Proceedings of the
44th IEEE Conference on Decision and Control, and
the European Control Conference 2005
Seville, Spain, December 12-15, 2005

Model Orbit Robust Stabilization (MORS) of Pendubot with Application to Swing up Control

Yuri Orlov, Luis T. Aguilar and Leonardo Acho

Abstract— **Orbital stabilization of a simple underactuted manipulator, namely, two-link pendulum robot (Pendubot) is under study. Since underactuated systems cannot be stabilized by means of smooth feedback, the solution to the stabilization problem is sought within switched control methods. The quasi-homogeneous control synthesis is utilized to design a switched controller that drives the Pendubot to its zero dynamics in finite time and maintains it there in sliding mode. The constructed controller is such that the Pendubot zero dynamics is generated by a modified Van der Pol oscillator, being viewed as a reference model. The closed-loop system is thus capable of moving from one orbit to another by simply changing the parameters of the proposed modification of the Van der Pol oscillator. Performance issues of the controller constructed are illustrated in a simulation study of the swing up control problem of moving the Pendubot from its stable downward position to the unstable inverted position and stabilizing it about the vertical.**

I. Introduction

Motivated by applications where the natural operation mode is periodic [6], orbital stabilization of underactuated systems, enforced by fewer actuators than degrees of freedom, presents a challenging problem. As well known (see, e.g., [4], [24]), these systems possess nonholonomic properties, caused by nonintegrable differential constraints, and therefore, they cannot be stabilized by means of smooth feedback. With this in mind, the solution to the orbital stabilization problem is sought within switched control methods. Capability of switched systems to generate steady state periodic solutions has recently been analyzed in [5].

For underactuted manipulators the orbital stabilization paradigm, referred to as periodic balancing [6], [21], differs from typical formulations of output tracking where the reference trajectory to follow is known a priori. The control objective for the periodic balancing, e.g., a walking rabbit [7] is to result in the closed-loop system that generates its own periodic orbit similar to that produced by a nonlinear oscillator. Apart from this, the closed-loop system should be capable of moving from one orbit to another by simply modifying the orbit parameters such as frequency and/or amplitude.

In the present paper, the following model orbit-based approach is explored to solve a periodic balancing problem for a simple underactuated mechanical manipulator, Pendulum robot (typically abbreviated as Pendubot), whose first link (shoulder) is actuated whereas the second one (elbow) is not actuated. Throughout, the positions of both links and their angular velocities are assumed to be available for measurements.

We demonstrate, thus contributing to the area, that a recently developed quasihomogeneous controller [15], [16], [17], [18] drives the Pendubot state to the desired zero dynamics in finite time and maintains it there in the so-called sliding mode of the second order [14], even in the presence of the input disturbances with an a priori known magnitude bound. Due to these features, the controller turns out to be extremely suited for addressing the problem in question. A modification of the Van der Pol equation, proposed in [19], is accepted as a reference model. This modification still possesses a stable limit cycle, being expressible in an analytical form (as opposed to that of the standard Van der Pol equation!).

Effectiveness of the developed synthesis procedure is illustrated in a simulation study of the swing up control problem where the Pendubot is required to move from its stable downward position to the unstable upright position and be stabilized about the vertical.

The swinging controller is composed by an inner loop controller, partially linearizing the Pendubot, and a model orbit quasihomogeneous outer loop controller that completes the generation of the swing up motion. The locally stabilizing controller is obtained by applying the nonlinear $\mathcal{H}_\infty$-synthesis from [1], being robust due to its nature.

In contrast to the energy-based approach [8], resulting in the Pendubot rotation along homoclinic orbits, the orbital stabilization strategy aims to balance the actuated link of the Pendubot in a vicinity of its upright position. Switching from the swinging controller to the stabilizing one, when the Pendubot enters the attraction basin of the latter, completes a unified framework for the MORS of the Pendubot around its unstable equilibrium. This framework presents an interesting alternative to the energy-based approach.

The paper is organized as follows. Section 2 is focused on the redesign of the Van der Pol dynamics to be used in Section 3 where the quasihomogeneity-based controller is synthesized for the Pendubot orbital stabilization. Simulation results on application to the swing up control problem is given in Section 4. Section 5 finalizes the paper with some conclusions.

Yuri Orlov is with CICESE Research Center, P.O. BOX 434944, San Diego, CA, 92143-4944, yorlov@cicese.mx

Luis T. Aguilar and Leonardo Acho are with CITEDI-IPN, 2498 Roll Dr. #757, Otay Mesa, San Diego, CA, 92154,laguilar{leonardo}@citedi.mx

The work was supported by CONACYT under grant number 45900

II. Modified Van der Pol Oscillator

The Van der Pol equation, whose general representation is given by the second order scalar nonlinear differential

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equation

$$\ddot{x} + \varepsilon[(x - x_0)^2 - \rho^2]\dot{x} + \mu^2(x - x_0) = 0 \qquad (1)$$

with positive parameters ε, ρ, μ, is a special case of the circuit equation (see, e.g., [12])

$$\ddot{v} + \varepsilon h'(v)\dot{v} + \mu^2 v = 0 \qquad (2)$$

where the function $h(v)$, characterizing the resistive element, satisfies the conditions

$$\begin{gathered} h(0) = 0, \quad h'(0) < 0 \\ \lim h(v)_{v \to -\infty} = -\infty, \quad \lim h(v)_{v \to \infty} = \infty. \end{gathered} \qquad (3)$$

The Van der Pol equation is a fundamental example in nonlinear oscillation theory. It possesses a periodic solution that attracts every other solutions except the unique equilibrium point $(x, \dot{x}) = (x_0, 0)$. Such a periodic solution is typically referred to as a stable limit cycle [12]. The parameter ρ controls the amplitude of this limit cycle, the parameter μ controls its frequency, the parameter ε controls the speed of the limit cycle transients, and the parameter x_0 is for the offset of x (see [26] for details).

For later use, we present the recently proposed modification of the Van der Pol equation [19]

$$\ddot{x} + \varepsilon[(x^2 + \frac{\dot{x}^2}{\mu^2}) - \rho^2]\dot{x} + \mu^2 x = 0 \qquad (4)$$

where in contrast to (1) no offset of x is admitted, i.e., the parameter $x_0 = 0$ is used, and the additional term $\frac{\varepsilon}{\mu^2}\dot{x}^3$ is involved. As opposed to the Van der Pol equation (1), the proposed modification (4) has nothing to do with the circuit equation (2).

In order to demonstrate that this modification still possesses a stable limit cycle, let us differentiate the positive definite function

$$V(x, \dot{x}) = \frac{1}{2}x^2 + \frac{1}{2\mu^2}\dot{x}^2 \qquad (5)$$

along the trajectories of (4):

$$\dot{V}(x, \dot{x}) = x\dot{x} + \frac{1}{\mu^2}\dot{x}\,\ddot{x} = \frac{\varepsilon}{\mu^2}[\rho^2 - (x^2 + \frac{\dot{x}^2}{\mu^2})]\dot{x}^2. \qquad (6)$$

It follows that

$$\dot{V}(x, \dot{x}) \begin{cases} > 0 & \text{if } (x^2 + \frac{\dot{x}^2}{\mu^2}) < \rho^2 \ \& \ \dot{x} \neq 0 \\ < 0 & \text{if } (x^2 + \frac{\dot{x}^2}{\mu^2}) > \rho^2 \ \& \ \dot{x} \neq 0 \\ = 0 & \text{if } [\rho^2 - (x^2 + \frac{\dot{x}^2}{\mu^2})]\dot{x} = 0 \end{cases} \qquad (7)$$

on the trajectories of equation (4). Since by inspection, the origin $x = \dot{x} = 0$ is a unique equilibrium point of (4) Poincare-Bendixson criterion [12, p. 61] is thus applicable to the modified Van der Pol equation (4). By applying Poincare-Bendixson criterion, the existence of a periodic orbit is concluded for this equation. Remarkably, such a periodic solution is expressed in an analytical form unlike that of the Van der Pol equation. This analytical representation comes from expression (7) of the time derivative of (5).

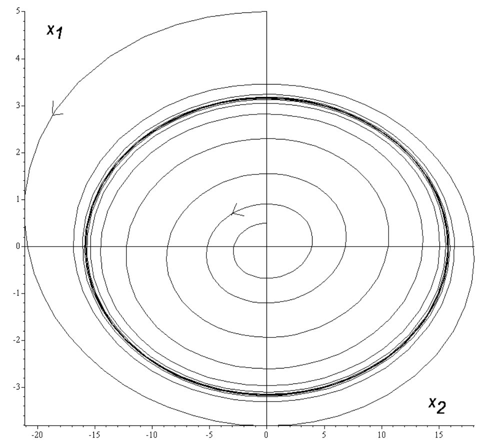

Fig. 1. Phase portrait of the modified Van der Pol equation

Indeed, by applying the invariance principle [12, Section 4.2] to (7), one concludes that a periodic solution of (4) has to oscillate within the set $\{(x, \dot{x}) : \dot{V}(x, \dot{x}) = 0\}$, i.e.,

$$[\rho^2 - (x^2 + \frac{\dot{x}^2}{\mu^2})]\dot{x} = 0. \qquad (8)$$

Since the origin is the unique equilibrium point of (4) all the trajectories of (4) cross the axis $\dot{x} = 0$ everywhere except the origin. Hence, the largest invariant manifold of set (8) coincides with the ellipse

$$x^2 + \frac{\dot{x}^2}{\mu^2} = \rho^2, \qquad (9)$$

and it remains to straightforwardly verify that (9) is a periodic orbit of the modified Van der Pol equation (4).

Now it becomes clear that in equation (4) the parameter ρ stands for the amplitude of the periodic orbit whereas μ is for its frequency.

According to (7), the norm of any trajectory of (4), initialized inside the periodic orbit (9), must grow with time. Conversely, the norm of any trajectory of (4), initialized outside the periodic orbit (9), must shrink with time. Thus, any trajectory of (4) except the equilibrium point $x = \dot{x} = 0$ is attracted by the periodic orbit. Phase portrait of equation (4) is shown in Fig. 1 for the parameter values $\varepsilon = 0.1$, $\rho^2 = 10$, and $\mu^2 = 1$.

In what follows the Van der Pol modification (4) is used as a reference model in the orbital Pendubot stabilization.

III. Orbital Pendubot Stabilization

A. Problem Statement

The state equation of the Pendubot, depicted in Fig. 2, is given by [25, p. 55]:

$$M(q)\,\ddot{q} + N(q, \dot{q}) = \tau + w \qquad (10)$$

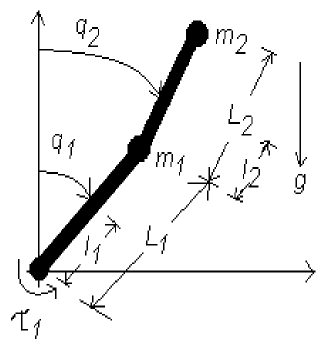

Fig. 2. Pendubot

where

$$M(q) = \begin{pmatrix} m_{11} & m_{12} \\ m_{12} & m_{22} \end{pmatrix}, N(q, \dot{q}) = \begin{pmatrix} N_1 \\ N_2 \end{pmatrix},$$

$$\tau = \begin{pmatrix} \tau_1 \\ 0 \end{pmatrix}, \ w = \begin{pmatrix} w_1 \\ 0 \end{pmatrix}$$

and

$$m_{11} = J_m + J_1 + m_1 l_1^2 + m_2 L_1^2$$
$$m_{12} = m_2 L_1 l_2 \cos(q_1 - q_2)$$
$$m_{22} = m_2 l_2^2 + J_2$$

$$\begin{aligned} N_1(q_1, q_2) &= m_2 L_1 l_2 \sin(q_1 - q_2)\dot{q}_2^2 \\ &\quad -g(m_1 l_1 + m_2 L_1)\sin(q_1), \\ N_2(q_1, q_2) &= -m_2 L_1 l_2 \sin(q_1 - q_2)\dot{q}_1^2 - m_2 g l_2 \sin(q_2). \end{aligned}$$

Here, m_1 is the mass of link 1, m_2 the mass of link 2, L_1 and L_2 are respectively the lengths of link 1 and link 2; l_1 and l_2 are the distances to the center of mass of link 1 and link 2; J_1 and J_2 are the moments of inertia of link 1 and link 2 about their centroids; J_m is the motor inertia, τ_1 is the control torque, w_1 is the to-be-attenuated input disturbance, and g is the gravity acceleration.

We assume throughout that the input disturbance is of class $L_\infty(0, \infty)$ with an *apriori* known norm bound $K > 0$, i.e.,

$$ess \sup_{t \in [0, \infty)} |w_1(t)| \leq K \tag{11}$$

Another assumption that has implicitly been made is that no mismatched disturbances affect the system. A reason for the latter assumption is that the attenuation level against mismatched disturbances is too large to be of practical interest for controlling underactuated systems.

From equation (10), one has

$$m_{11} \ddot{q}_1 + m_{12} \ddot{q}_2 + N_1 = \tau_1 + w_1 \tag{12}$$

$$m_{12} \ddot{q}_1 + m_{22} \ddot{q}_2 + N_2 = 0. \tag{13}$$

Now using equation (13), the following equation is derived

$$\ddot{q}_2 = -m_{22}^{-1}[m_{12} \ddot{q}_1 + N_2]. \tag{14}$$

Then substituting equation (14) into (12) yields

$$\begin{aligned} (m_{11} - m_{12} m_{22}^{-1} m_{12}) \ddot{q}_1 - m_{12} m_{22}^{-1} N_2 + N_1 \\ = \tau_1 + w_1. \end{aligned} \tag{15}$$

Finally, setting $|M| = m_{11} - m_{12} m_{22}^{-1} m_{12}$ and

$$\tau_1 = |M| u - m_{12} m_{22}^{-1} N_2 + N_1 \tag{16}$$

where u is the new control input, the partial linearization [23] is obtained:

$$\ddot{q}_1 = u + |M|^{-1} w_1 \tag{17}$$

$$\ddot{q}_2 = -m_{22}^{-1}[m_{12}(u + |M|^{-1} w_1) + N_2]. \tag{18}$$

In the above relations the positive definiteness of the inertia matrix $M(q)$ has been used to ensure that $|M| \neq 0$. Since system (17)-(18) describes the linearized actuated joint model it is referred to as *collocated linearization* [22].

Let the system output

$$y(t) = q_1(t) + x(t) \tag{19}$$

combine the actuated state $q_1(t)$ of the system and the reference variable $x(t)$ governed by the modified Van der Pol equation (4).

The control objective is to drive the system to the manifold $y = 0$ in finite time and maintains it there in spite of bounded external disturbances, affecting the system.

B. Switched Control Synthesis

Due to (4), (17), (19), the output dynamics is given by

$$\ddot{y} = u + |M|^{-1} w_1 - \varepsilon[(x^2 + \frac{\dot{x}^2}{\mu^2}) - \rho^2]\dot{x} - \mu^2 x. \tag{20}$$

The above objective is achieved with the following control law

$$\begin{aligned} u = \varepsilon[(x^2 + \frac{\dot{x}^2}{\mu^2}) - \rho^2]\dot{x} + \mu^2 x \\ -k_1 sign(y) - k_2 sign(\dot{y}) - hy - p\dot{y} \end{aligned} \tag{21}$$

if the gains are such that

$$h, p \geq 0, \ k_1 - k_2 > |M|^{-1} K. \tag{22}$$

Following the quasihomogeneous synthesis [18], the controller has been composed of the nonlinear compensator

$$u_c = \varepsilon[(x^2 + \frac{\dot{x}^2}{\mu^2}) - \rho^2]\dot{x} + \mu^2 x, \tag{23}$$

the homogeneous switching part (the so-called twisting controller from [9], [10])

$$u_h = -k_1 sign(y) - k_2 sign(\dot{y}),$$

and the linear remainder

$$u_l = -hy - p\dot{y}$$

that vanishes in the origin $y = \dot{y} = 0$. Thus, the closed-loop system (20), (21) is feedback transformed to the quasihomogeneous system

$$\ddot{y} = |M|^{-1}w_1 - k_1 sign(y) - k_2 sign(\dot{y}) - hy - p\dot{y}. \quad (24)$$

By Theorem 4.2 from [18] the quasihomogeneous system (24) with the parameter subordination (22) is finite time stable regardless of which external uniformly bounded disturbance subject to (11) affects the system.

Until recently, finite time stability of asymptotically stable (quasi)homogeneous systems has been well-recognized for only continuous vector fields [3], [11]. Extending this result to switched systems has required proceeding differently [16], [18] because a smooth (quasi)homogeneous Lyapunov function, whose existence was proven in [20] for continuous asymptotically stable (quasi)homogeneous vector fields, can no longer be brought into play.

The qualitative behavior of the quasihomogeneous system (22), (24) is as follows. The system trajectories rotate around the origin $y = \dot{y} = 0$, while approaching the origin in finite time. Thus, the system exhibits an infinite number of switches in a finite amount of time. This system do not generate sliding motions everywhere except the origin. If a trajectory starts there at any given finite time, there appears the so-called sliding mode of the second order (see [2], [9], [10] for advanced results on second order sliding modes).

So, starting from a finite time moment the Pendubot evolves in the second order sliding mode on the zero dynamics manifold $y = 0$. While being restricted to this manifold, the system dynamics is given by

$$\ddot{q}_2 = -\frac{m_{12}\{\varepsilon[(x^2 + \frac{\dot{x}^2}{\mu^2}) - \rho^2]\dot{x} + \mu^2 x\} + N_2(x, q_2)}{m_{22}}. \quad (25)$$

To formally derive (25) one should utilize the equivalent control method [25] and substitute the only solution u_{eq} of the algebraic equation

$$u + |M|^{-1}w_1 - \varepsilon[(x^2 + \frac{\dot{x}^2}{\mu^2}) - \rho^2]\dot{x} - \mu^2 x = 0$$

with respect to u (i.e., the equivalent control input u_{eq} that ensures equality $\ddot{y} = 0$) into (18). Phase portrait of the zero dynamics (25) is depicted in Fig. 3 for the parameter values $\varepsilon = 100$, $\rho = 0.01$, and $\mu = 10$.

Summarizing, the following result is obtained.

Theorem 1: Let the modified Van der Pol equation (4) with positive parameters ε, μ, ρ be a reference model of the Pendubot dynamics (10) and let the system output be given by (19). Then the quasihomogeneous controller (16), (21), (22) drives the Pendubot to the zero dynamics manifold $y = 0$ in finite time, uniformly in admissible disturbances (11). After that the actuated part $q_1(t)$ follows the output $x(t)$ of the modified Van der Pol equation (4) whereas the non-actuated part $q_2(t)$ is governed by the zero dynamics equation (25).

In the sequel, capabilities of the MORS procedure, constituted by Theorem 1, are analyzed via application to the swing up control problem.

IV. Swing up control and stabilization

In this section, a MORS is used to swing up the Pendubot from its downward position to the upright position and then it is switched to a nonlinear $\mathcal{H}_\infty$-controller, locally stabilizing the Pendubot about the vertical. The strategy is to select the amplitude ρ and the frequency μ of the model limit cycle (9) reasonably small and the parameter ε, controlling the speed of the limit cycle transient in the modified Van der Pol equation (4), reasonably large to ensure that the Pendubot driven by the developed controller (21) enters the attraction basin of a locally stabilizing nonlinear $\mathcal{H}_\infty$-controller. Proper switching from the swinging quasihomogeneous controller to the stabilizing one yields the generation of a swing up motion, stable about the vertical.

A. Simulated Pendubot model

In order to make physical sense the subsequent numerical study addresses the laboratory Pendubot presented in [13]. The following Pendubot parameters values, used in the simulations, are drown from [13]: $L_1 = 0.2030$ m, $L_2 = 0.2540$ m, $l_1 = 0.1574$ m, $l_2 = 0.1109$ m, $m_1 = 0.132$ Kg, $m_2 = 0.088$ Kg, $J_1 = 0.00362$ Kgm^2, $J_2 = 0.00114$ Kgm^2, and $J_m = 6 \times 10^{-5}$ Kgm^2.

B. Swinging controller design

The capability of the model orbit robust synthesis (4), (16), (21) to swing up the Pendubot is tested by means of simulations. The tuning of the parameters of the Van der Pol modification (4) is crucial to the achievement of a successful swing up. In the simulation runs the parameters were tuned to $\varepsilon = 100$, $\rho = 0.01$, $\mu = 10$ whereas the controller gains were set to $k_1 = 100$, $k_2 = 24$, $h, p = 0$.

C. Locally stabilizing controller design

A nonlinear H_∞-controller, locally stabilizing the Pendubot around its upright position, is derived according to the synthesis procedure developed in [1]. The procedure is based on a certain perturbation of the algebraic Riccati equation that appears in solving the standard $\mathcal{H}_\infty$- control problem for the linearized system. Since the local stabilizability is then ensured by the existence of a proper solution of the unperturbed algebraic Riccati equation (see [1] for details), an extra work on the stabilizability verification is thus obviated.

To apply the afore-mentioned procedure the Pendubot equations (17), (18) are represented in the form

$$\begin{aligned} \dot{x} &= f(x) + g_1(x)w_1 + g_2(x)u \\ z &= h_1(x) + k_{12}u \end{aligned} \quad (26)$$

where $x = [q_1, q_2, \dot{q}_1, \dot{q}_2]^T$ is the state space vector, $z = [z_1, z_2, z_3]^T$ is the unknown output to be controlled,

$$\begin{aligned} f(x) &= (x_3, x_4, 0, -m_{22}^{-1} N_2)^T, \\ g_1(x) &= (0, 0, |M|^{-1}, -m_{22}^{-1} m_{12} |M|^{-1})^T, \\ g_2(x) &= (0, 0, 1, -m_{22}^{-1} m_{12})^T, \\ h_1(x) &= (0, x_1, x_2)^T, \\ k_{12} &= (1, 0, 0)^T. \end{aligned} \tag{27}$$

Then a locally stabilizing nonlinear state feedback H_∞-controller is given by [1]

$$u = -g_2^T(x) P_\varepsilon x \tag{28}$$

where P_ε is a symmetric positive definite solution of the perturbed Riccati equation

$$P_\varepsilon A + A^T P_\varepsilon + C_1^T C_1 + P_\varepsilon \left[\gamma^{-2} B_1 B_1^T - B_2 B_2^T \right] P_\varepsilon = -\varepsilon I \tag{29}$$

with some positive ε and γ, and

$$A = \frac{\partial f}{\partial x}(0),\ B_1 = g_1(0),\ B_2 = g_2(0), C_1 = \frac{\partial h_1}{\partial x}(0). \tag{30}$$

To complete the nonlinear H_∞-synthesis the following positive definite solution

$$P_\varepsilon = \begin{bmatrix} 159.98 & 1573.5 & 172.1064 & 228.7793 \\ 1573.5 & 22788.0 & 1753.5 & 2808.1 \\ 172.1064 & 1753.5 & 186.6592 & 250.77 \\ 228.7793 & 2808.1 & 250.77 & 373.1426 \end{bmatrix}.$$

of the Riccati equation (29) has numerically been found for $\gamma = 200$ and $\varepsilon = 100$.

D. Variable Structure Swinging/Stabilizing Controller

In order to accompany swinging up the Pendubot by the subsequent stabilization around the upright position the model orbit swinging controller, presented in Subsection 4.B, is switched to the locally stabilizing H_∞-controller from Subsection 4.C whenever the Pendubot enters the basin of attraction, numerically found for the latter controller. While being not studied in details, the capability of the closed-loop system of entering the attraction basin of the locally stabilizing H_∞-controller is supported by simulation evidences. The problem of driving the Pendubot to the attraction domain is resolved via tuning the parameters of the Van der Pol modification (4). Various parameter scenarios are played for the Van der Pol modification in successive simulation runs to effectively determine these parameters. Appropriate numerical values of the parameters, carried out in the simulations, were presented in Subsection IV.B.

E. Simulation results

We used Simnon to produce our simulation experiments for the Variable Structure Swinging/Stabilizing Controller, proposed in Subsection 4.D. The initial conditions of the Pendubot position and that of the modified Van der Pol oscillator, selected for simulations, were $q_1(0) = 3.14$ Rad, $q_2(0) = 3.14$ Rad, and $x(0) = 0.011$ Rad, whereas all the velocity initial conditions were set to zero.

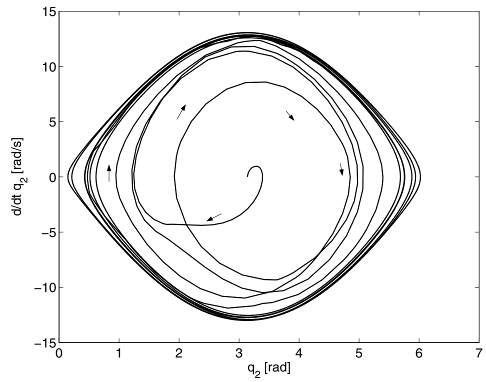

Fig. 3. Phase portrait of the zero dynamics.

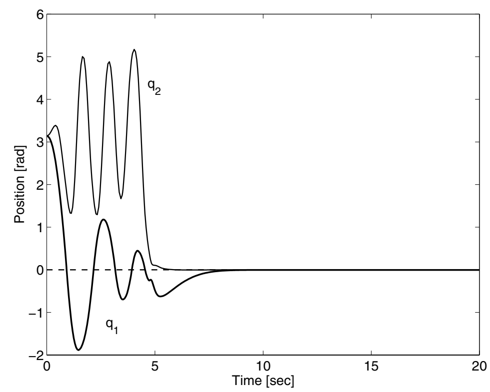

Fig. 4. MORS of the Pendubot under permanent disturbances: plots of shoulder position q_1 and elder position q_2.

Following the numerical procedure, described in Subsection 4.D, the time instant $t_s = 11.4$ sec was chosen for switching from the swinging controller to the locally stabilizing controller. Quite impressive simulation results were thus obtained for the Pendubot to swing up and be stabilized by the developed variable structure controller.

To better demonstrate attractive features of the proposed synthesis the Pendubot motion, enforced by the variable structure controller with the pre-specified switching time instant, was perturbed with the permanent input disturbance $w_1(t) = 0.1$ N-m. Simulation results for the feedback stabilization of the Pendubot, operating under the permanent input disturbance, are depicted in Figures 4-6. From these figures good performance of the perturbed closed-loop system is concluded.

The controller performance was additionally tested in an experimental study of a Pendubot model installed in the control laboratory of the CICESE research center. Because of space limitations the experimental results are not presented here to be published elsewhere.

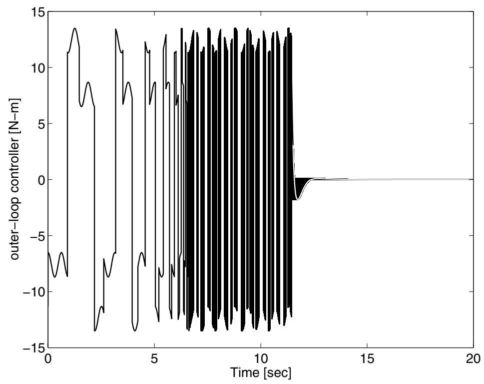

Fig. 5. MORS of the Pendubot under permanent disturbances: plot of the outer-loop controller u.

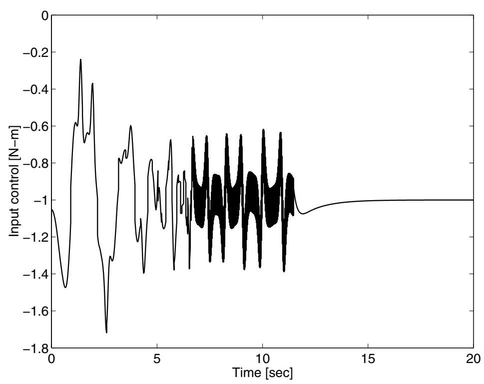

Fig. 6. MORS of the Pendubot under permanent disturbances: plot of the input torque τ_1.

V. CONCLUSIONS

Orbital stabilization of a Pendubot, presenting a simple underactuated (two degrees-of-freedom, one actuator) manipulator, is under study. Since underactuated systems cannot be stabilized by means of smooth feedback, a solution to the problem is proposed within switched control methods. The quasihomogeneity-based synthesis is utilized to design a switched controller that drives the Pendubot to its zero dynamics in finite time and maintains it there in sliding mode.

The controller proposed is such that the Pendubot zero dynamics is generated by a modified Van der Pol oscillator, viewed as a reference model. Since this modification still possesses a stable limit cycle the proposed synthesis constitutes the MORS of the Pendubot. The developed approach is hoped to suggest a practical framework for orbital stabilization of underactuated manipulators.

Capabilities of the approach and its robustness features are illustrated in a simulation study of the swing up control problem of moving the Pendubot from its stable downward position to the unstable inverted position and stabilizing it about the vertical.

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