Proceedings of the
44th IEEE Conference on Decision and Control, and
the European Control Conference 2005
Seville, Spain, December 12-15, 2005



# An Adaptive Control for Rotating Stall and Surge of Jet Engines - A Function Approximation Approach

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*Abstract*— **Compressor instabilities such as surge and rotating stall are highly unwanted phenomena in operation of jet engines. This is because these two instabilities reduce the performance and cause damage to aircraft engines. In this study, we design a model reference adaptive controller based on the function approximation technique to stabilize these two instabilities. Based upon this scheme, the controller parameters neither are restricted to be constant nor the bounds should be available a priori. The functions of the controller parameters are assumed to be piecewise continuous and satisfy the Dirichlet's conditions. Furthermore expressing these controller parameters in a finite-term Fourier series, they can be estimated by updating the corresponding Fourier coefficients. A Lyapunov stability approach is implemented to provide the update laws for the estimation of those time-invariant coefficients and guarantees the output error convergence. Therefore, the adaptive controller requires less model information and maintains consistent performance for the system when some controller parameters are disturbed.**

## I. Introduction

A compression system such as the gas turbine can exist two types of aerodynamic flow instabilities, which are known as surge and rotating stall. Surge and rotating stall restrict the efficiency and performance of the jet engine [1]. The problem of controlling surge and rotating stall in jet engines is important in preventing damage and lengthening the life of these components. Since 1986, Moore and Greitzer proposed a three-state nonlinear model (MG3) that can describe these phenomena [2], active control of surge and rotating stall has been investigated by a number of researchers, see e.g., [3–5].

The approach of active surge/rotating stall control aims at stabilizing some part of the unstable area in the compressor map using feedback [1]. Epstein et al. considered rotating stall and surge are initiated by small amplitude perturbations, and the active control was based on the feedback of small flow variations [6, 7]. Chen et al. used classical bifurcation theory to derive output feedback control laws in which throttle position is employed as actuator and pressure rise as output measurement based on a linearized model and both linear and nonlinear feedback control laws were shown to be effective in elimination hysteresis loop associated with rotating stall [8]. Krstić et al. used backstepping to avoid cancellation of useful nonlinearities and therefore use less control effort than feedback linearizing controllers, the resultant partial state controller requires minimal modeling information (rotating stall is stabilized without measuring its amplitude) [4]. The use of close-coupled valve (CCV) for backstepping control of compressor surge was studied in [9], the valve modifies the character of the compressor, and allows for stable operation beyond the original surge line.

From the above mentioned controllers, we observe that state feedback and output feedback control perform well for systems with precise parameter information. If the system parameters are not fully known beforehand, these control methodologies may not work well. To overcome this difficulty, an adaptive control may be used. From literature survey, only few existing adaptive control techniques were implemented on eliminating instabilities of the compression system. Among them, Diao et al. constructed a fault-tolerant controller based on stable adaptive fuzzy/neural control to improve the reliability and performance of a turbine engine [10]. In [11], Blanchini et al. based on Greitzer's model to propose a high-gain type adaptive control scheme for surge stabilizing in a compression system and both numerical simulation and experimental results were validated. Kristić et al. [12] considered a class of MIMO LTI with uncertain resonant modes and time delays, which arise in control of instabilities in jet engines, and used indirect adaption to develop an adaptive MIMO pole placement scheme for the system. Since the mathematical model is only an approximation of the real system, the simplified representation of the system behavior inevitably contains model inaccuracies. Because these inaccuracies may degrade the performance of the closed-loop system, any practical design should consider their effects. The inherent highly nonlinear coupling and model inaccuracies make the controller design for the compression system extremely difficult.

Traditional adaptive control is very effective in dealing with system unknown parameters defined in compact sets. However, if the bounds of the these parameters are not available a priori or the unknown parameters are not constant, the traditional adaptive control schemes would not work [13]. To overcome this difficulty, Huang et al. [13, 14] proposed a new adaptive sliding control scheme for non-linear systems containing time-varying uncertainties with unknown bounds. In this paper, we adopt the function approximation technique proposed by Huang to design an adaptive controller to stabilize the two instabilities of the compression system. The unknown controller parameters are transformed into finite term Fourier series, and these parameters can be estimated by updating the corresponding Fourier coefficients.

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## II. A Review of Function Approximation Technique

Here, we review some of the properties of the Fourier series and function approximation technique.

If a complete orthonormal set $\{q_i(t)\}$, $\forall i \in \mathbb{N}$ defined on $[t_1, t_2]$, then for any $f(t)$ satisfies Dirichlet's conditions can be expressed as [15]

$$f(t) = \sum_{i=1}^{\infty} w_i q_i(t), \tag{2.1}$$

where $w_i$ is the corresponding coefficient, and the mean square error of the series has the convergence property

$$\lim_{n\to\infty} \int_{t_1}^{t_2} \|\varepsilon_n(t)\|^2 dt = 0,$$

where $\varepsilon_n(t)$ is the truncation error and is defined as follows

$$\varepsilon_n(t) \triangleq f(t) - \sum_{i=1}^{n} w_i q_i(t) = \sum_{n+1}^{\infty} w_i q_i(t). \tag{2.2}$$

Furthermore, for every $\varepsilon_N > 0$, there exists $N(\varepsilon_N) > 0$ such that $|\varepsilon_n(t)| \leq \varepsilon_N$, for all $n \geq N(\varepsilon), t \in [t_1, t_2]$.

The Fourier series of $f$ is defined as

$$f(t) = a_0 + \sum_{i=1}^{\infty} (a_i \cos\omega_i t + b_i \sin\omega_i t), \tag{2.3}$$

where the frequencies of the sinusoidal function $\omega_i = \frac{2i\pi}{T}, \forall i \in \mathbb{N}$ and $T$ is the fundamental period of $f$. The values $a_0$, $a_i$ and $b_i$ are Fourier coefficients. Compare equation (2.1) with (2.3), and define the basis function vector

$$q(t) \triangleq [\ 1 \quad \cos\omega_1 t \quad \sin\omega_1 t \quad \ldots \quad \cos\omega_m t \quad \sin\omega_m t\ ]^{\mathrm{T}},$$

the coefficient vector

$$W \triangleq [\ a_0 \quad a_1 \quad b_1 \quad \ldots \quad a_m \quad b_m\ ]^{\mathrm{T}},$$

where $\omega_i = \frac{2i\pi}{T}$, $i = 1, \ldots, m$. Equation (2.3) can be written in the following vector form

$$f(t) = W^{\mathrm{T}} q(t) + \varepsilon_n(t), \tag{2.4}$$

where $\varepsilon_n(t)$, $n = 2m+1$, is the approximation error and satisfies

$$|\varepsilon_n| \leqslant \sum_{i>n} (|a_i| + |b_i|).$$

If the property of the signal to be approximated can be known beforehand, e.g., eveness, oddity and period, we may take the advantage of this information and consider, e.g., half-range cosine expansion. In this case, we have

$$q(t) \triangleq [\ 1 \quad \cos\omega_1 t \quad \cos\omega_2 t \quad \ldots \quad \cos\omega_m t\ ]^{\mathrm{T}},$$
$$W \triangleq [\ a_0 \quad a_1 \quad a_2 \quad \ldots \quad a_m\ ]^{\mathrm{T}},$$

where $\omega_i = \frac{i\pi}{T}$, $i = 1, \ldots, m$ and $n = m+1$.

An excellent property of (2.4) is its linear parametrization of the time-varying function $f(t)$ in a basis function vector and a time-invariant coefficient vector. Note that equation (2.4) is the Fourier series expansion of a periodic function. For a non-periodic function we may choose $T$ sufficiently large so that equation (2.4) is still capable of approximating the function.

## III. A Model Reference Adaptive Controller for the Compression System

### A. Problem Description

In this section, we adopt the following compression system model described by [4],

$$\begin{aligned} \dot{\Phi} &= -\Psi + \Psi_C(\Phi) - 3\Phi R, \\ \dot{\Psi} &= \frac{1}{\beta^2}(\Phi - \Phi_T(\Psi)), \\ \dot{R} &= \sigma R(1 - \Phi^2 - R), \quad R(0) \geq 0, \end{aligned} \tag{3.1}$$

where $\Phi$ represents the mass flow, $\Psi$ is the plenum pressure rise, $R \geq 0$ is the normalized stall cell squared amplitude, $\Phi_T(\Psi)$ is the mass flow through the throttle, and $\dot{()}$ denotes differentiation with respect to $\xi$, a dimensionless time [1]. The functions $\Psi_C(\Phi)$ and $\Phi_T(\Psi)$ are the compressor and throttle characteristics, respectively, and are defined as follows [2]:

$$\begin{aligned} \Psi_C(\Phi) &= \Psi_{C0} + 1 + \frac{3}{2}\Phi - \frac{1}{2}\Phi^3, \\ \Phi_T(\Psi) &= \gamma\sqrt{\Psi} - 1, \end{aligned}$$

where $\Psi_{C0}$ is the shut-off value of compressor characteristic, and $\gamma$ is the throttle opening, also the control input for the system. A set of typical value $\sigma = 7, \beta = \frac{1}{\sqrt{2}}$ and $\Psi_{C0} = 1.67$ are utilized in this paper [5]. The objective is to stabilize the compression system (3.1) around the critical equilibrium [5],

$$R^e = 0, \Phi^e = 1, \Psi^e = \Psi_C(\Phi^e) = \Psi_{C0} + 2,$$

which achieves the peak operation on the compressor characteristic. Shift the origin to the desired equilibrium by change of variables

$$\phi \triangleq \Phi - \Phi^e, \psi \triangleq \Psi - \Psi^e. \tag{3.2}$$

Then the compression system (3.1) can be further written as

$$\begin{aligned} \dot{R} &= -\sigma R^2 - \sigma R(2\phi + \phi^2),\ R(0) = R_0, \\ \dot{\phi} &= -\psi - \frac{3}{2}\phi^2 - \frac{1}{2}\phi^3 - 3R\phi - 3R,\ \phi(0) = \phi_0, \\ \dot{\psi} &= \frac{1}{\beta^2}(\phi - \gamma\sqrt{\psi + \Psi_{C0} + 2} + 2),\ \psi(0) = \psi_0. \end{aligned} \tag{3.3}$$

### B. A Model Reference Adaptive Controller Design

For convenience, let us write (3.3) as

$$\dot{x} = f(x) + g(x)u,$$

where the state vector $x$ and control input $u$ are defined by

$$x \triangleq \begin{bmatrix} R \\ \phi \\ \psi \end{bmatrix},\ u \triangleq \gamma\sqrt{\psi + \Psi_{C0} + 2} - 2, \tag{3.4}$$

the corresponding vector fields $f$ and $g$ are

$$f \triangleq \begin{bmatrix} -\sigma R^2 - \sigma R(2\phi + \phi^2) \\ -\psi - \frac{3}{2}\phi^2 - \frac{1}{2}\phi^3 - 3R\phi - 3R \\ \frac{1}{\beta^2}\phi \end{bmatrix}, g \triangleq \begin{bmatrix} 0 \\ 0 \\ -\frac{1}{\beta^2} \end{bmatrix}.$$

*Definition 3.1:* [16] A single-input nonlinear system in the form $\dot{x} = f(x) + g(x)u$, with $f(x)$ and $g(x)$ being smooth vector fields on $\mathbb{R}^n$, is said to be input-state linearizable if there exists a region $\Omega$ in $\mathbb{R}^n$, a diffeomorphism $\varrho : \Omega \to \mathbb{R}^n$, and a nonlinear feedback control law

$$u = \kappa(x) + \delta(x)\nu,$$

such that the new state variables $z = z(x)$ and the new input $\nu$ satisfy a linear time-invariant relation

$$\dot{z} = Az + b\nu,$$

where

$$A \triangleq \begin{bmatrix} 0 & 1 & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & 1 \\ 0 & 0 & \dots & 0 \end{bmatrix}, \; b \triangleq \begin{bmatrix} 0 \\ \vdots \\ 0 \\ 1 \end{bmatrix}.$$

The following theorems provide a method to check that if (3.3) is input-state linearizable. In the following equations the operator $\nabla(\cdot) \triangleq \frac{\partial}{\partial x}(\cdot)$.

*Theorem 3.1:* [16] The nonlinear system $\dot{x} = f(x) + g(x)u$, with $f(x)$ and $g(x)$ being smooth vector fields on $\mathbb{R}^n$, is input-state linearizable if and only if there exists a region $\Omega$ such that the following conditions hold:

1) the vector fields $\{g, \mathrm{ad}_f g, \dots, \mathrm{ad}_f^{n-1} g\}$ are linearly independent in $\Omega$.
2) the set $\{g, \mathrm{ad}_f g, \dots, \mathrm{ad}_f^{n-2} g\}$ is involutive in $\Omega$.

Then, let us check the controllability and the involutivity conditions. The controllability matrix is obtained by simple computation,

$$\begin{aligned} &\begin{bmatrix} g & \mathrm{ad}_f g & \mathrm{ad}_f^2 g \end{bmatrix} \\ &= \begin{bmatrix} 0 & 0 & -\frac{1}{\beta^2}\sigma R(2+2\phi) \\ 0 & -\frac{1}{\beta^2} & -\frac{1}{\beta^2}(3\phi + \frac{3}{2}\phi^2 + 3R) \\ -\frac{1}{\beta^2} & 0 & \frac{1}{\beta^4} \end{bmatrix}. \end{aligned}$$

It has rank 3 except at $\phi = -1$. Furthermore, the vector fields form an involutive set. Therefore, the system is input-state linearizable.

Now, let us find out the state transformation $z = z(x)$ and the input transformation $u = \kappa(x) + \delta(x)\nu$, so that input-state linearization can be achieved. The choice of $z_1$ can be selected as

$$z_1 = x_1 = R.$$

The other states can be obtained from $z_1$

$$\begin{aligned} z_2 &= \nabla z_1 f = -\sigma R^2 - \sigma R(2\phi + \phi^2). \\ z_3 &= \nabla z_2 f, \\ &= (-2\sigma R - 2\sigma\phi - \sigma\phi^2)(-\sigma R^2 - 2\sigma R\phi - \sigma R\phi^2) \\ &\quad + (-2\sigma R - 2\sigma R\phi)(-\psi - \frac{3}{2}\phi^2 - \frac{1}{2}\phi^3 - 3R\phi - 3R). \end{aligned}$$

Accordingly, the input transformation is

$$u = \frac{\nu - \nabla z_3 f}{\nabla z_3 g}, \tag{3.5}$$

or equivalently

$$\nu = a + bu, \tag{3.6}$$

where

$$\begin{aligned} a &\triangleq \nabla z_3 f, \\ &= [-2\sigma(-\sigma R^2 - 2\sigma R\phi - \sigma R\phi^2) \\ &\quad + (-2\sigma R - 2\sigma\phi - \sigma\phi^2)^2 + (-2\sigma - 2\sigma\phi) \\ &\quad \cdot (-\psi - \frac{3}{2}\phi^2 - \frac{1}{2}\phi^3 - 3R\phi - 3R) \\ &\quad + (-2\sigma R - 2\sigma R\phi)(-3\phi - 3)] \\ &\quad \cdot [-\sigma R^2 - \sigma R(2\phi + \phi^2)] \\ &\quad + [(-2\sigma - 2\sigma\phi)(-\sigma R^2 - 2\sigma R\phi - \sigma R\phi^2) \\ &\quad + (-2\sigma R - 2\sigma\phi - \sigma\phi^2)(-2\sigma R - 2\sigma R\phi) \\ &\quad - 2\sigma R(-\psi - \frac{3}{2}\phi^2 - \frac{1}{2}\phi^3 - 3R\phi - 3R) \\ &\quad + (-2\sigma R - 2\sigma R\phi)(-3\phi - \frac{3}{2}\phi^2 - 3R)] \\ &\quad \cdot (-\psi - \frac{3}{2}\phi^2 - \frac{1}{2}\phi^3 - 3R\phi - 3R) \\ &\quad + \frac{(2\sigma R + 2\sigma R\phi)}{\beta^2\phi}, \\ b &\triangleq \nabla z_3 g = -\frac{(2\sigma R + 2\sigma R\phi)}{\beta^2}. \end{aligned} \tag{3.7}$$

Note that functions $a$ and $b$ are quite involved and inevitably contain system parameters while these parameters may not be known precisely in advance. Hence, we assume that functions $a$ and $b$ are unknown even though the states can be measured. We end up with the following set of linear equations

$$\dot{z} = Az + B\nu, \tag{3.8}$$

where

$$A = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}, \quad B = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}.$$

Next, we consider the reference model,

$$\dot{z}_m = A_m z_m + B r_m, \tag{3.9}$$

where $r_m$ is the reference input, and

$$A_m \triangleq \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ a_{m1} & a_{m2} & a_{m3} \end{bmatrix}.$$

The $A_m$ is obtained by assigning the poles of the reference model. For notation convenience, let us define

$$a_m^{\mathrm{T}} \triangleq \begin{bmatrix} a_{m1} & a_{m2} & a_{m3} \end{bmatrix}.$$

Now rewrite (3.8) as

$$\dot{z} = Az + B[a + (b - \hat{b})u + \hat{b}u], \tag{3.10}$$

where $\hat{b}$ is the estimate of the unknown function $b$. Let $\hat{a}$ be the estimate of the unknown function of $a$, the controller can be chosen as

$$u = \frac{1}{\hat{b}}[a_m^{\mathrm{T}} z - \hat{a} + r_m]. \tag{3.11}$$

For the control law to be feasible, some measure should be taken, e.g., projection, so that $\hat{b}$ does not vanish [17], i.e.,

$|\hat{b}| \geq \hat{b}_{\min}$, where $\hat{b}_{\min}$ is a positive finite number. Substitute (3.11) into (3.10), we obtain

$$\begin{aligned}\dot{z} &= Az + B[a + (b-\hat{b})u + (a_m^{\mathrm{T}} z - \hat{a} + r_m)],\\ &= A_m z + B[(a-\hat{a}) + (b-\hat{b})u + r_m].\end{aligned}$$

From the behavior of the states $R, \sigma$ and $\psi$, it is reasonable to assume that $a$, $\hat{a}$, $b$, and $\hat{b}$ are all bounded piecewise continuous functions of time and satisfy the Dirichlet's conditions; therefore, they can be represented as

$$\begin{aligned}a &= W_a^{\mathrm{T}} q_a(t) + \varepsilon_a,\ \hat{a} = \hat{W}_a^{\mathrm{T}} q_a(t) + \varepsilon_{\hat{a}},\\ b &= W_b^{\mathrm{T}} q_b(t) + \varepsilon_b,\ \hat{b} = \hat{W}_b^{\mathrm{T}} q_b(t) + \varepsilon_{\hat{b}},\end{aligned} \tag{3.12}$$

where $\varepsilon_a$, $\varepsilon_{\hat{a}}$, $\varepsilon_b$, $\varepsilon_{\hat{b}}$ are the bounded approximation errors satisfying

$$|\varepsilon_a| < \bar{\varepsilon}, |\varepsilon_{\hat{a}}| < \bar{\varepsilon}, |\varepsilon_b| < \bar{\varepsilon}, |\varepsilon_{\hat{b}}| < \bar{\varepsilon}, \tag{3.13}$$

and the basis function

$$\begin{aligned}&q_a(t) = q_b(t)\\ &\quad \triangleq \begin{bmatrix}1 & \cos\omega_1 t & \sin\omega_1 t & \ldots & \cos\omega_m t & \sin\omega_m t\end{bmatrix}^{\mathrm{T}},\end{aligned}$$

in which $\omega_i = \frac{2i\pi}{T}$, $i = 1, \ldots, m$, and $T$ is the fundamental period of the estimated function. Define the error variable

$$e \triangleq z - z_m. \tag{3.14}$$

It is easy to see that the error dynamic is the following:

$$\begin{aligned}\dot{e} &= A_m(z - z_m) + B(a-\hat{a}) + B(b-\hat{b})u,\\ &= A_m e + B\tilde{W}_a^{\mathrm{T}} q_a + B\tilde{W}_b^{\mathrm{T}} q_b u + B\varepsilon,\end{aligned}$$

where

$$\varepsilon \triangleq \varepsilon_a - \varepsilon_{\hat{a}} + (\varepsilon_b - \varepsilon_{\hat{b}})u, \tag{3.15}$$

and the error coefficient vectors

$$\tilde{W}_a \triangleq W_a - \hat{W}_a, \tilde{W}_b \triangleq W_b - \hat{W}_b.$$

Note that $W_a$ and $W_b$ are constants.

Define the Lyapunov function candidate as

$$V(e, \tilde{W}_a, \tilde{W}_b) \triangleq e^{\mathrm{T}} P e + \tilde{W}_a^{\mathrm{T}} Q_a \tilde{W}_a + \tilde{W}_b^{\mathrm{T}} Q_b \tilde{W}_b, \tag{3.16}$$

where $P \in \mathbb{R}^{3\times 3}$, $Q_a \in \mathbb{R}^{n_a \times n_a}$, and $Q_b \in \mathbb{R}^{n_b \times n_b}$ are symmetric positive definite matrices. Take the time derivative of the Lyapunov function along the system trajectories, we have

$$\dot{V} = \dot{e}^{\mathrm{T}} P e + e^{\mathrm{T}} P \dot{e} + 2\tilde{W}_a^{\mathrm{T}} Q_a \dot{\tilde{W}}_a + 2\tilde{W}_b^{\mathrm{T}} Q_b \dot{\tilde{W}}_b. \tag{3.17}$$

Since $W_a$ and $W_b$ are both constant vectors, we have

$$\dot{\tilde{W}}_a = -\dot{\hat{W}}_a, \dot{\tilde{W}}_b = -\dot{\hat{W}}_b.$$

Therefore, equation (3.17) becomes

$$\begin{aligned}\dot{V} &= e^{\mathrm{T}}(A_m^{\mathrm{T}} P + P A_m)e + 2\tilde{W}_a^{\mathrm{T}}(q_a e^{\mathrm{T}} P B - Q_a \dot{\hat{W}}_a)\\ &\quad + 2\tilde{W}_b^{\mathrm{T}}(q_b u e^{\mathrm{T}} P B - Q_b \dot{\hat{W}}_b) + 2e^{\mathrm{T}} P B \varepsilon.\end{aligned}$$

Since $a_m$ can be selected so that the matrix $A_m$ is Hurwitz, for chosen $Q = Q^{\mathrm{T}} > 0$, $Q \in \mathbb{R}^{n\times n}$, there exists positive definite $P$ such that

$$A_m^{\mathrm{T}} P + P A_m + Q = 0.$$

Hence, we have

$$\begin{aligned}\dot{V} &= -e^{\mathrm{T}} Q e + 2\tilde{W}_a^{\mathrm{T}}(q_a e^{\mathrm{T}} P B - Q_a \dot{\hat{W}}_a)\\ &\quad + 2\tilde{W}_b^{\mathrm{T}}(q_b u e^{\mathrm{T}} P B - Q_b \dot{\hat{W}}_b) + 2e^{\mathrm{T}} P B \varepsilon.\end{aligned} \tag{3.18}$$

From (3.18), it is naive to choose the update law of $\hat{W}_a$ and $\hat{W}_b$ as

$$\dot{\hat{W}}_a = Q_a^{-1} q_a e^{\mathrm{T}} P B,\ \dot{\hat{W}}_b = Q_b^{-1} q_b u e^{\mathrm{T}} P B. \tag{3.19}$$

After substitution of (3.11), (3.14) and (3.15), it can be shown that

$$\begin{aligned}\dot{V} &= -e^{\mathrm{T}} Q e + 2e^{\mathrm{T}} P B \varepsilon,\\ &= -e^{\mathrm{T}} Q e + 2e^{\mathrm{T}} P B \left[(\varepsilon_a - \varepsilon_{\hat{a}}) + (\varepsilon_b - \varepsilon_{\hat{b}})u\right],\\ &\leq -\min(\lambda(Q))\|e\|^2 + 4\bar{\varepsilon}\Gamma\|e\|\|PB\| + \frac{2\bar{\varepsilon}\Lambda}{|\hat{b}|_{\min}}\|e\|^2,\end{aligned}$$

where $\lambda(\cdot)$ stands for the eigenvalue of related matrix, and

$$\begin{aligned}\Gamma &\triangleq 1 + \sup\left(\frac{1}{|\hat{b}|}|a_m^{\mathrm{T}} z_m - \hat{a} + r_m|\right),\\ \Lambda &\triangleq \max\left(\left|\lambda(PBa_m^{\mathrm{T}} + a_m B^{\mathrm{T}} P)\right|\right),\end{aligned}$$

which renders that if

$$e \in E \triangleq \left\{\delta \in \mathbb{R}^1 \,\middle|\, \|\delta\| \geq \frac{4\bar{\varepsilon}\Gamma\|PB\|}{\lambda_{\min}(Q) - \frac{2\bar{\varepsilon}\Lambda}{|\hat{b}|_{\min}}}\right\}, \tag{3.20}$$

then $\dot{V} \leq 0$. This further implies that the state error is ultimately bounded [18].

## IV. Numerical Examples

*Example 4.1:* For comparison purpose, we consider the state feedback controller proposed by Maggiore et al. which makes the origin of (3.3) an asymptotically stable equilibrium point with a global domain of attraction $\{(R, \phi, \psi) \in \mathbb{R}^3 | R \geq 0\}$ [5]. The choice of the control law was

$$\gamma = \frac{2 + (1 - \beta^2 k_1 k_2)\phi + \beta^2 k_2 \psi + 3\beta^2 k_1 R\phi}{\sqrt{\psi + \Psi_{C0} + 2}}, \tag{4.1}$$

with $k_1 = 100$, $k_2 = 100$ and $\beta = \frac{1}{\sqrt{2}}$ and initial conditions $[\ R_0 \quad \phi_0 \quad \psi_0\ ]^{\mathrm{T}} = [\ 1 \quad -0.8 \quad 1\ ]^{\mathrm{T}}$. Simulation results are depicted in Fig. 1 and 2, in which $u$ is defined by (3.4). It is obvious that the three states are regulated to the equilibrium point in about $\xi = 8$. However, it demands large control effort and may not be implemented in practical situation. To see the robustness provided by (4.1) with the same parameter values mentioned above, let the parameter $\beta$ be perturbed to 1.16 for illustration purpose. Fig. 3 depicts the simulation results. It can be observed that the system transient performance is almost the same, the robustness of the state feedback controller is preserved.

Next, we implement the proposed model reference adaptive controller on system (3.3). The parameter chosen for all simulations are $T = 73$, and 5 terms Fourier series basis are used to approximate the unknown functions $a$ and $b$ given by (3.7) with $Q_a = 1.8I$, $Q_b = 3.0I$ where $I \in \mathbb{R}^{5\times 5}$ is the identity matrix, and the initial estimated Fourier

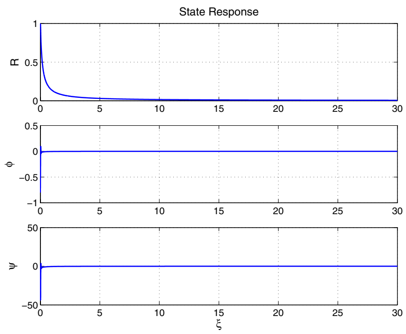


Fig. 1. State response for the closed loop compression system with state feedback controller.

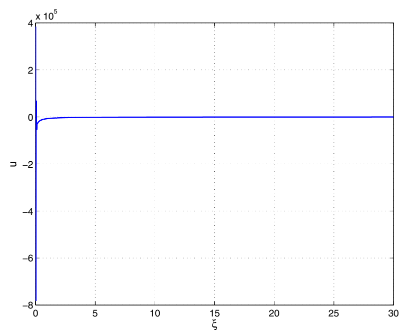


Fig. 2. Control input of the state feedback controller.

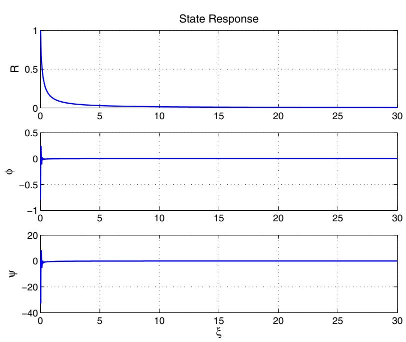


Fig. 3. State response for the closed loop compression system with state feedback controller (model parameter was perturbed).

series coefficient $\hat{W}_a(0) = [\begin{array}{ccccc} -0.5 & 0 & 0 & 0 & 0 \end{array}]^{\mathrm{T}}$, and $\hat{W}_b(0) = [\begin{array}{ccccc} -3.2 & 0 & 0 & 0 & 0 \end{array}]^{\mathrm{T}}$. The poles of the reference model are assigned at $-3$, $-4$ and $-5$, therefore $a_m = [\begin{array}{ccc} -60 & -47 & -12 \end{array}]$. Although by varying the desired poles one can improve the system performance, it is out of the scope of this paper. The initial condition for reference model is $z_m(0) = [\begin{array}{ccc} 0 & 0 & 0 \end{array}]^{\mathrm{T}}$, and the reference input $r_m = 0$. The parameter of the Lyapunov equation can be selected as

$$Q = \begin{bmatrix} 3 & 0 & 0 \\ 0 & 92 & 0 \\ 0 & 0 & 1 \end{bmatrix},$$

which yields

$$P = \begin{bmatrix} 70.889 & 8.6095 & 0.025 \\ 8.6095 & 20.427 & 1.1619 \\ 0.025 & 1.1619 & 0.13849 \end{bmatrix}.$$

Fig. 4 is the state response of the compression system with model reference adaptive controller, it shows that the three state can be regulated to zero in around $\xi = 12$, it is obvious that the system performance is similar to that of existing state feedback controller (4.1). While our proposed controller needs no information on the system parameter $\beta$. Fig. 5 depicts the control input of the proposed controller, which needs less control effort than that of the state feedback controller (4.1). Fig. 6 and 7 show that the estimated functions may not approach true values, but the tracking errors still converge, there are some deep insights remained to be explored. Table I shows the system performance index

$$I \triangleq \int_0^{\xi_f} \sqrt{R^2 + \phi^2 + \psi^2} d\xi,$$

with respect to different choice of $T$ for $\xi_f = 30$, so we can observe that when $T$ is selected in the range between 30 and 70, the performance index $I$ is relatively small. In order to see the robustness of our proposed controller, we also perturb the system parameter $\beta$ to 1.16. Simulation results are shown in Fig. 8, which is similar to Fig. 4. Hence, we can conclude that the robustness of our proposed controller is also preserved.

For experimental purpose, when the system is subject to input saturation, say $-10 \leq u \leq 10$, the system still provides satisfactory performance. Owing to the space limitation, the results are not shown here.

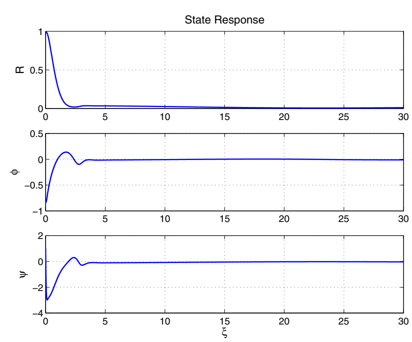


Fig. 4. State response for the closed loop system with adaptive controller.

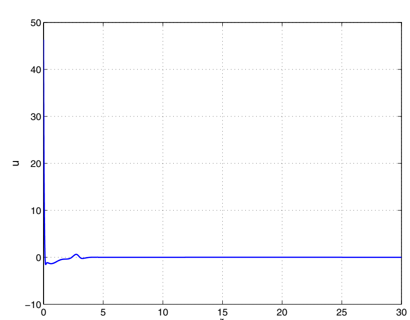


Fig. 5. Control input.

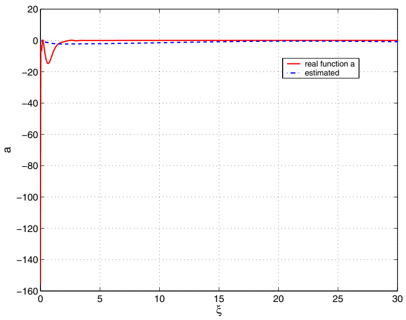


Fig. 6. The actual function of $a$ and $\hat{a}$.

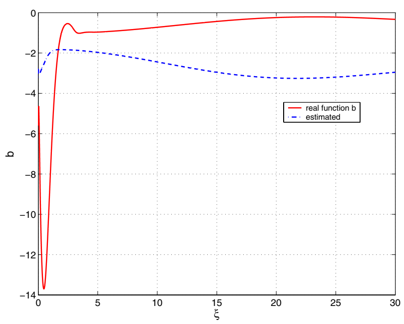


Fig. 7. The actual function of $b$ and $\hat{b}$.

TABLE I

THE PERFORMANCE INDEX WITH RESPECT TO DIFFERENT CHOICE OF $T$.

| $T$ | $I$ |
|---|---|
| 10 | 6.0597 |
| 20 | 4.8676 |
| 30 | 4.6157 |
| 40 | 4.5249 |
| 50 | 4.6447 |
| 60 | 4.6472 |
| 70 | 4.6744 |
| 80 | 4.7564 |

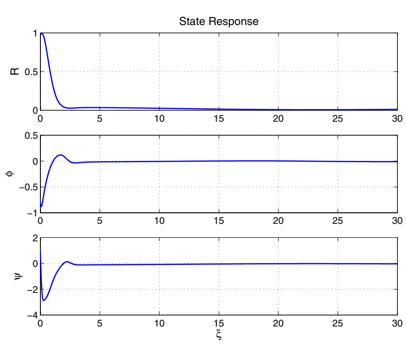


Fig. 8. State response for the closed loop compression system with model reference adaptive controller (model parameter is perturbed).

## V. CONCLUSION

Traditional adaptive controllers work well for systems with time-invariant or slow time-varying unknown parameters. For systems containing time-varying uncertainties or parameters with unknown bounds, most of the conventional adaptive control technique may not be applicable. In this paper, we present a model reference adaptive controller (MRAC) for compression system, where a lumped unknown controller parameters was estimated based on the function approximation technique and the unknown parameters are not restricted to time-invariant or known bound. The key idea is to transform the lumped unknown parameters of the controller into a finite term of Fourier series. Then the coefficients of these Fourier series can be updated based on Lyapunov approach.

From the simulation results, we observe that the system performance of model reference adaptive controller is similar to that of existing state feedback controller. However, the main advantage of the proposed MRAC is that it needs less model information than state feedback control does, and with on-line parameter estimation it can maintain consistent performance of the compression system.

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