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A fault tolerant sliding mode controller for accommodating actuator failures.

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Abstract— **This paper addresses the actuator failure compensation problem, and considers an uncertain linear plant which is supposed to undergo unknown failures causing the plant input components to be stuck at some uncertain but bounded time functions. A control policy, based on sliding mode, is presented, which guarantees the detection of the fault and the identification of the failed component by means of a suitable test input. Once the failed component has been identified, the control law is reconfigured, redistributing the control activity among the controllers still working. The proposed controller has been tested by simulation on a benchmark problem.**

Keywords: Actuator Failure Compensation Control, Sliding Mode Control, Switching Control, Robust Control.

I. Introduction

Control of systems with component failures is a long-standing and challenging problem, which has been given a number of possible, though partial, solutions along years. As is well known, in fact, system deterioration caused by actuator failures may produce catastrophic effects (e.g. in flight control systems), since it is not known either when and how many failures will occur, or even the failed actuators output.

A number of results are available in the literature on actuator failure compensation control systems, exploring different directions. Indeed, one of the major research thrusts relies on adaptive control (see e.g. [13] [11] [12] [15] [16] and references therein), where both direct and indirect control schemes have been proposed. Neural Networks have been used as well in direct adaptive control schemes [2]. Another approach for controlling systems with actuator or component failures, which received recently some attention, is based on the multiple models, switching and tuning methodology [6] [1] [10], where a number of different identification models are run in parallel and the model closest to the current operating point is sought and switched to. On the contrary, few results using Variable Structure Control (VSC) [14] are available in the literature relative to actuator failure compensation [7], [3], though the extensive adoption of this technique in robust control design is widely recognized.

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The scenario addressed in this paper contains an uncertain linear plant undergoing unknown failures, which cause the plant input components to be stuck at some uncertain but bounded time functions [8] [9]. The presented control law, based on VSC, guarantees robust asymptotic stability in the presence of actuator failures, and allows to reduce control efforts distributing them on the actuators still working.

The control policy presented, able to detect, identify and compensate faults in the actuators, is constituted by distinct phases. First, the occurrence of a fault is detected simply monitoring sliding surfaces: when the state leaves the sliding hyperplane, it means that a fault has occurred in one of the actuators components. Once the presence of a fault has been detected, a particular test input is applied to the plant, in order to identify in which actuator and in which component the fault has occurred. It is worth noting that the test input does not compromise stability, since it guarantees sliding mode existence except in the case when we try to apply it to the failed component, and in this case, due to sliding mode properties, it is applied only for an arbitrarily small time interval. Finally, when the failed component is identified, a fault accommodating control law is activated. It distributes the control input among the remaining working actuators components, in order to reduce control efforts and ensure stability. The effectiveness of the proposed control law has been tested by simulation on an IFAC benchmark problem of a mechanical system, providing satisfactory results.

The paper is organized as follows. Section II contains the problem statement, while Section III provides a very conservative solution to the addressed control problem. This first solution, though conservative, is useful for the developments contained in Section IV, where a fault detection and identification device is proposed. The fault accommodating controller is finally presented in Section V, and theoretical results described are validated by simulation in Section VI using an IFAC benchmark problem.

II. Problem statement

Consider the following continuous-time, time invariant, uncertain MIMO plant $S_{\mathbf{q}} \stackrel{\text{def}}{=} \{\hat{\mathbf{A}}(\mathbf{q}),\, \hat{\mathbf{D}},\, \hat{\mathbf{C}}\}$ described by:

$$S_{\mathbf{q}} : \begin{cases} \dot{\hat{\mathbf{x}}} = \hat{\mathbf{A}}(\mathbf{q})\hat{\mathbf{x}} + \hat{\mathbf{D}}\mathbf{u} \\ \hat{\mathbf{y}} = \hat{\mathbf{C}}\hat{\mathbf{x}} \end{cases} \tag{1}$$

where: $\hat{\mathbf{x}} = [\hat{x}_1 \cdots \hat{x}_n]^T \in \mathbb{R}^n$ is the state vector available for measurement; $\mathbf{u} = [\mathbf{u}_1^T\ \mathbf{u}_2^T\ \ldots\ \mathbf{u}_s^T]^T$ is the control input whose components $\mathbf{u}_j \in \mathbb{R}^p$, $j = 1 \ldots s$, represent the s available actuators, and $\hat{\mathbf{A}}(\mathbf{q}) : \mathbb{R}^l \rightarrow \mathbb{R}^{n\times n}$ is the uncertain state matrix. Note that each entry $[\mathbf{u}_j]_i$, $i = 1 \ldots p$, of the j-th component, $j = 1 \dots s$ may fail during system operation[1]. The vector of uncertain parameters $\mathbf{q}$ takes values in a closed set $Q \subset \mathbb{R}^l$ with known bounds. Given a nominal value $\bar{\mathbf{q}}$ of $\mathbf{q}$, define $\mathbf{\Delta}\hat{\mathbf{A}}(\mathbf{q},\bar{\mathbf{q}}) \stackrel{\text{def}}{=} \hat{\mathbf{A}}(\mathbf{q}) - \hat{\mathbf{A}}(\bar{\mathbf{q}})$, hence $\hat{\mathbf{A}}(\mathbf{q}) = \hat{\mathbf{A}}(\bar{\mathbf{q}}) + \mathbf{\Delta}\hat{\mathbf{A}}(\mathbf{q},\bar{\mathbf{q}})\ \forall \mathbf{q} \in Q$. Accordingly, it is useful to rewrite the system $S_{\mathbf{q}}$ as:

$$\dot{\hat{\mathbf{x}}} \;=\; [\hat{\mathbf{A}}(\bar{\mathbf{q}}) + \mathbf{\Delta}\hat{\mathbf{A}}(\mathbf{q},\bar{\mathbf{q}})]\hat{\mathbf{x}} + \hat{\mathbf{D}}\mathbf{u} \tag{2}$$

where the term $\mathbf{\Delta}\hat{\mathbf{A}}(\mathbf{q},\bar{\mathbf{q}})$ is assumed to represent a matching uncertainty term. The considered type of actuator failures is the following:

$$[\mathbf{u}_k(t)]_i = \tilde{u}_k^{(i)}(t),\ \ \forall\, t \geq t_k^{(i)},\ \ k \in \{1 \dots s\},\ i \in \{1 \dots p\} \tag{3}$$

where $\tilde{u}_k^{(i)}(t)$ is uncertain but bounded:

$$|\tilde{u}_k^{(i)}(t)| \leq \bar{u}, \quad k \in \{1 \dots s\}, \quad i \in \{1 \dots p\} \tag{4}$$

and the failure times $t_k^{(i)}$ are unknown. In other words, abrupt failures are considered [11], causing the actuator output components to be stuck at some uncertain but bounded time functions at unknown time instants.

Analogously to [11], the following assumption is introduced:

Assumption 2.1: A number of failures occur for the actuators $\mathbf{u}_j \in \mathbb{R}^p$, $j = 1 \dots s$ such that at least p fault-free components exist, as to guarantee controllability of the nominal system.

Being the plant matrix $\hat{\mathbf{A}}(\mathbf{q})$ and the actuator failure $\tilde{u}_k^{(i)}(t)$ uncertain, $k = 1 \dots s$, $i = 1 \dots p$, and considering unknown failure pattern and time, the task of the actuator failure compensation technique addressed in this paper is to provide a feedback control law $\mathbf{u}$ such that asymptotical stabilization is achieved when the number of faults occurred in the elements of each actuator $\mathbf{u}_j \in \mathbb{R}^p$, $j = 1 \dots s$ fulfills Assumption 2.1.

Therefore, the addressed control problem can be summarized as follows:

Problem 1: This problem is finding a state feedback controller $\mathbf{u}$ ensuring the global asymptotic stabilization of the plant $S_{\mathbf{q}}$ independently of $\mathbf{q} \in Q$, in the presence of actuator failures (3)-(4) of unknown pattern and time.

A simple sliding mode controller will be presented first, designed in the worst case condition when the actuators have undergone the maximum number of failures. This very conservative solution, which is expected to produce an unnecessarily high control activity, will be successively improved adopting a fault identification scheme, of arbitrary duration. Once the failed actuator has been identified, a "tailored" controller guaranteeing recovery from failure will be proposed, producing reduced control efforts.

III. An over-conservative controller for actuator failure compensation.

In this section, a control law solving Problem 1 will be proposed. Indeed, it provides a very conservative solution to the addressed control problem. Its statement, however, is useful for the developments to follow.

Matrix $\hat{\mathbf{D}}$ in (2) can be given the following block structure:

$$\hat{\mathbf{D}} = \left[\hat{\mathbf{B}}\ \hat{\mathbf{B}}_1 \dots \hat{\mathbf{B}}_{s-1}\right], \tag{5}$$

where $\hat{\mathbf{B}} \in \mathbb{R}^{n \times p}$ is full rank ($p < n$), and $\hat{\mathbf{B}}_j \in \mathbb{R}^{n \times p}$, $j = 1 \dots s-1$, are full rank, too, with $\hat{\mathbf{B}}_j \in span\ \left(\hat{\mathbf{B}}\right)$.

Remark 3.1: Assuming the structure (5) obeys to simple considerations. In a fault tolerant control system, the plant is usually equipped with redundant actuators. It is likely that extra actuators are equal or, at least, parallel to the main one, all of them sharing a similar matrix structure.

In view of (5), due to Assumption 2.1 and to Remark 3.1, one can write $\hat{\mathbf{B}}_j = \hat{\mathbf{B}}\mathbf{K}_j$, $j = 1, \dots, s-1$ for some nonsingular $\mathbf{K}_j \in \mathbb{R}^{p \times p}$. Without loss of generality, it will be assumed in the following that $\mathbf{K}_j = \alpha_i \mathbf{I}_p$, $j = 1 \dots s-1$, being $\mathbf{I}_p$ the p dimensional identity matrix. Therefore matrix $\hat{\mathbf{D}}$ is given by:

$$\hat{\mathbf{D}} = \left[\hat{\mathbf{B}}\ \ \alpha_1\hat{\mathbf{B}} \dots\ \alpha_{s-1}\hat{\mathbf{B}}\right], \tag{6}$$

Consider the following system $\Sigma_{\mathbf{q}} = (\hat{\mathbf{A}}(\mathbf{q}), \hat{\mathbf{B}})$, driven by the j-th actuator only (and hereafter named Single Actuator System (SAS)):

$$\Sigma_{\mathbf{q}}: \quad \dot{\hat{\mathbf{z}}} = [\hat{\mathbf{A}}(\bar{\mathbf{q}}) + \mathbf{\Delta}\hat{\mathbf{A}}(\mathbf{q},\bar{\mathbf{q}})]\hat{\mathbf{z}} + \hat{\mathbf{B}}\mathbf{v} \tag{7}$$

with $\mathbf{v} = \alpha_{j-1}\mathbf{u}_j \in \mathbb{R}^p$, $\alpha_{j-1} \in \mathbb{R}$, $j = 1 \dots s$, $\alpha_0 = 1$, which fulfills the controllability property $\forall \mathbf{q} \in Q$ by Assumption 2.1. Under this hypothesis, it is known that there exists a smooth change of coordinates: $\mathbf{z} = \begin{bmatrix} \mathbf{z}_1 \\ \mathbf{z}_2 \end{bmatrix} = \mathbf{M}\hat{\mathbf{z}}$ such that $\mathbf{z}_1 \in \mathbb{R}^{n-p}$, $\mathbf{z}_2 \in \mathbb{R}^p$ and $\mathbf{M}\hat{\mathbf{B}} = \begin{bmatrix} \mathbf{0} \\ \mathbf{b} \end{bmatrix}$, with $\mathbf{b} \in \mathbb{R}^{p \times p}$. Applying this transformation, and assuming that uncertainties satisfy matching conditions, system (7) (now denoted $\Sigma'_{\mathbf{q}}$) becomes:

$$\Sigma'_{\mathbf{q}}: \quad \dot{\mathbf{z}} = \mathbf{A}(\mathbf{q})\mathbf{z} + \mathbf{B}\mathbf{v} = [\mathbf{A}(\bar{\mathbf{q}}) + \mathbf{\Delta}\mathbf{A}(\mathbf{q})]\,\mathbf{z} + \mathbf{B}\mathbf{v} \tag{8}$$

where $\mathbf{A}(\mathbf{q}) = \mathbf{M}\hat{\mathbf{A}}(\mathbf{q})\mathbf{M}^{-1}$, $\mathbf{\Delta}\mathbf{A}(\mathbf{q}) = \mathbf{M}\mathbf{\Delta}\hat{\mathbf{A}}(\mathbf{q},\bar{\mathbf{q}})\mathbf{M}^{-1}$, $\mathbf{B} = \mathbf{M}\hat{\mathbf{B}} = \begin{bmatrix} \mathbf{0} \\ \mathbf{b} \end{bmatrix}$, and the couple $(\mathbf{A}_{11}, \mathbf{A}_{12})$ is controllable by assumption. As well known [5], a matrix $\mathbf{R} = [\mathbf{R}_1\ \mathbf{R}_2] \in \mathbb{R}^{p \times n}$ can be chosen such that, when a sliding motion is achieved on the following sliding surface:

$$\bar{\sigma} = (\mathbf{R}\mathbf{B})^{-1}\mathbf{R}\mathbf{z} = \mathbf{0} \tag{9}$$

the reduced order system ($\mathbf{z}_1$) has assigned stable eigenvalues [5], and, as a consequence, system $\Sigma'_{\mathbf{q}}$ is stable, too.

This result can be easily extended to the original system $S_{\mathbf{q}}$, i.e. to the system driven by the entire set of actuators. Applying the same state transformation: $\mathbf{x} = \begin{bmatrix} \mathbf{x}_1 \\ \mathbf{x}_2 \end{bmatrix} = \mathbf{M}\hat{\mathbf{x}}$,

[1]Given a matrix $\mathbf{V}$, its i-th row will be denoted hereafter by $[\mathbf{V}]_i$, and its (i,j)-th entry by $[\mathbf{V}]_{ij}$.

the system $S'_{\mathbf{q}}$, transformed from $S_{\mathbf{q}}$, is given by:

$$S'_{\mathbf{q}}: \begin{array}{l} \dot{\mathbf{x}} = \mathbf{A}(\mathbf{q})\mathbf{x} + \mathbf{B}\left(\sum_{j=1}^{s} \alpha_{j-1}\mathbf{u}_j\right) = \\ \left[\mathbf{A}(\bar{\mathbf{q}}) + \mathbf{\Delta A}(\mathbf{q})\right]\mathbf{x} + \mathbf{B}\left(\sum_{j=1}^{s} \alpha_{j-1}\mathbf{u}_j\right) \end{array} \quad (10)$$

Moreover, analogously to (9), the following sliding surface can be introduced:

$$\sigma = (\mathbf{RB})^{-1}\mathbf{Rx} = \mathbf{0} \quad (11)$$

and it is straightforward to show that the control law described in the following Lemma guarantees the asymptotic stabilization of system $S'_{\mathbf{q}}$ (and, as a consequence, of $S_{\mathbf{q}}$) when all the actuators are working (i.e. with no faults occurring).

Lemma 3.1: Consider the system $S'_{\mathbf{q}}$ described by (10) with $\mathbf{q} \in Q$ and with all the actuators working. The following control law:

$$\begin{aligned} \mathbf{u} = \mathbf{u}^{eq} + \mathbf{u}^{n} &= \left[(\mathbf{u}_1^{eq})^T \ (\mathbf{u}_2^{eq})^T \ \ldots \ (\mathbf{u}_s^{eq})^T\right]^T + \\ &+ \left[(\mathbf{u}_1^{n})^T \ (\mathbf{u}_2^{n})^T \ \ldots \ (\mathbf{u}_s^{n})^T\right]^T \end{aligned} \quad (12)$$

where:

$$\alpha_{j-1}\left[\mathbf{u}_j^{eq}\right] = -\frac{(\mathbf{RB})^{-1}\mathbf{RA}(\bar{\mathbf{q}})\mathbf{x}}{s} \quad (13)$$

$$\alpha_{j-1}\left[\mathbf{u}_j^{n}\right]_i = -\frac{G_M||\mathbf{x}|| + \eta}{s} sign\left([\sigma]_i\right), \quad (14)$$

with $j = 1 \ldots s$, $i = 1 \ldots p$, $\eta \geq 0$ and:

$$G_M \stackrel{\text{def}}{=} \max_{\mathbf{q}\in Q}\left\{||(\mathbf{RB})^{-1}\mathbf{R\Delta A}(\mathbf{q})||\right\} \quad (15)$$

guarantees the global asymptotic stabilization $\forall \mathbf{q} \in Q$.
The proof of the Lemma is straightforward, since it can be easily verified that the sliding mode existence condition is fulfilled by control law (13)-(14) [14]. The design constant η, if chosen strictly positive, can be used to guarantee the achievement of the sliding motion within a finite time [14].

Taking a more conservative worst case, it is easy to determine a suitable controller managing the case when a failure of the form (3) has occurred at an unknown time instant $t_k^{(i)}$, as stated in the following result.

Lemma 3.2: Consider the system $S'_{\mathbf{q}}$ described by (10), with $\mathbf{q} \in Q$, subject to actuator failures (3)-(4) of unknown pattern and time. The following control law:

$$\begin{array}{l} \alpha_{j-1}\left[\mathbf{u}_j\right]_i = -\left(H_M||\mathbf{x}|| + \eta + \bar{\alpha}(s-1)\bar{u}\right) sign\left([\sigma]_i\right), \\ \quad j = 1 \ldots s, \ i = 1 \ldots p \end{array} \quad (16)$$

with $\bar{\alpha} \stackrel{\text{def}}{=} \max_{i=1\ldots s} |\alpha_{i-1}|$, and:

$$\begin{aligned} H_M &\stackrel{\text{def}}{=} \max_{\mathbf{q}\in Q}\left\{||(\mathbf{RB})^{-1}\mathbf{RA}(\mathbf{q})||\right\} \\ &= \max_{\mathbf{q}\in Q}\left\{\left|\left|(\mathbf{RB})^{-1}\mathbf{R}\left[\mathbf{A}(\bar{\mathbf{q}}) + \mathbf{\Delta A}(\mathbf{q})\right]\right|\right|\right\} \end{aligned}$$

guarantees the global asymptotic stabilization $\forall \mathbf{q} \in Q$.
The proof is omitted for brevity.

IV. FAILURE DETECTION AND IDENTIFICATION

As already discussed, the controller (16) is expected to produce an unnecessarily high control activity, having been designed taking a very "pessimistic" worst case. It paves the way, however, to the design of a fault detection and identification device, based on the check of the variables $|[\sigma]_i|$, $i = 1 \ldots p$, at arbitrary time instants, and the successive supply of a suitable test input, of arbitrary duration as well.

Indeed, when sliding mode control is used, one can consider the increase of any component of the sliding surface $|[\sigma]_i|$, $i = 1 \ldots p$ as symptomatic of the occurrence of a failure. In fact, it corresponds to a violation of inequalities ensuring the existence of a sliding motion, and of course mean that the sliding mode controller is not effective as a consequence of a fault. Though the presence of a failure does not necessarily produce an (even temporary) increase of the quantities $|[\sigma]_i|$, $i = 1 \ldots p$, one should keep in mind that, if this were the case, such a fault would not be "serious enough" to affect controller performances, as control requirements were still satisfied along with inequalities ensuring the existence of the sliding motion itself.

The task of identifying which component of which actuator has undergone a failure is more involved. Correct identification is however fundamental for the design of a suitable failure compensation controller. To pursue this goal, we propose here a simple procedure based on the sequential check of the actuator components using the test input defined in the following Theorem. It should be noticed that such test input is applied for a time interval of arbitrary duration, thus making the entire identification phase of arbitrary duration as well.

Before stating the Theorem, the following useful quantities are defined:

$$[\Gamma]_i = \left[(\mathbf{RB})^{-1}\mathbf{RA}(\bar{\mathbf{q}})\mathbf{x}\right]_i \quad (17)$$

$$\bar{\delta}(\mathbf{x}) = \frac{[\Gamma]_i + (s-2)(\bar{\alpha}\bar{u} + G_M||\mathbf{x}|| - \eta)}{s-1} \quad (18)$$

$$\delta_1(\mathbf{x}) = G_M||\mathbf{x}|| + \bar{\alpha}\bar{u} + \eta \quad (19)$$

$$[\delta_2(\mathbf{x})]_i = \frac{-[\Gamma]_i + \bar{\alpha}\bar{u} + (s-2)(\eta - G_M||\mathbf{x}||)}{s-1} \quad (20)$$

$$+ \frac{\varepsilon}{s-2} \quad (21)$$

$$[\delta_3(\mathbf{x})]_i = \frac{s[\Gamma]_i}{(s-1)(s-2)} \quad (22)$$

$$+ \frac{\bar{\alpha}\bar{u} + (s-2)\eta + sG_M||\mathbf{x}||}{s-1} + \frac{\varepsilon}{s-2} \quad (23)$$

$$i = 1, \ldots, p \quad s > 2$$

Theorem 4.1: Consider the system $S'_{\mathbf{q}}$ described by (10), with $\mathbf{q} \in Q$ and $s > 2$. Assume that the entry $[\mathbf{u}_k]_\ell$ (i.e. the ℓ-th component of $\mathbf{u}_k$) underwent a failure of the type (3), and that it has been detected at $t = T$. The failed actuator component can be identified within a finite arbitrary time by applying the following iterative procedure.

1) Initialize $r = 1$.

2) Apply the following control input for an arbitrary time interval ΔT_r:

$$\begin{array}{rcl}\mathbf{u}_r & = & \mathbf{u}^{test} \\ \mathbf{u}_j & = & \hat{\mathbf{u}}_j^{eq} + \hat{\mathbf{u}}_j^n, \quad j = 1\dots s,\ j \neq r\end{array} \tag{24}$$

where:

$$\begin{array}{rcl}\alpha_{j-1}\left[\mathbf{u}^{test}\right]_i & = & -\left\{[\delta(\mathbf{x})]_i + \bar{\delta}(\mathbf{x}) + \varepsilon\right\} sign([\sigma]_i) \\ & & i = 1\dots p \quad \varepsilon > 0 \qquad (25) \\ {[\delta(\mathbf{x})]_i} & = & \theta_1 \max\left\{\delta_1(\mathbf{x}), [\delta_2(\mathbf{x})]_i, [\delta_3(\mathbf{x})]_i\right\} \\ & & i = 1\dots p \quad \theta_1 > 1 \qquad (26) \\ \alpha_{j-1}\hat{\mathbf{u}}_j^{eq} & = & -\dfrac{(\mathbf{RB})^{-1}\mathbf{RA}(\bar{\mathbf{q}})\mathbf{x} + \alpha_{r-1}\mathbf{u}^{test}}{s-2}, \\ & & j = 1\dots s,\ j \neq r \qquad (27) \\ \alpha_{j-1}\left[\hat{\mathbf{u}}_j^n\right]_i & = & -\dfrac{[\delta(\mathbf{x})]_i}{s-2} sign\left([\sigma]_i\right), \\ & & j = 1\dots s,\ j \neq r,\ i = 1\dots p \qquad (28)\end{array}$$

3) Check the variables $|[\sigma]_i|$, $i = 1\dots p$ at $t = T + \Delta T_r$. If $|[\sigma]_i|$ is increasing, then stop the procedure since the failure occurred in the i-th component of the r-th actuator; otherwise increase the index $r = r + 1$ and repeat the test (go to step 2), until $r = s$.

Proof. Only a short sketch of the proof will be presented below, for the sake of brevity. To show that the application of the above procedure after the occurrence of a fault ensures the failure identification within a finite time, it is enough to analyze the effects of the controller (24) on the fulfillment of the sliding mode existence condition for $[\sigma]_i$, $i = 1, \dots, p$. Four different cases can occur. It can be shown that the adoption of the control law (24) induces the increase of $|[\sigma]_\ell|$ only, i.e. an increase of some $[\sigma]_i$, $i = 1, \dots, p$ can be produced by the test function $[\mathbf{u}^{test}]$ only when we try to apply it to the failed ℓ-th component of the actuator $\mathbf{u}_k$. ■

Remark 4.1: The previous result has been given with reference to the occurrence of the first failure, for the sake of simplifying notation. However, it is straightforward to extend Theorem 4.1 in order to cope with multiple faults occurring in sequence (not simultaneously). Moreover, the apparent singularity of Theorem 4.1 for $s = 2$ can be easily removed since it can be restated (and simplified) in the case $s = 2$.

Remark 4.2: It is worthwhile to notice that the test input of Theorem 4.1 can be theoretically applied for an arbitrarily short time, hence fault identification can be performed within an interval arbitrarily short. In practice, however, the presence of a suitable dwell time will be probably needed.

Remark 4.3: Note that during the application of $\mathbf{u}^{test}$ for $r = 1\dots s$, quantities $[\sigma]_i$ converge to zero, except for $r = k$, $i = \ell$. That means that the failed component search does not compromise system stability. Furthermore the test input is applied for an arbitrarily short time ΔT_r.

Remark 4.4: It could be argued that is difficult to guarantee, in practice, that the state leaves the sliding hyperplane when a fault has occurred in one of the actuators, as conservative theoretical bounds are usually given for the magnitude of the switched term required to attain and maintain sliding. Indeed, if the fault does not perturb the sliding motion, this simply means that it is not "serious enough" to affect the attainment of control requirements. Moreover, external disturbance signals or system uncertainties have the potential to let the state leave the sliding hyperplane, but this cannot occur if they have been accounted for during design.

V. A FAULT ACCOMMODATING CONTROLLER

Once a number n_f of failed components have been detected, the following p sets can be defined:

$$\mathcal{K}_i = \{k \in [1\dots s] : [\mathbf{u}_k]_i \quad failed\,\} \quad i = 1\dots p \tag{29}$$

Denote with f_i, $i = 1\dots p$, $0 \leq f_i \leq s$, the cardinality of $\mathcal{K}_i$, i.e. the number of failed components having the i-th position. A proper fault accommodating control law can be easily derived analogously to Lemma 3.1 and Lemma 3.2, taking into account the sets (29), as stated in the following Corollary.

Corollary 5.1: Consider the system $S'_{\mathbf{q}}$ described by (10), with $\mathbf{q} \in Q$, and assume that a number of faults (3)-(4) have occurred, properly detected and described by (29). The following control law:

$$\mathbf{u} = \mathbf{u}^{eq} + \mathbf{u}^n$$

with:

$$\begin{array}{rcl}\alpha_{j-1}\left[\mathbf{u}_j^{eq}\right]_i & = & -\dfrac{\left[(\mathbf{RB})^{-1}\mathbf{RA}(\bar{\mathbf{q}})\mathbf{x}\right]_i}{s - f_i} \\ & & j = 1\dots s,\ i = 1\dots p\end{array} \tag{30}$$

and:

$$\begin{array}{rcl}\alpha_{j-1}\left[\mathbf{u}_j^n\right]_i & = & -\dfrac{G_M\|\mathbf{x}\| + \eta + \bar{\alpha} f_i \bar{u}}{s - f_i} sign\left([\sigma]_i\right), \\ & & j = 1\dots s,\ i = 1\dots p\end{array} \tag{31}$$

guarantees the global asymptotic stabilization $\forall \mathbf{q} \in Q$. The proof is omitted for brevity.

VI. SIMULATION RESULTS

Theoretical results described in Section III and IV have been validated by simulation using the IFAC benchmark problem #90-09 [4]. It describes a linearized model of a cascade of i ($i = 2$ in our case) inverted pendula, where all nominal point masses and links are $m_i = 1\ Kg$, $l_i = 1\ m$, respectively, and the input vector $\mathbf{u}$ denotes torques about the respective pivots. The system state space representation is given by equation (2) where:

$$\hat{\mathbf{A}}(\mathbf{q}) = \begin{bmatrix} 0 & 1 & 0 & 0 \\ p_1 & 0 & -p_1 & 0 \\ 0 & 0 & 0 & 1 \\ -p_1 & 0 & p_2 & 0 \end{bmatrix}$$

$$\hat{\mathbf{B}} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & -2 & 1 & -2 & 1 & -2 & 1 & -2 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ -2 & -5 & -2 & -5 & -2 & -5 & -2 & -5 \end{bmatrix}$$

and $\mathbf{q} = [p_1\ p_2]^T$ with nominal values $\mathbf{q} : p_1 = 9.8$, $p_2 = 29.40$ and variation ranges: $p_1 \in [8, 11]$, $p_2 \in [28, 31]$.

To validate the approach proposed here, and differently from what is reported in [4], four replicas of the original input distribution matrix, ensuring redundancy for reliability reasons, are assumed to be available. Note that the plant is "highly unstable and difficult to control" [4] even in the nominal case. Simulation results are reported for the values $p_1 = 9$, $p_2 = 28.6$.

The sliding surface (9) has been designed with $\mathbf{R}_1 = diag(1, 2)$, $\mathbf{R}_2 = \mathbf{I}$. The following initial condition has been considered: $\mathbf{x}(0) = \left[\ 0.2\ 0\ 0.2\ 0\ \right]^T$, and a boundary layer of width 0.01 has been used to avoid chattering. The following failures have been considered to occur:

$$\begin{array}{l} \left[\mathbf{u}_3\right]_1(t) = 3, \quad \forall\, t \geq 0.05\ s, \\ \left[\mathbf{u}_1\right]_1(t) = 3, \quad \forall\, t \geq 0.15\ s. \end{array}$$

Fault identification has been performed applying recursively the test input (25) for $\Delta T = 0.005\ s$. Results obtained with the fault accommodating controller reported in this paper are reported in Fig. 1-4, showing the evolution of the four state variables, in Fig.5, reporting the control input $\left[\mathbf{u}_3\right]_1$ (only this control variable has been shown for the sake of brevity) and in Fig.6, where the components of sliding surface (9) are plotted.

The simulation results reported clearly show the control policy adopted by the fault tolerant controller. During the first phase (up to about $0.051\ \ s$), the decreasing of the sliding surface prevents the controller from detecting a fault. After the failure on $\left[\mathbf{u}_3\right]_1$ occurs, however, the sliding surface stops decreasing, and the failure is properly detected. As a consequence, the failure detection procedure of Theorem 4.1 begins and the test input is applied, causing a temporary further increase of the sliding surface. Once the failed actuator (nearly at $0.08\ \ s$) is found, and the fault accommodating controller is supplied, the sliding surface begins to decrease again.

A similar behavior is repeated again after the failure on $\left[\mathbf{u}_1\right]_1$ occurs, as shown by the reported simulations. This second fault causes a strong increase in the sliding surface (affecting also the state variables), nevertheless also this failure is properly managed by the fault tolerant controller of Corollary 5.1, and finally stabilization is achieved using the only two actuators still available.

VII. Conclusions

The actuator failure compensation problem has been addressed in this paper. A linear uncertain plant has been considered, in the presence of actuator failures of uncertain pattern and time.

A state feedback fault tolerant controller, based on sliding mode control, has been proposed, and has been shown to guarantee the global asymptotic stability of the uncertain system subject to unknown failures. The fault accommodating controller is preceded by a failure identification phase when, after a failure has been detected, a test input is applied for an arbitrarily short time. Theoretical results have been supported by simulation of a mechanical system used as a benchmark problem.

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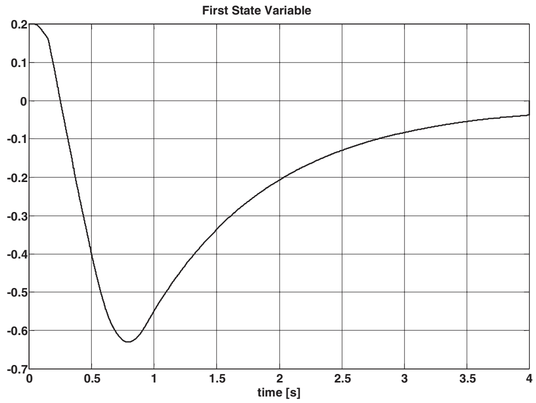

Fig.1 - **State variable** x_1

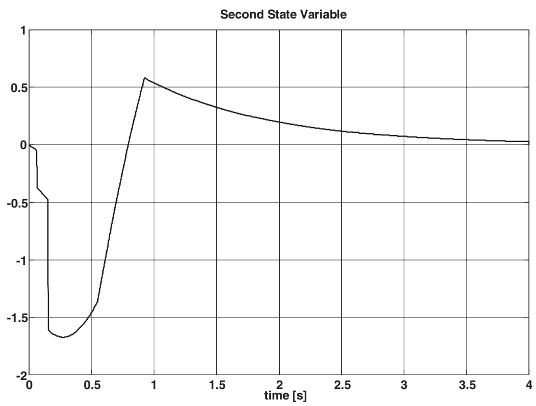

Fig.2 - **State variable** x_2

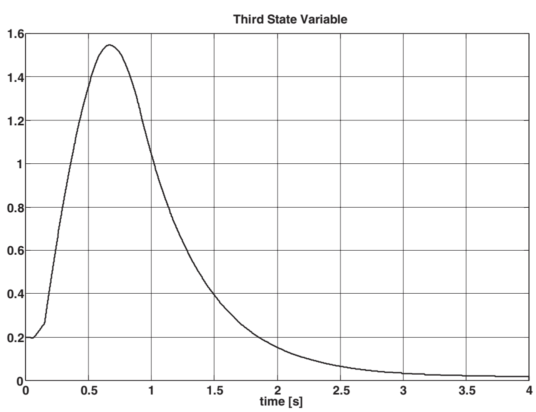

Fig.3 - **State variable** x_3

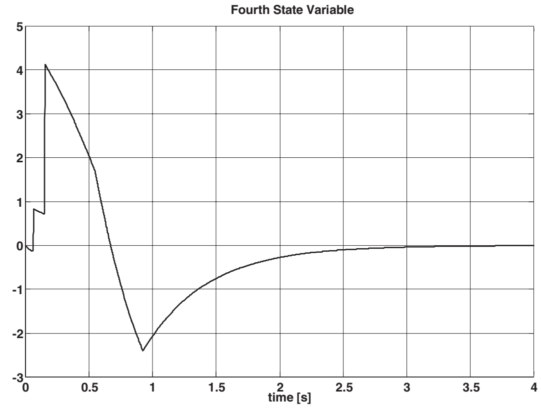

Fig.4 - **State variable** x_4

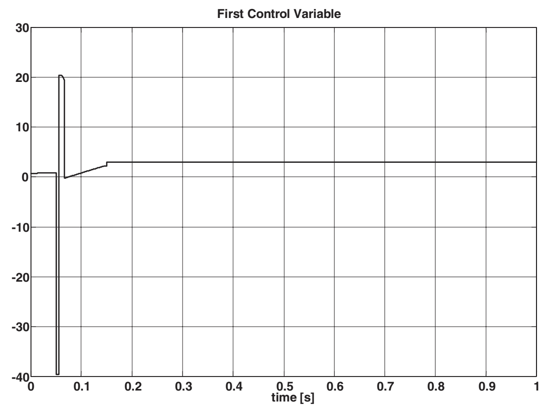

Fig.5 - **Control input** $[\mathbf{u}_1]_1$

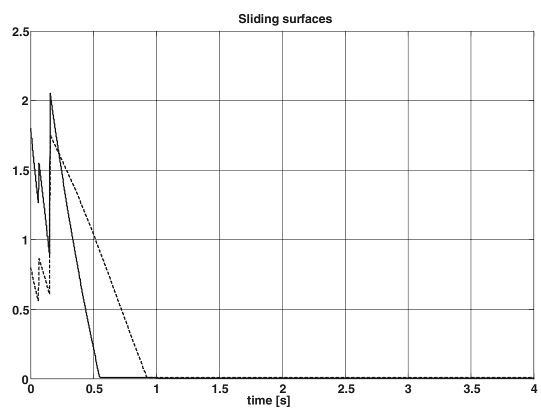

Fig.6 - **Sliding surfaces** $[\sigma]_1$ **(continuous line) and** $[\sigma]_2$ **(dashed line)**