_{il})Q_ee_l(t-\tau_{il})\Big) \quad (26)$$

$$\begin{aligned}\dot{V}_{\eta i}|_{(16)} =& 2\sum_{q\neq j_1,\ldots,j_g}^{m_i}\left|\theta^*_{oriq}\right|^{-1}\tilde{\theta}_{fiq}(t)^T\Gamma^{-1}_{i1q}\dot{\tilde{\eta}}_{iq} \\ &+ 2\sum_{q\neq j_1,\ldots,j_g}^{m_i}\left|\theta^*_{oriq}\right|^{-1}\eta^{*T}_{iq}\Gamma^{-1}_{i1q}\dot{\tilde{\eta}}_{iq} \\ &+ 2\sum_{q\neq j_1,\ldots,j_g}^{m_i}\left|\theta^*_{oriq}\right|^{-1}\Big(\eta_{iq}(t) + \eta_{iq}(t-h_i)\Big)^T\Gamma^{-1}_{i1q}\dot{\tilde{\eta}}_{iq} \\ &+ \eta^T_{iq}(t)\Gamma^{-1}_{i1q}\eta_{iq}(t) - \eta^T_{iq}(t-h_i)\Gamma^{-1}_{i1q}\eta_{iq}(t-h_i)\end{aligned} \quad (27)$$

Combining (23) and (27) it follow that

$$\begin{aligned}\dot{V}_{\eta i}|_{(16)} =& -2\sum_{q\neq j_1,\ldots,j_g}^{m_i}\left|\theta^*_{oriq}\right|^{-1}\eta^{*T}_{iq}\Gamma^{-1}_{i1q}\eta_{iq} \\ &- 2\sum_{q\neq j_1,\ldots,j_g}^{m_i}\left|\theta^*_{oriq}\right|^{-1}\Big(\eta_{iq}(t) + \eta_{iq}(t-h_i)\Big)^T\Gamma^{-1}_{i1q}\eta_{iq} \\ &+ \eta^T_{iq}(t)\Gamma^{-1}_{i1q}\eta_{iq}(t) - \eta^T_{iq}(t-h_i)\Gamma^{-1}_{i1q}\eta_{iq}(t-h_i)\end{aligned} \quad (28)$$

and view of (20) we obtain

$$\begin{aligned}\dot{V}_{\eta i}|_{(16)} =& -r_0\sum_{q\neq j_1,\ldots,j_g}^{m_i} e_i(t)^TP_ib_{ri}b_{ri}^TP_ie_i(t) \\ &- \sum_{q\neq j_1,\ldots,j_g}^{m_i}\left|\theta^*_{oriq}\right|^{-1}\Big(\eta_{iq}(t) + \eta_{iq}(t-h_i)\Big)^T\Gamma^{-1}_{i1q}\times \\ &\Big(\eta_{iq}(t) + \eta_{iq}(t-h_i)\Big)\end{aligned} \quad (29)$$

Similarly we have

$$\begin{aligned}\dot{V}_{\eta il}|_{(16)} =& -\sum_{q\neq j_1,\ldots,j_g}^{m_i}\left|\theta^*_{oriq}\right|^{-1}\Big(\eta_{ilq}(t) + \eta_{ilq}(t-h_i)\Big)^T\Gamma^{-1}_{i2q}\times \\ &\Big(\eta_{ilq}(t) + \eta_{ilq}(t-h_i)\Big)\end{aligned} \quad (30)$$

Then using (19), (29) and (30) we get

$$\begin{aligned}\dot{V}_i|_{(16)} \leq& -\tfrac{1}{M}\sum_{l=1}^{M} e_l^T(t-\tau_{il})Q_ee_l(t-\tau_{il}) \\ &+ \sum_{l=1}^{M}\sum_{q=1}^{m_i} 2e_i^T(t)P_ib_{ri}\theta^{*-1}_{oriq}a^{*T}_{ilq}e_l(t-\tau_{il}) \\ &- r_0\sum_{q\neq j_1,\ldots,j_g}^{m_i} e_i(t)^TP_ib_{ri}b_{ri}^TP_ie_i(t) \\ \leq& \sum_{l=1}^{M}\sum_{q=1}^{m_i} 2e_i^T(t)P_ib_{ri}\theta^{*-1}_{oriq}a^{*T}_{ilq}e_l(t-\tau_{il}) \\ &- \tfrac{1}{M}\sum_{l=1}^{M} e_l^T(t-\tau_{il})Q_ee_l(t-\tau_{il}) \\ &- r_0\tfrac{\beta}{M}\sum_{l=1}^{M}\sum_{q=1}^{m_i} e_i(t)^TP_ib_{ri}b_{ri}^TP_ie_i(t), \quad \beta > 0\end{aligned} \quad (31)$$

After completing the squares we have

$$\dot{V}_i|_{(16)} \leq - e_i^T(t)Q_i e_i(t) - \frac{1}{M}\sum_{l=1}^{M} e_l^T(t-\tau_{il})Q_e e_l(t-\tau_{il}) + \frac{1}{r_0}\sum_{l=1}^{M} e_l(t-\tau_{il})^T \Psi_{ilg} e_l(t-\tau_{il}) \quad (32)$$

where $\Psi_{ilg} = \sum_{q=1}^{m_i} \frac{M}{\beta\theta^{*2}_{oriq}} a^*_{ilq} a^{*T}_{ilq}$

Let us $Q_e = Q_{e1} + Q_{e2}$ with $Q_{e1} = Q_{e1}^T$ and $Q_{e2} = Q_{e2}^T$ are some matrices.

If we select the value $r_0$ from the inequality $r_0 > \frac{\lambda_{max}(\Psi_{ilg})}{\lambda_{min}(Q_{e1})}$ from (32) where $\lambda_{max}(.)$ and $\lambda_{min}(.)$ are the maximum and minimum eigenvalues of (.), we obtain from (32)

$$\dot{V}|_{(16)} \leq - \sum_{i=1}^{M}\Big(e_i^T(t)Q_i e_i(t) + \frac{1}{M}\sum_{l=1}^{M}\left[e_l^T(t-\tau_{il})Q_{e2}e_l(t-\tau_{il})\right]\Big) \leq 0 \quad (33)$$

for $t \in (T_i, T_{i+1}$, i.e. $\dot{V}|_{(16)}$ is negative semi-definite. Thus [20], we have proved that the adaptive control (10) and the update algorithms (21) and (23) guarantee that $V(t)$ and, therefore, $e_i(t)$, $\tilde{\theta}_{fiq} \in \mathsf{L}_\infty$. The remainder of the stability analysis follows directly using the steps in [21].

We have thus obtained the following result

*Theorem 1:* Consider system (1) and the reference model (3). Then the adaptive control (10) with update laws (21), (23) and actuator failures model (2) assures that the closed loop signals are bounded and that the tracking error $e(t)$ converges to zero asymptotically.

*Remark 2:* We note that the coefficient matrices $Q_i$, $Q_e$ and the scalar $r_0$ are used only for analysis and do not influence the control law. Decentralized controller gains adjust automatically to counter the non-desirable effects of delayed interconnections, actuator failures and parameter uncertainties.

## V. Simulation

To illustrate the application of the proposed adaptive scheme, let us consider a plant with two subsystems described as

$$\begin{aligned}
\dot{x}_1(t) &= \begin{bmatrix} 0 & 1 \\ 1 & 2 \end{bmatrix} x_1(t) + \begin{bmatrix} 0 & 0 \\ 1 & 2 \end{bmatrix} u_1(t) \\
&\quad + \begin{bmatrix} 0 & 0 \\ 1 & 2 \end{bmatrix} x_1(t-\tau_1) + \begin{bmatrix} 0 & 0 \\ 2 & 3 \end{bmatrix} x_2(t-\tau_{12}) \\
\dot{x}_2(t) &= \begin{bmatrix} 0 & 1 \\ -1 & 2 \end{bmatrix} x_2(t) + \begin{bmatrix} 0 & 0 \\ 1 & 3 \end{bmatrix} u_2(t) \\
&\quad + \begin{bmatrix} 0 & 0 \\ 2 & 4 \end{bmatrix} x_2(t-\tau_2) + \begin{bmatrix} 0 & 0 \\ 3 & 2 \end{bmatrix} x_1(t-\tau_{21}) \\
x_1(0) &= x_2(0) = [0 \quad 0]^T. \qquad (34)
\end{aligned}$$

We choose the reference model as

$$\begin{aligned}
\dot{x}_{r1}(t) &= \begin{bmatrix} 0 & 1 \\ -1 & -3 \end{bmatrix} x_{r1}(t) + \begin{bmatrix} 0 \\ 1 \end{bmatrix} r_1(t) \\
\dot{x}_{r2}(t) &= \begin{bmatrix} 0 & 0 \\ -1 & -1 \end{bmatrix} x_{r2}(t) + \begin{bmatrix} 0 \\ 1 \end{bmatrix} r_2(t) \\
x_{r1}(0) &= x_{r2}(0) = \begin{bmatrix} 0.5 \\ -0.5 \end{bmatrix}. \qquad (35)
\end{aligned}$$

All parameters except the time delays ($\tau_1 = 5$, $\tau_{12} = 5$, $\tau_2 = 4$, $\tau_{21} = 4$) are unknown to the controller.

The adaptation algorithms (21) in our simulation are

$$\theta_{iq}(t) = -PITD\Big(\omega_i(t)e_i^T(t)P_i b_{ri}\Big),\ i,q,l = 1,2;\ l \neq i \quad (36)$$

where $PITD(\star)$ is the operator form for $PITD\,(Z_i(t)) = k_I\int_0^t Z_i(s)ds + k_P Z_i(t) + k_D Z_i(t-h_i)$, where the parameter values were chosen as $h_1 = h_2 = 1$, $k_I = 1$, $k_P = 0.5$, $k_D = 0.05$, with $\omega_i = [e_i(t)\, x_{ri}(t)\, x_{ri}(t-\tau_i)\, r_i(t)\; 1\; x_{ril}(t-\tau_{il})]^T$, and $P_i$ from (19)

$$P_1 = \begin{bmatrix} 9.1667 & 2.5000 \\ 2.5000 & 1.6667 \end{bmatrix}, \quad P_2 = \begin{bmatrix} 7.5000 & 2.5000 \\ 2.5000 & 5.0000 \end{bmatrix}.$$

The input signals of the reference models $r_1$ and $r_2$ are $r_1 = r_2 = 2sin(t)$.

In the simulation study, we suppose that the second control input of the subsystem 1 fails at $t = 30$ seconds and the second control input of the subsystem 2 fails at $t = 50$ seconds.

Simulation results are found in Figures 1–3.

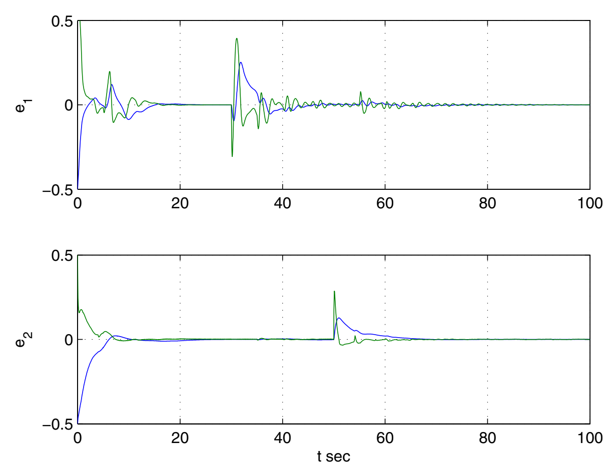


Fig. 1. The graphs show the time history of the tracking errors.

All simulation results verified the desired system performance. At the time instant when one of the actuators fails, there is a transient system response in the tracking error, and as time goes on, the tracking errors starting from a transient value, converge to zero, i.e. asymptotic tracking is achieved despite the unknown actuators failures and unknown system parameters. The values of controller parameters $\theta_i$ also jump when one actuator failure occurs, and then converge to constant values.

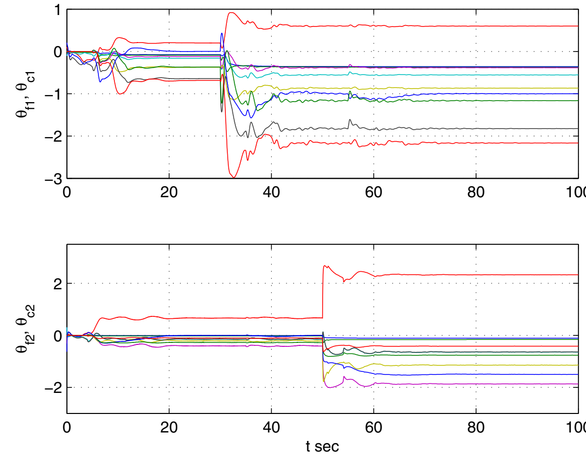


Fig. 2. The graphs show the time history of the controller parameters.

For comparison, the tracking errors for a nominal problem with a decentralized adaptive controller when there is no actuator failure shown in Figure 3.

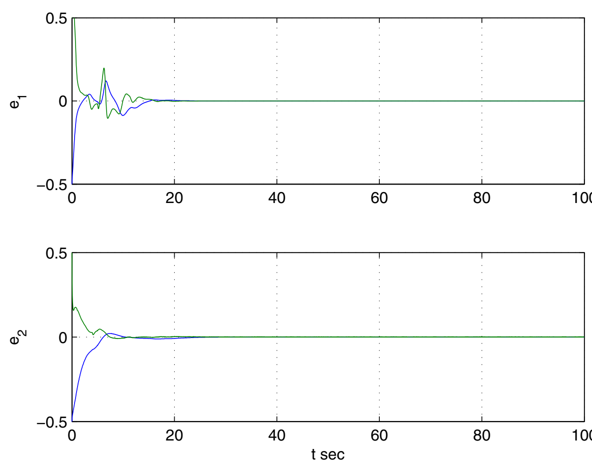


Fig. 3. The graphs show the time history of the tracking errors $e_1$ and $e_2$ for the nominal case.

## VI. Conclusion

We have developed a coordinated adaptive decentralize control scheme for state delayed systems with unknown actuator failures. This scheme ensures asymptotic exact tracking in the presence of unknown plant parameters and unknown actuator failure parameters. Simulation results verified the desired performance of the developed coordinated adaptive decentralized controller. For the updating of the decentralized controller parameters, we develop a proportional, integral, time delayed (PITD) adaptation mechanism which possesses a better adaptation performance than the traditional I and PI schemes. A special Lyapunov-Krasovskii functional is introduced to design the update mechanism for the controller parameters and prove stability.

## References


[1] A. T. Vemuri and M. M. Polycarpou, "Robust nonlinear fault diagnosis in input-output systems," *International Journal of Control*, vol. 68, pp. 343–360, 1997.

[2] H. Wang, Z. J. Huang, and S. Daley, "On the use of adaptive updating rules for actuator and sensor fault diagnosis," *Automatica*, vol. 33, no. 2, pp. 217–225, 1997.

[3] ——, "Detection, estimation, and accommodation of loss of control effectiveness," *Inter. Journal of Adaptive Control and Signal Processing*, vol. 14, pp. 775–795, 2000.

[4] F. Liao, J. L. Wang, and G.-H. Yang, "Reliable robust flight tracking control: an lmi approach," *IEEE Transactions on Control Systems Technology*, vol. 10, no. 1, pp. 76–89, 2002.

[5] J. Bošković and R. Mehra, "Stable multiple model adaptive flight control for accommodation of a large class of control effector failures," in *Proc. American Control Conference*, 1999, p. 19201924.

[6] ——, "A decentralized scheme for accommodation of multiple simultaneous actuator failures," in *Proc. American Control Conference*, Anchorage, AK, 2002, pp. 5098–5103.

[7] M. L. Corradini and G. Orlando, "A sliding mode controller for actuator failure compensation," in *Proc. 42nd IEEE Conference on Decision and Control*, Denver, Colorado, June 25-27, 2003, pp. 4255–4260.

[8] M. Bodson and J. E. Groszkiewich, "Multivariable adaptive algorithms for reconfigurable flight control," *IEEE Transactions on Control Systems Technology*, vol. 5, no. 2, pp. 217–229, 1997.

[9] J. Bošković, S.-H. Yu, and R. Mehra, "A stable scheme for automatic control reconfiguration in the presence of actuator failures," in *Proc. American Control Conference*, Philadelphia, Pennsylvania, June 24–26, 1998, pp. 2455–2459.

[10] G. Tao, S. M. Joshi, and X. Ma, "Adaptive state feedback and tracking control of systems with actuator failures," *IEEE Transactions on Automatic Control*, vol. 46, no. 1, pp. 78–95, 2001.

[11] A. Taware and G. Tao, *Control of Sandwich Nonlinear Systems*. Berlin: Springer-Verlag, 2003.

[12] G. Tao, S. Chen, X. Tang, and S. Joshi, *Adaptive Control of Systems with Actuator Failures*. London: Springer-Verlag, 2004.

[13] W. Chen and M. Saif, "Fault detection and accommodation in nonlinear time-delay systems," in *Proc. American Control Conference*, Maui, Hawaii USA, December 9-12, 2003, pp. 4291–4296.

[14] Q. Zhao and C. Cheng, "State feedback control for time-delayed systems with actuator failures," in *Proc. American Control Conference*, Denver, Colorado, June 25-27, 2003, pp. 827–832.

[15] B. M. Mirkin and P. O. Gutman, "Model reference adaptive control of state delayed system with actuator failures," *International Journal of Control*, 2005, appear.

[16] B. M. Mirkin, "Comments on "Exact output tracking in decentralized adaptive control"," *IEEE Transactions on Automatic Control*, vol. 48, no. 2, pp. 348–350, 2003.

[17] B. M. Mirkin and P. O. Gutman, "Decentralized output-feedback MRAC of linear state delay systems," *IEEE Transactions on Automatic Control*, vol. 48, no. 9, pp. 1613–1619, 2003.

[18] B. M. Mirkin, "Adaptive decentralized control with model coordination," *Automation and Remote Control*, vol. 60, no. 1, pp. 73–81, 1999, (Translated from Avtomatika i telemekhanika, no. 1, 1999, pp. 90–100).

[19] I. Landau, R. Lozano, and M. M'Saad, *Adaptive Control*. London: Springer-Verlag, 1998.

[20] J. K. Hale and S. M. V. Lunel, *Introduction to Functional Differential Equations*. New York: Springer-Verlag, 1993.

[21] P. A. Ioannou and J. Sun, *Robust Adaptive Control*. New Jersey: Prentice-Hall, 1996.