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Analysis and control design of MIMO systems based on symmetry properties

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Abstract— **In this work the design advantages derived from certain symmetry properties inherent to different class of systems are explored. In particular, some class of MIMO systems which can be separated into different and independent SISO problems, thus making easier the analysis and control design, are addressed. Also, it is shown how the symmetry properties give a direct way for open-loop identification, valid also in unstable cases, using closed-loop data. As a real-world application example, the proposed method is applied to a magnetic bearing system having the required symmetry properties.**

Index Terms— **Symmetry, control systems, MIMO systems, magnetic bearings.**

I. Introduction

The analysis and control of MIMO systems is a very active research field. The problem has been approached from a great variety of viewpoints, starting either from state-space or transfer matrices representations and using many different control philosophies, from classical time and frequency domain methods to newer approaches such as robust QFT [1], H_2 and H_∞ [2], geometry-based control [6] or hamiltonian methods [7], among others. However, one characteristic of all current MIMO methodologies is that they are very difficult to apply in practice, at least with respect to the easy use of SISO techniques.

On the other hand, many common physical systems are ruled by mechanical or electromagnetic laws which are known to have remarkable symmetry properties. This can result in a highly symmetrical structure in the mathematical models of very usual electromechanical systems. A good example of this can be found in rotary machinery [3].

In this work, the symmetry properties in physical MIMO systems are exploited in order to simplify its analysis and control, splitting the MIMO problem into different and independent SISO problems, thanks to the plant symmetry. In addition, in this paper the proposed methodology is applied to a particular electromechanical device having the mentioned symmetry properties, which consists of a spindle supported by magnetic levitation by two magnetic bearings. This application example has been selected by its special practical importance in mechanical manufacturing industry, where high speed and high precision machine tools can take advantage of non-contact magnetic suspension technology [3].

The paper is organized as follows. In section II, the separation of the analysis and control design of a symmetrical MIMO system into independent SISO problems is presented. This symmetry is considered in the next section to perform the open-loop identification process using closed-loop data. This is specially useful for studying unstable plants. In section IV, the proposed approach is applied to a magnetic bearing system, concretely the MBC500 Rotor Dynamics system, [4], [5], in order to validate the theoretical proposition. Finally, the conclusions are presented in section V.

II. Problem statement

In order to simplify the problem, consider a 2-inputs 2-outputs MIMO system described by the following transfer matrix:

$$\begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = \begin{bmatrix} A & B \\ B & A \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \end{bmatrix}$$

Note the symmetrical relation between the input and output signals, u_i and y_i, respectively. The characteristic equation of the system can be obtained easily from the following expression:

$$det \begin{pmatrix} A & B \\ B & A \end{pmatrix} = A^2 - B^2 = (A+B)(A-B) = 0$$

Then, the characteristic equation can be separated into two independent and more simple characteristic equations. This property is conserved through feedback if the controller structure is symmetric. Effectively, considering as example the trivial feedback loop $\begin{bmatrix} u_1 \\ u_2 \end{bmatrix} = \begin{bmatrix} r_1 \\ r_2 \end{bmatrix} - \begin{bmatrix} y_1 \\ y_2 \end{bmatrix}$, the closed-loop characteristic equation is

$$\begin{aligned} det \begin{pmatrix} 1+A & B \\ B & 1+A \end{pmatrix} &= 1 + A^2 - B^2 + 2A \\ &= (1+(A+B))\,(1+(A-B)) \\ &= 0 \end{aligned}$$

and this can be again splitted into two characteristic equations.

A. Symmetric controller structure

In order to take advantage of the property presented in the above section, the following symmetric controller structure is proposed:

$$\mathbf{G_c} = \begin{bmatrix} C & D \\ D & C \end{bmatrix}$$

Using this controller the closed-loop characteristic equation results:

$$
\begin{aligned}
&det\begin{pmatrix} 1+CA+DB & CB+DA \\ CB+DA & 1+CA+DB \end{pmatrix} \\
&= det\begin{pmatrix} 1+A' & B' \\ B' & 1+A' \end{pmatrix} = 0 \\
&= 1+A'^2-B'^2+2A' = 0 \\
&= (1+(A'+B'))\,(1+(A'-B')) = 0 \\
&= (1+(C+D)(A+B))(1+(C-D)(A-B)) = 0
\end{aligned}
$$

In conclusion, the MIMO system can splitted into two different control design SISO problem, simplifying the overall control procedure:

$$
\begin{aligned}
(1+C_1(A+B)) &= 0 \\
(1+C_2(A-B)) &= 0
\end{aligned}
$$

leading to the MIMO system controller

$$
\begin{array}{l} C+D=C_1 \\ C-D=C_2 \end{array} \;\Rightarrow\; \begin{array}{l} \dfrac{C_1+C_2}{2}=C \\ \dfrac{C_1-C_2}{2}=D \end{array} \tag{1}
$$

It is important to note that this procedure is independent of the control design technique used for the two SISO equivalent problems, so separate and different control methodologies can be applied in each case, depending on the required closed-loop specifications.

B. Extension to other MIMO systems

The proposed approach has been presented for a 2 inputs-2 outputs MIMO system, but this can be easily extended to higher order systems under certain symmetric properties. The most evident case results from a 3 inputs-3 outputs MIMO system with the following structure, using the external representation

$$
\begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix} = \begin{bmatrix} A & B & B \\ B & A & B \\ B & B & A \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \\ u_3 \end{bmatrix}
$$

In this case, the characteristic equation can be again separated in three different simplified characteristic equations:

$$
\begin{aligned}
&det\begin{pmatrix} A & B & B \\ B & A & B \\ B & B & A \end{pmatrix} \\
&= det\begin{pmatrix} A+2B & A+2B & A+2B \\ B & A & B \\ B & B & A \end{pmatrix} \\
&= (A+2B)det\begin{pmatrix} 1 & 1 & 1 \\ B & A & B \\ B & B & A \end{pmatrix} \\
&= (A+2B)det\begin{pmatrix} 1 & 0 & 0 \\ B & A-B & 0 \\ B & 0 & A-B \end{pmatrix} \\
&= (A+2B)(A-B)^2 = 0
\end{aligned}
$$

So, in the general case with n inputs and n outputs, following the same procedure the following relations are obtained:

$$
det\begin{pmatrix} A & B & \dots & B \\ B & A & & \vdots \\ \vdots & & \ddots & B \\ B & B & B & A \end{pmatrix} = (A+(n-1)B)(A-B)^{n-1}
$$

Thus, this class of n inputs-n outputs MIMO system can be easily represented by n SISO systems. However, note that n-1 of these SISO systems are the same, so the control design of this type of symmetric systems is relatively straightforward.

III. CLOSED-LOOP IDENTIFICATION

Consider the open-loop system shown in Fig. 1, being the input vector $u = [u_1,\, u_2]^T$,the output vector $y = [y_1,\, y_2]^T$, the transfer matrix $G_c = \begin{bmatrix} A & B \\ B & A \end{bmatrix}$and the disturbances $n = [n_1, n_2]$.

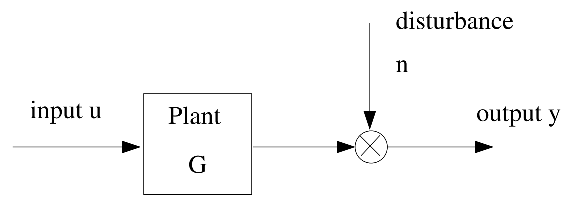

Fig. 1. Open loop system with noise

If the disturbance is a noise signal, the plant identification can be easily performed by the estimation of the following expression:

$$
\begin{aligned}
\frac{y_i}{u_i} &= A + \frac{n_i}{u_i} \quad (\text{with } u_j = 0) \\
\frac{y_j}{u_i} &= B + \frac{n_j}{u_i} \quad (\text{with } u_j = 0)
\end{aligned}
$$

Since in general n has zero mean and the input and the noise are uncorrelated, the terms $\frac{n_i}{u_i}$ and $\frac{n_j}{u_i}$ will average to zero. Then, collecting data over an extended time period, it is possible to obtain an approximate system matrix transfer identifying the average ratios

$$
\begin{aligned}
\frac{y_i}{u_i} &\simeq A \quad (\text{with } u_j = 0) \\
\frac{y_j}{u_i} &\simeq B \quad (\text{with } u_j = 0)
\end{aligned}
$$

Note that if the input u_i and the noise signals n_i are not uncorrelated the previous approximation is no more valid and the noise can lead to an incorrect system identification. This happens actually in a closed-loop configuration and it is problematic when the plant is unstable, so a controller is needed in order to perform data acquisition and system identification. In this case, the symmetry of the transfer matrix gives a straightforward way for identifying the system.

In order to study the closed-loop case, consider the previous system adding a feedback loop, as it is shown in Fig. 2, where $\mathbf{r} = [r_1, r_2]^T$, $\mathbf{u} = [u_1, u_2]^T$, $\mathbf{y} = [y_1, y_2]^T$, $\mathbf{n} = [n_1, n_2]$, $\mathbf{G} = \begin{bmatrix} A & B \\ B & A \end{bmatrix}$, and $\mathbf{G_c} = \begin{bmatrix} C & D \\ D & C \end{bmatrix}$.

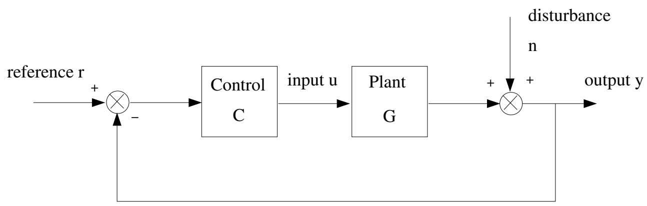

Fig. 2. Closed-loop system with noise

The system output is

$$\begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = \mathbf{M}^{-1} \begin{bmatrix} AC+BD & AD+BC \\ AD+BC & AC+BD \end{bmatrix} \begin{bmatrix} r_1 \\ r_2 \end{bmatrix} + \mathbf{M}^{-1} \begin{bmatrix} n_1 \\ n_2 \end{bmatrix}$$

and the control signal

$$\begin{bmatrix} u_1 \\ u_2 \end{bmatrix} = \mathbf{M}^{-1}\mathbf{G_c} \begin{bmatrix} r_1 \\ r_2 \end{bmatrix} - \mathbf{G_c}\mathbf{M}^{-1} \begin{bmatrix} n_1 \\ n_2 \end{bmatrix}$$

with $\mathbf{M} = \begin{bmatrix} 1+AC+BD & AD+BC \\ AD+BC & 1+AC+BD \end{bmatrix}$. In order to take advantage of the system symmetry, a symmetrical reference signal can be considered. For $r_1 = r_2 = r$

$$\begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = \begin{bmatrix} \frac{C_1(A+B)}{1+C_1(A+B)} r \\ \frac{C_1(A+B)}{1+C_1(A+B)} r \end{bmatrix} + \mathbf{M}^{-1} \begin{bmatrix} n_1 \\ n_2 \end{bmatrix}$$

and

$$\begin{bmatrix} u_1 \\ u_2 \end{bmatrix} = \begin{bmatrix} \frac{C_1}{1+C_1(A+B)} r \\ \frac{C_1}{1+C_1(A+B)} r \end{bmatrix} - G_c M^{-1} \begin{bmatrix} n_1 \\ n_2 \end{bmatrix}$$

with $C_1 = C + D$. Then, supposing that the reference input and the noise are uncorrelated, the averaged ratios fulfills

$$\frac{y_i}{u_i} = \frac{y_i/r}{u_i/r} \simeq A + B$$

Again , considering $r_1 = -r_2 = r$

$$\begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = \begin{bmatrix} \frac{C_2(A-B)}{1+C_2(A-B)} r \\ \frac{C_2(A-B)}{1+C_2(A-B)} r \end{bmatrix} + \mathbf{M}^{-1} \begin{bmatrix} n_1 \\ n_2 \end{bmatrix}$$

and

$$\begin{bmatrix} u_1 \\ u_2 \end{bmatrix} = \begin{bmatrix} \frac{C_2}{1+C_2(A-B)} r \\ \frac{C_2}{1+C_2(A-B)} r \end{bmatrix} - \mathbf{G_c}\mathbf{M}^{-1} \begin{bmatrix} n_1 \\ n_2 \end{bmatrix}$$

with $C_2 = C - D$. As in the previous case,

$$\frac{y_i}{u_i} \simeq A - B$$

In conclusion, the system symmetry allows the straightforward estimation of $A + B$ and $A - B$ (i.e., A and B), using closed-loop data.

IV. Application Example

In this section the proposed control technique is applied to a mechatronic system of special interest: a magnetic bearing system. This is an active industrial and research field since the magnetic levitation and, in particular, active magnetic bearings (AMB) provide important advantages to high speed rotary systems, specially due to the absence of friction. There are different applications as in vacuum techniques, energy storage, machine tools, isolation of vibrations, [3], where high speed is needed and the possibility of active control of vibrations, without mechanical worn out, is very interesting. This characteristic makes magnetic bearings very attractive for high performance machining, where traditional technology presents problems by the mechanical stoking which is caused by the mechanical worn out at high speeds.

However, since magnetic bearings are open-loop unstable systems and present lower damping that traditional bearings, the controller design is a totally necessary requirement. In addition, this task must be carefully performed and it can be a cumbersome process.

In this section, the control process will be simplified taking advantage of the symmetry properties of rotary axis. First, a simplified system model is presented[1] and the design process is described.

Note that in the next subsection it is performed an analytical modelization of this systems leading to an open-loop unstable description. In the case of performing an identification process, the procedure described in section III can be followed.

A. System model

The system considered as application example is the MBC500 Rotor Dynamics, [4], [5], which is a magnetic bearing rotor device, and it is used as experimentation testbed in the authors' laboratory. The testbed is described in Fig. 3.

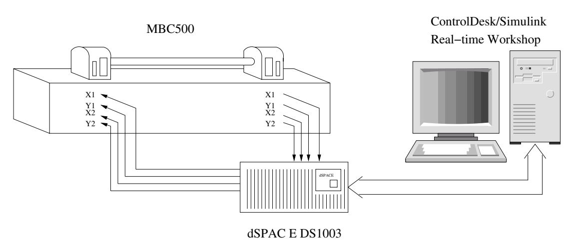

Fig. 3. MBC500 Rotor Dynamics based testbed

The MBC500 Rotor Dynamics system is composed of two AMB and a rotor which includes an air turbine drive allowing speeds up to 22000 RPM. The position of the beam can be measured by hall effect and eddy current sensors and the control action in the bearings is driven by voltage amplifiers (Fig. 4).

[1]Here, the system model includes only the rigid dynamics, without taking into account the flexible ones. The inclusion of this dynamics can be necessary, in general, but the methodology proposed is still valid since the symmetry properties hold.

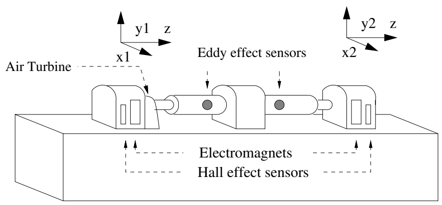

Fig. 4. MBC500 Rotor Dynamics

Taking into account the differential configuration of the eight poles symmetrically disposed in each of the bearings in the MBC500 Rotor Dynamics, Fig. 5, the forces over the x and y axis can be considered as decoupled for relatively slow rotatory speeds. Then, the modelization process can be performed independently for the two axis, simplifying it, since the motion of the two axis are equivalent except for the gravity action over y axis[2].

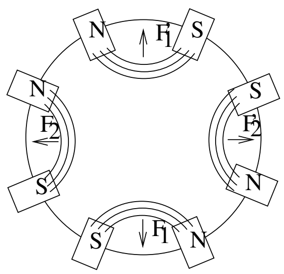

Fig. 5. Pole configuration

In order to simplify the subsequent discussion, in the modelization the rigid dynamics and the x axis will be only considered, using as output signals the voltages measured in the Hall sensors. This can be easily described by Newton's laws applied to mass center (M.C.) x_0 and the angle θ which is the beam angle respect to the z axis, as it is shown in Fig. 6. This description leads to the system (2), where the variables are:

$X_i \equiv$Hall sensor position
$x_i \equiv$bearing position
$F_i \equiv$Force applied to bearing i over the rotor
$L \equiv$Rotor length
$l \equiv$Distance between rotor end and bearing
$l_2 \equiv$Distance between rotor end and Hall effect sensor

$$\begin{aligned} F_1 + F_2 &= m\ddot{x}_0 \\ -\left(\frac{L}{2} - l\right) F_1 + \left(\frac{L}{2} - l\right) F_2 &= I_0 \ddot{\theta} \end{aligned} \tag{2}$$

The equivalent state-variables representation is:

[2]In this example, the effect of the gravity is not included, but it can be added easily later.

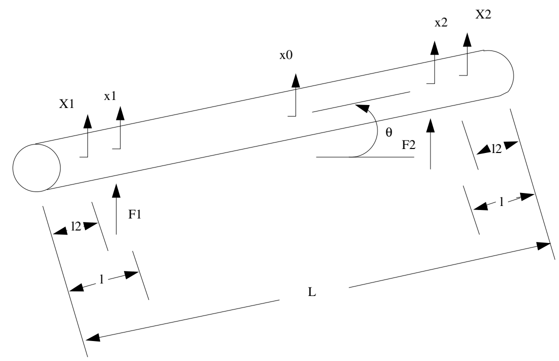

Fig. 6. Axis system

$$\begin{bmatrix} \dot{x}_0 \\ \ddot{x}_0 \\ \dot{\theta} \\ \ddot{\theta} \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_0 \\ \dot{x}_0 \\ \theta \\ \dot{\theta} \end{bmatrix} + \\ + \begin{bmatrix} 0 & 0 \\ \frac{1}{m} & \frac{1}{m} \\ 0 & 0 \\ -\frac{1}{I_0}\left(\frac{L}{2} - l\right) & \frac{1}{I_0}\left(\frac{L}{2} - l\right) \end{bmatrix} \begin{bmatrix} F_1 \\ F_2 \end{bmatrix} \tag{3}$$

In addition, the relationship between the position of the beam measured in the Hall sensors and the state variables is presented in (4). These signals are used for generating the control signal.

$$\begin{bmatrix} X_1 \\ X_2 \end{bmatrix} = \begin{bmatrix} x_0 & -\left(\frac{L}{2} - l_2\right) sin\theta \\ x_0 & +\left(\frac{L}{2} - l_2\right) sin\theta \end{bmatrix} \tag{4}$$

The Hall sensors give a voltage level depending on the following nonlinear equation:

$$\begin{aligned} V_{sensor_i} &= 10volts/mmX_i + 24volts/mm^3 X_i^3 \pm \\ &\pm \ 1volt\,offset \end{aligned} \tag{5}$$

On the other hand, the magnetic forces over the rotor, i.e., the system input, are related to the position x_i of the beam and the current i_i in the solenoid leading to (6).

$$F_i = K\frac{(i_i + i_0)^2}{(x_i - x_g)^2} - K\frac{(i_i - i_0)^2}{(x_i + x_g)^2} \tag{6}$$

where $K = 2.8 \times 10^{-7} Nw \cdot m^2/A^2$ is a geometric constant depending on the bearing, i_0 is a bias current and $x_g = 0.0004m$ is the mean distance between the bearing and the rotor. Note that (6) is not valid when the rotor is near the electromagnets since the attractive magnetic force saturates at low distances.

Finally, the positions of the bearings, referred to the rotor, are given as a function of the state variables x_0 and θ:

$$\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} x_0 & -\left(\frac{L}{2}-l\right)\sin\theta \\ x_0 & +\left(\frac{L}{2}-l\right)\sin\theta \end{bmatrix} \quad (7)$$

and the current in the solenoids are controlled by the input voltage in the power amplifier following the equation:

$$i_{control_i} = \frac{0.25}{(1+2.2\times10^{-4}s)} A/volt \times V_{control_i} \quad (8)$$

where $V_{control_i}$ are the input variables in the magnetic bearing system.

In order to make easier the analysis and design, the system is linearized around the operation point $(0,0)$. Developing a Taylor expansion in (6)

$$F_i = K_x x_x + K_i i_i \quad (9)$$

with $K_x = \frac{4Ki_0^2}{x_g^3}$ and $K_i = \frac{4Ki_0}{x_g^2}$. On the other hand, the sensor behaviour can be simplified by a constant gain

$$V_{sensor_i} = 10000 volts/mX_i \quad (10)$$

and applying geometrical approximations in (4) and (7), $sin\theta \simeq \theta$, the model describing the rigid dynamics of this system is obtained as:

$$\begin{bmatrix} \dot{x}_0 \\ \ddot{x}_0 \\ \dot{\theta} \\ \ddot{\theta} \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ \frac{2}{m}K_x & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 2K_xL_1^2 & 0 \end{bmatrix} \begin{bmatrix} x_0 \\ \dot{x}_0 \\ \theta \\ \dot{\theta} \end{bmatrix} + \frac{1}{4} \begin{bmatrix} 0 & 0 \\ \frac{1}{m}K_i & \frac{1}{m}K_i \\ 0 & 0 \\ -\frac{K_i}{I_0}L_1 & \frac{K_i}{I_0}L_1 \end{bmatrix} \begin{bmatrix} V_{cont_1} \\ V_{cont_2} \end{bmatrix} \quad (11)$$

$$\begin{bmatrix} V_{sensor_1} \\ V_{sensor_2} \end{bmatrix} = 10^4 \begin{bmatrix} 1 & 0 & -\left(\frac{L}{2}-l_2\right) & 0 \\ 1 & 0 & \left(\frac{L}{2}-l_2\right) & 0 \end{bmatrix} \begin{bmatrix} x_0 \\ \dot{x}_0 \\ \theta \\ \dot{\theta} \end{bmatrix}$$

where $L_1 = \frac{L}{2} - l$.

Translating the system model to a external representation, this can be expressed by the following transfer matrix:

$$\begin{bmatrix} V_{sensor_1} \\ V_{sensor_2} \end{bmatrix} = \begin{bmatrix} A & B \\ B & A \end{bmatrix} \begin{bmatrix} V_{control_1} \\ V_{control_2} \end{bmatrix} \quad (12)$$

where in this case:

$$A = \frac{1104992.6792(s+337.6)(s-337.6)}{(s+396.5)(s+290.9)} \frac{1}{(s-396.5)(s-290.9)(2.2\,10^{-4}s+1)} \quad (13)$$

$$B = \frac{18424.2345(s-124)(s+124)}{(s+396.5)(s+290.9)} \frac{1}{(s-396.5)(s-290.9)(2.2\,10^{-4}s+1)} \quad (14)$$

In conclusion, this system holds the symmetry properties considered in the previous sections, so the proposed control procedure can be followed.

B. Controller design

Following the procedure presented above and considering the scheme shown in Fig. 9 , the control design is separated into two SISO independent problems:

$$\begin{aligned} 1 + C_1(A+B) &= 0 \\ 1 + C_2(A-B) &= 0 \end{aligned}$$

where

$$A + B = \frac{768644545.4633}{(s+290.9)(s-290.9)(s+4545)} \quad (15)$$

$$A - B = \frac{1697115821.6852}{(s+396.5)(s-396.5)(s+4545)} \quad (16)$$

In this particular case, the two resultant SISO problems are quite similar and then it is possible (depending on the design requirements) to use the same controller for each subsystem ($C_1 = C_2$). Then, the MIMO controller has also a symmetrical structure:

$$\mathbf{G_c}(s) = \begin{bmatrix} C & 0 \\ 0 & C \end{bmatrix} \quad (17)$$

However, two independent designs have been performed, using two different techniques, for the sake of generality. First, it is considered the design of a phase compensator in the frequency domain, being the main design restriction a phase margin greater than 40 degrees and low high frequency gain. Fig. 7 shows the frequency response of $A+B$ with the lead-lag compensator (18) and without it.

$$C_1(s) = \frac{1+0.05s}{1+0.06745s} \frac{1+0.01207s}{1+0.002071s} \quad (18)$$

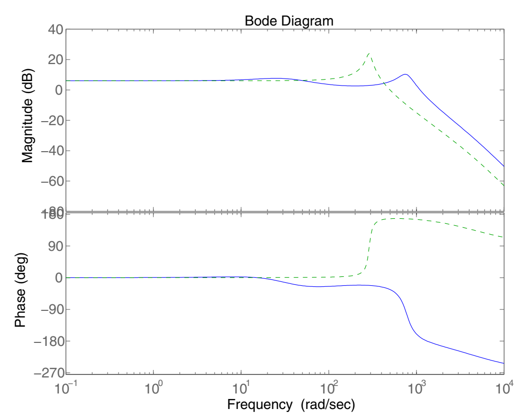

Fig. 7. Bode diagram A+B: __ line with compensator, - - line without compensator

For the subsystem represented by $A - B$, the root locus methodology is used, considering a proportional-derivative controller (19)[3]. In Fig. 8, the root locus including the controller and the location of the selected closed-loop poles are shown.

$$C_2(s) = \frac{1.25(1 + s/308)}{1 + s/6000} \tag{19}$$

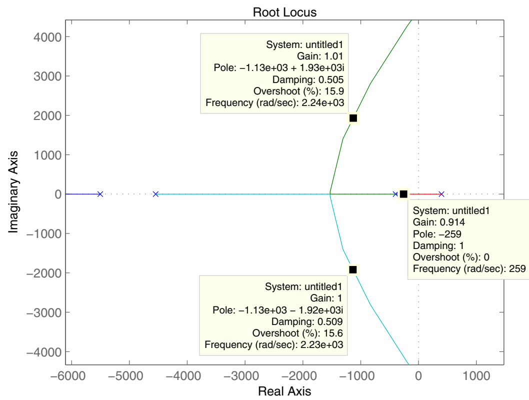

Fig. 8. Root locus of $C_2(A - B)$

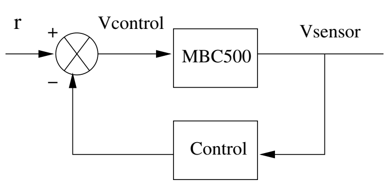

Fig. 9. Control scheme

Finally, the real MIMO controller must be introduced applying (1). In this case, the following transfer matrix is obtained

$$\mathbf{G_c} = \begin{pmatrix} \frac{C_1+C_2}{2} & \frac{C_1-C_2}{2} \\ \frac{C_1-C_2}{2} & \frac{C_1+C_2}{2} \end{pmatrix}$$

being $C_1(s)$and $C_2(s)$ the previous obtained controllers, that is, $C = \frac{13.242(s+1432)(s+140.2)(s+16.6)}{(s+482.8)(s+5504)(s+14.83)}$ and $D = \frac{-8.9213(s+8.189)(s^2-365.1s+6.484e04)}{(s+482.8)(s+5504)(s+14.83)}$. The result of the application of this controller is shown in Fig. 10. As it can be observed from the shown step responses, the control design performs reasonably well in this simulation test.

V. Conclusions

In this work, a control technique valid for a class of symmetrical MIMO systems is described. The main characteristic of the approach is that the MIMO problem is splitted into independent SISO systems, which makes easier the study and design of the original MIMO system. Moreover, the symmetry property can be also used to facilitate the identification process of a open-loop unstable system from closed-loop data.

In addition, as an application example, the methodology is applied to a magnetic bearing system (MBC500 Rotor Dynamics). First, the model of the plant is obtained, describing the rigid dynamics, linearized around an operation point. The obtained model has the symmetrical structure required so that the presented methodology can be applied. By following the proposed method a controller design useful for practical uses, is presented.

An interesting characteristic of the proposed approach is that the control design method used in each SISO subsystem can be freely chosen. This fact further facilitates the whole design process.

[3]One pole is added to satisfy the controller causality.

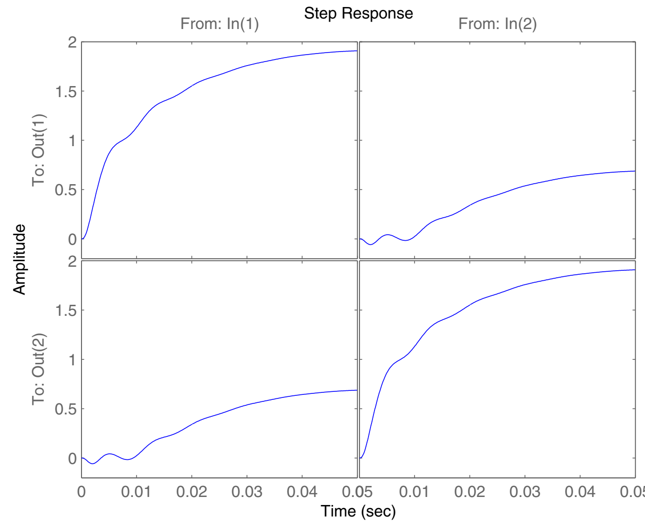

Fig. 10. Time response of the magnetic bearings system

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