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Nonparametric identification for the causal optimization of set point tracking

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Abstract— **The problem of identifying a nonparametric model of the closed loop part of a 2-d.o.f. control system, suitable for the design of the feedforward regulator, is addressed in this paper. The approach presented to describe the model relies on a finite set of 'control' and 'output base functions', that have to be experimentally determined. A thorough analysis of the methods used to determine these base functions is thus presented, focusing attention on simplicity, feasibility, noise and disturbance sensitivity, as the set point tracking performance of the feedforward regulator depends on the quality of the estimated model.**

I. INTRODUCTION

Given an LTI plant a basic control problem is the determination of an LTI regulator that ensures the stability of the closed loop system and meets various design objectives, e.g. set point tracking and disturbance rejection. In process control many approaches have been developed to design such a controller, emphasizing the importance of decoupling the problem of stability and disturbance rejection from that of set point tracking [1], [2]. This separation can be achieved through a two degrees of freedom (2-d.o.f.) control scheme, where the feedforward regulator is designed for optimal set point tracking.
However, despite of the great importance of the 2-d.o.f. structure, a very small number of tuning methodologies, suitable for industrial implementation [3], have been proposed. This lack of synthesis methods is even more significant if one focuses attention on methods that are suitable for embedded systems, or wherever the limits imposed by hardware and/or software architecture play an important role, and that does not require a precise knowledge of the closed loop part of the control system.

A method to synthesize the feedforward part of a generic LTI 2-d.o.f. controller has been proposed in [4]. This method is completely independent of the technique used to synthesize the feedback controller and of the aspect of the set point signal, does not require any parametric model of the control loop, can provide a reliable forecast of the obtained results, is computationally simple and based on a few, easily interpreted, design parameters. The key idea, preliminary introduced in [5] with exclusive reference to the PID case, is to formulate the problem as an optimization based on a nonparametric model of the control loop, that is easily identified on-line. Moreover, this model relies upon the concepts of 'output' and 'control base functions', i.e. the set point response of the controlled variable and the control signal in the closed loop system are vectors in a finite-dimensional functional space, where the output and control base functions play the role of a base (whence the name).
As a consequence, the problem of determining the base functions in a realistic noisy environment should be tackled with great care.

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In this paper, we address the problem of determining the base functions relative to a given set point signal *experimentally*. As the set point tracking performance of the feedforward regulator depends obviously on the quality of the estimated closed loop system, a thorough analysis of the methods used to determine the base functions is appropriate. In fact, the only error affecting the model is that due to disturbances and noise, i.e., with reference to Fig. 1, to the signals $q(t)$ and $n(t)$.
Moreover, if the base functions are relative to a particular set point, they have to be determined every time the set point changes, arising in an excessive computational burden or in an unnecessary waste of time. Thus, the problem of determining a 'general' set of base functions from which the base functions relative to any set point signal can be derived, without requiring a further experiment, will be also addressed.

The paper is organized as follows. In Section II the key idea of using a set of output and control base functions as a nonparametric model of the closed loop part of the system is introduced. Sections III and IV present two different methods to determine the base functions: direct algebraic methods and indirect filtering methods. Moreover the effects of noise and disturbances on the base functions determination are discussed in Section IV. Section V introduces a new method to determine a 'general' set of base functions, independent of the considered set point. A few application examples are presented in Section VI to illustrate the proposed methodologies. Finally, Section VII concludes the paper.

II. PROBLEM STATEMENT

Consider the class of 2-d.o.f. linear controllers described, in the frequency domain, by the general control law

$$D(s)U(s) = N_{FF}(s)Y^{\circ}(s) - N_{FB}(s)Y(s)$$

where $Y^{\circ}(s)$, $Y(s)$, and $U(s)$ are the Laplace transforms of the set point, the controlled variable and the control signal, respectively, while $N_{FF}(s)$, $N_{FB}(s)$ and $D(s)$ are three polynomials in the complex variable s expressed as

$$N_{FF}(s) = \sum_{k=0}^{n_N} n_k^{FF} s^k \qquad N_{FB}(s) = \sum_{k=0}^{n_N} n_k^{FB} s^k$$
$$D(s) = \sum_{k=0}^{n_D} n_k^D s^k \tag{1}$$

with $n_N \leq n_D$, $n_0^{FF} \neq 0$, $n_0^{FB} \neq 0$. Such controllers are implemented as shown in Fig. 1, where

$$R_{FF}(s) = \frac{N_{FF}(s)}{N_{FB}(s)} \qquad R_{FB}(s) = \frac{N_{FB}(s)}{D(s)}$$

and we have to assume that all the roots of $N_{FB}(s)$ lie in the open LHP. This loss of generality is of minimal importance in the application domain, however.

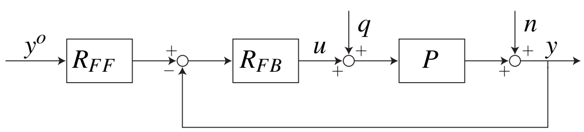


Fig. 1. Control scheme with a 2-d.o.f. regulator.

Suppose that a stabilizing feedback block $R_{FB}(s)$ has already been synthesized, so that the polynomials $N_{FB}(s)$ and $D(s)$ are fixed, and that the polynomial $N_{FF}(s)$ has been chosen equal to $N_{FB}(s)$, i.e., $R_{FF}(s) = 1$ (the problem of determining a convenient $N_{FF}(s)$ will be discussed later on). We can now state the following

Lemma 1. The response $y(t)$ of the controlled variable in the control system of Fig. 1 forced by a Laplace-transformable set point $y^\circ(t)$ can be expressed as a weighed sum of $n_N + 1$ functions $y_k^b(t)$, the weights being the coefficients of $N_{FF}(s)$, i.e.,

$$y(t) = \sum_{k=0}^{n_N} n_k^{FF} y_k^b(t) \tag{2}$$

having expressed $N_{FF}(s)$ as in (1). The functions $y_k^b(t)$ do not depend on the coefficients of $N_{FF}(s)$, and will be termed the control system's 'output base functions' relative to the set point signal $y^\circ(t)$.

Proof. Define the complementary sensitivity of the closed loop part of the control system as

$$T(s) = \frac{R_{FB}(s)P(s)}{1 + R_{FB}(s)P(s)}$$

where P(s) is the transfer function of the process. It follows immediately that

$$Y(s) = \frac{N_{FF}(s)}{N_{FB}(s)} T(s) Y^\circ(s) \tag{3}$$

Hence, recalling (1),

$$Y(s) = \sum_{k=0}^{n_N} n_k^{FF} \frac{s^k T(s)}{N_{FB}(s)} Y^\circ(s) \tag{4}$$

and, in force of the Laplace transform linearity, the thesis follows by setting

$$y_k^b(t) = \mathscr{L}^{-1}\left[\frac{s^k T(s)}{N_{FB}(s)} Y^\circ(s)\right] \tag{5}$$

Considering now the control signal, we can also state the following

Lemma 2. The response $u(t)$ of the control signal in the control system of Fig. 1 forced by a Laplace-transformable set point $y^\circ(t)$ can be expressed as a weighed sum of $n_N + 1$ functions $u_k^b(t)$, the weights being the coefficients of $N_{FF}(s)$, i.e.,

$$u(t) = \sum_{k=0}^{n_N} n_k^{FF} u_k^b(t)$$

having expressed $N_{FF}(s)$ as in (1). The functions $u_k^b(t)$ do not depend on the coefficients of $N_{FF}(s)$, and will be termed the control system's 'control base functions' relative to the set point signal $y^\circ(t)$.

Proof. The proof is analogous to that of Lemma 1, and leads to

$$u_k^b(t) = \mathscr{L}^{-1}\left[\frac{s^k T(s)}{N_{FB}(s)P(s)} Y^\circ(s)\right]$$

These first results can be interpreted as follows. Each of the closed loop set point responses $y(t)$ and $u(t)$ of the control system is a vector in a finite-dimensional functional space, where we can take the functions $y_k^b(t)$, or $u_k^b(t)$, as a base (which motivates the name). The base functions allow to compute the set point responses of the control system for any value of the coefficients n_k^{FF}, and the relationship between the coefficients and the responses is linear. Therefore, the base functions are a nonparametric model of the control system, independent of the structure of the process dynamics and particularly suitable for tuning the feedforward regulator [4].

III. DETERMINATION OF THE BASE FUNCTIONS: DIRECT (ALGEBRAIC) METHODS

By definition, the i-th output (control) base function relative to a set point signal is the output (control) response to that signal obtained with $n_i^{FF} = 1$ and $n_k^{FF} = 0$, $k \neq i$. Therefore, the set of base functions can be obtained directly with $n_N + 1$ experiments, setting in each of them $N_{FF}(s) = s^i$, $i = 0 \ldots n_N$. This is the crudest idea, and the simplest direct method. An apparent drawback of this method is that, in the general case, the control signal in the $n_N + 1$ experiments may have very different behaviours and amplitudes. As a result, some experiments may excite the possible control system's nonlinearities more than others, and the signal to noise ratio in the experiments may be very different—two facts that clearly adversely affect the base functions' estimation. A remedy can be devised by observing that, denoting by $y^{(i)}(t)$, $u^{(i)}(t)$ the output and control response obtained in the i-th experiment, one can write

$$\begin{bmatrix} y^{(0)}(t) \\ \vdots \\ y^{(n_N)}(t) \end{bmatrix} = M_{FF} \begin{bmatrix} y_0^b(t) \\ \vdots \\ y_{n_N}^b(t) \end{bmatrix}$$
$$\begin{bmatrix} u^{(0)}(t) \\ \vdots \\ u^{(n_N)}(t) \end{bmatrix} = M_{FF} \begin{bmatrix} u_0^b(t) \\ \vdots \\ u_{n_N}^b(t) \end{bmatrix} \tag{6}$$

where

$$M_{FF} = \begin{bmatrix} n_0^{FF(0)} & \dots & n_{n_N}^{FF(0)} \\ \vdots & & \vdots \\ n_0^{FF(n_N)} & \dots & n_{n_N}^{FF(n_N)} \end{bmatrix} \tag{7}$$

$n_k^{FF(i)}$ being the k-th coefficient of $N_{FF}(s)$ in the i-th experiment. Provided that the coefficients $n_k^{FF(i)}$ are chosen so that M_{FF} be nonsingular, the set of base functions can be computed from the algebraic systems (6), whence the second name of these methods. Direct methods can be considered as parameterized by matrix M_{FF}, in that a specific value of M_{FF} defines a particular direct method (e.g., that with $N_{FF}(s) = s^i$, $i = 0 \dots n_N$ corresponds to $M_{FF} = I$).

Direct methods are conceptually simple, but *extremely* impractical unless n_N is very small. The main reason is that it is not easy do derive a criterion for the selection of M_{FF}. The aim of such a criterion should be to make the output of $R_{FF}(s)$, and therefore the control signal, have 'similar excitation characteristics' in the n_N+1 experiments, but it is difficult to state precisely what such a similarity is. If the feedback controller $R_{FB}(s)$ makes the closed loop behave almost linearly in the vicinity of the operating point from which the experiments start, it is enough that the output of $R_{FF}(s)$ in the experiments have similar peak amplitudes, and that their energy is significantly greater than that of noise. If (almost) nothing is known on the process nonlinearity, however, things get much more complex. Since the only way to obtain a good linear model is to remain near the starting operating point, a reliable *a priori* determination of M_{FF} is too difficult for practical applications, and computing M_{FF} by means of iterations makes the base functions' determination procedure long and potentially cumbersome.

In one word, the only advantage of direct methods is their computational simplicity, but the selection of M_{FF} makes them impractical. To obtain a tuning method applicable in a reasonably wide variety of cases with acceptable effort, another solution to determine the base functions is in order.

IV. DETERMINATION OF THE BASE FUNCTIONS: INDIRECT (FILTERING) METHODS

A more interesting solution to the problem of determining the base functions comes from the following

Remark 1. Denoting by $Y_k^b(s)$ the Laplace transform of $y_k^b(t)$, on the basis of (5) and (3) it is immediate to write

$$Y_k^b(s) = F_k^b(s)Y(s) \qquad 0 \le k \le n_N \tag{8}$$

where

$$F_0^b(s) = \frac{1}{N_{FF}(s)} \qquad F_{k+1}^b(s) = sF_k^b(s)$$

Similarly, for the control base functions, with obvious notation it turns out that

$$U_k^b(s) = F_k^b(s)U(s) \tag{9}$$

Notice that the transfer functions $F_k^b(s)$ are the same for the output and control base functions, and depend only on $N_{FF}(s)$. Therefore, these transfer functions will be termed the 'base filters' relative to $N_{FF}(s)$.

Remark 1 shows that one can obtain the base functions relative to a given set point signal with a single experiment: in principle, it suffices to choose as $N_{FF}(s)$ an arbitrary polynomial $N_{FF}^e(s)$, apply the set point signal of interest to the control system, and then filter the obtained responses $y^e(t)$ and $u^e(t)$ as dictated by (8) and (9). Unfortunately, this would be correct only if the system were disturbance- and noise-free (i.e., with reference to Fig. 1, $q(t) = n(t) = 0$) and there were no limits on the admissible control effort. Since these hypotheses are unrealistic, $N_{FF}^e(s)$ cannot be chosen arbitrarily, but has to be determined according to the criteria derived in the following.

A. Choice of $N_{FF}^e(s)$ in filtering methods

To choose $N_{FF}^e(s)$, three facts must be considered.

1. Stability and causality. A first criterion is that all the n_N+1 base filters $F_k^b(s)$ must be stable (quite an obvious requirement, especially in the presence of noise), and causal. Thus, $N_{FF}^e(s)$ must be of degree n_N, and its roots must all lie in the open LHP.

2. Effect of disturbance and noise. Suppose that the control system is subject to a load disturbance and an additive output noise with Laplace transforms $Q(s)$ and $N(s)$, respectively, as shown in Fig. 1. Both the disturbance and the noise deteriorate the estimation of the base functions. To quantify this deterioration, recall that

$$Y(s) = \frac{R_{FF}(s)R_{FB}(s)P(s)Y^\circ(s) + P(s)Q(s) + N(s)}{1 + R_{FB}(s)P(s)}$$

and

$$U(s) = R_{FB}(s)\frac{R_{FF}(s)Y^\circ(s) - P(s)Q(s) - N(s)}{1 + R_{FB}(s)P(s)}$$

Hence, the estimates $\hat{Y}_k^b(s)$ and $\hat{U}_k^b(s)$ of $Y_k^b(s)$ and $U_k^b(s)$, obtained in the presence of disturbance and noise with a certain $N_{FF}^e(s)$, are

$$\hat{Y}_k^b(s) = \frac{s^k P(s)Y^\circ(s)}{D(s) + N_{FB}(s)P(s)} + \frac{s^k D(s)\,(P(s)Q(s) + N(s))}{N_{FF}^e(s)\,(D(s) + N_{FB}(s)P(s))}$$

and

$$\hat{U}_k^b(s) = \frac{s^k Y^\circ(s)}{D(s) + N_{FB}(s)P(s)} - \frac{s^k N_{FB}(s)\,(P(s)Q(s) + N(s))}{N_{FF}^e(s)\,(D(s) + N_{FB}(s)P(s))}$$

that clearly approach $Y_k^b(s)$ and $U_k^b(s)$ as $Q(s)$ and $N(s)$ vanish. The important fact is that the choice of $N_{FF}^e(s)$ does not affect the contribution of $Y^\circ(s)$ to the base function estimates. On the other hand, the larger $|N^e_{FF}(j\omega)|$ is at a given frequency ω, the less a noise or disturbance component at that frequency will deteriorate the estimates. Note that $N^e_{FF}(s)$ is only used in the experiment, so there is no particular reason why $N^e_{FF}(0)$ should equal $N_{FB}(0)$. Therefore, a second criterion is that any polynomial $\underline{N}^e_{FF}(s)$ with complex roots is surely suboptimal for noise rejection, because there exists another polynomial $\bar{N}^e_{FF}(s)$ with real roots such that $|\bar{N}^e_{FF}(j\omega)| \geq |\underline{N}^e_{FF}(j\omega)|$, $\forall\omega$.

3. Control limits. From the scheme of Fig. 1 it follows immediately that

$$U(s) = \frac{N_{FF}(s)}{D(s) + N_{FB}(s)P(s)} Y^\circ(s)$$

thus that increasing $|N^e_{FF}(j\omega)|$ makes the control effort in the experiment bigger. An excessively nervous control signal may be undesirable *per se*, but an even more serious fact is that if some value or rate saturation is hit the control system's behaviour ceases to be linear, which is not acceptable in an experiment devoted to determining the base functions. A third criterion, then, is that $N^e_{FF}(s)$ must be chosen so that the control signal does not exceed any value or rate limit during the experiment.

V. DETERMINATION OF A 'GENERAL' SET OF BASE FUNCTIONS

The methods described in Section III and IV lead to the determination of the base functions relative to a given set point signal. We now address the more ambitious problem of determining a 'general' set of base functions, i.e. a set of base functions suitable for any set point signal.

Suppose that a set of base functions relative to a given set point signal has already been experimentally determined. When the set point changes a new experiment is required and the feedforward controller has to be tuned again, to ensure the best tracking performance. In practice, however, this could arise in an excessive computational burden or in an unnecessary waste of time. To avoid this bottleneck a different procedure is proposed, with the aim of determining a 'general' set of base functions from which the base functions relative to a given set point can be derived without requiring a further set of experimental data.

Assume that a set of output base functions $y^b_k(t)$, $k = 0 \dots n_N$ relative to the set point signal $y^o(t)$ is known. From equation (4) it follows

$$Y^b_k(s) = \frac{s^k T(s)}{N_{FB}(s)} Y^o(s) \quad k = 0 \dots n_N \tag{10}$$

Multiplying now both sides of (10) by $\tilde{Y}^o(s)/Y^o(s)$ yields

$$Y^b_k(s)\frac{\tilde{Y}^o(s)}{Y^o(s)} = \frac{s^k T(s)}{N_{FB}(s)}\tilde{Y}^o(s) = \tilde{Y}^b_k(s) \quad k = 0 \dots n_N \tag{11}$$

where $\tilde{y}^b_k(t)$ are the output base functions relative to the set point signal $\tilde{y}^o(t)$ (and $\tilde{Y}^b_k(s)$, $\tilde{Y}^o(s)$ their Laplace transforms). Then, from (11) it follows immediately that

$$\tilde{y}^b_k(t) = \mathscr{L}^{-1}\left[Y^b_k(s)\frac{\tilde{Y}^o(s)}{Y^o(s)}\right] \tag{12}$$

Equations (11) and (12) show that one can obtain the output base functions $\tilde{y}^b_k(t)$ relative to a set point $\tilde{y}^o(t)$, assuming that a set of output base functions $y^b_k(t)$ relative to a set point $y^o(t)$ is known, simply applying to each element of the set $y^b_k(t)$ the filter $\tilde{Y}^o(s)/Y^o(s)$. Unfortunately, this filter exists if and only if $\tilde{\Omega} \subset \Omega$, where

$$\Omega = \{\omega \in \mathbf{R}^+ : Y^o(j\omega) \neq 0\}$$
$$\tilde{\Omega} = \{\omega \in \mathbf{R}^+ : \tilde{Y}^o(j\omega) \neq 0\}$$

being $Y^o(j\omega)$ and $\tilde{Y}^o(j\omega)$ the discrete Fourier transforms of $y^o(t)$ and $\tilde{y}^o(t)$, respectively. Therefore, this procedure can be applied only when the spectral components of the set point $\tilde{y}^o(t)$ are a subset of the previous one. However, as can be easily guessed, the determination of a procedure of general validity is a very complex problem and is out of the scope of this work.
In practice, a reasonable and computationally simple solution can be devised in the context of discrete time systems.

Consider a discrete time set point signal $\tilde{y}^o(h)$ of finite duration N, it can be expressed as

$$\tilde{y}^o(h) = \sum_{l=0}^{N} \delta_l \, \mathrm{sca}(h-l) \tag{13}$$

with $\delta_l = \tilde{y}^o(l+1) - \tilde{y}^o(l)$.
Let $y^{bs}_k(h)$, $k = 0 \dots n_N$ be the set of output base functions relative to a unit-step set point signal. Then, the response of the closed loop system to the l-th addendum of (13) is given by

$$\delta_l \sum_{k=0}^{n_N} n^{FF}_k y^{bs}_k(h-l)$$

and the system output, generated by $\tilde{y}^o(h)$, is thus

$$y(h) = \sum_{l=0}^{N} \delta_l \sum_{k=0}^{n_N} n^{FF}_k y^{bs}_k(h-l) = \sum_{k=0}^{n_N} n^{FF}_k \sum_{l=0}^{N} \delta_l \, y^{bs}_k(h-l) \tag{14}$$

As a consequence, recalling (2), from (14) it follows

$$\tilde{y}^b_k(h) = \sum_{l=0}^{N} \delta_l \, y^{bs}_k(h-l)$$

being $\tilde{y}^b_k(h)$ the k-th output base function relative to the set point signal $\tilde{y}^o(h)$.

Similarly, for the control base functions, with obvious notation it turns out that

$$u(h) = \sum_{k=0}^{n_N} n^{FF}_k \sum_{l=0}^{N} \delta_l \, u^{bs}_k(h-l)$$

being $u^{bs}_k(h)$, $k = 0 \dots n_N$ the set of control base functions relative to a unit-step set point signal. As a consequence, the k-th control base function relative to the set point signal $\tilde{y}^o(h)$ is given by

$$\tilde{u}^b_k(h) = \sum_{l=0}^{N} \delta_l \, u^{bs}_k(h-l)$$

We can now state the following

Theorem 1. The output and control base functions $y_k^b(h)$ and $u_k^b(h)$, $k=0\dots n_N$, relative to a discrete time set point signal $y^o(h)$ of finite duration N, can be determined as follows

$$y_k^b(h)=\sum_{l=0}^{N}\delta_l\, y_k^{bs}(h-l)$$
$$u_k^b(h)=\sum_{l=0}^{N}\delta_l\, u_k^{bs}(h-l)$$

being $y_k^{bs}(h)$ and $u_k^{bs}(h)$, $k=0\dots n_N$ the output and control base functions relative to a unit-step set point signal, respectively and $\delta_l=y^o(l+1)-y^o(l)$, $l=0\dots N$.

VI. APPLICATION EXAMPLES

In this Section two examples will be presented to show how the choice of a different $N_{FF}^e(s)$ affects the determination of the control and output base functions, and in which way the approach described in Section V can be exploited to derive the base functions relative to a given set point from the unit-step base functions, without making a new experiment. Note that only the filtering methods will be taken into account in the following, as they represent the better solution, applicable in a reasonably wide variety of actual cases.

Consider the plant with transfer function

$$P(s)=\frac{1}{s^3+2s^2+2s+1}$$

controlled by a feedback regulator with the following structure

$$R_{FB}(s)=\frac{s^2+s+1}{s(1+0.01s)}$$

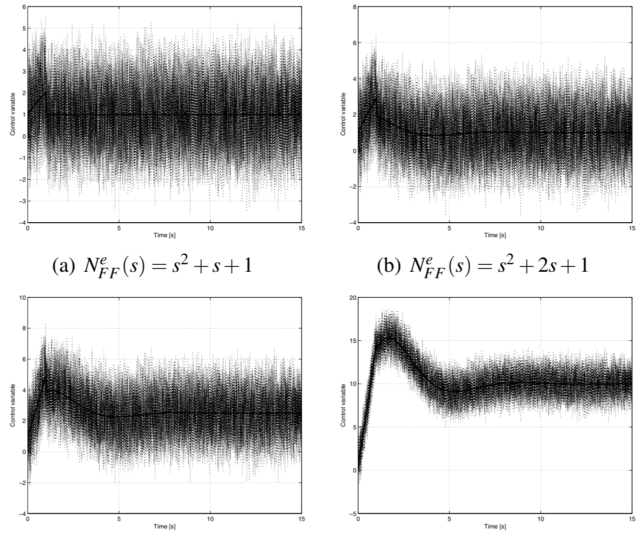
A. Choice of $N_{FF}^e(s)$

Consider the following choices for the polynomial $N_{FF}^e(s)$

$$N_{FF}^e(s)=\begin{cases} s^2+s+1\\ s^2+2s+1\\ s^2+3.5s+2.5\\ s^2+11s+10\end{cases} \tag{15}$$

The first one corresponds to the standard choice $N_{FF}^e(s)=N_{FB}(s)$. The polynomial $N_{FB}(s)$, however, has a couple of complex roots and, recalling the criterion expressed in Section IV-A, there exists another polynomial with real roots, the second one of (15), which is better for noise and disturbance rejection, i.e. whose frequency response has a larger magnitude. In the last two choices the magnitude of $N_{FF}^e(s)$ is gradually increased to study how this change affects the noise and disturbance rejection, at the expense however of an increasing control effort.

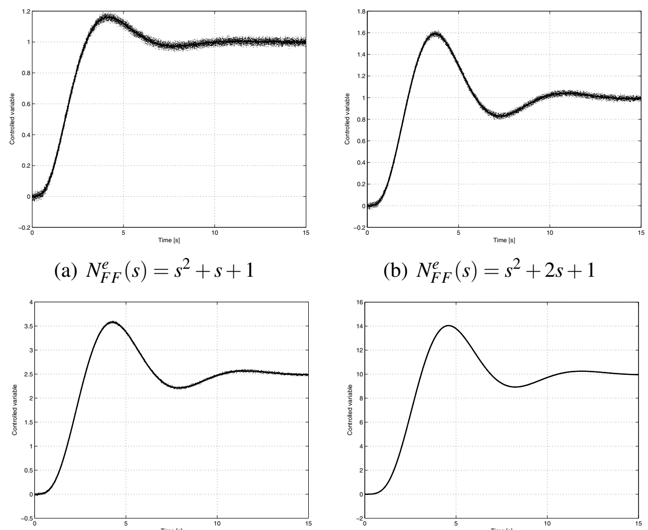
Consider a first situation in which $n(t)$ is a uniform white noise of amplitude ± 0.05, spanning a frequency range of about 0 to 670 rad/s, and $q(t)=0$, $\forall t$. Figs. 2 and 3 compare the responses of the controlled variable and the control signal to a saturated ramp set point (dotted line), reconstructed from the output and control base functions, with the output and control signal simulated in a noise free condition (solid line).



(a) $N_{FF}^e(s)=s^2+s+1$ (b) $N_{FF}^e(s)=s^2+2s+1$

(c) $N_{FF}^e(s)=s^2+3.5s+2.5$ (d) $N_{FF}^e(s)=s^2+11s+10$

Fig. 2. Control signal estimated from the control base functions.



(a) $N_{FF}^e(s)=s^2+s+1$ (b) $N_{FF}^e(s)=s^2+2s+1$

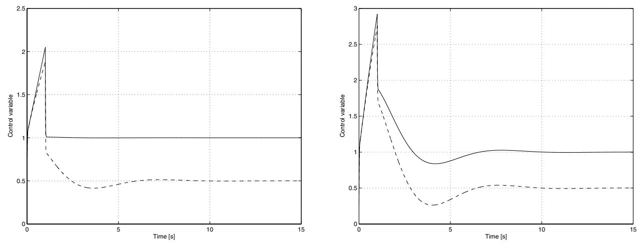
(c) $N_{FF}^e(s)=s^2+3.5s+2.5$ (d) $N_{FF}^e(s)=s^2+11s+10$

Fig. 3. Controlled variable estimated from the output base functions.

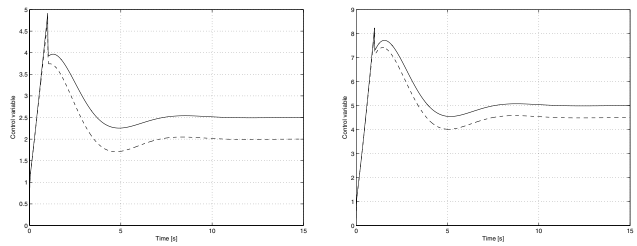
As can be seen from Figs. 2 and 3 the larger $|N_{FF}^e(j\omega)|$ is, the less the noise deteriorates the estimate. This improvement is at expense, however, of an increasing control effort (Fig. 2), as discussed in Section IV-A.

Consider now a different situation, in which $q(t)$ is a step disturbance of amplitude 0.5 and $n(t)=0$, $\forall t$. Again Figs. 4 and 5 compare the responses of the controlled variable and the control signal to a saturated ramp set point (dotted line), reconstructed from the output and control base functions, with the output and control signal simulated without the presence of the load disturbance (solid line).

As previously noticed, the larger $|N_{FF}^e(j\omega)|$ is the less the load disturbance affects the base functions' estimation (Figs.

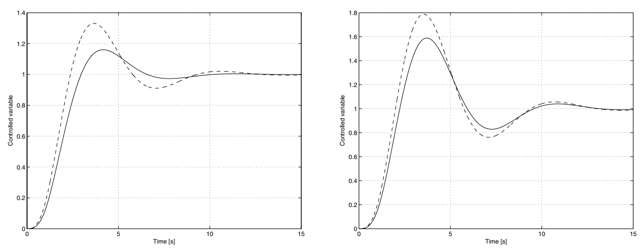


(a) $N_{FF}^{e}(s) = s^2 + s + 1$ (b) $N_{FF}^{e}(s) = s^2 + 2s + 1$

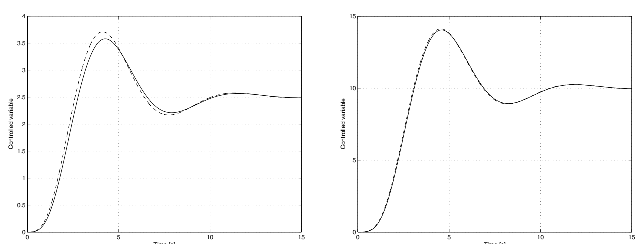


(c) $N_{FF}^{e}(s) = s^2 + 3.5s + 2.5$ (d) $N_{FF}^{e}(s) = s^2 + 11s + 10$

Fig. 4. Control signal estimated from the control base functions.



(a) $N_{FF}^{e}(s) = s^2 + s + 1$ (b) $N_{FF}^{e}(s) = s^2 + 2s + 1$



(c) $N_{FF}^{e}(s) = s^2 + 3.5s + 2.5$ (d) $N_{FF}^{e}(s) = s^2 + 11s + 10$

Fig. 5. Controlled variable estimated from the output base functions.

4 and 5). Again, it is clear from Fig. 4 that the improvement of the estimations is achieved increasing the control effort.

B. A 'general' set of base functions

We address now the problem of determining a set of output and control base functions, relative to a set point $y^o(t)$ given by

$$y^o(t) = 0.001t^3 - 0.002e^{0.5t} \tag{16}$$

starting from the unit-step base functions and applying Theorem 1. For the sake of clarity, in this case a noise and disturbance free environment will be considered.

Fig. 6 demonstrates the effectiveness of the approach proposed in Section V: one cannot distinguish between the output base functions reconstructed using Theorem 1 and the ones determined directly from set point (16).

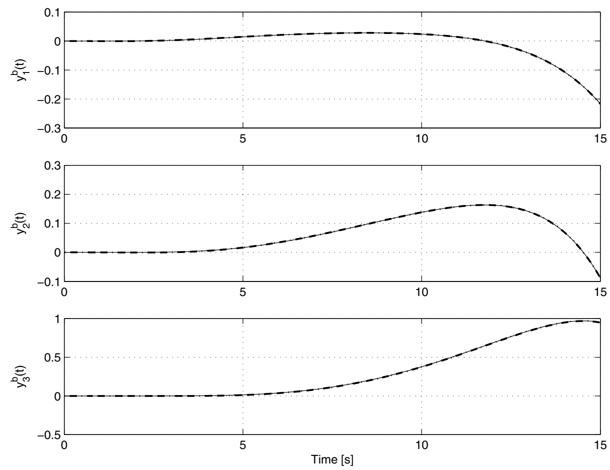


Fig. 6. Output base functions derived from the unit-step base functions (dash-dot line) and determined from $y^o(t)$ (solid line).

VII. CONCLUSIONS

The key idea of the method proposed in [4] for the synthesis of the feedforward part of a 2-d.o.f. controller is to formulate the problem as an optimization based on a nonparametric model of the control loop. As a consequence, the problem of determining this model in a realistic noisy environment plays a crucial role.

A few approaches to determine a set of base functions, i.e. a nonparametric model of the closed loop part of the system, experimentally have been discussed in this paper. The effects of noise and disturbances on the quality of the estimated model have been analyzed, given suitable criteria to optimize the estimation procedure. Finally, an approach to determine a 'general' set of base functions with a unique experiment, independent of the aspect of the set point signal, has been devised.

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