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A Case Study for the Delay-type Nehari Problem

Qing-Chang Zhong

***Abstract*— In this paper, a case study is given for the delay-type Nehari problem so that it can be better understood.**

***Index Terms*— Delay-type Nehari problem, algebraic Riccati equation, H^∞ control, Smith predictor**

I. Introduction

The H^∞ control of processes with delay(s) has been an active research area since the mid 80's. It is well known that the Nehari problem [1] played an important role in the development of H^∞ control theory. This is still true in the case for systems with delay(s) [2]. Some papers, e.g. [3], [4], were devoted to calculate the infimum of the delay-type Nehari problem in the stable case. It was shown in [3] that this problem in the stable case is equivalent to calculating an $L_2[0,h]$-induced norm. For the unstable case, Tadmor [5] presented a state-space solution using the differential/algebraic matrix Riccati equation-based method and Zhong [6] proposed a frequency-domain solution using the J-spectral factorization. The solvability condition of the delay-type Nehari problem is formulated in terms of the non-singularity of a delay-dependent matrix in [6]. The optimal value γ_{opt} is the maximal $\gamma \in [0,\ \infty)$ such that this matrix becomes singular when γ decreases from ∞. All sub-optimal compensators are parameterized in a transparent structure incorporating a modified Smith predictor. The solution is mathematically elegant. However, it seems not trivial to find the solution for a given system, even for a simple system. In order to better understand this problem, a case study for a first-order system is given here.

II. Summary of the Theoretical Results

The Delay-type Nehari Problem(NP$_\text{h}$) is described as follows. Given a minimal state-space realization $G_\beta(s) \doteq \left[\begin{array}{c|c} A & B \\ \hline -C & 0 \end{array}\right] = -C(sI-A)^{-1}B$, which is not necessarily stable and $h \geq 0$, characterize the optimal value

$$\gamma_{opt} = \inf\{\left\|G_\beta(s) + e^{-sh}K(s)\right\|_{L_\infty} :\ K(s) \in H^\infty\} \quad (1)$$

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Q.-C. Zhong is with the Department of Electrical Engineering and Electronics, The University of Liverpool, Brownlow Hill, Liverpool L69 3GJ, UK. Email: zhongqc@ieee.org, URL: http://come.to/zhongqc.

and for a given $\gamma > \gamma_{opt}$, parameterize the suboptimal set of proper $K(s) \in H^\infty$ such that

$$\left\|G_\beta(s) + e^{-sh}K(s)\right\|_{L_\infty} < \gamma. \quad (2)$$

The result about this problem is summarized as follows. See [6], [2] for more details.

Assume that G_β has neither $j\omega$-axis zeros nor $j\omega$-axis poles. Define the following two Hamiltonian matrices:

$$H_c = \begin{bmatrix} A & \gamma^{-2}BB^* \\ 0 & -A^* \end{bmatrix}, \qquad H_o = \begin{bmatrix} A & 0 \\ -C^*C & -A^* \end{bmatrix}$$

and

$$H = \begin{bmatrix} A & \gamma^{-2}BB^* \\ -C^*C & -A^* \end{bmatrix},$$

$$\Sigma = \begin{bmatrix} \Sigma_{11} & \Sigma_{12} \\ \Sigma_{21} & \Sigma_{22} \end{bmatrix} \doteq \Sigma(h) = e^{Hh}, \quad (3)$$

The algebraic Riccati equations (ARE)

$$\begin{bmatrix} -L_c & I \end{bmatrix} H_c \begin{bmatrix} I \\ L_c \end{bmatrix} = 0 \quad (4)$$

and

$$\begin{bmatrix} I & -L_o \end{bmatrix} H_o \begin{bmatrix} L_o \\ I \end{bmatrix} = 0 \quad (5)$$

always have unique solutions $L_c \leq 0$ and $L_o \leq 0$ such that $A + \gamma^{-2}BB^*L_c = \begin{bmatrix} I & 0 \end{bmatrix} H_c \begin{bmatrix} I \\ L_c \end{bmatrix}$ and $A + L_oC^*C = \begin{bmatrix} I & -L_o \end{bmatrix} H_o \begin{bmatrix} I \\ 0 \end{bmatrix}$ are stable, respectively.

Then, the optimal value γ_{opt} is given by

$$\gamma_{opt} = \max\{\gamma :\ \det \hat{\Sigma}_{22} = 0\},$$

where

$$\hat{\Sigma}_{22} = \begin{bmatrix} -L_c & I \end{bmatrix} \Sigma \begin{bmatrix} L_o \\ I \end{bmatrix}. \quad (6)$$

Furthermore, for a given $\gamma > \gamma_{opt}$, the suboptimal set of proper $K \in H^\infty$ such that (2) holds is given by

$$K = \mathcal{H}_r(\begin{bmatrix} I & 0 \\ Z & I \end{bmatrix} W^{-1},\ Q) \quad (7)$$

where $\|Q(s)\|_{H^\infty} < \gamma$ is a free parameter and

$$Z = -\pi_h\{\mathcal{F}_u(\begin{bmatrix} G_\beta & I \\ I & 0 \end{bmatrix}, \gamma^{-2}G_\beta^{\sim})\}, \quad (8)$$

$$W^{-1} = \left[\begin{array}{c|cc} A+\gamma^{-2}BB^*L_c & \hat{\Sigma}_{22}^{-*}(\Sigma_{12}^* + L_o\Sigma_{11}^*)C^* & -\hat{\Sigma}_{22}^{-*}B \\ \hline -C & I & 0 \\ \gamma^{-2}B^*(\Sigma_{21}^* - \Sigma_{11}^* L_c) & 0 & I \end{array}\right]. \tag{9}$$

Here, π_h, called the completion operator, is defined as

$$\pi_h(G) \doteq \left[\begin{array}{c|c} A & B \\ \hline Ce^{-Ah} & 0 \end{array}\right] - e^{-sh}\left[\begin{array}{c|c} A & B \\ \hline C & D \end{array}\right] \tag{10}$$

for $G = \left[\begin{array}{c|c} A & B \\ \hline C & D \end{array}\right] = D + C(sI-A)^{-1}B$.

It is easy to see that γ_{opt} satisfies

$$\left\|\Gamma_{G_\beta}\right\| \le \gamma_{opt} \le \|G_\beta(s)\|_{L_\infty} \tag{11}$$

because, on one hand, the delay-type Nehari problem is solvable [7] iff

$$\gamma > \gamma_{opt} \doteq \left\|\Gamma_{e^{sh}G_\beta}\right\|, \tag{12}$$

where Γ denotes the Hankel operator and, on the other hand, it can be seen from (1) that

$$\gamma_{opt} \le \|G_\beta(s)\|_{L_\infty}$$

(at least K can be chosen as 0). The symbol $e^{sh}G_\beta$ in (12) is non-causal and, possibly, unstable. Hence, $\gamma_{opt} \ge \left\|\Gamma_{G_\beta}\right\|$.

III. A Case Study

Consider

$$G_\beta(s) = -\frac{1}{s-a}$$

with a minimal realization

$$G_\beta = \left[\begin{array}{c|c} a & 1 \\ \hline -1 & 0 \end{array}\right],$$

i.e., $A = a$, $B = 1$ and $C = 1$. According to the assumptions, $a \ne 0$. This gives

$$H_c = \begin{bmatrix} a & \gamma^{-2} \\ 0 & -a \end{bmatrix}, \quad H_o = \begin{bmatrix} a & 0 \\ -1 & -a \end{bmatrix}$$

and

$$H = \begin{bmatrix} a & \gamma^{-2} \\ -1 & -a \end{bmatrix}.$$

The ARE (4) and (5) can be re-written as

$$L_c^2\gamma^{-2} + 2aL_c = 0 \quad \text{and} \quad L_o^2 + 2aL_o = 0.$$

When $a < 0$, i.e., G_β is stable, the stabilizing solutions are

$$L_c = 0 \quad \text{and} \quad L_o = 0;$$

When $a > 0$, i.e., G_β is unstable, these are[1]

$$L_c = -2a\gamma^2 \quad \text{and} \quad L_o = -2a.$$

[1]In this case, $A + \gamma^{-2}BB^*L_c = a - 2a = -a$ and $A + L_oC^*C = a - 2a = -a$ are stable.

It is easy to find that the eigenvalues of H are $\lambda_{1,2} = \pm\lambda$ with $\lambda = \sqrt{a^2 - \gamma^{-2}}$. Note that λ may be an imaginary number. Assume that $\gamma \ne 1/|a|$ temporarily. With two similarity transformations with

$$S_1 = \begin{bmatrix} 1 & -a+\lambda \\ 0 & 1 \end{bmatrix} \quad \text{and} \quad S_2 = \begin{bmatrix} 1 & 0 \\ -\frac{1}{2\lambda} & 1 \end{bmatrix},$$

H can be transformed into $\begin{bmatrix} \lambda & 0 \\ 0 & -\lambda \end{bmatrix}$, i.e.,

$$S_2^{-1}S_1^{-1}HS_1S_2 = \begin{bmatrix} \lambda & 0 \\ 0 & -\lambda \end{bmatrix}.$$

Hence, the Σ-matrix defined in (3) is

$$\begin{aligned} \Sigma &= e^{Hh} \\ &= S_1S_2\begin{bmatrix} e^{\lambda h} & 0 \\ 0 & e^{-\lambda h} \end{bmatrix} S_2^{-1}S_1^{-1} \\ &= S_1\begin{bmatrix} e^{\lambda h} & 0 \\ -\frac{1}{2\lambda}(e^{\lambda h} - e^{-\lambda h}) & e^{-\lambda h} \end{bmatrix} S_1^{-1} \\ &= \begin{bmatrix} e^{\lambda h} + \frac{a-\lambda}{2\lambda}(e^{\lambda h} - e^{-\lambda h}) & \frac{a^2-\lambda^2}{2\lambda}(e^{\lambda h} - e^{-\lambda h}) \\ -\frac{1}{2\lambda}(e^{\lambda h} - e^{-\lambda h}) & e^{-\lambda h} - \frac{a-\lambda}{2\lambda}(e^{\lambda h} - e^{-\lambda h}) \end{bmatrix}. \end{aligned}$$

When $\gamma = 1/|a|$, i.e., $\lambda = 0$, the above Σ still holds if the limit for $\lambda \to 0$ is taken on the right-hand side, which gives

$$\Sigma|_{\lambda=0} = \begin{bmatrix} 1+ah & a^2h \\ -h & 1-ah \end{bmatrix}.$$

In the sequel, Σ is assumed to be defined as above for $\lambda = 0$.

A. The stable case ($a < 0$)

In this case, G_β is stable and $L_c = L_o = 0$. Hence, $\hat{\Sigma}_{22}$ defined in (6) is

$$\hat{\Sigma}_{22} = \Sigma_{22} = e^{-\lambda h} - \frac{a-\lambda}{2\lambda}(e^{\lambda h} - e^{-\lambda h}).$$

When $\gamma \ge 1/|a|$, the number λ is positive and hence the eigenvalues of H are real. It is easy to see that $\hat{\Sigma}_{22}$ is always positive (nonsingular)[2]. According to Section II, there is

$$\gamma_{opt} < 1/|a|.$$

When $0 < \gamma < 1/|a|$, the number $\lambda = \omega i$, where $\omega = \sqrt{\gamma^{-2} - a^2}$, is an imaginary and hence the eigenvalues of H are imaginaries. However, $\hat{\Sigma}_{22}$ is still a real number because

$$\begin{aligned} \hat{\Sigma}_{22} &= e^{-\omega hi} - \frac{a-\omega i}{2\omega i}(e^{\omega hi} - e^{-\omega hi}) \\ &= \cos(\omega h) - \frac{a}{\omega}\sin(\omega h). \end{aligned}$$

Substitute $\omega = \sqrt{\gamma^{-2} - a^2}$ into it, then

[2]$\hat{\Sigma}_{22} = 1 - ah$ when $\gamma = 1/|a| = -1/a$.

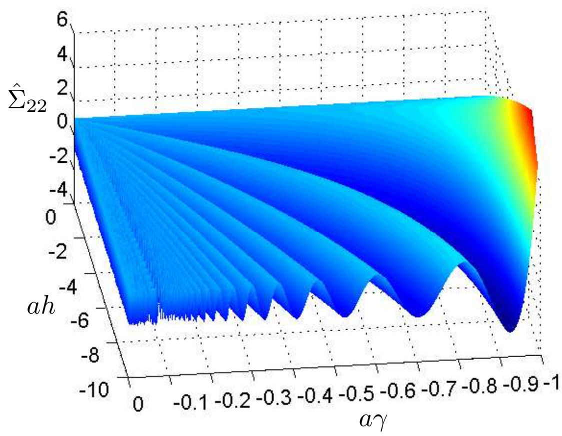

Fig. 1. The surface of $\hat{\Sigma}_{22}$ with respect to ah and $a\gamma$ ($a < 0$)

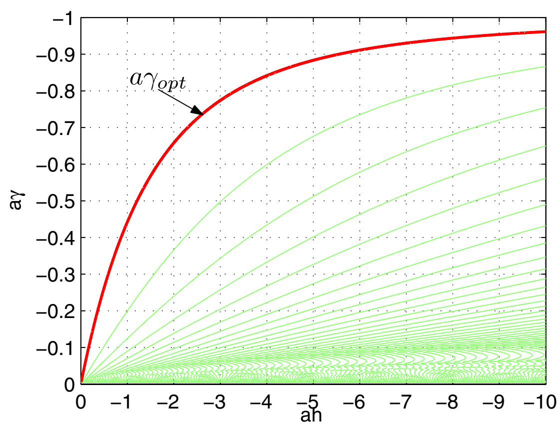

Fig. 2. The contour $\hat{\Sigma}_{22} = 0$ on the ah-$a\gamma$ plane ($a < 0$)

$$\hat{\Sigma}_{22} = \cos(\frac{ah}{a\gamma}\sqrt{1-a^2\gamma^2}) - \frac{a\gamma}{\sqrt{1-a^2\gamma^2}}\sin(\frac{ah}{a\gamma}\sqrt{1-a^2\gamma^2}).$$

This can be shown as the surface in Fig. 1, with respect to the normalized delay ah and the normalized performance index $a\gamma$. This surface crosses the plane $\hat{\Sigma}_{22} = 0$ many times, as can be seen from the contours of $\hat{\Sigma}_{22} = 0$ shown in Fig. 2. The top curve in Fig. 2 characterizes the normalized optimal performance index $a\gamma_{opt}$ with respect to the normalized delay ah. On this curve, $\hat{\Sigma}_{22}$ becomes singular the first time when γ decreases from $+\infty$ (or actually, $\|G_\beta\|_{L_\infty}$) to 0. Since $\left\|\Gamma_{G_\beta}\right\| = 0$ and $\|G_\beta\|_{L_\infty} = 1/|a|$, the optimal value γ_{opt} satisfies $0 \le \gamma_{opt} \le 1/|a|$, i.e., $-1 \le a\gamma_{opt} \le 0$. This coincides with the curve $a\gamma_{opt}$ shown in Fig. 2.

Now discuss the (sub-optimal) compensator for $\gamma > \gamma_{opt}$.

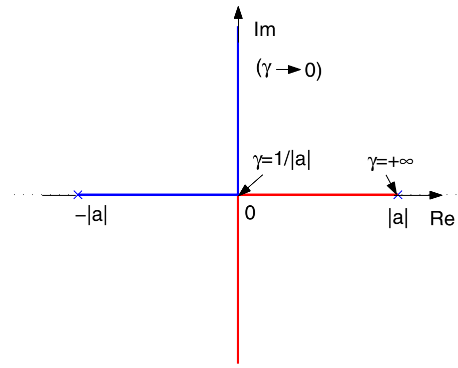

Fig. 3. The locus of the hidden poles of Z v.s. γ

According to (8) and (10), there is

$$\begin{aligned} Z(s) &= -\pi_h(\left[\begin{array}{cc|c} a & \gamma^{-2} & 0 \\ -1 & -a & 1 \\ \hline 0 & \gamma^{-2} & 0 \end{array}\right]) \\ &= \gamma^{-2}\frac{\gamma^{-2}\Sigma_{21}^* + (e^{-sh} - \Sigma_{11}^*)(s-a)}{s^2+\gamma^{-2}-a^2}. \quad (13) \end{aligned}$$

The locus of the hidden poles of $Z(s)$, i.e. the eigenvalues of H, is shown in Fig. 3. When $\gamma \ge 1/|a|$, $Z(s)$ has two hidden real poles symmetric to the $j\omega$-axis. When $0 < \gamma < 1/|a|$, $Z(s)$ has a pair of hidden imaginary poles. In either case, the implementation of Z needs to be careful; see [8].

The W^{-1} given in (9) is well defined for $\gamma > \gamma_{opt}$ as

$$W^{-1} = \left[\begin{array}{c|cc} a & \Sigma_{22}^{-*}\Sigma_{12}^* & -\Sigma_{22}^{-*} \\ \hline -1 & 1 & 0 \\ \gamma^{-2}\Sigma_{21}^* & 0 & 1 \end{array}\right]$$

and $\Pi_{22} = \left[\begin{array}{cc} Z & I \end{array}\right] W^{-1} \left[\begin{array}{c} 0 \\ I \end{array}\right]$ is

$$\begin{aligned} \Pi_{22} &= \left[\begin{array}{cc} Z & 1 \end{array}\right] \left[\begin{array}{c|c} a & -\Sigma_{22}^{-*} \\ \hline -1 & 0 \\ \gamma^{-2}\Sigma_{21}^* & 1 \end{array}\right] \\ &= 1 + \frac{\Sigma_{22}^{-*}}{s-a}(Z - \gamma^{-2}\Sigma_{21}^*) \\ &= 1 + \gamma^{-2}\Sigma_{22}^{-*}\frac{e^{-sh} - \Sigma_{11}^* - \Sigma_{21}^*(s+a)}{s^2+\gamma^{-2}-a^2}. \end{aligned}$$

It is easy to see that Π_{22} is stable. As a matter of fact, as required, Π_{22} is bistable for $\gamma > \gamma_{opt}$. The Nyquist plots of Π_{22} for different values of $a\gamma$ are shown in Fig. 6 for $ah = -1$. The optimal value γ_{opt} is between $-0.44/a$ and $-0.45/a$. This corresponds to the transition from Fig. 6(b) to Fig. 6(c), in which the number of encirclements changes accordingly to the change of the bi-stability of Π_{22}: the

Nyquist plot encircles the origin when $\gamma < \gamma_{opt}$ and hence Π_{22} is not bistable but the Nyquist plot does not encircle the origin when $\gamma > \gamma_{opt}$ and hence Π_{22} is bistable.

B. The unstable case ($a > 0$)

In this case, G_β is unstable and $L_c = -2a\gamma^2$, $L_o = -2a$. Hence, $\hat{\Sigma}_{22}$ defined in (6) is

$$\begin{aligned}\hat{\Sigma}_{22} &= e^{-\lambda h} - 4a^2\gamma^2 e^{\lambda h} + (e^{\lambda h} - e^{-\lambda h}) \cdot \\ &\quad \frac{-2a\gamma^2\lambda^2 - 2a^3\gamma^2 + 4\lambda a^2\gamma^2 + a + \lambda}{2\lambda} \\ &= \frac{-2a\gamma^2\lambda^2 - 2a^3\gamma^2 + a + \lambda - 4\lambda a^2\gamma^2}{2\lambda} e^{\lambda h} - \\ &\quad \frac{-2a\gamma^2\lambda^2 - 2a^3\gamma^2 + a - \lambda + 4\lambda a^2\gamma^2}{2\lambda} e^{-\lambda h} \\ &= \frac{-2a\gamma^2\lambda^2 - 2a^3\gamma^2 + a}{2\lambda}(e^{\lambda h} - e^{-\lambda h}) + \\ &\quad \frac{e^{\lambda h} + e^{-\lambda h}}{2}(1 - 4a^2\gamma^2) \\ &= \frac{-4a^3\gamma^2 + 3a}{2\lambda}(e^{\lambda h} - e^{-\lambda h}) + \\ &\quad \frac{e^{\lambda h} + e^{-\lambda h}}{2}(1 - 4a^2\gamma^2). \qquad (14)\end{aligned}$$

$\hat{\Sigma}_{22}$ can be re-arranged as

$$\begin{aligned}\hat{\Sigma}_{22} &= -\frac{4a(a^2\gamma^2 - 1) + a}{2\lambda}(e^{\lambda h} - e^{-\lambda h}) - \\ &\quad \frac{4(a^2\gamma^2 - 1) + 3}{2}(e^{\lambda h} + e^{-\lambda h}).\end{aligned}$$

Hence, $\hat{\Sigma}_{22}$ is always negative (nonsingular)[3] when $\gamma \geq 1/|a|$, noting that a and λ are positive. According to Section II, the optimal performance index is less than $1/|a|$, i.e.,

$$\gamma_{opt} < 1/|a|.$$

When $0 < \gamma < 1/|a|$, the eigenvalues $\pm\lambda$ of H are on the $j\omega$-axis. Substitute $\lambda = \omega i$ with $\omega = \sqrt{\gamma^{-2} - a^2}$ into (14), then

$$\hat{\Sigma}_{22} = (1 - 4a^2\gamma^2)\cos(\omega h) + (3 - 4a^2\gamma^2)\frac{a}{\omega}\sin(\omega h).$$

Similarly, with the substitution of $\omega = \sqrt{\gamma^{-2} - a^2} = \gamma^{-1}\sqrt{1 - a^2\gamma^2}$, then

$$\begin{aligned}\hat{\Sigma}_{22} &= (1 - 4a^2\gamma^2)\cos(\frac{ah}{a\gamma}\sqrt{1 - a^2\gamma^2}) + \\ &\quad \frac{a\gamma(3 - 4a^2\gamma^2)}{\sqrt{1 - a^2\gamma^2}}\sin(\frac{ah}{a\gamma}\sqrt{1 - a^2\gamma^2}).\end{aligned}$$

[3] $\hat{\Sigma}_{22} = -ah - 3$ when $\gamma = 1/|a| = 1/a$.

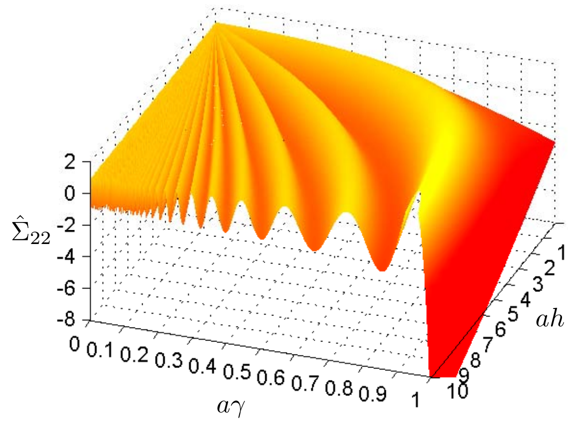

Fig. 4. The surface of $\hat{\Sigma}_{22}$ with respect to ah and $a\gamma$ ($a > 0$)

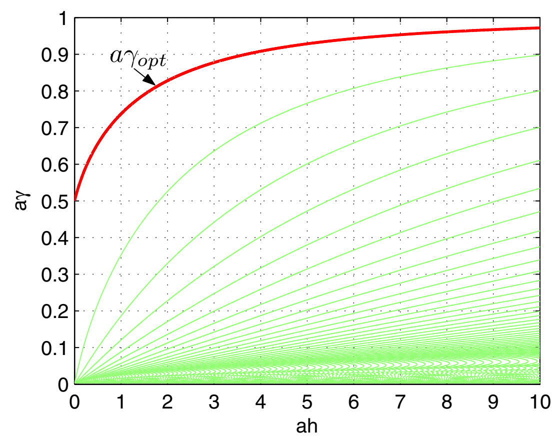

Fig. 5. The contour $\hat{\Sigma}_{22} = 0$ on the ah-$a\gamma$ plane ($a > 0$)

This surface is shown in Fig. 4 and the contours of $\hat{\Sigma}_{22} = 0$ on the ah-$a\gamma$ plane are shown in Fig. 5. The top curve in Fig. 5 characterizes the normalized optimal performance index $a\gamma_{opt}$ with respect to the normalized delay ah. On this curve, $\hat{\Sigma}_{22}$ becomes singular the first time when γ decreases from $+\infty$ to 0.

Since $I - L_cL_o = 1 - 4a^2\gamma^2$, there is $\left\|\Gamma_{G_\beta}\right\| = \frac{1}{2a}$. As a result, the optimal value γ_{opt} satisfies $\frac{1}{2a} \leq \gamma_{opt} \leq \frac{1}{a}$, i.e., $0.5 \leq a\gamma_{opt} \leq 1$. This coincides with the curve $a\gamma_{opt}$ shown in Fig. 5.

Now discuss the (sub-optimal) compensator for $\gamma > \gamma_{opt}$.

In this case, the FIR block Z remains the same as in (13) and the form is not affected because of the sign of

a. However, W^{-1} is changed to

$$W^{-1}=\left[\begin{array}{c|cc} -a & \hat{\Sigma}_{22}^{-*}(\Sigma_{12}^{*}-2a\Sigma_{11}^{*}) & -\hat{\Sigma}_{22}^{-*} \\ \hline -1 & 1 & 0 \\ \gamma^{-2}\Sigma_{21}^{*}+2a\Sigma_{11}^{*} & 0 & 1 \end{array}\right]$$

and $\Pi_{22}=\left[\begin{array}{cc} Z & I \end{array}\right] W^{-1}\left[\begin{array}{c} 0 \\ I \end{array}\right]$ is

$$\begin{aligned}
\Pi_{22} &= \left[\begin{array}{cc} Z & 1 \end{array}\right]\left[\begin{array}{c|c} -a & -\hat{\Sigma}_{22}^{-*} \\ \hline -1 & 0 \\ \gamma^{-2}\Sigma_{21}^{*}+2a\Sigma_{11}^{*} & 1 \end{array}\right] \\
&= 1+\frac{\hat{\Sigma}_{22}^{-*}}{s+a}(Z-\gamma^{-2}\Sigma_{21}^{*}-2a\Sigma_{11}^{*}) \\
&= 1+\gamma^{-2}\hat{\Sigma}_{22}^{-*}\cdot\frac{\frac{s-a}{s+a}e^{-sh}-(s-a)\Sigma_{21}^{*}+(2a^2\gamma^2-1-2a\gamma^2 s)\Sigma_{11}^{*}}{s^2+\gamma^{-2}-a^2} \\
&= 1+\gamma^{-2}\hat{\Sigma}_{22}^{-*}\cdot\frac{\frac{s-a}{s+a}e^{-sh}-(s-a)(\Sigma_{21}^{*}+2a\gamma^2\Sigma_{11}^{*})-\Sigma_{11}^{*}}{s^2+\gamma^{-2}-a^2}.
\end{aligned}$$

It is easy to see that Π_{22} is stable and invertible. As a matter of fact, as required, Π_{22} is bistable for $\gamma>\gamma_{opt}$. The Nyquist plots of Π_{22} for different values of $a\gamma$ are shown in Fig. 7 for $ah=1$. The optimal value γ_{opt} is between $0.73/a$ and $0.74/a$. This corresponds to the transition from Fig. 7(b) to Fig. 7(c), in which the number of encirclements changes accordingly to the change of the bi-stability of Π_{22}.

IV. Conclusions

In this paper, a case study is given to show the delay-type Nehari problem using a first-order system. The stable case and the unstable case are all discussed. The system is normalized so that the (normalized) optimal value can be shown as a function of the delay. It has been shown that solving the problem involving only a first-order system is not easy at all. When the system order gets higher, the computation of the optimal value becomes very complicated. One might have to use (11) for a compromise.

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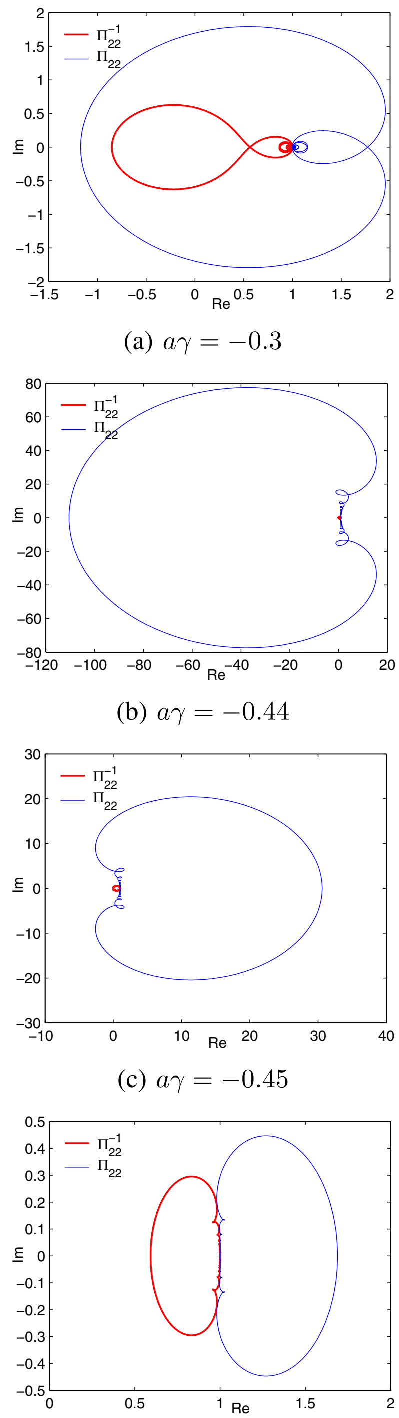


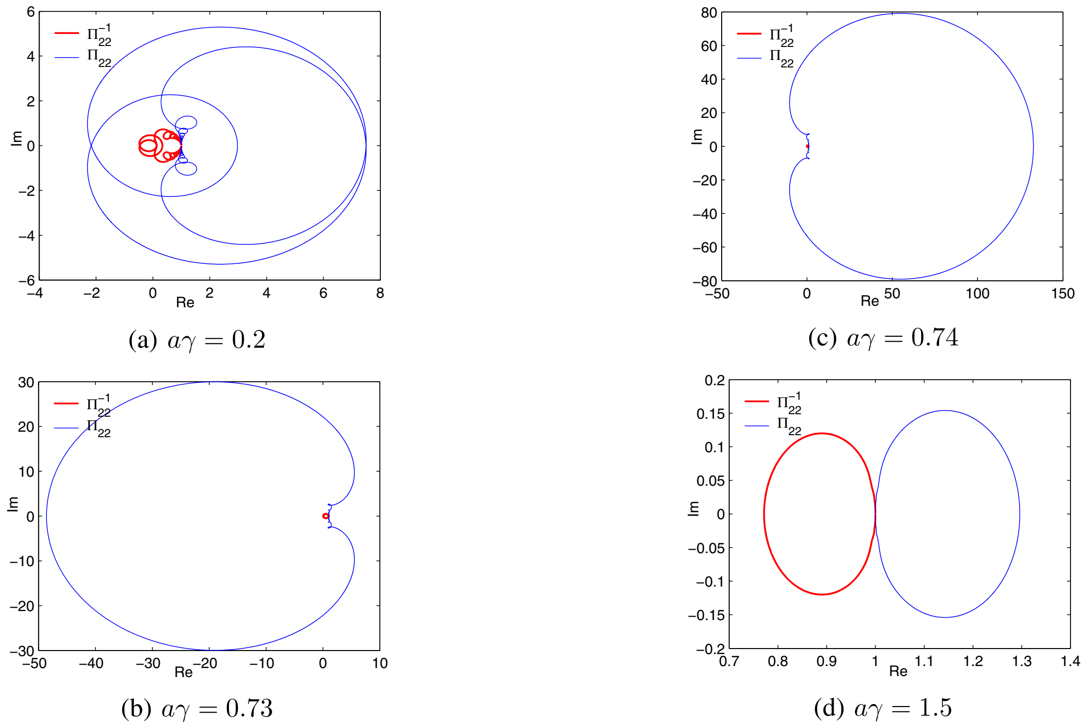
(a) $a\gamma=-0.3$

(b) $a\gamma=-0.44$

(c) $a\gamma=-0.45$

(d) $a\gamma=-0.7$

Fig. 6. The Nyquist plot of Π_{22} ($a<0$ and $ah=-1$)



(a) $a\gamma = 0.2$

(b) $a\gamma = 0.73$

(c) $a\gamma = 0.74$

(d) $a\gamma = 1.5$

Fig. 7. The Nyquist plot of Π_{22} ($a > 0$ and $ah = 1$)