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Causal design methodology for optimal tracking in 2-d.o.f. industrial controllers

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Abstract— **This paper presents and discusses a causal method to design the feedforward part of 2-d.o.f. industrial controllers for optimal tracking, independently of the structure of the feedback controller, of the way it was designed, and of the characteristics of the set point signal. The method is particularly well suited for embedded systems, or wherever the limits imposed by the hardware and/or software architecture play a significant role; it is based on a nonparametric model of the control loop, that can be easily identified on-line, and allows to cast various tracking problems into a unified optimization framework. An experimental application example demonstrates the method's effectiveness, simplicity, and practical usefulness.**

I. INTRODUCTION

In many industrial controls, the use of two degrees of freedom (2-d.o.f.) regulators is highly beneficial, to separate - as much as possible - the problem of stability and disturbance rejection from that of set point tracking. However, despite the great importance of the 2-d.o.f. structure, see e.g. [1], [2] and a number of subsequent works, a very small number of tuning methodologies have been proposed that allow to synthesize such a control structure in a systematic (and possibly automated) way, and are suitable for industrial implementation (see, for example, the extensive review reported in [3] for PI-PID controllers). The mentioned lack of 'application oriented' synthesis methods turns into an almost total absence if one focuses attention on methods that are *particularly simple*, i.e., suitable for embedded systems, or wherever the limits imposed by the hardware and/or software architecture play a significant role.

To witness the statement above, one may analyze the vast literature that addresses the matter, typically from a quite control-theoretical point of view. Though a complete review would stray from the scope of this paper, such an analysis would reveal that many proposed approaches refer to conceptually complex situations like nonlinear, nonminimum-phase and/or time-varying systems and lead to correspondingly complex methods [4]; others are relative to specific applications [5], [6] or architectures [7], or are basically heuristic extensions of 1-d.o.f. methods [8]; others again are based on the inversion of the closed loop part of the control system and leads to excellent results, but requires a *noncausal* inversion.

From the application standpoint, the main problem (apart from computational complexity) is that most methods are limited to particular classes of problems (e.g., minimum-phase plants), or require that the set point enjoy some properties (see e.g. the concept of 'smooth output transition' involved in [9]). A key fact is that almost the totality of the presented methods are based on a parametric model of the closed loop part of the control system, and are therefore obliged to trust that model up to a very significant extent, since the improvement of set point tracking inherently involves open-loop actions.

This paper proposes a method to synthesize the feedforward part of a generic LTI 2-d.o.f. controller. The method is completely independent of the technique used to synthesize the feedback controller and of the aspect of the set point signal, does not require any parametric model of the control loop, can provide a reliable forecast of the obtained results, is computationally simple and based on a few, easily interpreted design parameters.

The key idea, preliminary introduced in [10] with exclusive reference to the PID case, is to formulate the problem as an (off-line) optimization based on a nonparametric model of the control loop, that is easily identified on-line. The method allows for a closed-form solution for some typical tracking problems, allowing for *explicit* tuning formulae. In addition, if one accepts to give up the closed-form solution and resort to numerical optimization, virtually any tracking specification and/or constraint can be accommodated for.

The paper is organized as follows. In Section II the 2-d.o.f. control scheme is introduced and the key idea of 'output' and 'control base functions' is presented. Section III describes a procedure to experimentally determine a set of base functions relative to a given set point signal. In Section IV a causal method to design a feedforward controller to achieve optimal set point tracking, based on the base functions approach, is presented. An experimental application example is discussed in Section V to demonstrate the method's effectiveness. Finally, Section VI concludes the paper with a concise overview of interesting open problems.

II. PROBLEM STATEMENT

Consider the class of 2-d.o.f. linear controllers described, in the frequency domain, by the general control law

$$D(s)U(s) = N_{FF}(s)Y^{\circ}(s) - N_{FB}(s)Y(s) \qquad (1)$$

where $Y^{\circ}(s)$, $Y(s)$, and $U(s)$ are the Laplace transforms of the set point, the controlled variable and the control signal, respectively, while $N_{FF}(s)$, $N_{FB}(s)$ and $D(s)$ are three

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TABLE I

POLYNOMIAL COEFFICIENTS FOR AN ISA PID STRUCTURE.

k	0	1	2
n_k^{FF}	K	$K(bT_i+T_d)/N$	$KT_iT_d(c+b/N)$
n_k^{FB}	K	$K(T_i+T_d)/N$	$KT_iT_d(1+1/N)$
n_k^{D}	0	T_i	T_iT_d/N

polynomials in the complex variable s expressed as

$$N_{FF}(s)=\sum_{k=0}^{n_N} n_k^{FF}s^k \qquad N_{FB}(s)=\sum_{k=0}^{n_N} n_k^{FB}s^k$$
$$D(s)=\sum_{k=0}^{n_D} n_k^{D}s^k \tag{2}$$

with $n_N \leq n_D$, $n_0^{FF} \neq 0$, $n_0^{FB} \neq 0$. Such controllers are implemented as shown in Fig. 1, where

$$R_{FF}(s)=\frac{N_{FF}(s)}{N_{FB}(s)} \qquad R_{FB}(s)=\frac{N_{FB}(s)}{D(s)}$$

and we have to assume that all the roots of $N_{FB}(s)$ lie in the open LHP. This loss of generality is of minimal importance in the application domain, however.

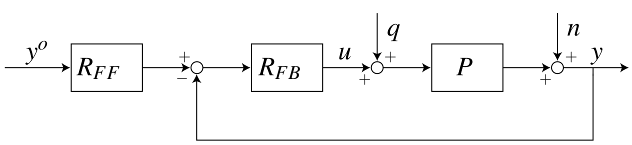


Fig. 1. Control scheme with a 2-d.o.f. regulator.

Notice that (1) is a generalization of several control laws widely used in the industrial domain, as it can represent any linear 2-d.o.f. controller where the transfer functions from $Y^\circ(s)$ to $Y(s)$ and from $U(s)$ to $Y(s)$ differ only in the numerator, i.e., any controller adopting the well known 'set point weighing' technique. For example, (1) corresponds to the 2-d.o.f. ISA PID control law [2]

$$U=K\left[bY^\circ-Y+\frac{1}{sT_i}(Y^\circ-Y)+\frac{sT_d}{1+sT_d/N}(cY^\circ-Y)\right]$$

by setting $n_N=2$, $n_D=2$ and the polynomial coefficients as in Table I.

Suppose that a stabilizing feedback block $R_{FB}(s)$ has already been synthesized, so that the polynomials $N_{FB}(s)$ and $D(s)$ are fixed, and the problem is to determine the polynomial $N_{FF}(s)$ so that convenient set-point tracking properties (discussed later on) are enjoyed by the control system. Note that if the feedback regulator $R_{FB}(s)$ has an integral action, i.e., if $D(0)=0$, n_0^{FF} must equal n_0^{FB} to preserve zero steady-state error. More in general, it is common (and very reasonable) practice that $R_{FF}(s)$ has unity gain. As such, the polynomial $N_{FF}(s)$ resulting from the proposed synthesis method will have $n_0^{FF}=n_0^{FB}$. We can now state the following

Lemma 1. The responses $y(t)$ and $u(t)$ of the controlled variable and the control signal forced, in the control system of Fig. 1, by a Laplace-transformable set point $y^\circ(t)$ can be expressed as a weighed sum of n_N+1 functions $y_k^b(t)$ and $u_k^b(t)$, respectively, the weights being the coefficients of $N_{FF}(s)$, i.e.,

$$y(t)=\sum_{k=0}^{n_N} n_k^{FF}y_k^b(t)$$

$$u(t)=\sum_{k=0}^{n_N} n_k^{FF}u_k^b(t)$$

where $N_{FF}(s)$ is expressed as in (2). The functions $y_k^b(t)$ and $u_k^b(t)$ do not depend on the coefficients of $N_{FF}(s)$, and will be termed the control system's 'output base functions' and 'control base functions' relative to the set point signal $y^\circ(t)$.

The proof is omitted for brevity, see [10] for a discussion. Note that the output and the control base functions can be expressed as

$$y_k^b(t)=\mathscr{L}^{-1}\left[\frac{s^kT(s)}{N_{FB}(s)}Y^\circ(s)\right] \tag{3}$$

$$u_k^b(t)=\mathscr{L}^{-1}\left[\frac{s^kT(s)}{N_{FB}(s)P(s)}Y^\circ(s)\right] \tag{4}$$

and interpreted as follows. Each closed loop set point response $y(t)$ or $u(t)$ of the control system is a vector in a finite-dimensional functional space, where we can take the functions $y_k^b(t)$, or $u_k^b(t)$, as a base (whence the name). The base functions allow to compute the set point response of the control system for any value of the coefficients n_k^{FF}, and the relationship between the coefficients and the responses is linear. Therefore, the base functions are a nonparametric model of the control system, particularly suitable for tuning $N_{FF}(s)$. Such a model does not imply any assumption on the structure of the process dynamics.

III. DETERMINATION OF THE BASE FUNCTIONS

We now address the problem of determining the base functions relative to a given set point signal experimentally. An interesting solution comes from the following

Remark 1. Denoting by $Y_k^b(s)$ the Laplace transform of $y_k^b(t)$, on the basis of (3) it is immediate to write

$$Y_k^b(s)=F_k^b(s)Y(s) \qquad 0\leq k\leq n_N \tag{5}$$

where

$$F_0^b(s)=\frac{1}{N_{FF}(s)} \qquad F_{k+1}^b(s)=sF_k^b(s)$$

Similarly, for the control base functions, with obvious notation it turns out that

$$U_k^b(s)=F_k^b(s)U(s) \tag{6}$$

Notice that the transfer functions $F_k^b(s)$ are the same for the output and control base functions, and depend only on $N_{FF}(s)$. Therefore, these transfer functions will be termed the 'base filters' relative to $N_{FF}(s)$.

Remark 1 shows that one can obtain the base functions relative to a given set point signal with a single experiment: in principle, it suffices to choose as $N_{FF}(s)$ an arbitrary polynomial $N^e_{FF}(s)$, ensuring that the n_N+1 base filters $F^b_k(s)$ are causal and stable, apply the set point signal of interest to the control system, and then filter the obtained responses $y^e(t)$ and $u^e(t)$ as dictated by (5) and (6). Unfortunately, this would be correct only if the system were disturbance- and noise-free (i.e., with reference to Fig. 1, $q(t) = n(t) = 0$) and there were no limits on the admissible control effort. Since these hypotheses are unrealistic, $N^e_{FF}(s)$ cannot be chosen arbitrarily. However, as can be easily guessed, even formalizing the problem of choosing $N^e_{FF}(s)$ is very complex, and practically impossible without a quite reliable model of the process. Moreover, assuming the availability of such a model is in contrast with the context of this work.

As such, for the choice of $N^e_{FF}(s)$ the following (heuristic) method can be applied: set $N^e_{FF}(s) = N_{FB}(s)$, so that $R^e_{FF}(s) = 1$. This solution does not involve any unstable pole-zero cancelation, since the roots of $N_{FB}(s)$ have been assumed belonging to the open LHP and ensures the causality and stability of the base filters.

IV. USE OF THE BASE FUNCTIONS FOR TRACKING OPTIMIZATION

Set point weighting was originally introduced in PID controllers [1] to reduce overshoots in the closed loop set point step response. The choice of the weights, however, is not an easy task. Sometimes the closed loop response is very sensitive to them, while in other cases their effect is almost negligible. Which of the two situations occurs depends on the process dynamics and on the approach adopted for the synthesis of the feedback part of the regulator. This is probably the main reason why the rules for tuning the weights proposed up to now are empirical and generally deduced from extensive simulations.

As long as a parametric model of the loop is used, this bottleneck cannot be eliminated. It is apparently impossible to assess the characteristics of the closed loop response unless some assumptions on the structure of the process dynamics and on the tuning method used for the feedback regulator are made.

The method proposed herein adopts a different and more global point of view: the basic idea is to choose the weights, i.e., the polynomial $N_{FF}(s)$, so as to make the closed loop set point responses of the controlled variable and/or the control signal *as similar as possible to reference ones*. This is a very natural way to express a set point tracking request, while maintaining the possibility of penalizing the control effort. To formalize the method, then, it suffices to specify the response(s) taken as reference and the criterion used to assess how close an actual response is to a reference one. This can be done in several ways, each one leading to an optimization problem with a different goal. The base functions' approach allows to state all of these problems within a unitary framework, so that a common (and very efficient) solution procedure can be devised. Defining

$$\theta = \left[n_0^{FF}\; n_1^{FF} \ldots n_{n_N}^{FF}\right]'$$

$$y^b(t) = \left[y_0^b(t)\; y_1^b(t) \ldots y_{n_N}^b(t)\right]'$$

$$u^b(t) = \left[u_0^b(t)\; u_1^b(t) \ldots u_{n_N}^b(t)\right]'$$

allows to express the responses estimated with the base functions as

$$\hat{y}(\theta,t) = \theta' y^b(t) \qquad \hat{u}(\theta,t) = \theta' u^b(t)$$

Now, denote with $y_d(t)$, $u_d(t)$ the desired responses of the controlled variable and the control signal. These desired responses are *de facto* the design parameters, and will be discussed later. To achieve the goal of making $\hat{y}(\theta,t)$, $\hat{u}(\theta,t)$ resemble $y_d(t)$, $u_d(t)$, it is convenient to minimize the cost function

$$J(\theta, y_d(\cdot), u_d(\cdot), \alpha) = \int_0^T \Big(\alpha\,(\hat{y}(\theta,t) - y_d(t))^2 + (1-\alpha)\,(\hat{u}(\theta,t) - u_d(t))^2\Big)\,dt$$

where $0 < T \leq +\infty$, while in this research we are only interested in the finite duration case; α ($0 \leq \alpha \leq 1$) is a further design parameter, used to balance the penalization of the deviations of $\hat{y}(\theta,t)$ and $\hat{u}(\theta,t)$ from the respective desired responses. Notice that $J \geq 0$ for any admissible α.

Defining a symmetric matrix A, a column vector b and a scalar c as

$$A(\alpha) = \{a_{ij}(\alpha)\}$$
$$b(\alpha, y_d(\cdot), u_d(\cdot)) = \{b_i(\alpha, y_d(\cdot), u_d(\cdot))\}$$

with $i, j = 0 \ldots n_N$, and

$$a_{ij}(\alpha) = a_{ji}(\alpha) = \alpha \int_0^T y_i^b(t) y_j^b(t)dt + (1-\alpha)\int_0^T u_i^b(t) u_j^b(t)dt$$

$$b_i(\alpha, y_d(\cdot), u_d(\cdot)) = \alpha \int_0^T y_i^b(t) y_d(t)dt + (1-\alpha)\int_0^T u_i^b(t) u_d(t)dt$$

$$c(\alpha, y_d(\cdot), u_d(\cdot)) = \alpha \int_0^T (y_d(t))^2 dt + (1-\alpha)\int_0^T (u_d(t))^2 dt$$

Dropping unnecessary notation, J can be written as

$$J(\theta) = \theta' A \theta - 2b'\theta + c \qquad (7)$$

The cost function (7) can have no minimum, a unique minimum or infinite minima, according to the rank of $A(\alpha)$. We can thus state the following

Theorem 1. The cost function J is convex and a solution of the optimization problem

$$\theta^o = \arg\min J(\theta)$$

always exists. Moreover the following facts can be stated:

1) if $A(\alpha)$ is nonsingular, J has a unique minimum, and the corresponding (optimal) value of θ is

$$\theta^o(\alpha, y_d(\cdot), u_d(\cdot)) = A^{-1}(\alpha) b(\alpha, y_d(\cdot), u_d(\cdot)) \quad (8)$$

2) if $A(\alpha)$ is singular, denoting its nullity by ν_A, J has ∞^{ν_A} minima that correspond to ∞^{ν_A} optimal values of θ, which form a linear variety in $\mathbf{R}^{n_N+1}$, of dimension ν_A, and parallel to the null space of $A(\alpha)$
3) all the minima are global.

Proof. Suppose that $\bar{\theta}$ exists such that $\bar{\theta}' A(\alpha) \bar{\theta} < 0$, i.e. $A(\alpha) < 0$, and define $\bar{\bar{\theta}} = \lambda \bar{\theta}$, with $\lambda > 0$. This implies that

$$J = \lambda^2 \left(\bar{\bar{\theta}}' A(\alpha) \bar{\bar{\theta}} \right) - \lambda \left(2b' \bar{\bar{\theta}} \right) + c < 0$$

as the quadratic term dominates the linear term, for λ sufficiently large. As a consequence, since $J \geq 0$, $A(\alpha) \geq 0$ for any admissible α and the cost function is convex, being $\nabla^2 J = 2A \geq 0$.

On the other hand

$$c(\alpha, y_d(\cdot), u_d(\cdot)) \geq 0 \qquad \forall \alpha, y_d(\cdot), u_d(\cdot)$$

thus

$$\theta' A(\alpha) \theta - 2b' \theta \geq -c$$

i.e. the cost function is lower bounded. This implies that a solution of the aforementioned optimization problem always exists.

Finally, the optimal values of θ are given by the following equation

$$\nabla J = 2(A\theta - b) = 0 \quad (9)$$

Thus, if $A(\alpha)$ is nonsingular (9) has a unique solution given by (8). On the other hand, if $\text{rank}(A) = \tilde{n}_N < n_N$, equation (9) has ∞^{ν_A} solutions, with $\nu_A = n_N - \tilde{n}_N$, that correspond to ∞^{ν_A} optimal values of θ.

Finally, being the cost function J convex, all the solutions of (9) are global minima.

In practice, if A is nonsingular all the coefficients n_k^{FF} are uniquely determined. In the opposite case, ν_A of them can be chosen freely. Notice that the elements of A and b are affine in α, and that the elements of A do not depend on $y_d(t)$ and $u_d(t)$. This means that, if several optimizations are tried, no integral must be recomputed if α is changed, while only $n_N + 1$ integrals must be recomputed if $y_d(t)$ or $u_d(t)$ are modified.

In many practical situations some constraints on the coefficients n_k^{FF} have to be taken into account. For example, when the control loop includes an integral action, in the process or in the controller, i.e. when $T(0) = 1$, n_0^{FF} must equal n_0^{FB} to preserve zero steady-state error.

Assume that n_C constraints are introduced on the coefficients n_k^{FF} as follows

$$n_i^{FF} = \bar{n}_i^{FF} \quad i \in \mathbf{N}_C \subset \mathbf{N}_N = \{1, \ldots, n_N\}, |\mathbf{N}_C| = n_C$$

and define a new vector of parameters

$$\tilde{\theta} = \begin{bmatrix} \bar{n}^{FF} \\ \theta_{FF} \end{bmatrix}$$

where $\bar{n}^{FF}$ is the vector of fixed parameters, while θ_{FF} contains the $n_N - n_C$ independent variables.

Consequently, the rows and columns of matrix A and vector b can be rearranged obtaining

$$\tilde{A} = \begin{bmatrix} \bar{A} & d \\ d' & A_{FF} \end{bmatrix} \quad \tilde{B} = \begin{bmatrix} \bar{b} \\ b_{FF} \end{bmatrix}$$

where

$$\begin{aligned} \bar{A} &= \{a_{ij}\} & & i, j \in \mathbf{N}_C \\ A_{FF} &= \{a_{ij}\} & & i, j \in \mathbf{N}_N \setminus \mathbf{N}_C \\ d &= \{a_{ij}\} & & i \in \mathbf{N}_C, j \in \mathbf{N}_N \setminus \mathbf{N}_C \\ \bar{b} &= b_i & & i \in \mathbf{N}_C \\ b_{FF} &= b_i & & i \in \mathbf{N}_N \setminus \mathbf{N}_C \end{aligned}$$

Therefore, equation (7) becomes

$$\tilde{J}(\theta_{FF}) = \begin{bmatrix} \bar{n}^{FF} \\ \theta_{FF} \end{bmatrix}' \begin{bmatrix} \bar{A} & d \\ d' & A_{FF} \end{bmatrix} \begin{bmatrix} \bar{n}^{FF} \\ \theta_{FF} \end{bmatrix} - 2 \begin{bmatrix} \bar{b} \\ b_{FF} \end{bmatrix}' \begin{bmatrix} \bar{n}^{FF} \\ \theta_{FF} \end{bmatrix} + c$$

or equivalently

$$\tilde{J}(\theta_{FF}) = \theta_{FF}' A_{FF} \theta_{FF} - 2\beta' \theta_{FF} + \gamma$$

where

$$\begin{aligned} \beta &= b_{FF}' - d' \bar{n}^{FF} \\ \gamma &= \bar{n}^{FF'} \bar{A} \bar{n}^{FF} - 2\bar{b}' \bar{n}^{FF} + c \end{aligned}$$

As can be easily seen, the cost function $\tilde{J}(\theta_{FF})$ has the same form of $J(\theta)$. Moreover, $\tilde{J}(\theta_{FF}) \geq 0$ and this implies $A_{FF} \geq 0$ and $\gamma \geq 0$. As a consequence, Theorem 1 holds in this case as well.

The general problem stated so far can be specialized to pursue different goals, depending on the choice of $y_d(t)$, $u_d(t)$, and α. Four cases of interest are illustrated in the following.

Ideal tracking with costless control. If $\alpha = 1$, there is no control penalization. The most demanding (ideal) solution in that case is to select the reference response $y_d(t)$ as the set point signal $y^\circ(t)$ of interest itself. The problem is then

$$\begin{aligned} \theta_o &= \arg\min J_{itcc}(\theta) \\ J_{itcc}(\theta) &= J(\theta, y^\circ(\cdot), \cdot, 1) \end{aligned}$$

Ideal tracking with ideal control penalization. The simplest way to penalize the control action is to set $u_d(t) = 0$ and employ parameter α to modulate the penalization.

Selecting $y_d(t) = y^\circ(t)$ as in the previous case, the problem is

$$\theta_o = \arg\min J_{itcp}(\theta,\alpha)$$
$$J_{itcp}(\theta,\alpha) = J(\theta, y^\circ(\cdot), 0, \alpha)$$

Referenced tracking with ideal costless control. When no control penalization is required, a more realistic solution is to select the reference response $y_d(t)$ as the response of a reference model $M(\theta_M, s)$, vector θ_M being its parameters, to the set point signal $y^\circ(t)$ of interest. The problem is then

$$\theta_o = \arg\min J_{rtcc}(\theta,\theta_M)$$
$$J_{rtcc}(\theta,\theta_M) = J(\theta, y_d(\theta_M,\cdot), \cdot, 1)$$

Referenced tracking with control penalization. Reasoning as in the previous case, but in the presence of control penalization, the problem becomes

$$\theta_o = \arg\min J_{rtcp}(\theta,\theta_M,\alpha)$$
$$J_{rtcp}(\theta,\theta_M,\alpha) = J(\theta, y_d(\theta_M,\cdot), 0, \alpha)$$

Remark 2. Applying the Parseval theorem, the general referenced tracking problem without control penalization can be interpreted as a model matching one in the 2-norm, because

$$J = \frac{1}{2\pi}\left\|\left(\frac{N_{FF}(s)}{D(s)}T(s) - M(s)\right)Y^\circ(s)\right\|_2^2$$

To solve this problem satisfactorily, an accurate process model is needed. It is not possible to use the same model employed for the synthesis of $R_{FB}(s)$, whatever tuning method is considered. The base functions framework allows to operate directly in the time domain, thus avoiding the mismatch problem.

V. AN EXPERIMENTAL MOTOR VELOCITY CONTROL

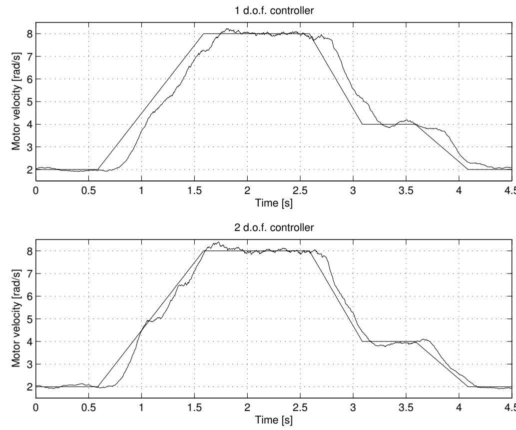
The proposed method has been applied to an experimental test bed for velocity control, composed of a DC motor, a flexible transmission, a mechanical load, and a tacho generator. The control variable is the motor voltage, and the controlled variable is the load velocity. The example aims at proving the feasibility of base function determination in a (noisy) physical environment, and at demonstrating the method's effectiveness with different regulator structures. For brevity, only the ideal tracking problem with costless control is considered.

Fig. 2 depicts the results obtained with the PI regulator

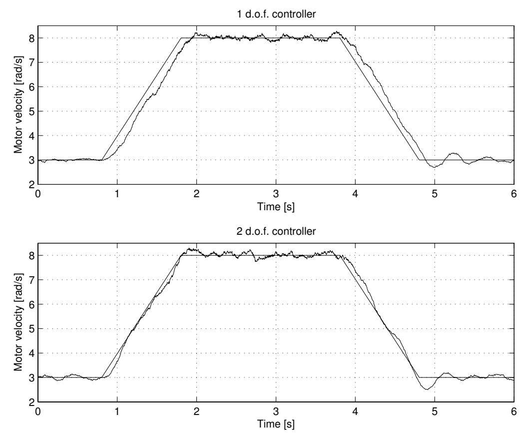
$$R_{FB}(s) = \frac{0.03\ s + 0.5}{0.06\ s}$$

and the third order regulator

$$R_{FB}(s) = \frac{10\ s^3 + 400\ s^2 + 10000\ s + 100000}{s(s^2 + 200\ s + 10000)}$$



(a) PI controller



(b) Third order controller

Fig. 2. Tracking of a trapezoidal velocity profile.

Recall that for the presented research the tuning method used for $R_{FB}(s)$ is irrelevant. The solutions yielded by the proposed method in the two cases are

$$R_{FF}(s) = \frac{0.0678\ s + 0.5}{0.03\ s + 0.5}$$

and

$$R_{FF}(s) = \frac{25\ s^3 + 671\ s^2 + 17610\ s + 100000}{10\ s^3 + 400\ s^2 + 10000\ s + 100000}$$

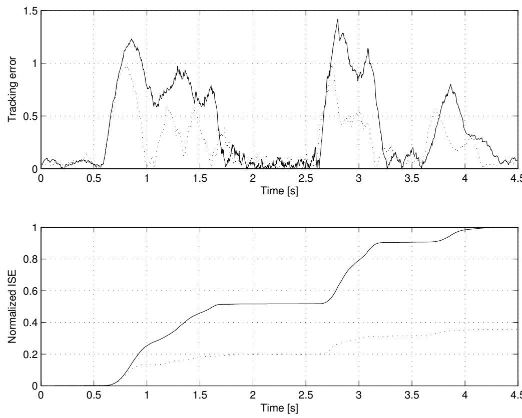
respectively.

Fig. 3 shows the tracking error and normalized ISE with a 1-d.o.f. regulator and with the 2-d.o.f. one provided by the proposed method. The achieved improvement is apparent, and does not result in an excessive control effort (Fig. 4).

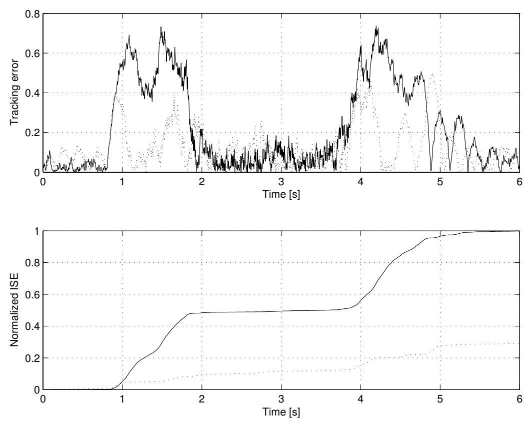
VI. CONCLUSIONS AND FUTURE WORKS

A. Conclusions

A computationally simple method to design the feedforward part of a 2-d.o.f. controller has been presented in this paper. The method is based on the 'base functions' approach, i.e., a nonparametric model of the control system, and is completely independent of the technique used to design the feedback controller and to the aspect of the set-point signal. Moreover, a simple procedure to experimentally determine

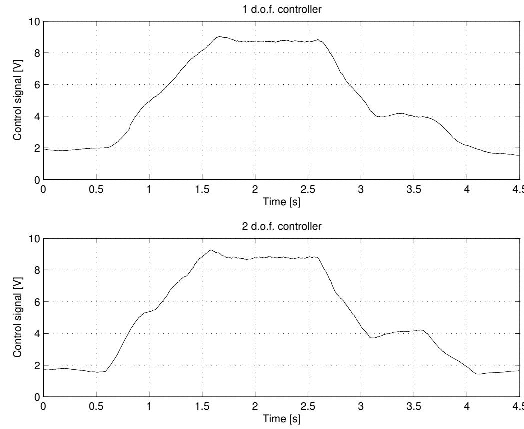


(a) PI controller

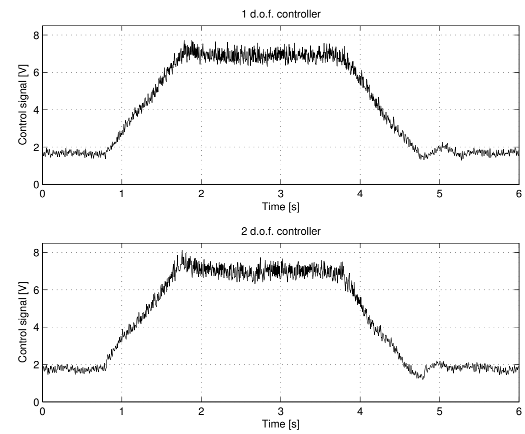


(b) Third order controller

Fig. 3. Tracking error and normalized ISE: 1-d.o.f. (solid line), 2-d.o.f. (dotted line).



(a) PI controller



(b) Third order controller

Fig. 4. Control signal.

the base functions relative to a given set-point signal has been introduced.

Finally, an experimental application example has been conceived to prove the feasibility of the base function determination in a noisy physical environment and to demonstrate the method's effectiveness with different structures of the feedback regulator.

B. Future Works

In view of the mentioned lack of "application oriented" synthesis techniques, further extensions of the proposed method will be considered in the near future. For example, the study of specific optimization problems could be of great interest to industrial applications, e.g. in CNC controllers the feedback loop could be tuned with the aim of maximizing the closed loop bandwidth and disturbance rejection, leaving to the feedforward controller the set-point tracking task. Moreover, when periodic set-point signals are of concern, as it frequently happens in motion control applications, an iterative procedure that optimizes the feedforward controller weights at each period could enhance the set-point tracking performances of the system.

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