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Compensation terms to improve fault detection in multivariate auto-correlated processes

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Abstract— **This paper analyses the use of ARMA filters for detecting abnormal conditions in complex processes. Such filters were recently introduced in the multivariate statistical process control (MSPC) framework to address the issue of auto-correlation in the recorded variables [1]. While these filters can indeed remove auto-correlation from the associated MSPC monitoring scheme, this paper shows that their application influences the sensitivity for fault detection. A compensation term is introduced here to correctly identify the magnitude of abnormal conditions.**

Index Terms— **Autoregressive moving average processes, fault diagnosis, correlation, monitoring, statistical.**

I. Introduction

The important task of detecting abnormal behavior in industrial multivariate processes has led to the evolution of a range of statistically based monitoring techniques, collectively referred to as multivariate statistical process control (MSPC). One of the most important MSPC tools is principal component analysis (PCA) which aims to remove redundancy in the data by defining a reduced variable set [2]. Utilising this reduced set, univariate statistics can then be used for on-line process monitoring. To detect abnormal behavior, confidence limits can be determined for these univariate statistics.

A recent study by Kruger et al. [1], however, demonstrated that strongly auto-correlated process variables may produce false alarms, consequently invalidating the on-line monitoring approach. Controller feedback or unmeasured disturbances cause the process variables to move around the steady-state condition and exhibit some degree of auto-correlation [3].

To monitor auto-correlated process variables, Ku et al. [3] proposed the inclusion of time-series structures into the PCA analysis. Although this dynamic PCA approach leads to an increased sensitivity for detecting disturbances, the influence of auto-correlation is not removed, and may therefore lead to false alarms [1].

To avoid this, Kruger et al. [1] showed that ARMA filters can remove auto-correlation from the MSPC monitoring model. The contribution of the presented paper is to show that the application of such filters changes the sensitivity for detecting abnormal conditions. In previous work, Harris and Ross [4] showed that positively auto-correlated AR models tend to be insensitive to step type faults. For a single auto-correlated variable, Wardell et al. [5] extended this analysis to an ARMA filter and showed that an inverse ARMA filter may converge to a steady-state level differing from the magnitude of the step fault. This was investigated further by Dyer et al. [6]. However, previous work did not answer a number of questions: What is this steady-state value produced by an inverse ARMA filter in this case? Furthermore, what impact has an inverse ARMA filter upon a ramp type fault e.g. estimated slope? These questions are addressed here for one analysed variable. The paper also proposes the incorporation of compensation terms to correctly identify the magnitude of fault conditions. These are introduced for both step (e.g. sensor failure) and ramp (e.g. performance deterioration of process units) type faults. The results are finally applied to a multivariate example to enhance the work by Kruger et al. [1].

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II. Process monitoring preliminaries

The principles of PCA, one of the most popular MSPC techniques, are now briefly reviewed. The statistical monitoring charts are then defined.

A. The core PCA algorithm

The application of PCA involves the construction of a reduced set of principal components (PCs) describing significant variations in the steady-state operation of the monitored process variables. The values of the PCs for the k^{th} sampling point are obtained as follows:

$$\mathbf{t}_k = \mathbf{P}^T \mathbf{z}_k, \tag{1}$$

where $\mathbf{t}_k \in \mathbb{R}^n$ is the vector of PCs, or scores, $\mathbf{P} \in \mathbb{R}^{N \times n}$ is a transformation, or loading matrix, where each column vector is denoted as a loading vector. Finally $\mathbf{z}_k \in \mathbb{R}^N$ is the vector of monitored variables. Here N denotes the number of variables included in the PCA analysis and n represents the number of retained PCs.

The PCs chosen represent the significant process variation. The mismatch between the measured and predicted values of the process variables is $\mathbf{f}_k$ given by:

$$\mathbf{f}_k = \mathbf{z}_k - \mathbf{P}\mathbf{t}_k = \left[\mathbf{I} - \mathbf{P}\mathbf{P}^T\right] \mathbf{z}_k, \tag{2}$$

where the predicted values are estimated by a model based on the PCs

B. Univariate statistics

A univariate statistic, T^2, can be established from the score variables as:

$$T_k^2 = \mathbf{t}_k^T \mathbf{\Lambda}^{-1} \mathbf{t}_k, \tag{3}$$

where $\mathbf{\Lambda}$ is a diagonal matrix of the variances of the scores. The time variation in the T^2 statistic can be used for monitoring, with confidence limits obtained as discussed in Jackson [7].

A second univariate statistic, relating to the residuals of the PCA model prediction, can also be defined. However, this statistic is not considered here since the residuals represent measurement uncertainty which is assumed to be stationary and normally distributed as required in MSPC based process monitoring.

III. Application of ARMA filters

As proposed by Kruger et al. [1], inverse ARMA filters are applied to remove auto-correlation. Given an input function, $v(z^{-1})$, an output function, $e(z^{-1})$ and a filter, $e(z^{-1}) = R^{-1}(z^{-1})v(z^{-1})$, the inverse filter, R^{-1}, is employed to filter $v(z^{-1})$ to produce a normally distributed white noise sequence, $e(z^{-1})$. Here z^{-1} is the backshift operator and $R(z^{-1})$ the transfer function of the ARMA filter. More precisely, the calculation of $e(z^{-1})$ using the filter $R^{-1}(z^{-1})$ at the k^{th} instance is given by:

$$e_k = v_k + \sum_{i=1}^{q} b_i v_{k-i} - \sum_{i=1}^{p} a_i e_{k-i}, \tag{4}$$

where p is the number of moving average terms and q denotes the number of autoregressive terms.

To determine an ARMA based monitoring model, the first step is to record reference data from the process. Such data has to be selected with care to ensure that abnormal process behaviour is avoided. Conversely, if the size of this reference data is too small, then normal variation within the process will not be adequately represented [8]. A PCA model is then established. After defining the order of the incorporated AR structure using parallel analysis, as proposed by Ku et al. [3], Jackson [7] showed that the loading vectors are the dominant eigenvectors of the correlation matrix, constructed from the reference data set. The number of retained PCs can also be determined by a parallel analysis, including a subsequent correlation analysis to ensure that PCs which capture significant process variation are not discarded [3]. The score variables are then determined using Equation (1), and the number of AR and MA terms needed to remove the auto-correlation from these variables is found as discussed in [1].

IV. Analysis of ARMA filters

This section address the outstanding issues raised in the introduction and analyses ARMA filters in a condition monitoring context using one variable. Based on a simplified example, it is shown that an ARMA filter can influence the magnitude of a fault condition. Furthermore, the influence of the ARMA filter upon the multivariate T^2 statistic, in terms of the sensitivity in detecting faults, is also discussed. Step and ramp type faults are considered: The former resulting typically from sensor failures, whilst ramp faults characterise performance deterioration of process units.

A. *Effect of step type changes*

An ARMA filter for removing auto-correlation from a PC is defined by

$$\frac{v(z^{-1})}{e(z^{-1})} = \frac{1 + \sum_{i=1}^{p} a_i z^{-i}}{1 + \sum_{i=1}^{q} b_i z^{-i}}. \tag{5}$$

If it is assumed that a PC takes the form of a unit step, e.g. a sensor failure then, e_∞, the steady-state value of e, is given by [9]:

$$e_\infty = \frac{1 + \sum_{i=1}^{q} b_i}{1 + \sum_{i=1}^{p} a_i}. \tag{6}$$

For a stable filter and also stable inverse filter, which has no roots in the nominator and denominator polynomials outside the unit circle, two cases can be established:

(i) $1 + \sum_{k=1}^{p} a_k \gg 1 + \sum_{k=1}^{q} b_k \Rightarrow e_\infty \simeq 0$; and
(ii) $1 + \sum_{k=1}^{p} a_k \ll 1 + \sum_{k=1}^{q} b_k \Rightarrow e_\infty \gg 0$.

These describe two possible extremes; the first that the fault impact has been considerably reduced, which may render it undetected, the second that the monitoring model is very sensitive.

The influence of the ARMA filter upon the magnitude of the detected fault is now illustrated using a simple single-input, single-output (SISO) example. Here, 1000 data points that describe a mean-centered, normally-distributed sequence with unit variance, ($e \in \mathcal{N}\{0, 10^{-4}\}$), are filtered as:

$$v_k = -bv_{k-1} + e_k + ae_{k-1}, \tag{7}$$

where $a = 0.6$, $b = -0.4$, and v denotes the output variable. An offset of $f = 2$ was then added to each data point of v_k in the second half of the filtered data set. According to Equation (6), the ARMA filtering operation in Equation (7) will influence the magnitude of the step. Instead of the correct step size of 2, the estimate of e_∞ is equal to 0.75 which can be seen from Figure 1. This problem can be overcome by introducing a compensation term, as discussed later in Section V.

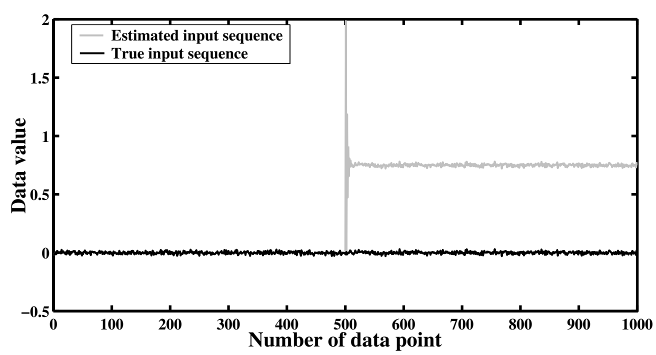

Fig. 1. Estimated input sequence without the compensation scheme applied

B. *Effect of ramp type changes*

The influence of an ARMA filter on statistical monitoring models is now investigated for a ramp type fault. To our knowledge, such an analysis has not yet been conducted.

This time, a data set containing 2500 samples was again generated using Equation (7). A ramp with a gradient $h = 0.0025$ was then added to the output sequence starting at the 500^{th} sample. The resultant faulty sequence then deviated by a total of 5 units at sample 2500.

For simplicity, the influence of the ARMA filter on a ramp fault is analysed for a first-order filter, i.e. $p = 1$ and $q = 1$. The change in slope provided by the ARMA filter is given by [9]:

$$\triangle h_k = h\left[\frac{1-(-a)^k}{1+a} - \frac{b}{a}\left(\frac{1-(-a)^k}{1+a} - 1\right)\right]. \tag{8}$$

Using this equation the estimated slope for each sample can be obtained, where $|a| < 1$ for a stable filter. After convergence, $k \to \infty$, the steady state-value of the slope is [9]:

$$\triangle h_\infty = h\left[\frac{1}{1+a} - \frac{b}{a}\left(\frac{1}{1+a} - 1\right)\right]. \tag{9}$$

Using the same parameters as in Equation (7), the final value of the slope is now 0.00094, rather than the correct value of 0.0025. Note that two extreme cases may again emerge:

(i) $b \gg a \Rightarrow \triangle h_\infty \gg 0$; and
(ii) $b \ll a \Rightarrow \triangle h_\infty \simeq 0$.

The first case implies that the filter gives rise to a very sensitive condition monitoring. In contrast, the second describes the possibility that the fault impact has been considerably reduced. This may render it unnoticed over a longer period of time.

For the above example, the input sequence determined without compensation, which shows an incorrect result and the prediction error, $e - \hat{e}$, are shown in Figure 2. Note

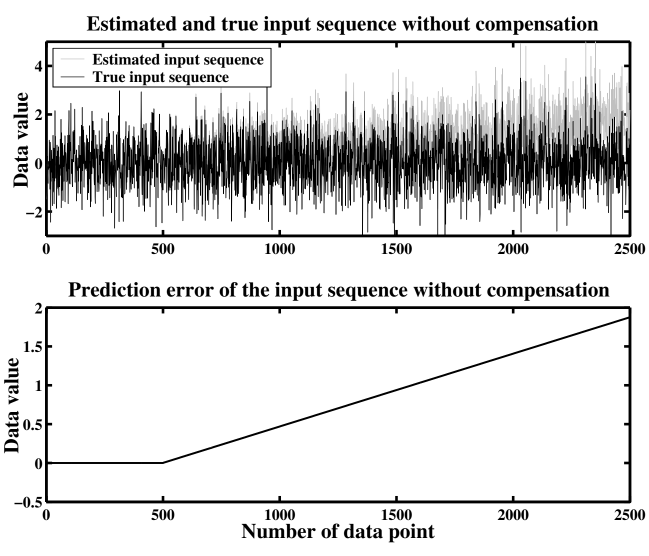

Fig. 2. Estimated input sequence the prediction error without compensation at the ramp fault

that the inverse ARMA filter estimates at the 2500^{th} sample an offset of 1.88 units which coincides with the prediction from Equation (9), i.e. $0.00094 \times 2000 = 1.88$. However, the correct value was 5, showing that the ramp magnitude was incorrect. This problem can be overcome by a compensation term as described in Section V.

C. Effect on the Hotelling T^2 statistic

An abnormal condition affects the Hotelling's T^2 statistic for example a step change in one variable will influence the scores as follows:

$$\triangle \mathbf{t} \quad = \quad \mathbf{P}^T \triangle \mathbf{z}, \tag{10}$$

where $\triangle \mathbf{z}$ denotes the offset in one variable. The resultant T^2 statistic is then equal to

$$\begin{aligned} T_k^2 \quad = \quad & \mathbf{t}_k^T \mathbf{\Lambda}^{-1} \mathbf{t}_k + 2\mathbf{t}_k^T \mathbf{\Lambda}^{-1} \triangle \mathbf{t} \\ & + \triangle \mathbf{t}^T \mathbf{\Lambda}^{-1} \triangle \mathbf{t}. \end{aligned} \tag{11}$$

Here the second and third terms describe the changes produced by the step fault. Given the results of the previous two subsections, the Hotelling's T^2 statistic will also be incorrect since the score variables are filtered using ARMA filters.

V. Compensation Scheme

This section introduces a compensation scheme to identify the correct magnitude of fault conditions.

A. Compensation for step faults

From Subsections IV-A and IV-B it follows, that inverse ARMA filters may lead to a considerable increase or reduction in sensitivity for detecting faults. The compensation for a step fault is based on the definition of an inverse ARMA filter:

$$e_k = v_k + \mathbf{b}^T \mathbf{v}_{k-1} - \mathbf{a}^T \mathbf{e}_{k-1} \tag{12}$$

where $\mathbf{e}_{k-1}^T = \begin{pmatrix} e_{k-1} & \cdots & e_{k-p} \end{pmatrix}$, $\mathbf{v}_{k-1}^T = \begin{pmatrix} v_{k-1} & \cdots & v_{k-q} \end{pmatrix}$ and $\mathbf{a}$, $\mathbf{b}$ are vectors containing the ARMA parameters. The occurrence of a step fault is characterised by the superposition of a constant to v_k.

To correct for the effect of adding this constant to the input, the compensation schemed is defined as follows:

$$\bar{e}_{k-1} = \sum_{i=j}^{k-1} \frac{e_i}{k-j}. \tag{13}$$

Here j is the sample where the step change first occurred, which has to be provided by the current fault detection scheme. This compensation term, $\bar{e}_k$, is now subtracted from the output sequence v_k and the input sequence e_k:

$$\begin{aligned} e_k \quad &= \quad v_k + \mathbf{b}^T \left(\mathbf{v}_{k-1} - \mathbf{1}_q \bar{e}_{k-1}\right) - \mathbf{a}^T \left(\mathbf{e}_k - \mathbf{1}_p \bar{e}_{k-1}\right), \\ e_k \quad &= \quad v_k + \mathbf{b}^T \hat{\mathbf{v}}_{k-1} - \mathbf{a}^T \hat{\mathbf{e}}_k \end{aligned} \tag{14}$$

Here, $\mathbf{1}_p$ and $\mathbf{1}_q$ are vectors of ones, of dimension p and q, respectively. The compensation term isolates the influence of the step from the ARMA filter. Consequently, if a new sample becomes available the departure of this sample from the prediction of the "corrected" inverse ARMA filter converges to the correct fault size. Noted that the fault can be detected using the filtered sequence of the ARMA filter, i.e. the integer value j can be determined on the occurrence of a step-type fault. As discussed in Section IV, however, the fault magnitude may be altered without application of the compensation term above.

The compensation term is now applied to the SISO example given in Equation (7). The correct fault magnitude is estimated as follows:

$$\begin{aligned} e_k &= v_k + b\hat{v}_{k-1} - a\hat{e}_{k-1}, & (15)\\ \bar{e}_k &= \frac{k-j}{k-j+1}\bar{e}_{k-1} + \frac{e_k}{k-j+1}, & (16)\\ \hat{v}_k &= v_k - \bar{e}_k, & (17)\\ \hat{e}_k &= e_k - \bar{e}_k, & (18) \end{aligned}$$

Figure 3 shows the estimated input sequence of the ARMA filter with the compensation scheme applied. Note that the correct step size of the fault, i.e. $e_\infty = 2$, has been identified.

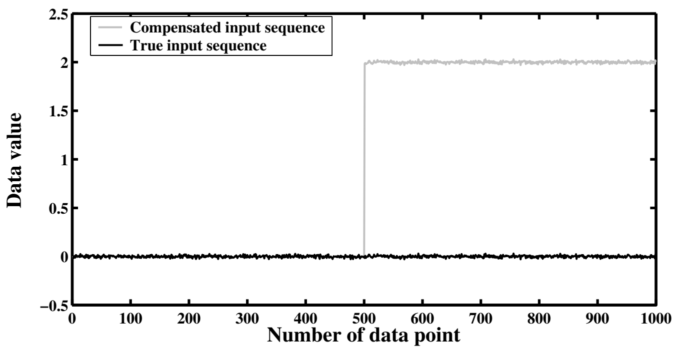

Fig. 3. Estimated input sequence with the compensation scheme applied

B. Compensation for ramp type faults

Compensation for the effect of the inverse ARMA filters in determining the slope of ramp-type faults, is carried out here using a least-squares approach. More precisely, a least-squares analysis is applied to the filter residuals inside a moving window. As before, the aim of the compensation term is to remove the signature of the ramp-type fault from past values of the sequences v_k and e_k. This produces 'corrected' sequences where the effect of the ramp fault has been removed. The predictions of the inverse ARMA filter consequently produce the correct fault magnitude.

A ramp is characterised by the offset term d_1 and a slope d_2. The least-squares estimate $\mathbf{d}^T = \begin{pmatrix} d_1 & d_2 \end{pmatrix}$ is therefore given by:

$$\begin{aligned} \mathbf{X}^T &= \begin{bmatrix} 1 & 1 & \cdots & 1 \\ 1 & 2 & \cdots & 100 \end{bmatrix} & (19)\\ \mathbf{d} &= \left[\mathbf{X}^T\mathbf{X}\right]^{-1}\mathbf{X}^T \begin{pmatrix} e_{k-1} \\ \vdots \\ e_{k-100} \end{pmatrix}, & (20) \end{aligned}$$

where $\mathbf{X}$ contains offset and position of the data point inside of a window of size 100 here. After the ramp parameters have been computed, v_k and e_k are compensated as follows:

$$\begin{aligned} \Delta\bar{e}_k &= \mathbf{d}^T \begin{pmatrix} 1 \\ e_k \end{pmatrix} & (21)\\ \hat{v}_k &= v_k - \Delta\bar{e}_k & (22)\\ \hat{e}_k &= e_k - \Delta\bar{e}_k. & (23) \end{aligned}$$

After compensation has been carried out and the next sample becomes available, the above procedure is repeated. Figure 4 shows the results of applying this compensation to the example in Subsection V-B. The compensation scheme converges to the correct estimate of the fault slope as seen by the deviation of 5 units at sample 2500^{th}. The compensation scheme in fact had started to converge after about 300 samples. This is to be expected since the first few samples were not affected much by the ramp fault and the window size was selected to be 100.

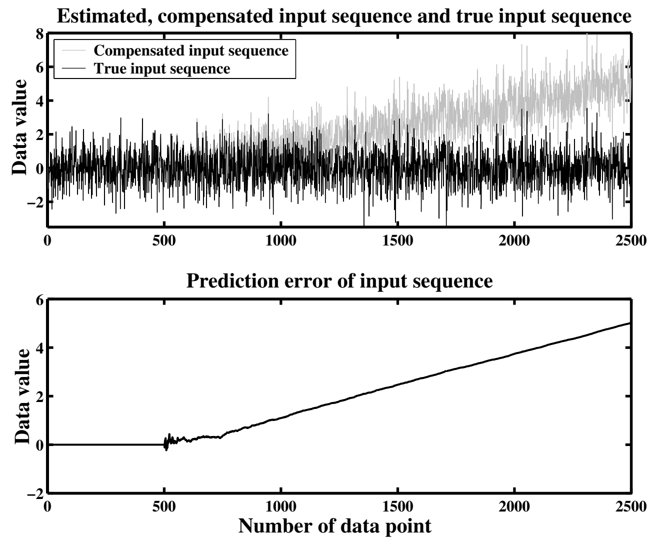

Fig. 4. Estimated input sequence and the prediction error with compensation applied

VI. Application Example

To present a more complex multivariate application example, the simulated two-input, two-output dynamic process by Ku et al. [3] was utilised. The inputs were ARMA sequences and a total of 2500 samples of all process variables were recorded prior to the introduction of uncorrelated measurement noise.

The first 500 samples were selected as the reference data set. The remaining 2000 samples served as a test data set for evaluating the performance of the monitoring scheme in terms of its sensitivity in detecting a step-type fault of size 3. This was introduced to the last 1000 samples of the second input variable.

For this process, Kruger et al. [1] showed that the "true" values of the output variables are linear combinations of previous "true" values of the input and output variables and hence, a total number of 4 PCs were retained. The two discarded PCs represent the measurement noise.

A. Application of ARMA filter

A PCA model was first established from the first 500 samples and the ARMA filter for the four retained PCs were then identified [1]. The variation in the T^2 statistic, based on the filtered score variables, is shown in Figure (5). As expected, the T^2 statistic showed a considerable number of violations with respect to its 99% confidence limit after the first 1000 samples. Although these violations indicate abnormal process behaviour, they do not reflect the correct fault magnitude.

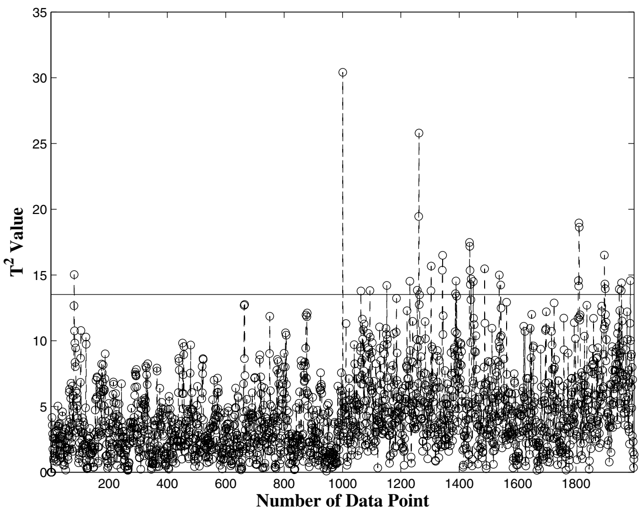

Fig. 5. T^2 statistic based on ARMA fi ltered scores

B. Compensated ARMA filter

The compensation scheme for step type faults, discussed in Subsection V-A, was now applied to the four ARMA filters. Figure (6) shows the resultant "corrected" Hotelling's T^2 statistic based on the filtered PCs. Note that, compared to

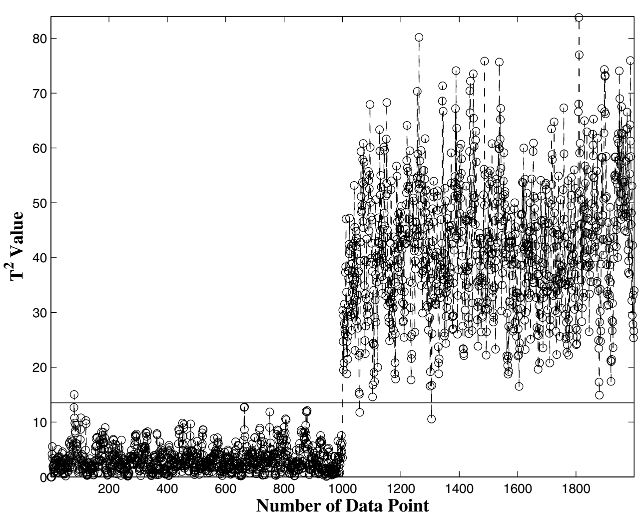

Fig. 6. T^2 statistic of the compensated ARMA process

Figure (5), now the step fault was not reduced by filtering and correctly indicates the fault in the second half of the data set.

VII. Conclusions and discussion

This paper showed that filtering strongly auto-correlated variables using an inverse ARMA filter may alter the magnitude of step and ramp type faults.

To correctly identify the magnitude of step and ramp type faults, compensation terms were introduced that isolate the impact of such faults from the inverse ARMA filter. This was demonstrated using a simple ARMA process with one process variable and a more complex simulation of a process with two input and two output variables. In each example, the "uncorrected" inverse ARMA filter led to incorrect estimation of the size of a step fault and the slope of a ramp fault. Inclusion of the compensation terms correctly identified these fault parameters.

The compensation term is triggered when the univariate statistic is violating its corresponding confidence limit. This was demonstrated through the analysis of the single variable in Section V and the multivariate example in Section VI.

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