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A Direct Method for Robust Adaptive Nonlinear Control with Unknown Hysteresis

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***Abstract*— It is usually difficult and challenging to develop a general control framework for systems in the presence of unknown hysteresis nonlinearities. Based on the Prandtl-Ishlinskii model and the direct method for robust adaptive design given in [1], this paper deals with robust adaptive control of a class of uncertain nonlinear systems preceded by unknown hysteresis nonlinearity. By utilizing the Prandtl-Ishlinskii model and a neural networks approximator, the robust adaptive control developed ensures that all the close-loop system signals are bounded, and the tracking error converges to a set of adjustable neighborhood of zero independent of initial conditions.**

I. INTRODUCTION

Hysteresis nonlinearities occurring in a wide range of physical systems usually cause undesirable inaccuracies or oscillations and even instabilities [2], [3], [4]. Various control techniques have been developed to mitigate the effects of hysteresis for decades and have recently re-attracted significant attentions from both industry and academic research. Much of these renewed interests are multi-folds: (i) hysteresis occurs in many different engineering systems, (ii) its control problem is academically interested and academically challenging because of its inherent high nonlinearity, and (iii) existing traditional control methods are not effective and adequate in dealing with *non-smooth* and multi-valued hysteresis nonlinearities.

To address such a challenge, it necessitates the development of mathematical models to characterize hysteresis nonlinearities sufficiently accurate. Such models should also be amenable to control design for nonlinearity compensation and be efficient to use in real-time applications [5]. Several control models have been widely accepted and found applications in different areas. The reader may refer to [6], [7], [8], [9], [10], [11], [12], [13], and [14] for a recent review. At present, Preisach models have been widely utilized in modelling hysteresis phenomena appeared in piezoelectric, magnetostrictive and shape memory alloy actuators [15], [16], [17].

In past decades, robust adaptive control of uncertain nonlinear dynamics has undergone rapid developments. Systematic controller design procedures are now available for reasonably large classes of nonlinear systems in the presence of parametric uncertainties and unknown nonlinear functions, see [1], [18], [19], [20], [21], [22] and the references therein. In our recent research, through investigating the Prandtl-Ishlinskii model, a subclass of Preisach type model, we found that the Prandtl-Ishlinskii model can be connected to these robust adaptive control methods [23]. Thus, system stability and asymptotic behavior analysis can be conducted. In paper [24], we have fused the Prandtl-Ishlinskii model with the adaptive controller structures for a class of nonlinear systems with parametric uncertainties to guarantee global stability and the tracking accuracy.

In this paper, we consider a class of systems with unknown nonlinear functions. The systems are also subjected to unknown hysteresis input and bounded disturbances representing the system uncertainties such as the external disturbances and modelling errors. Based on the direct method for robust adaptive design given in [1], by utilizing the Prandtl-Ishlinskii model and a neural networks approximator, a robust adaptive control scheme is developed, which ensures that all the close-loop system signals are bounded, and the tracking error converges to a set of adjustable neighborhood of zero independent of initial conditions.

II. HYSTERESIS MODELS

In this paper the hysteresis nonlinearity is represented by the Prandtl-Ishlinskii model. Using this model, it is possible to integrate the hysteresis model with available adaptive control structures to mitigate the effects of the hysteresis.

A. Stop and Play Operators

Before presenting the Prandtl-Ishlinskii model, we shall introduce two essential yet well known hysteresis operators first. For detailed discussion, see monographs [7], [8], [11].

1) Stop Operator: The first useful operator is the stop operator, $w(t) = E_r[v](t)$, with threshold r. Analytically, suppose that $C_m[0, t_E]$ is the space of piece-wise monotone continuous functions, a function $e_r : R \to R$ is defined by

$$e_r(v) = \min(r, \max(-r, v)) \tag{1}$$

with $r \geq 0$. For any input $v(t) \in C_m[0, t_E]$, and any initial value $w_{-1} \in R$, the stop operator E_r can be given by the inductive definition

$$\begin{aligned} &E_r[v; w_{-1}](0) = e_r(v(0) - w_{-1}), \\ &E_r[v; w_{-1}](t) = e_r(v(t) - v(t_i) + E_r[v; w_{-1}](t_i)), \quad \text{for} \\ &t_i < t \leq t_{i+1} \text{ and } 0 \leq i \leq N-1 \end{aligned} \tag{2}$$

where $0 = t_0 < t_1 < \cdots < t_N = t_E$ is a partition of $[0, t_E]$, such that the function v is monotone on each of the sub-intervals $[t_i, t_{i+1}]$. The argument of the operator is written in square brackets to indicate the functional dependence, since

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it maps a function to a function. The stop operator however is mainly characterized by its threshold parameter r which determines the height of the hysteresis region in the (v, w) plane.

2) Play Operator: The second basic yet important hysteresis operator is the play operator: For $r \geq 0$ and a general initial value $w_{-1} \in R$, the play operator $F_r[.; w_{-1}] : C_m[0, t_E] \times R \rightarrow C_m[0, t_E]$ with threshold r is then inductively defined by

$$\begin{array}{l} F_r[v; w_{-1}](0) = f_r(v(0), w_{-1}), \\ F_r[v; w_{-1}](t) = f_r(v(t), F_r[v; w_{-1}](t_i)), \quad \text{for} \\ t_i < t \leq t_{i+1} \text{ and } 0 \leq i \leq N-1, \end{array} \tag{3}$$

with

$$f_r(v, w) = \max(v - r, \min(v + r, w)). \tag{4}$$

where $0 = t_0 < t_1 < \cdots < t_N = t_E$ is a partition of $[0, t_E]$, such that the function v is monotone on each of the sub-intervals $[t_i, t_{i+1}]$.

¿From the definitions given in (2) and (3), it can be proved that the operator F_r is the complement of E_r, i.e., they are closely related through the equation

$$E_r[v; w_{-1}](t) + F_r[v; w_{-1}](t) = v(t), \tag{5}$$

for any piece-wise monotone input function v and $r \geq 0$[7].

In the sequel, we will simply write $F_r[v]$ instead of $F_r[v; w_{-1}]$, whenever where is no ambiguity. Due to the nature of play and stop operators, above discussions are defined on the space $C_m[0, t_E]$ of continuous and piecewise monotone functions; however, they can also be extended to the space $C[0, t_E]$ of continuous functions.

B. Prandtl-Ishlinskii Model

We are ready to introduce the Prandtl-Ishlinskii model defined by the stop or play hysteresis operators. The Prandtl-Ishlinskii model was introduced to formulate the elastic-plastic behavior through a weighted superposition of basic elastic-plastic elements $E_r[v]$, or **stop** as follows:

$$w(t) = \int_0^R p(r) E_r[v](t) dr, \tag{6}$$

where $p(r)$ is a given density function, satisfying $p(r) \geq 0$ with $\int^{\infty} r p(r) dr < \infty$, and is expected to be identified from experimental data. With the defined density function, this operator maps $C[t_0, \infty)$ into $C[t_0, \infty)$, i.e., Lipschitz continuous inputs will yield Lipschitz continuous outputs [8]. Since the density function $p(r)$ vanishes for large values of r, the choice of $R = \infty$ as the upper limit of integration in the literature is just a matter of convenience [7].

It can be seen that the stop operator E_r serves as the building element in the Prandtl-Ishlinskii model (6). We should mention that stop play operators are rate-independent, so is the Prandtl-Ishlinskii model (6). As an illustration, Fig. 1 shows $w(t)$ generated by model given in (6), with $p(r) = e^{-0.067(r-1)^2}$, $r \in [0, 10]$, and input $v(t) = 7sin(3t)/(1+t)$, $t \in [0, 2\pi]$ with $w_{-1} = 0$. This numerical result shows that

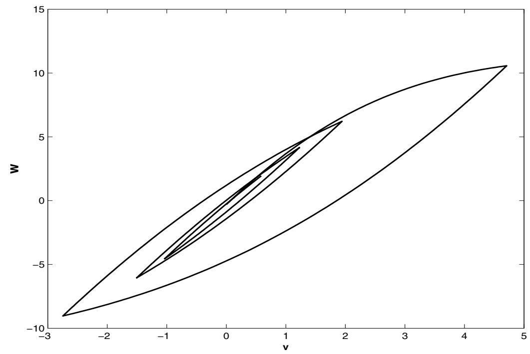

Fig. 1. Hysteresis curves given by (6)

the Prandtl-Ishlinskii model (6) indeed generates the hysteresis curves and is well-suited to model the rate-independent hysteretic behavior.

Since the operator F_r is the complement of E_r, the Prandtl-Ishlinskii model can also be expressed through the play operator. Using Equation (5) and substituting E_r in (6) by F_r, the Prandtl-Ishlinskii model defined by the play hysteresis operator is expressed as follows:

$$w(t) = p_0 v(t) - \int_0^R p(r) F_r[v](t) dr, \tag{7}$$

where $p_0 = \int_0^R p(r) dr$ is constant which depends on the density function.

It should be noted that Equation (7) decomposes the hysteresis behavior into two terms. The first term describes the linear reversible part and the second term describes the nonlinear hysteretic part. This decomposition is crucial since it facilitates the utilization of the currently available robust adaptive control techniques for the controller design.

III. Problem Statement

Consider a controlled system consisting of a nonlinear plant preceded by an actuator with hysteresis nonlinearity, that is, the hysteresis is presented as an input of the nonlinear plant. The nonlinear dynamic system is in the form as

$$\begin{array}{rcl} \dot{x}_i &=& x_{i+1}, \qquad i = 1, 2, \cdots, n-1 \\ \dot{x}_n &=& a(x) + b(x) w(t) + d_e(t) \\ y &=& x_1 \end{array} \tag{8}$$

where $x = [x_1, x_2, \cdots, x_n]^T \in R^n$ is the system state; $a(x)$ and $b(x)$ are unknown smooth functions; $d_e(t)$ represents the system uncertainties such as the external disturbances and modelling errors bounded by a known constant $d_0 > 0$, that is, $|d_e(t)| \leq d_0$. $w(t)$ is the hysteresis nonlinearity represented by the Prandtl-Ishlinskii model with a play operator given in (7),

$$w(t) = p_0 v(t) - d[v](t) \tag{9}$$

where

$$d[v](t) = \int_0^R p(r) F_r[v](t) dr \tag{10}$$

with $p_0 = \int_0^R p(r)dr$. For convenience, $F_r[v, w_{-1}]$ is denoted by $F_r[v]$ for any given hysteresis initial state $w_{-1} \in R$. Using this hysteresis model, system (8) becomes

$$\begin{aligned} \dot{x}_i &= x_{i+1}, \qquad i = 1, 2, \cdots, n-1 \\ \dot{x}_n &= a(x) + b(x)p_0 v(t) - b(x)d[v](t) + d_e(t) \\ y &= x_1 \end{aligned} \tag{11}$$

In this paper, we study the adaptive control problem of the physical plants operating in bounded regions, the state variable belongs to a compact set $\Omega_x \subset R^n$. The objective is to design a stable control law $v(t)$, to force the state vector $x = [x_1, x_2, \cdots, x_n]^T \in \Omega_x$ to follow a specified desired trajectory $\mathbf{x}_d = [x_d, \dot{x}_d, ..., x_d^{(n-1)}]^T$ as close as possible.

Throughout the paper, the following assumptions are made:

Assumption 1: The sign of $b(x)$ is known and there exists a constant $b_0 > 0$, $b_0 < |b(x)|$, $\forall x \in \Omega_x$. Since the sign of $b(x)$ is known and $b(x)$ is not equal to zero, we may assume that $b(x) > 0$.

Assumption 2: There exists a smooth function $\bar{b}(x)$ such that $|b(x)| \leq \bar{b}(x)$ and $b(x)/\bar{b}(x)$ is independent of the state x_n, $\forall x \in \Omega_x \subset R^n$.

Assumption 3: The desired trajectory $\mathbf{x}_d \in C^{(n)}(R)$ is available and $\mathbf{x}_d \in \Omega_d \subset R^n$ with Ω_d a compact set.

Assumption 4: There exist a known constant $p_{0min} > 0$ and a known function $p_{max}(r)$, such that $p_0 > p_{0min}$, and $p(r) \leq p_{max}(r)$ for all $r \in [0, R]$.

Remark: Assumption 1 and 3 are generally adopted for the design of tracking controller. As mentioned in [1], Assumption 2 imposes an additional restriction on the class of systems. However, many physical systems possess such a property, such as, pendulum plants and magnetic levitation systems. As for Assumption 4, based on the properties of the density function $p(r)$, it is reasonable to set an upper bound p_{max} for $p(r)$. Here $p_{0min} > 0$ must be satisfied, otherwise $p_0 = 0$ implies $w(t) = 0$.

To simplify the notification, let

$$g(x) = b(x)p_0/\bar{b}(x)p_{0max}$$

where $p_{0max} = \int_0^R p_{max}(r)dr$, from Assumption 1, 2 and 3, $g(x)$ is independent of x_n and $0 < g(x) \leq 1$.

Define the tracking error vector $\tilde{\mathbf{x}}$ as

$$\tilde{\mathbf{x}} = \mathbf{x} - \mathbf{x}_d,$$

and a filtered tracking error as

$$s(t) = (\frac{d}{dt} + \lambda)^{(n-1)}\tilde{x}_1(t), \qquad \lambda > 0 \tag{12}$$

$s(t)$ can be rewritten as $s(t) = (\Lambda^T, 1)\tilde{\mathbf{x}}(t)$ with $\Lambda^T = [\lambda^{(n-1)}, (n-1)\lambda^{(n-2)}, ..., (n-1)\lambda]$.

It has been shown in [25] that the definition given in (12) has the following properties: (i) the equation $s(t) = 0$ defines a time-varying hyperplane in R^n on which the tracking error vector $\tilde{\mathbf{x}}(t)$ decays exponentially to zero, (ii) if $\tilde{\mathbf{x}}(0) = 0$ and $|s(t)| \leq \epsilon$, where ϵ is a constant, then $\tilde{\mathbf{x}}(t) \in \Omega_\epsilon \triangleq \{\tilde{\mathbf{x}}(t) \mid |\tilde{\mathbf{x}}_i| \leq 2^{i-1}\lambda^{i-n}\epsilon, i = 1, ..., n\}$ for $\forall t \geq 0$, and (iii) if $\tilde{\mathbf{x}}(0) \neq 0$ and $|s(t)| \leq \epsilon$, then $\tilde{\mathbf{x}}(t)$ will converge to Ω_ϵ within a time-constant $(n-1)/\lambda$.

IV. Controller Design

In this section, we first assume that nonlinear functions $a(x)$, $b(x)$ are known exactly, hysteresis weight function $p(r)$ is available, and the system uncertainty $d_e(t) = 0$. Notice that the Prandtl-Ishlinskii model (9) decomposes the hysteresis behavior into two terms: the linear reversible component $p_0 v(t)$ and the nonlinear hysteretic component $d[v](t)$. If $d[v](t) = 0$, the system input is $w(t) = p_0 v(t)$, there exists an ideal feedback control v^* as suggested in [1]. Under this control the state vector $\mathbf{x}$ will follows the desired trajectory $\mathbf{x_d}$ asymptotically.

Consider the state feedback control

$$v^*(t) = \frac{1}{\bar{b}(x)p_{0max}} v_n^*(t) \tag{13}$$

with

$$v_n^*(t) = -\frac{1}{g(x)}[a(x) + \mu] - s[\frac{1}{\delta g(x)} + \frac{1}{\delta g^2(x)} - \frac{\dot{g}(x)}{2g^2(x)}], \tag{14}$$

where $\delta > 0$ is a constant and

$$\mu = (0, \Lambda^T)\tilde{\mathbf{x}} - y_d^{(n)}$$

By definition (12), the time derivative of s with the input $v^*(t)$ for the system (11) can be written as

$$\dot{s}(t) = -[\frac{1}{\delta} + \frac{1}{\delta g(x)} - \frac{\dot{g}(x)}{2g(x)}]s. \tag{15}$$

Define a Lyapunov function candidate $V_1 = \frac{1}{2g(x)}s^2$, the time derivative of V_1 along (15) equals

$$\dot{V}_1(t) = -\frac{2}{\delta}[1 + \frac{1}{g(x)}]\frac{1}{2g(x)}s^2. \tag{16}$$

Since $0 < g(x) \leq 1$, it follows that

$$\dot{V}_1(t) \leq -\frac{4}{\delta}V_1. \tag{17}$$

The solution of the above inequality satisfies

$$V_1(t) \leq e^{-\frac{4}{\delta}(t-t_0)}V_1(t_0), \quad \forall \ t \geq t_0 \tag{18}$$

$|b(x)| \geq b_0 > 0$, $\lim_{t\to\infty} V_1(t) = 0$ implies $\lim_{t\to\infty} s = 0$. Furthermore, using properties of s, $\lim_{t\to\infty} \|\tilde{\mathbf{x}}\| = 0$.

We have proved that when functions $a(x)$ and $b(x)$ are known, the hysteresis weight function $p(r)$ is available, using the control input v^* defined in (13), the tracking error vector $\tilde{\mathbf{x}} = \mathbf{x} - \mathbf{x}_d$ converges asymptotically to zero if $d[v](t) = 0$ and the system uncertainty $d_e(t) = 0$.

When $a(x)$, $b(x)$, and $p(r)$ are unknown, the controller v^* given in (13) cannot be implemented. A reasonable idea is to use estimated v to approximate $v^*(t)$. From previous discussion, v_n^* exists. Under Assumptions 1-2, $a(x)$ and $b(x)$ are continuous functions of x, v_n^* is continuous respect to $x(t)$ and $\mathbf{x}_d$. It has been assumed that $\mathbf{x}_d$ is continuous on the compact set Ω_d and $x(t)$ takes values in compact set Ω_x, all conditions for the Universal Approximation theorem are satisfied. Therefore, function approximation methods such as neural networks or fuzzy systems can be applied. In what follows, neural networks will be used to approximate v_n^*.

Consider v_n^* is a function of $z = (x^T, s, \mu)^T$, z belongs to a compact set

$$\Omega_z = \{(x^T, s, \mu) | x \in \Omega_x, x_d \in \Omega_d\}$$

For any arbitrary constant ϵ_0, there exists an integer l^*, such that for all $l \geq l^*$, the following approximation holds:

$$v_n^*(t) \quad = \quad \theta^{*T}\Phi(z) + \epsilon_l \qquad \forall z \in \Omega_z \tag{19}$$

where l is the number of neural networks node, $\Phi(z) \in R^l$ is the basis function vector, the approximation error ϵ_l satisfies $|\epsilon_l| \leq \epsilon_0$. θ^* is the ideal weight defined as

$$\theta^* = arg\ min_{\theta \in R^l}\{sup_{z\in\Omega_z}|\theta^T\Phi(z) - v_n^*(t)|\} \tag{20}$$

Now, the unknown nonlinearity problem transformed into a problem to estimate the idea parameter vector θ^*. Let $\hat{\theta}$ be an estimate of the ideal neural networks weight θ^*, and the controller $v_n(t)$ is chosen as

$$v_n(t) = \hat{\theta}^T\Phi(z) \tag{21}$$

with the adaptation law

$$\dot{\hat{\theta}} = -\Gamma[\Phi(z)s + \sigma\hat{\theta}] \tag{22}$$

where $\Gamma, \sigma > 0$ are adaptive gains.

In order to cancel the effect caused by the term $d[v](t)$, we notice that $d[v](t)$ is determined by the weight function $p(r)$, $p(r)$ is not a function of t. Thus it can be considered as a parameter for each fixed $r \in [0, R]$ and adjusted by adaptation law. Let $\hat{p}(t, r)$ be the estimate of $p(r)$ at any $r \in [0, R]$, define

$$v_h(t) = \begin{cases} 0 & \text{if } s\int_0^R p(r)F_r[v](t)dr \geq 0; \\ \int_0^R \frac{\hat{p}(t,r)}{p_{0min}}F_r[v](t)dr & \text{otherwise} \end{cases} \tag{23}$$

with adaptation law

$$\frac{\partial}{\partial t}\hat{p}(t,r) = \begin{cases} -\gamma\eta\hat{p}(t,r), & \text{if } s\int_0^R p(r)F_r[v](t)dr \geq 0; \\ -\eta\frac{\bar{b}(x)p_{0max}}{p_{0min}}F_r[v](t)s - \gamma\eta\hat{p}(t,r), & \text{otherwise} \end{cases} \tag{24}$$

where $\gamma > 0$ and $\eta > 0$ are adaptive parameters.

Now, the adaptive controller is defined as following

$$v(t) = \frac{1}{\bar{b}(x)p_{0max}}v_n(t) + v_h(t) \tag{25}$$

where v_n and v_h are given by (21) and (23). Substituting $v(t)$ into system (11), the time derivative of s can be rewritten as

$$\begin{aligned} \dot{s}(t) \quad &= \quad [a(x) + \mu] + g(x)v_n + b(x)p_0v_h(t) \\ &\quad -b(x)\int_0^R p(r)F_r[v](t)dr + d_e(t) \end{aligned} \tag{26}$$

To establish global boundedness, let

$$\tilde{\theta} = \hat{\theta} - \theta^*, \tag{27}$$

$$\tilde{p}(t,r) = \hat{p}(t,r) - p(r), \qquad \forall\ r \in [0, R] \tag{28}$$

Choose the Lyapunov function candidate as

$$V(t) = \frac{1}{2g(x)}s^2 + \frac{1}{2}\tilde{\theta}^T\Gamma^{-1}\tilde{\theta} + \frac{1}{2\eta}\int_0^R \tilde{p}^2(t,r)dr \tag{29}$$

The time derivative of V is

$$\begin{aligned} \dot{V} \quad &= \quad \frac{1}{g(x)}s\dot{s} - \frac{\dot{g}(x)}{2g^2(x)}s^2 + \tilde{\theta}^T\Gamma^{-1}\dot{\tilde{\theta}} \\ &\quad +\frac{1}{\eta}\int_0^R \tilde{p}(t,r)\frac{\partial}{\partial t}\tilde{p}(t,r)dr \\ &= \quad -[\frac{1}{\delta g(x)} + \frac{1}{\delta g^2(x)}]s^2 - \epsilon_l s + \frac{d_e(t)}{g(x)}s \\ &\quad +\tilde{\theta}^T[\Phi(z)s + \Gamma^{-1}\dot{\hat{\theta}}] + \frac{1}{\eta}\int_0^R \tilde{p}(t,r)\frac{\partial}{\partial t}\tilde{p}(t,r)dr \\ &\quad +\bar{b}(x)p_{0max}s[v_h(t) - \int_0^R \frac{p(r)}{p_0}F_r[v](t)dr] \end{aligned} \tag{30}$$

Utilizing adaptation law (22), the fourth term in the above equation becomes

$$\tilde{\theta}^T[\Phi(z)s + \Gamma^{-1}\dot{\hat{\theta}}] = -\sigma\tilde{\theta}^T\hat{\theta} \tag{31}$$

Next, let us simplify the last two terms in (30). For $s\int_0^R p(r)F_r[v](t)dr \geq 0$, from definition (23) and adaptation law (24), we have

$$v_h(t) = 0$$

$$\frac{\partial}{\partial t}\tilde{p}(t,r) = -\gamma\eta\hat{p}(t,r)$$

which lead to

$$\begin{aligned} &\bar{b}(x)p_{0max}s[v_h(t) - \frac{1}{p_0}\int_0^R p(r)F_r[v](t)dr] \\ &\quad +\frac{1}{\eta}\int_0^R \tilde{p}(t,r)\frac{\partial}{\partial t}\tilde{p}(t,r)dr \\ &= -\frac{\bar{b}(x)p_{0max}}{p_0}s\int_0^R p(r)F_r[v](t)dr \\ &\qquad -\gamma\int_0^R \tilde{p}(t,r)\hat{p}(t,r)dr \\ &\leq -\gamma\int_0^R \tilde{p}(t,r)\hat{p}(t,r)dr \end{aligned} \tag{32}$$

For $s\int_0^R p(r)F_r[v](t)dr < 0$, from (23), (24) and $p_{0min} \leq p_0$, we have

$$\begin{aligned} &\bar{b}(x)p_{0max}s[v_h(t) - \frac{1}{p_0}\int_0^R p(r)F_r[v](t)dr] \\ &\quad +\frac{1}{\eta}\int_0^R \tilde{p}(t,r)\frac{\partial}{\partial t}\tilde{p}(t,r)dr \\ &\leq \bar{b}(x)p_{0max}s[v_h(t) - \frac{1}{p_{0min}}\int_0^R p(r)F_r[v](t)dr] \\ &\quad +\frac{1}{\eta}\int_0^R \tilde{p}(t,r)\frac{\partial}{\partial t}\tilde{p}(t,r)dr \end{aligned}$$

$$
\begin{aligned}
&= \frac{\bar{b}(x)p_{0max}}{p_{0min}} \int_0^R \tilde{p}(t,r) F_r[v](t) s dr \\
&\quad + \frac{1}{\eta} \int_0^R \tilde{p}(t,r) \frac{\partial}{\partial t} \tilde{p}(t,r) dr \\
&\leq -\gamma \int_0^R \tilde{p}(t,r) \hat{p}(t,r) dr
\end{aligned}
$$

Therefore,

$$
\begin{aligned}
\dot{V}(t) \quad \leq \quad & -[\frac{1}{\delta g(x)} + \frac{1}{\delta g^2(x)}] s^2 - \epsilon_l s + \frac{d_e(t)}{g(x)} s \\
& -\sigma \tilde{\theta}^T \hat{\theta} - \gamma \int_0^R \tilde{p}(t,r) \hat{p}(t,r) dr
\end{aligned} \tag{33}
$$

Furthermore, using the following inequalities

$$
-\sigma \tilde{\theta}^T \hat{\theta} \leq -\frac{\sigma}{2} \|\tilde{\theta}\|^2 + \frac{\sigma}{2} \|\theta^*\|^2 \tag{34}
$$

$$
|\epsilon_l s| \leq \sqrt{2} |\epsilon_l s| \leq \frac{1}{\delta g(x)} s^2 + \frac{\delta g(x)}{2} \epsilon_l^2 \tag{35}
$$

$$
|\frac{d_e(t)}{g(x)} s| \leq \frac{1}{2\delta g^2(x)} s^2 + \frac{\delta}{2} d_e^2(t) \tag{36}
$$

$$
-\tilde{p}(t,r) \hat{p}(t,r) \leq -\frac{1}{2} \tilde{p}^2(t,r) + \frac{1}{2} p^2(r) \tag{37}
$$

and noticing that $0 < g(x) \leq 1$, $|\epsilon_l| \leq \epsilon_0$, and $|d_e(t)| \leq d_0$,

$$
\begin{aligned}
\dot{V}(t) \quad \leq \quad & -\frac{1}{2\delta g^2(x)} s^2 - \frac{\sigma}{2} \|\tilde{\theta}\|^2 - \frac{\gamma}{2} \int_0^R \tilde{p}^2(t,r) dr \\
& + \frac{\delta g(x)}{2} \epsilon_l^2 + \frac{\delta}{2} d_e^2(t) + \frac{\sigma}{2} \|\theta^*\|^2 + \frac{\gamma}{2} \int_0^R p^2(r) dr \\
\leq \quad & -\frac{1}{2\delta g(x)} s^2 - \frac{\sigma}{2} \|\tilde{\theta}\|^2 - \frac{\gamma}{2} \int_0^R \tilde{p}^2(t,r) dr \\
& + \frac{\delta}{2} \epsilon_0^2 + \frac{\delta}{2} d_0^2 + \frac{\sigma}{2} \|\theta^*\|^2 + \frac{\gamma}{2} \int_0^R p^2(r) dr
\end{aligned} \tag{38}
$$

letting

$$
\tau = min(\frac{1}{\delta}, \frac{\sigma}{\lambda_{max}}, \gamma\eta) \tag{39}
$$

where λ_{max} is the largest eigenvalue of Γ^{-1}, we have

$$
\dot{V}(t) \quad \leq \quad -\tau V + \frac{c}{2} \tag{40}
$$

with

$$
c = \delta \epsilon_0^2 + \delta d_0^2 + \sigma \|\theta^*\|^2 + \gamma \int_0^R p^2(r) dr \tag{41}
$$

Accordingly,

$$
\begin{aligned}
V(t) \quad & \leq \quad e^{-\tau(t-t_0)} V(t_0) + \frac{c}{2} \int_{t_0}^t e^{-\tau(t-\nu)} d\nu \\
& \leq \quad e^{-\tau(t-t_0)} V(t_0) + \frac{c}{2\tau}
\end{aligned} \tag{42}
$$

¿From the definition of V, we conclude that s, $\tilde{\theta}$, and $\tilde{p}$ are bounded. Explicitly,we have

$$
|s(t)| \leq \sqrt{2V(t)} \quad \leq \quad \sqrt{2V(t_0)} e^{-\frac{\tau(t-t_0)}{2}} + \sqrt{\frac{c}{\tau}} \tag{43}
$$

Noticing that the bound for the filtered tracking error in (43) is a function of t depending on the initial value $V(t_0)$. Using the same method as in [25], we can prove that the tracking error vector $\tilde{\mathbf{x}}$ converges to a set, which is independent of the initial condition $V(t_0)$. Let $p = d/dt$ be the Laplace operator,

$$
\begin{aligned}
y_1(p) \quad & = \quad \frac{1}{p+\lambda} s(p) \\
y_i(p) \quad & = \quad \frac{1}{p+\lambda} y_{i-1}, \qquad i = 1, 2, \cdots, n-1
\end{aligned} \tag{44}
$$

¿From (43), $y_1(t)$ is bounded by

$$
\begin{aligned}
|y_1(t)| \quad & \leq \quad \int_{t_0}^t e^{-\lambda(t-\alpha)} |s(\alpha)| d\alpha \\
& \leq \quad \begin{cases} \frac{1}{\lambda}\sqrt{\frac{c}{\tau}} & + \sqrt{2V(t_0)} e^{-\lambda(t-t_0)} (t-t_0) \\ & \text{if } \lambda = \tau/2; \\ \\ \frac{1}{\lambda}\sqrt{\frac{c}{\tau}} & + \frac{2\sqrt{2V(t_0)}}{2\lambda-\tau} \{ e^{-\frac{\tau}{2}(t-t_0)} - e^{-\lambda(t-t_0)} \} \\ & \text{otherwise} \end{cases}
\end{aligned} \tag{45}
$$

By integrating inequality $|y_i(t)| \leq \int_{t_0}^t e^{-\lambda(t-\alpha)} |y_{i-1}(t)| d\alpha$ from $i = 2$ to $i = n-1$, for $n \geq 3$, we have

$$
|y_{n-1}(t)| \quad \leq \quad \int_{t_0}^t e^{-\lambda(t-\alpha)} |y_{n-2}(t)| d\alpha \tag{46}
$$

$$
\leq \quad \begin{cases} \frac{1}{\lambda^{n-1}}\sqrt{\frac{c}{\tau}} & + \frac{\sqrt{2V(t_0)}}{(n-1)!} (t-t_0)^{n-1} e^{-\lambda(t-t_0)} \\ & \text{if } \lambda = \tau/2; \\ \\ \frac{1}{\lambda^{n-1}}\sqrt{\frac{c}{\tau}} & + \sqrt{2V(t_0)} \{ (\frac{2}{2\lambda-\tau})^{n-1} e^{-\frac{\tau}{2}(t-t_0)} \\ -\sum_{i=1}^{n-1} & (\frac{2}{2\lambda-\tau})^{n-i} \frac{(t-t_0)^{i-1}}{(i-1)!} e^{-\lambda(t-t_0)} \} \\ & \text{otherwise} \end{cases} \tag{47}
$$

Since $\tilde{x}_1(t) = y_{n-1}(t)$, $\tilde{x}_1(t)$ satisfies the above inequality, and $\lim_{t\to\infty} \tilde{x}_1(t) = \frac{1}{\lambda^{n-1}}\sqrt{\frac{c}{\tau}}$, where c and τ are existing constants given in (39) and (41). The upper bounds for $|\tilde{x}_1(t)|$, $\forall t \geq t_0$ are also given as

$$
|\tilde{x}_1(t)| \leq \begin{cases} \frac{1}{\lambda^{n-1}}\sqrt{\frac{c}{\tau}} & + \frac{\sqrt{2V(t_0)}}{(n-1)!} (\frac{n-1}{\lambda})^{n-1} e^{-(n-1)} \\ & \text{if } \lambda = \tau/2; \\ \\ \frac{1}{\lambda^{n-1}}\sqrt{\frac{c}{\tau}} & + \sqrt{2V(t_0)} \{ (\frac{2}{2\lambda-\tau})^{n-1} e^{-\frac{\tau}{2\lambda}(i-1)} \\ -\sum_{i=1}^{n-1} & (\frac{2}{2\lambda-\tau})^{n-i} \frac{(i-1)^{i-1}}{(i-1)!\lambda^{i-1}} e^{-(i-1)} \} \\ & \text{otherwise} \end{cases} \tag{48}
$$

they are reached at $t = \frac{n-1}{\lambda} + t_0$ for $\lambda = \tau/2$ and $t = \frac{i-1}{\lambda} + t_0$ otherwise.

Similarly, for $\tilde{x}_i(t), i = 2, \cdots, n-1$, let

$$
\begin{aligned}
y_1(p) \quad & = \quad \frac{1}{p+\lambda} s(p) \\
y_j(p) \quad & = \quad \frac{1}{p+\lambda} y_{j-1}(p), \qquad j = 1, 2, \cdots, n-i-1 \\
z_1(p) \quad & = \quad y_{n-i-1}(p) \\
z_j(p) \quad & = \quad \frac{s}{s+\lambda} z_{j-1}(p), \qquad j = 2, \cdots, i
\end{aligned} \tag{49}
$$

Since $\tilde{x}_i(t) = z_i(t)$, using previous results we can prove that

$$\lim_{t\to\infty} \tilde{x}_i(t) = \lim_{t\to\infty} z_i(t) = 2^{i-1}\lambda^{i-n}\sqrt{c/\tau}$$

details are tedious to be omitted.

In conclusion, we summarize the above discussion in the following theorem:

Theorem: Consider nonlinear system (8) with the hysteresis as an input represented by the Prandtl-Ishlinskii model (7) satisfying Assumptions 1)-4), if the robust adaptive controller is specified by (25) with adaptation laws (22) and (24), then for any bounded initial conditions, all closed-loop signals are bounded and the state vector $x(t)$ converges to

$$\Omega_\epsilon = \{x(t) |\ |\tilde{x}_i| \le 2^{i-1}\lambda^{i-n}\sqrt{c/\tau}, i = 1, ..., n\}$$

where τ and c are constants given in(39) and (41).

Remark : We have to point out that, in the above theorem, the bound for the converging set Ω_ϵ is determined by τ and c. Since τ is a constant decided by controller parameters δ, σ, λ_{max}, γ, and η. Constant c depends on controller parameters and the properties of the plant, such as weight function of the hysteresis $p(r)$, the bound of disturbances d_0, and the approximator to be used to estimate the unknown nonlinear functions $a(x)$ and $b(x)$. The bound can be made small by choosing suitable parameters for system properties ϵ_0, d_0, θ^*, and $p(r)$. We give an example to make it clear. Suppose that we take $\tau = \gamma\eta$, from (39) and (41), we have

$$\gamma\eta \le \frac{1}{\delta} \quad , \qquad \gamma\eta \le \frac{\sigma}{\lambda_{max}}$$
$$c/\tau = \frac{\delta}{\gamma\eta}\{\epsilon_0^2 + d_0^2\} + \frac{\sigma}{\gamma\eta}\|\theta^*\|^2 + \frac{1}{\eta}\int_0^R p^2(r)dr \qquad (50)$$

Assume that based on estimates of ϵ_0, d_0, $\|\theta^*\|$, and $\int_0^R p^2(r)dr$, we set

$$\frac{\delta}{\gamma\eta} = 0.1\ , \quad \frac{\sigma}{\gamma\eta} = 0.1 \quad and \quad \frac{1}{\eta} = 0.01$$

then the parameters should satisfy

$$\eta = 100\ ; \qquad 0 < \gamma \le \frac{1}{10\sqrt{10}}\ ;$$
$$0 < \lambda_{max} \le 0.1\ ; \quad \delta = \sigma = 10\gamma \qquad (51)$$

There are two inequalities for γ and λ_{max} leaving room for further adjustment.

V. Conclusion

In this paper, a robust adaptive control architecture has been proposed for a general class of continuous-time nonlinear dynamic systems preceded by a hysteresis nonlinearity with bounded disturbances. By utilizing the Prandtl-Ishlinskii model and neural networks approximator, the robust adaptive control scheme ensures that all the close-loop system signals are bounded, and the tracking error converges to a adjustable neighborhood of zero which is independent of the initial conditions.

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