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Adaptive Fuzzy Sliding Mode Control For Time-Delay Uncertain Large-Scale Systems

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Abstract—**In this paper, a class of uncertain large-scale systems with time-delay interconnections is represented by an equivalent Takagi-Sugeno type fuzzy model. In general, the existence of time-delays and perturbations is common in control practice. Therefore the problem of controlling such systems becomes more complex. For the design of the sliding mode control of time-delay uncertain large-scale systems, we propose the adaptive fuzzy sliding mode control schemes, and the fuzzy approximator is used to approximate the upper bound of the uncertainties which combine time-delay interconnections with perturbations in the system. Furthermore, based on the Lyapunov stability theorem, asymptotic stability results can be obtained. Finally, some simulation results are illustrated to demonstrate the effectiveness of the proposed method in this paper.**

I. Introduction

The main feature of variable structure control [1], [2] is to employ a discontinuous control law to drive the state from an arbitrarily initial state in the state space to a desired state along a prespecified trajectory. Moreover, it has been shown that variable structure control possesses several advantages, e.g., robustness of stability, insensitivity to the matching parameter variations, and external disturbances [1]-[3]. In recent years, more research and development in this area has been done world wide [4]-[18].

For large-scale systems, a few research results in which variable structure control technique is employed exist due to the complexity of control systems and the effects of interconnections. In such case, the main difficulty is in handling the interconnected terms among each subsystem. Richter *et al* .[7] and Matthews and DeCarlo [8] used variable structure control approach to control a large-scale system; however, all the local system's parameters have to be known exactly. Several researchers [9]-[11] also presented robust decentralized variable structure control for large-scale systems with the requirement of information of upper bound of perturbations. Without knowing the exact values of the coupling parameters, a decentralized variable structure control approach was employed in [12] to handle the interconnected terms. However, the exact value of the parameter about the input gain must be known [13]. In order to relax the condition that the exact value of the parameter about the input gain must be known, another model-reference control was proposed in [13] for a large-scale variable structure adaptive control system. It is noted that all the approaches mentioned above do not consider time-delay in the interconnections.

It is well known that time-delay large-scale dynamical systems are essential features of our modern society. For instance, transportation systems, power systems, computer communication systems, economic systems and so on, can be considered as such a class of dynamical systems. Generally, a time-delay large-scale system is often considered as a set of interconnected systems, and referred to as large-scale systems with time-delay interconnections. The advantage of this aspect in controller design is to reduce complexity and this therefore allows the control implementation to be feasible. Therefore, the problem of decentralized control of time-delay large-scale systems has received considerable attention, and many approaches have been developed to investigate the stability and stabilization of time-delay large-scale systems. The task of effectively controlling large-scale system with time-delay interconnections is still one of the most interesting and challenging control problems [25]-[27].

During the past several years, fuzzy-rule-based modeling has become an active research field because of its unique merits in solving complex nonlinear system identification and control problems. In attempt to obtain more flexibility and more effective capability of handling and processing uncertainties in complicated and ill-defined systems, Zadeh [29] proposed a linguistic approach as the model of human thinking, which introduced the fuzziness into systems theory [30]. Unlike conventional modeling, fuzzy-rule-based modeling is essentially a multimodel approach in which individual rules are combined to describe the global behavior of the system [31]. Fuzzy logic control is generally applicable to plants that are mathematically poorly modeled and where experienced operators are available for providing qualitative guidance.

In this paper, a systematic analysis and a simple design method for a class of uncertain large-scale systems with time-delay interconnections are proposed. Using the T-S fuzzy model, it is to combine some simple local linear delayed systems with their linguistic description to represent original nonlinear delayed systems. In this paper, fuzzy approximator and sliding mode control scheme are considered. That is, the fuzzy logic systems are used here as a tool to approximate the upper bound of the uncertainties which combine time-delay interconnections with perturbations in the time-delay large-scale system. Furthermore, on the basis of the Lyapunov stability theorem, we prove that by employing the proposed controller, the solutions of the resulting adaptive closed-loop large-scale

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time-delay system can be guaranteed to be asymptotic stability in the presence of delayed state perturbations in the interconnections.

II. Problem formulation

This paper focuses on the design of adaptive fuzzy sliding mode controller for a class of large-scale systems with time-delay interconnections. In general, the equation of i th subsystem is given as

$$\dot{x}_i(t)=A_i x_i(t)+B_i u_i(t)+\sum_{\substack{j=1\\ j\neq i}}^{N} A_{ij} x_j\left(t-\tau_{ij}(t)\right)+f_i\left(x_i(t),t\right), \tag{1}$$

where $x_i(t)=\left[x_{i1}(t),\cdots,x_{in_i}(t)\right]^T$ for $i=1,2\cdots,N$

with $\dot{x}_{i1}(t)=x_{i2}(t)$, $\dot{x}_{i2}(t)=x_{i3}(t),\cdots,\ \dot{x}_{in_i-1}(t)=x_{in_i}(t)$, $u_i\in R$ is the manipulated input of i th subsystem, $\tau_{ij}>0$ is a time delay in the interconnections. The matrices A_i and B_i denote the system matrix and input matrix, respectively. The A_{ij} represent the interconnection matrix between the i th and j th subsystems, and the vector $f_i(x_i(t),t)$ is a nonlinear perturbation.

This time-delay large-scale system can be approximated by T-S fuzzy model, which combines the fuzzy inference rule and the local linear model. The l th rule of the T-S fuzzy model, representing the i th subsystem (1) of a time-delay uncertain large-scale system, is the following:

R_i^l : if z_{i1} is F_{i1}^l and $\cdots z_{in_i}$ is $F_{in_i}^l$

then

$$\dot{x}_i(t)=A_i^l x_i(t)+B_i^l u_i(t)+\sum_{\substack{j=1\\ j\neq i}}^{N} A_{ij}^l x_j\left(t-\tau_{ij}^l(t)\right)+f_i^l\left(x_i(t),t\right) \tag{2}$$

where $z_i(t)=[\,z_{i1},z_{i2},\cdots,z_{in_i}\,]$ are some measurable premise variables for subsystem, R_i^l ($l=1,2,\cdots,r_i$) denotes the l th fuzzy inference rule of i th subsystem, r_i is the number of rules, F_{iq}^l ($q=1,2,\cdots,n_i$) are fuzzy set, $A_i^l\in R^{n_i\times n_i}$ and $B_i^l\in R^{n_i\times 1}$ are constant matrices in controllability canonical form and given by

$$A_i^l=\begin{bmatrix}0&1&0&\cdots&0\\0&0&1&\cdots&0\\ \vdots&\vdots&\vdots&\ddots&\vdots\\0&0&0&\cdots&1\\a_{i1}^l&a_{i2}^l&a_{i3}^l&\cdots&a_{in_i}^l\end{bmatrix},\quad B_i^l=\begin{bmatrix}0\\0\\ \vdots\\0\\b_{in_i}^l\end{bmatrix} \tag{3}$$

$A_{ij}^l\in R^{n_i\times n_j}$ and $f_i^l(x_i(t),t)$, which represent interconnection matrix and the nonlinear perturbation, respectively.

$$A_{ij}^l=\begin{bmatrix}0&0&0&\cdots&0\\0&0&0&\cdots&0\\ \vdots&\vdots&\vdots&\ddots&\vdots\\0&0&0&\cdots&0\\a_{ij1}^l&a_{ij2}^l&a_{ij3}^l&\cdots&a_{ijn_j}^l\end{bmatrix},\quad f_i^l(x_i(t),t)=\begin{bmatrix}0\\0\\0\\ \vdots\\ \Delta f_{in_i}^l\end{bmatrix} \tag{4}$$

where $a_{ij1}^l,a_{ij2}^l,\cdots,a_{ijn_i}^l$ and $\Delta f_{in_i}^l$ are unknown.

Assumption 1: All pairs $\left(A_i^l,B_i^l\right)$, $i=1,2,\cdots,N$, are controllable.

By using the fuzzy inference method with a singleton fuzzifier, product inference, and center average defuzzifier, the fuzzy model (2) can be expressed as following global model:

$$\begin{cases}\dot{x}_{i1}(t)=x_{i2}(t)\\ \dot{x}_{i2}(t)=x_{i3}(t)\\ \quad\vdots\\ \dot{x}_{in_i}(t)=\sum_{l=1}^{r_i} h_i^l[\left(a_{i1}^l x_{i1}(t)+a_{i2}^l x_{i2}(t)+\cdots+a_{in_i}^l x_{in_i}(t)\right)+b_{in_i}^l u_i(t)\\ \qquad+\sum_{\substack{j=1\\ j\neq i}}^{N} A_{ij}^l x_j\left(t-\tau_{ij}^l(t)\right)+f_i^l\left(x_i(t),t\right)]\end{cases} \tag{5}$$

where

$$h_i^l\left(z_{iq}(t)\right)=\frac{w_i^l\left(z_{iq}(t)\right)}{\sum_{l=1}^{r_i} w_i^l\left(z_{iq}(t)\right)}\ ;\ w_i^l\left(z_{iq}(t)\right)=\prod_{iq=1}^{n_i} F_{iq}^l\left(z_{iq}(t)\right)$$

and $F_{iq}^l(z_{iq}(t))$ is the grade of membership of $z_{iq}(t)$ in F_{iq}^l. It is seen that

$$w_i^l\left(z_{iq}(t)\right)\geq 0\ ;\ \sum_{l=1}^{r_i} w_i^l\left(z_{iq}(t)\right)\geq 0,\ l=1,\ 2,\ \cdots,\ r_i$$

for all t . Therefore,

$$h_i^l\left(z_{iq}(t)\right)\geq 0;\ \sum_{l=1}^{r_i} h_i^l\left(z_{iq}(t)\right)=1 \quad \text{for all } t.$$

Accordingly, the main control objective of this paper is to utilize the fuzzy control $u_i(t)$ such that the perturbed closed-loop fuzzy time-delay large-scale system is asymptotically stable.

III. Decentralized Adaptive fuzzy sliding mode controller design

In this section, we synthesize the decentralized adaptive fuzzy sliding mode controller to deal with the control problem of large-scale systems with time-delay interconnections and perturbations.

First, let

$$\sum_{\substack{j=1\\ j\neq i}}^{N} A_{ij}^l x_j\left(t-\tau_{ij}^l(t)\right)+f_i^l\left(x_i(t),t\right)=\sum_{j=1}^{N}\Delta_{ij}^l=\begin{bmatrix}0\\0\\0\\ \vdots\\ \sum_{j=1}^{N}\overline{\Delta}_{ij}^l\end{bmatrix}\in R^{n_i\times 1}$$

Then it yields

$$\begin{cases} \dot{x}_{i1}(t) = x_{i2}(t) \\ \dot{x}_{i2}(t) = x_{i3}(t) \\ \vdots \\ \dot{x}_{in_i}(t) = \sum_{l=1}^{r_i} h_i^l [\left(a_{i1}^l x_{i1}(t) + a_{i2}^l x_{i2}(t) + \cdots + a_{in_i}^l x_{in_i}(t)\right) \\ \qquad + b_{in_i}^l u_i(t) + \sum_{j=1}^{N} \overline{\Delta}_{ij}^l] \end{cases} \tag{6}$$

Define the controller as the following form:

$$u_i(t) = \frac{1}{\sum_{l=1}^{r_i} h_i^l b_{in_i}^l} \left[-\sum_{l=1}^{r_i} h_i^l \left(a_{i1}^l x_{i1}(t) + a_{i2}^l x_{i2}(t) + \cdots + a_{in_i}^l x_{in_i}(t)\right) + u_{si}(t) \right] \tag{7}$$

Substituting (7) into (6), we have

$$\begin{cases} \dot{x}_{i1}(t) = x_{i2}(t) \\ \dot{x}_{i2}(t) = x_{i3}(t) \\ \vdots \\ \dot{x}_{in_i}(t) = u_{si}(t) + \sum_{l=1}^{r_i} \sum_{j=1}^{N} h_i^l \overline{\Delta}_{ij}^l \end{cases} \tag{8}$$

The control problem is to drive the state $x_i(t)$ to track a specific desired state $x_{di}(t) = [x_{di1}(t), x_{di2}(t), \cdots, x_{din_i}(t)]^T$, where $\dot{x}_{di\alpha-1} = x_{di\alpha}$ for $\alpha = 1,2,\cdots,n_i$.

Assumption 2: The desired state $x_{di}(t) = [x_{di1}(t), x_{di2}(t), \cdots, x_{din_i}(t)]^T$ for the i th subsystem is continuous and available, and $\|x_{di}(t)\| \le P_i$ with P_i being a known bound.

Let the tracking error of the ith subsystem, $e_i(t) = [e_{i1}(t), e_{i2}(t), \cdots, e_{in_i}(t)]^T$ be

$$e_{i\beta}(t) = x_{i\beta}(t) - x_{di\beta}(t) \tag{9}$$

where $\beta = 1,2,\cdots n_i$, and $i = 1,2,\cdots,N$.

Then, from (7), (8), and (9), the error dynamic system of the ith subsystem can be expressed as

$$\begin{cases} \dot{e}_{i1}(t) = e_{i2}(t) \\ \dot{e}_{i2}(t) = e_{i3}(t) \\ \vdots \\ \dot{e}_{in_i-1}(t) = e_{in_i}(t) \\ \dot{e}_{in_i}(t) = u_{si}(t) + \sum_{l=1}^{r_i} \sum_{j=1}^{N} h_i^l \overline{\Delta}_{ij}^l - \dot{x}_{din_i}(t) \end{cases} \tag{10}$$

In general, the design procedure of variable structure controller can be divided into two phase. First, it is necessary to design a sliding hyperplane for each subsystem, so that not only the sliding motion can occur on that manifold, but also the controlled system will yield the desired dynamic performance. The composite switching hyperplane of the proposed control scheme is defined by letting the composite switching vector $S = [S_1\ S_2 \cdots S_N]^T = 0$, where the sliding functions of each subsystem are selected as the following form:

$$S_i = e_{in_i} + c_{i1} e_{i(n_i-1)} + \cdots + c_{i(n_i-2)} e_{i2} + c_{i(n_i-1)} e_{i1} \tag{11}$$

where $c_k > 0$ for $1 \le k \le n_i - 1$, and c_k, $k = 1,2,\cdots,n_i - 1$ are chosen so that the following polynomial:

$$L(s) = s^{n_i-1} + c_{i1} s^{n_i-2} + \cdots + c_{i(r-2)} s + c_{i(r-1)} \tag{12}$$

is Hurwitz and s is Laplace opertor. Thus, when the state error trajectory reaches the sliding surface $S_i = 0$ and slide along the surface i.e.,

$$e_1^{(n_i-1)} + c_{i1} e_1^{(n_i-2)} + \cdots + c_{i(\gamma-2)} \dot{e}_1 + c_{i(\gamma-1)} = 0 \tag{13}$$

which implies that tracking error tends to zero as $t \to \infty$.

After designing the sliding hyperplane, the next phase of the traditional variable structure controller is to design an appropriate control law such that the sliding hyperplane will attract the trajectories of (10), and these trajectories will remain on the sliding hyperplane for all subsequent time.

Assumption 3: The result after combining time-delay interconnections with perturbation in the system are bounded by

$$\sum_{l=1}^{r_i} \sum_{j=1}^{N} h_i^l \overline{\Delta}_{ij}^l \le \zeta_{i0} + \sum_{j=1}^{N} \zeta_{ij}(x_j) \tag{14}$$

where ζ_{i0} are unknown constants and smooth functions $\zeta_{ij}(x_j)$ are unknown smooth functions with $\zeta_{ij}(0) = 0$.

However, in the (14) that constants ζ_{i0} and smooth functions $\zeta_{ij}(x_j)$ are unknown. To solve these situations, in the following, we employ an adaptive gain $\hat{\zeta}_{i0}$ to adapt unknown constant ζ_{i0} and the fuzzy logic system $\hat{\zeta}_{ij}(x_j|\theta_{ij})$ will be applied for approximating unknown functions $\zeta_{ij}(x_j)$ respectively.

In this case, we replace $\zeta_{ij}(x_j)$ by the fuzzy logic system $\hat{\zeta}_{ij}(x_j|\theta_{ij})$. Choose $\hat{\zeta}_{ij}(x_j|\theta_{ij})$ to be fuzzy systems with singleton fuzzifier, center average defuzzifier, and product inference are the following form:

$$\hat{\zeta}_{ij}(x_j|\theta_{ij}) = \frac{\sum_{l=1}^{r_i} \theta_{ij} \left[\prod_{i=1}^{N} \mu_{F_i^l}(x_j)\right]}{\sum_{l=1}^{r_i} \left[\prod_{i=1}^{N} \mu_{F_i^l}(x_j)\right]} \tag{15}$$

Define fuzzy basis function as

$$\xi^l(x_j) = \frac{\prod_{i=1}^{N} \mu_{F_i^l}(x_j)}{\sum_{l=1}^{r_i} \left(\prod_{i=1}^{N} \mu_{F_i^l}(x_j)\right)} \tag{16}$$

where $\mu_{F_i^l}(x_j)$ are Triangular membership functions.

Then the fuzzy logic system (16) is equivalent to a fuzzy basis function expansion

$$\hat{\zeta}_{ij}(x_j|\theta_{ij}) = \theta^T \xi(x_j) \tag{17}$$

where $\xi(x_j) = [\xi^1(x_j), \cdots, \xi^{r_i}(x_j)]^T$ is a regressive vector with regressor $\xi^M(x_j)$ which is defined as a fuzzy basis function, $\theta = [\theta_{ij}^1, \theta_{ij}^2, \cdots, \theta_{ij}^{r_i}]^T$ are the corresponding parameter vectors of the fuzzy logic system. It has been shown in [32] that fuzzy systems is capable of uniformly approximating any nonlinear function over U to any degree of accuracy if U is compact.

Remark 1: For any given real continuous function $\zeta_{ij}(x_j)$ on a compact set $U \in R^{n_i}$ and arbitrary $\varepsilon > 0$, there exist a fuzzy logic system $\zeta_{ij}(x_j|\theta_{ij})$ such that

$$\sup_{x_j \in U}\left|\zeta_{ij}(x_j) - \zeta_{ij}(x_j|\theta_{ij})\right| < \varepsilon \tag{18}$$

and we can fix all the parameter in $\xi^M(x_j)$ at beginning of the fuzzy basis function expansion design procedure, so that the only free design parameters are θ_{ij}.

Define the optimal parameter vector of fuzzy logic system θ_{ij}^* and the minimum approximation error as

$$\theta_{ij}^* = \arg\min_{\theta_{ij}\in\Omega_{ij}}\left\{\sup_{x_j\in R^{n_i}}\left|\zeta_{ij}(x_j) - \hat{\zeta}_{ij}(x_j|\theta_{ij})\right|\right\} \tag{19}$$

$$\omega_i = \sum_{J=1}^{N}\zeta_{ij}(x_j) - \sum_{j=1}^{N}\hat{\zeta}_{ij}(x_j|\theta_{ij}^*) \tag{20}$$

where Ω_{ij} is the convex compact set, which contain feasible parameter set for θ_{ij}^*, and $\tilde{\theta}_{ij} = \theta_{ij}^* - \theta_{ij}$, denotes the parameter estimation error.

Then, we adapt minimum approximation error ω_i of utilizing the adaptive gain $\hat{\omega}_i$, therefore, we can define adaptation error of minimum approximation error as

$$\tilde{\omega}_i = \omega_i - \hat{\omega}_i \tag{21}$$

and the adaptation error of adaptive gain $\hat{\zeta}_{i0}$ is defined as

$$\tilde{\zeta}_{i0} = \zeta_{i0} - \hat{\zeta}_{i0} \tag{22}$$

By taking the time derivative of both sides of (11), we obtain

$$\begin{aligned}\dot{S}_i &= e_{in_i} + c_1\dot{e}_{in_i-1} + \cdots + c_{n_i-2}\dot{e}_{i2} + c_{n_i-1}\dot{e}_{i1} \\ &= u_{si}(t) + \sum_{l=1}^{r_i}\sum_{j=1}^{N}h_i^l\overline{\Delta}_{ij}^l - \dot{x}_{din_i}(t) + c_1\dot{e}_{in_i-1} + \cdots + c_{n_i-2}\dot{e}_{i2} + c_{n_i-1}\dot{e}_{i1}\end{aligned} \tag{23}$$

Now, the control law can be chosen according to (7) with $u_{si}(t)$ given by:

$$u_{si}(t) = -K_iS_i - \hat{\zeta}_{i0} - \sum_{j=1}^{N}\zeta_{ij}(x_j|\theta_{ij}) + \dot{x}_{din_i} - c_1\dot{e}_{in_i-1}\cdots - c_{n_i-2}\dot{e}_{i2} - c_{n_i-1}\dot{e}_{i1} - \hat{\omega}_i \tag{24}$$

with $K_i > 0$.

Consider the following parameter update laws:

$$\dot{\hat{\omega}}_i = \gamma_1 S_i \tag{25}$$

$$\dot{\theta}_{ij} = \gamma_2 S_i \xi(x_j) \tag{26}$$

$$\dot{\hat{\zeta}}_{i0} = \gamma_3 S_i \tag{27}$$

where γ_1, γ_2, and γ_3 are positive constants specified by the designer. The proposed control law will guarantee the asympotical stability for the error dynamics of (10), and it will be proved in the following theorem.

Theorem 1: For the subsystems consisting of (2), the decentralized adaptive fuzzy control law is chosen as (7) with (24), and consider the adaptation laws (25)-(27). If *Assumptions 1-3* are satisfied, then the following properties are guaranteed:

(i) All the signals in the closed-loop system are bounded.
(ii) The tracking error $e_i(t)$ decreases asymptotically to zero.

Proof: Consider the Lyapunov function candidate for the time-delay large-scale system (2) as

$$V = \sum_{i=1}^{N}V_{1i} + \sum_{i=1}^{N}V_{2i}$$

where $V_{1i} = \frac{1}{2}S_i^2$, $V_{2i} = \left[\frac{1}{\gamma_1}\tilde{\omega}_i^2 + \frac{1}{\gamma_2}\sum_{j=1}^{N}\tilde{\theta}_{ij}^T\tilde{\theta}_{ij} + \frac{1}{\gamma_3}\tilde{\zeta}_{i0}^2\right]$.

Using the control of $u_{si}(t)$, the sliding surface may be expressed as

$$\begin{aligned}\dot{S}_i &= u_{si}(t) + \sum_{l=1}^{r_i}\sum_{j=1}^{N}h_i^l\overline{\Delta}_{ij}^l - \dot{x}_{din_i}(t) + c_1\dot{e}_{in_i-1} + \cdots + c_{n_i-2}\dot{e}_{i2} + c_{n_i-1}\dot{e}_{i1} \\ &\le u_{si}(t) + \zeta_{i0} + \sum_{j=1}^{N}\zeta_{ij}(x_j) - \dot{x}_{din_i} + c_1\dot{e}_{in_i-1} + \cdots + c_{n_i-2}\dot{e}_{i2} + c_{n_i-1}\dot{e}_{i1} \\ &= -K_iS_i + \tilde{\zeta}_{i0} + \tilde{\omega}_i + \sum_{j=1}^{N}\tilde{\theta}_{ij}^T\xi(x_j)\end{aligned} \tag{28}$$

First, we compute time derivative of V_{1i}, we have

$$\dot{V}_{1i} \le S_i\left(-K_iS_i + \tilde{\zeta}_{i0} + \tilde{\omega}_i + \sum_{j=1}^{N}\tilde{\theta}_{ij}^T\xi(x_j)\right) \tag{29}$$

Thus, we get

$$\sum_{i=1}^{N}\dot{V}_{1i} \le \sum_{i=1}^{N}S_i\left(-K_iS_i + \tilde{\zeta}_{i0} + \tilde{\omega}_i + \sum_{j=1}^{N}\tilde{\theta}_{ij}^T\xi(x_j)\right) \tag{30}$$

then, we compute time derivative of V_{2i}, we have

$$\dot{V}_{2i} = \frac{1}{\gamma_1}\tilde{\omega}_i\dot{\tilde{\omega}}_i + \frac{1}{\gamma_2}\sum_{j=1}^{N}\tilde{\theta}_{ij}^T\dot{\tilde{\theta}}_{ij} + \frac{1}{\gamma_3}\tilde{\zeta}_{i0}\dot{\tilde{\zeta}}_{i0}$$

By the fact $\dot{\tilde{\omega}}_i = \dot{\hat{\omega}}_i$, $\dot{\tilde{\theta}}_{ij} = \dot{\theta}_{ij}$ and $\dot{\tilde{\zeta}}_{i0} = \dot{\hat{\zeta}}_{i0}$, the above equation becomes

$$\dot{V}_{2i} = -\frac{1}{\gamma_1}\tilde{\omega}_i\dot{\hat{\omega}}_i - \frac{1}{\gamma_2}\sum_{j=1}^{N}\tilde{\theta}_{ij}^T\dot{\theta}_{ij} - \frac{1}{\gamma_3}\tilde{\zeta}_{i0}\dot{\hat{\zeta}}_{i0} \tag{31}$$

so

$$\sum_{i=1}^{N}\dot{V}_{2i} = \sum_{i=1}^{N}\left[-\frac{1}{\gamma_1}\tilde{\omega}_i\dot{\hat{\omega}}_i - \frac{1}{\gamma_2}\sum_{J=1}^{N}\tilde{\theta}_{ij}^T\dot{\theta}_{ij} - \frac{1}{\gamma_3}\tilde{\zeta}_{i0}\dot{\hat{\zeta}}_{i0}\right] \tag{32}$$

Then

$$\begin{aligned}\dot{V} \le \sum_{i=1}^{N}\Bigg[&-K_iS_i^2 + \tilde{\zeta}_{i0}\left(S_i - \frac{1}{\gamma_3}\dot{\hat{\zeta}}_{i0}\right) + \sum_{j=1}^{N}\tilde{\theta}_{ij}^T\left(S_i\xi(x_j) - \frac{1}{\gamma_2}\dot{\theta}_{ij}\right) \quad (2.24) \\ &+ \tilde{\omega}_i\left(S_i - \frac{1}{\gamma_1}\dot{\hat{\omega}}_i\right)\Bigg]\end{aligned} \tag{33}$$

with adaptations (25)-(27), $\dot{V}$ is given by

$$\dot{V} \le -\sum_{i=1}^{N}K_iS_i^2 \le 0 \tag{34}$$

Thus, we know that $\lim_{t\to\infty}V(t)$ exists, i.e. $V(\infty)$ exists. It is easy to show that $\int_0^\infty \dot{V}(t)dt$ exists. Hence,

$$\int_0^\infty\sum_{i=1}^{N}K_iS_i^2 \le -\int_0^\infty\dot{V}dt = V(0) - V(\infty) < \infty \tag{35}$$

Since $\{V(t)\}$ is convergent, from the above analysis, we have the solutions S_i, $\dot{\hat{\omega}}_i$, θ_{ij}, and $\hat{\zeta}_{ij}$ are bound. Because of the boundedness of all the signals, from (28), we have that $\dot{S}_i$ is bounded.

From (35) and based on the above discussion, this implies that $S_i \in L_2$. Then, according to Barbalat's Lemma, we can get $\lim_{t\to\infty} S_i(t)=0$ and $e_i(t)$ tend to zero at $t \to \infty$. Thus, we conclude that the asymptotic state tracking $x_i \to x_{di}$ can be achieved.

IV. AN EXAMPLE AND SIMULATION RESULTS

In this section, in order to illustrate the above design approach, we give an example to show the effectiveness and feasibility of the proposed method. Now, we consider a large-scale system T which is composed of two fuzzy subsystems T_i as follows:

Subsystem 1:

Rule 1:

If $x_{11}(t)$ is about 0 and $x_{12}(t)$ is about 0,

Then

$$\begin{bmatrix}\dot{x}_{11}(t)\\ \dot{x}_{12}(t)\end{bmatrix}=\begin{bmatrix}0 & 1\\ 0 & -0.1\end{bmatrix}\times\begin{bmatrix}x_{11}(t)\\ x_{12}(t)\end{bmatrix}+\begin{bmatrix}0\\ 1\end{bmatrix}\times u_1+\sum_{j\neq 1}^{2} A_{1j}^1 x_j\left(t-\tau_{ij}(t)\right)+f_1^1(x_i(t),t)$$

Rule 2:

If $x_{11}(t)$ is about ± 1 and $x_{12}(t)$ is about 0,

Then

$$\begin{bmatrix}\dot{x}_{11}(t)\\ \dot{x}_{12}(t)\end{bmatrix}=\begin{bmatrix}0 & 1\\ -3 & -0.1\end{bmatrix}\times\begin{bmatrix}x_{11}(t)\\ x_{12}(t)\end{bmatrix}+\begin{bmatrix}0\\ 1\end{bmatrix}\times u_1+\sum_{j\neq 1}^{2} A_{1j}^2 x_j\left(t-\tau_{ij}(t)\right)+f_1^2(x_i(t),t)$$

Rule 3:

If $x_{11}(t)$ is about ± 2 and $x_{12}(t)$ is about 0,

Then

$$\begin{bmatrix}\dot{x}_{11}(t)\\ \dot{x}_{12}(t)\end{bmatrix}=\begin{bmatrix}0 & 1\\ -12 & -0.1\end{bmatrix}\times\begin{bmatrix}x_{11}(t)\\ x_{12}(t)\end{bmatrix}+\begin{bmatrix}0\\ 1\end{bmatrix}\times u_1+\sum_{j\neq 1}^{2} A_{1j}^3 x_j\left(t-\tau_{ij}(t)\right)+f_1^3(x_i(t),t)$$

Subsystem 2:

Rule 1:

If $x_{21}(t)$ is about 0 and $x_{22}(t)$ is about 0,

Then

$$\begin{bmatrix}\dot{x}_{21}(t)\\ \dot{x}_{22}(t)\end{bmatrix}=\begin{bmatrix}0 & 1\\ 0 & -1.2\end{bmatrix}\times\begin{bmatrix}x_{21}(t)\\ x_{22}(t)\end{bmatrix}+\begin{bmatrix}0\\ 1\end{bmatrix}\times u_2+\sum_{j\neq 1}^{2} A_{2j}^1 x_j\left(t-\tau_{ij}(t)\right)+f_2^1(x_i(t),t)$$

Rule 2:

If $x_{21}(t)$ is about ± 1 and $x_{22}(t)$ is about -1,

Then

$$\begin{bmatrix}\dot{x}_{21}(t)\\ \dot{x}_{22}(t)\end{bmatrix}=\begin{bmatrix}0 & 1\\ -2 & -1.2\end{bmatrix}\times\begin{bmatrix}x_{21}(t)\\ x_{22}(t)\end{bmatrix}+\begin{bmatrix}0\\ 1\end{bmatrix}\times u_2+\sum_{j\neq 1}^{2} A_{2j}^2 x_j\left(t-\tau_{ij}(t)\right)+f_2^2(x_i(t),t)$$

The membership functions for $x_{11}(t)$, $x_{12}(t)$, $x_{21}(t)$, and $x_{22}(t)$ are shown in Fig. 1-2. Moreover, the interconnections and perturbations among two subsystems are given as

$$\sum_{j\neq 1}^{2} A_{1j}^1 x_j\left(t-\tau_{ij}(t)\right)+f_1^1(x_i(t),t)=0.42\times x_{21}(t-2)+0.15\times x_{22}(t-2)$$
$$+0.45\times\sin(x_{11}(t))+0.45\times\sin(x_{12}(t))+1.8$$

$$\sum_{j\neq 1}^{2} A_{1j}^2 x_j\left(t-\tau_{ij}(t)\right)+f_1^2(x_i(t),t)=0.54\times x_{21}(t-2)+0.1\times x_{22}(t-2)$$
$$+0.45\times\sin(x_{11}(t))+0.45\times\sin(x_{12}(t))+1.8$$

$$\sum_{j\neq 1}^{2} A_{1j}^3 x_j\left(t-\tau_{ij}(t)\right)+f_1^3(x_i(t),t)=0.38\times x_{21}(t-2)+0.15\times x_{22}(t-2)$$
$$+0.45\times\sin(x_{11}(t))+0.45\times\sin(x_{12}(t))+1.8$$

$$\sum_{j\neq 1}^{2} A_{2j}^1 x_j\left(t-\tau_{ij}(t)\right)+f_2^1(x_i(t),t)=0.28\times x_{11}(t-2)+0.15\times x_{12}(t-2)$$
$$+0.38\times\sin(x_{21}(t))+0.38\times\sin(x_{22}(t))+2.2$$

$$\sum_{j\neq 1}^{2} A_{2j}^2 x_j\left(t-\tau_{ij}(t)\right)+f_2^2(x_i(t),t)=0.33\times x_{11}(t-2)+0.27\times x_{12}(t-2)$$
$$+0.38\times\sin(x_{21}(t))+0.38\times\sin(x_{22}(t))+2.2$$

The desired state are selected as $x_{d11}(t)=\sin(t)$, $x_{d12}(t)=\cos(t)$, $x_{d21}(t)=\sin(t)$, and $x_{d22}(t)=\cos(t)$. The main objective is to use the proposed control strategy to design a controller $u_i(t)$ such that the state trajectory $x_i(t)$ of each subsystem can track the desired state $x_{di}(t)$.

In according with (11), the decentralized switching manifolds for the two subsystems are chosen as follows:

$$S_1 = e_{12}(t)+c_{11}e_{11}(t)$$
$$S_2 = e_{22}(t)+c_{21}e_{21}(t)$$

where $c_{11}=c_{21}=8$. The decentralized controller $u_i(t)$ of each subsystem is designed in accordance with (7) and (24), where we set $K_1=K_2=9$. The adaptive laws are governed by (25), (26), and (27), then the designed parameters are set to $\gamma_1=1$, $\gamma_2=9$, and $\gamma_3=2$.

The results of simulation are shown in Figs. 3-4 with $x_1(0)=[x_{11}(0)\ \ x_{12}(0)]=[2\ \ 0]$, $x_2(0)=[x_{21}(0)\ \ x_{22}(0)]=[1\ \ 0]$, and $\hat{\zeta}_{10}^1=\hat{\zeta}_{10}^2=\hat{\zeta}_{10}^3=\hat{\zeta}_{20}^1=\hat{\zeta}_{20}^2$=0. Figs. 3-4 show the trajectories of the states $x_1(t), x_2(t)$ and desired states $x_{d1}(t), x_{d2}(t)$.

V. CONCLUSIONS

This paper has presented a stability analysis and design method for a class of time-delay large-scale systems based on Takagi-Sugeno fuzzy modeling and control approach. The proposed controller guarantees that the tracking error decreases asymptotically to zero and all the signals in the closed-loop system are bounded. Finally, an example and simulation results are provided to illustrate the versatility and performance of the proposed method.

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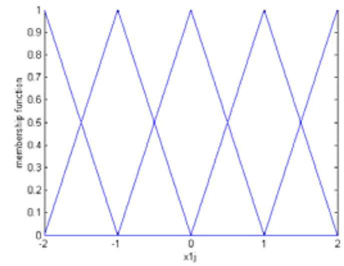

Fig. 1. The membership function of x_{11} and x_{12}

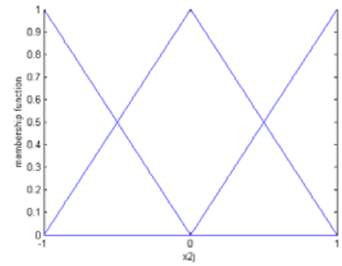

Fig. 2. The membership function of x_{21} and x_{22}

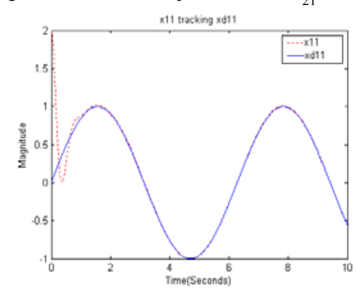

Fig. 3. Trajectory of the state $x_{11}(t)$ and desired state $x_{d11}(t)$

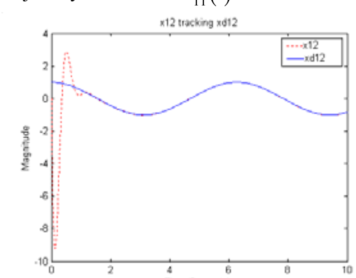

Fig. 4. Trajectory of the state $x_{12}(t)$ and desired state $x_{d12}(t)$