Proceedings of the
44th IEEE Conference on Decision and Control, and
the European Control Conference 2005
Seville, Spain, December 12-15, 2005

Modeling and control of a small autonomous aircraft having two tilting rotors

Farid Kendoul, Isabelle Fantoni, Rogelio Lozano

Laboratoire Heudiasyc, UMR CNRS 6599, Université de Technologie de Compiègne, 60200 Compiègne, France
Email: {fkendoul, ifantoni, rlozano}@hds.utc.fr

Abstract— **This paper presents recent work concerning a small tiltrotor aircraft with a reduced number of rotors. After several attempts, the final design consists of two propellers mounted laterally. The direction of the thrust can be redirected by tilting the propellers laterally and longitudinally. A theoretical analysis of this mechanism proves its effectiveness and the experimental results show that this aerodynamical configuration is very promising. A model of the full birotor dynamics is also presented in this paper and a controller based on the backstepping procedure is synthesized performing also an autonomous flight.**

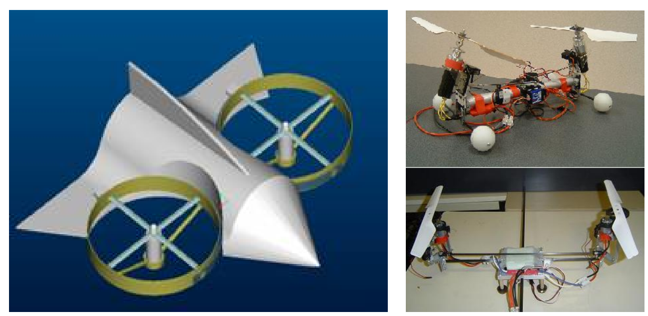
I. INTRODUCTION

Since the beginning of the 20^{th} century, research efforts have continually pushed the limits to create effective flying machines that offer many capabilities. The tiltrotor aircraft configuration [1] has the potential to revolutionize air transportation by providing an economical combination of vertical take-off and landing capability with efficient, high-speed cruise flight. This makes advanced technology helicopters and civil tiltrotors extremely attractive as corporate/executive transport aircraft and open the opportunity for designing and commercializing tiltrotor UAVs. Indeed, The Bell Eagle Eye UAV [2] which is based on tiltrotor technology has a large success in the civil and military domains.

However, this design brings its own problems, since the degradation in stability is usually observed in high speed forward flight (airplane mode) [3]. Moreover, the involved equations of motion are highly coupled and nonlinear. Gary Gress [4] discovered that the tiltrotor-based mechanism can provide hover stability of a small UAV by using the gyroscopic nature of two tilting rotors.

In this paper, we propose an alternative configuration/system, called *BIROTAN*[1] *(BI-ROtors with tilting propellers in TANdem)* where the center of mass of the UAV is located below the tilting axes, resulting in a significant pitching moment.

We have constructed two prototypes of a birotor rotorcraft as shown in Figure 1(b), inspired by G. Gress's mechanism. The experimental results showed that this aerodynamical configuration is very promising. This configuration is adapted for the miniaturization of the UAV, and it results in a simple mechanical realization. Unlike the full-scale tiltrotors, the propellers can tilt in two directions providing also stability and control in hover. By using this original structure inspired by the ingenious mechanism of Gary Gress [4], required lift and control moments are obtained. Therefore, no helicopter-type cyclic controls are needed, nor are any other reactive devices. The roll movement is obtained by differential propeller speeds. The yaw angle can be controlled through differential longitudinal tilting. The gyroscopic moments issued from opposed lateral tilting, added to the torque generated by the collective longitudinal tilting allow to obtain a significant pitching moment. In addition, our main contribution is to provide a complete model of the birotor-rotorcraft and to present a control algorithm for stabilization and trajectory tracking based on backstepping methodology ([5], [6], [7]). This control strategy turned out to be methodologically simple for the proposed system.



(a) The birotor rotorcraft (b) Our prototypes

Fig. 1. Tiltrotor-based mechanism

In this paper, we first motivate the use of tilting rotors, and describe such a mechanism in Section II. Next, in Sections III and IV we present the model of the birotor rotorcraft whose dynamical model is obtained via a Newton approach. Finally, we address a control strategy for stabilization and trajectory tracking in Section V. Furthermore, robustness with respect to parameter uncertainty and unmodeled dynamics has been observed in simulation as shown in Section VI.

II. TILTROTOR-BASED MECHANISM

In this section, we describe and show briefly the capabilities of the tiltrotor-based mechanism for providing the three required moments (roll, pitch and yaw), which will

[1]This work was supported by the ONERA (French aeronautics and space research centre), the DGA (French Arms Procurement Agency of the Ministry of Defence) and the French Picardie Region Council.

be detailed separately below. We will focus on certain basic aspects of the proposed configuration, namely: our aircraft model is distinguished from the one proposed by G. Gress by its center of mass which is moved below the tilt axis. This arrangement has the advantage to provide a significant pitching moment. But this modification results in complex dynamical model of the vehicle. This issue is resolved by synthesizing an appropriate controller which takes into account these complications.

For more details on system design and technical analysis, refer to ([4], [8]).

Differential propeller speeds: Since it is possible to modify independently the speed of each propeller, then the rolling moment can be given by

$$\tau_\phi = l(P_1 - P_2) \tag{1}$$

where l is the distance from the motors to the aerodynamic center O (see Figure 2). P_1 and P_2 are the lifts generated by each propeller.

Longitudinal Tilting (LT): The longitudinal tilting of the propellers is controlled by two inboard servos. The yaw moment is obtained by differential longitudinal tilting of the rotors. Indeed, the difference of lift longitudinal components creates a yawing moment with respect to G as follows

$$\tau_\psi = l(P_1 \sin\alpha_1 - P_2 \sin\alpha_2) \tag{2}$$

where α_1 and α_2 are the longitudinal tilt angles.

Since the center of gravity G is located below the tilt axes, thus, the sum of longitudinal components of P_1 and P_2 generates a pitching moment with respect to G. It can be expressed as

$$\tau_{\theta_\alpha} = h(P_1 \sin\alpha_1 + P_2 \sin\alpha_2) \tag{3}$$

where h is the distance from G to the tilt axes.

It seems that the longitudinal tilting is sufficient to achieve the control of yaw and pitch movements. Unfortunately, collective tilting of the propellers creates an adverse reaction moment $M_{a.r}$ (see equation (19)). At the exact instant servo-arm rotation is initiated, tilt acceleration relative to the airframe is non-zero, thereby creating a momentary, unbalanced and opposite pitching moment on the aircraft due to the inertia I_t of the propeller about its tilt axis. This moment acts on the airframe, causing loss of control authority and resulting in instability. Moreover, when the propellers have an angle of attack in longitudinal flight, they have a severe tendency to pitch up. They create substantial positive pitching moments.

We conclude that the method by itself offers no practical means for countering these effects, and stability can not be obtained using only longitudinal tilting of the propellers. These issues may be resolved by combining this *LT* with an opposed lateral tilting.

Opposed Lateral Tilting (OLT): Referring to Figure 2, forcing propellers to precess laterally in opposite directions will create gyroscopic moments $(M_{1\beta}, M_{2\beta})$ which are perpendicular to their respective spin and tilt axes [8],

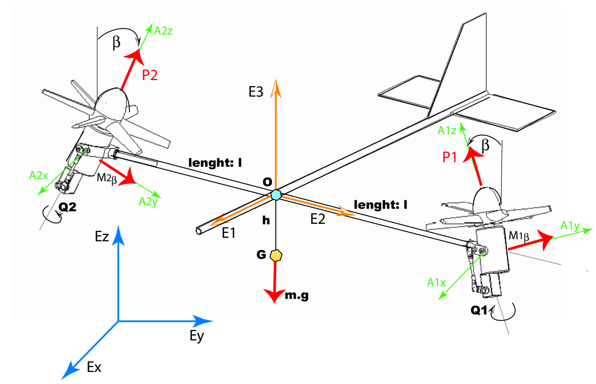


Fig. 2. Opposed lateral tilting and the associated moments

and directed as shown in Figure 2. Their total magnitude, calculated in Section III, is given by

$$M_\beta = M_{1\beta} + M_{2\beta} = I_r \dot{\beta}(\omega_1 + \omega_2)\cos\beta \tag{4}$$

with I_r is the fan inertia moment about spin axis.

Also, a non-zero tilt angle β will generate a component of the propeller torque vector along the lateral axis E_2, creating a pitching moment on the aircraft in the same direction as M_β. Then, the pitching moment created by *OLT* is the sum of the components of the gyroscopic moments and fan torques (Q_1,Q_2) resolved along the E_2 axis:

$$\tau_{\theta_\beta} = I_r \dot{\beta}(\omega_1 + \omega_2)\cos\beta + (Q_1 + Q_2)\sin\beta \tag{5}$$

The experimental tests showed that the moment τ_{θ_β} caused by *OLT* is weak and not sufficient for controlling the pitch movement and establishing forward motion of the aircraft.

The key idea consists in using both *LT* and *OLT* for pitch/forward motion control. More precisely, the collective longitudinal tilting generates a pitching moment (3) by directing the combined thrust vector, and the *OLT* creates complementary pitching moment (τ_{θ_β}) that minimizes or eliminates the resulting adverse reaction $M_{a.r}$ and aerodynamic pitch-up. The *OLT* and the *LT* have to be programmed (assigned by the controller) in such manner to optimize the aircraft behavior at all operating points in the flight envelope.

III. Birotor Model

The complete dynamics of a helicopter is quite complex and somewhat unmanageable for the purpose of control. Therefore, we consider a helicopter model as a rigid body incorporating a force and moment generation process [5].

A. Rigid body dynamics

The equations of motion for a rigid body subject to body force $F_0 \in \mathbb{R}^3$ and torque $\tau \in \mathbb{R}^3$ applied at the center of mass and specified with respect to the body coordinate frame $\mathcal{A} = (E_1, E_2, E_3)$ (see Figure 2) are given by the Newton-Euler equations in $\mathcal{A}$, which can be written as

$$\begin{cases} m\dot{v}^{\mathcal{A}} + \Omega \times m v^{\mathcal{A}} = F_0 \\ J\dot{\Omega} + \Omega \times J\Omega = \tau \end{cases} \tag{6}$$

where $\upsilon^{\mathcal{A}} \in \mathbb{R}^3$ is the body velocity vector, $\Omega \in \mathbb{R}^3$ is the body angular velocity vector, $m \in \mathbb{R}$ specifies the mass, and $J \in \mathbb{R}^{3\times3}$ is an inertial matrix. F_0 combines the force of gravity and the lift vector F generated by the propellers.

The first equation in (6) can be also expressed in the inertial frame $\mathcal{I} = (E_x, E_y, E_z)$. By defining $\xi = (x, y, z)$ and $\upsilon^{\mathcal{I}}$ as the position and the velocity of the helicopter relative to $\mathcal{I}$, we can write

$$\begin{cases} \dot{\xi} = \upsilon^{\mathcal{I}}, \;\; m\dot{\upsilon}^{\mathcal{I}} = RF - mgE_z \\ J\dot{\Omega} + \Omega \times J\Omega = \tau \end{cases} \tag{7}$$

where $R \in SO(3)$ is the rotation matrix of the body axes relative to $\mathcal{I}$, satisfying $R^{-1} = R^T$ and $det(R) = 1$. It can be obtained based on Euler angles $\eta = (\psi, \theta, \phi)$ which are (yaw, pitch and roll) (cf. [5] and [9] for its expression).

B. Force/moment generation process

In the following, we express the force/moment pair (F, τ), exerted on the helicopter. To develop our analysis, we use two additional coordinate frames: $A_1 = (A_{1x}, A_{1y}, A_{1z})$ and $A_2 = (A_{2x}, A_{2y}, A_{2z})$ which are associated to the rotor $n^{\circ}1$ and rotor $n^{\circ}2$, respectively (see Figure 2). The tilt movement of the rotors is obtained by a rotation β around the axes A_{ix} and then a rotation α_i around the lateral axis E_2, $i = 1, 2$. Therefore, the orientation of the rotors $n^{\circ}1$, $n^{\circ}2$ with respect to $\mathcal{A}$ can be defined by the rotational matrices T_1, T_2 whose expressions are given, respectively, by

$$\begin{bmatrix} c_{\alpha_1} & s_{\alpha_1}s_\beta & s_{\alpha_1}c_\beta \\ 0 & c_\beta & -s_\beta \\ -s_{\alpha_1} & c_{\alpha_1}s_\beta & c_{\alpha_1}c_\beta \end{bmatrix}, \begin{bmatrix} c_{\alpha_2} & -s_{\alpha_2}s_\beta & s_{\alpha_2}c_\beta \\ 0 & c_\beta & s_\beta \\ -s_{\alpha_2} & -c_{\alpha_2}s_\beta & c_{\alpha_2}c_\beta \end{bmatrix} \tag{8}$$

Force vector: The force F generated by the rotorcraft is the resultant force of the thrusts generated by the two propellers.

$$\mathbf{P}_1^{A_1} = (0, 0, P_1)^T \;\;,\;\; \mathbf{P}_2^{A_2} = (0, 0, P_2)^T \tag{9}$$

P_i is the lift that the propeller i produces by pushing air in a direction perpendicular to its plane of rotation. This thrust can be modelled as follows: $P_i = C_l\omega_i^2$, where $C_l > 0$ is the lift coefficient depending on blade geometry and fluid density.
Therefore, the vector F expressed in $\mathcal{A}$ is given by

$$F = T_1\mathbf{P}_1^{A_1} + T_2\mathbf{P}_2^{A_2} = \begin{pmatrix} (P_1s_{\alpha_1} + P_2s_{\alpha_2})c_\beta \\ (P_2 - P_1)s_\beta \\ (P_1c_{\alpha_1} + P_2c_{\alpha_2})c_\beta \end{pmatrix} \tag{10}$$

Gyroscopic moments: As explained in Section II, tilting the propellers around the axes E_2 and A_{ix} creates gyroscopic moments which are perpendicular to these axes and to the spin axes (A_{iz}). Indeed, these moments are defined by the cross product of the kinetic moments $(I_r\omega_iA_{iz})$ of the propellers and the tilt velocity vector. They are first expressed in the rotor frames as [8]

$$M_{\alpha_1}^{A_1} = -I_r\omega_1\dot{\alpha_1}A_{1x} \;,\; M_{\alpha_2}^{A_2} = I_r\omega_2\dot{\alpha_2}A_{2x} \tag{11}$$

for the longitudinal tilting, and

$$M_{1\beta}^{A_1} = I_r\omega_1\dot{\beta}A_{1y} \;,\; M_{2\beta}^{A_2} = I_r\omega_2\dot{\beta}A_{2y} \tag{12}$$

for the lateral tilting.
In order to express these gyroscopic moments in the fixed-body frame $\mathcal{A}$, we should multiply the above equations by the rotational matrices T_1 and T_2. We therefore find

$$M_\alpha = \sum_{i=1}^{2} T_iM_{\alpha_i}^{A_i} = I_r\begin{pmatrix} -\omega_1\dot{\alpha_1}c_{\alpha_1} + \omega_2\dot{\alpha_2}c_{\alpha_2} \\ 0 \\ \omega_1\dot{\alpha_1}s_{\alpha_1} - \omega_2\dot{\alpha_2}s_{\alpha_2} \end{pmatrix} \tag{13}$$

$$M_\beta = \sum_{i=1}^{2} T_iM_{i\beta}^{A_i} = I_r\dot{\beta}\begin{pmatrix} (\omega_1s_{\alpha_1} - \omega_2s_{\alpha_2})s_\beta \\ (\omega_1 + \omega_2)c_\beta \\ (\omega_1c_{\alpha_1} - \omega_2c_{\alpha_2})s_\beta \end{pmatrix} \tag{14}$$

Fan torques: As the blades rotate, they are subject to drag forces which produce torques around the aerodynamic center O. These moments act in opposite direction relative to ω.

$$Q_1^{A_1} = (0, 0, -Q_1)^T \;\;,\;\; Q_2^{A_2} = (0, 0, Q_2)^T \tag{15}$$

The positive quantities Q_i can be written as a function of propeller speeds: $Q_i = C_t\omega_i^2$, $C_t > 0$.

Similarly, these torques can be written in $\mathcal{A}$ as

$$Q = \sum_{i=1}^{2} T_iQ_i^{A_i} = \begin{pmatrix} -(Q_1s_{\alpha_1} - Q_2s_{\alpha_2})c_\beta \\ (Q_1 + Q_2)s_\beta \\ -(Q_1c_{\alpha_1} - Q_2c_{\alpha_2})c_\beta \end{pmatrix} \tag{16}$$

Thrust vectoring moment: Denote O_1 (O_2) the application point of the thrust P_1 (P_2). From Figure 2, we can define $\mathbf{O_1G} = (0, -l, -h)^T$ and $\mathbf{O_2G} = (0, l, -h)^T$ as positional vectors expressed in $\mathcal{A}$.

Then, the moment exerted by the force F on the airframe is

$$M_F = (T_1\mathbf{P_1}) \times \mathbf{O_1G} + (T_2\mathbf{P_2}) \times \mathbf{O_2G} \tag{17}$$

After some computations/development, we obtain

$$M_F = \begin{pmatrix} l(P_1c_{\alpha_1} - P_2c_{\alpha_2})c_\beta + h(P_1 - P_2)s_\beta \\ h(P_1s_{\alpha_1} + P_2s_{\alpha_2})c_\beta \\ -l(P_1s_{\alpha_1} - P_2s_{\alpha_2})c_\beta \end{pmatrix} \tag{18}$$

Adverse reactionary moment: As described in Section II, this moment appears when forcing the rotors to tilt longitudinally. It depends especially on the propeller inertia I_t and on tilt accelerations. This moment acts as a pitching moment and can be expressed in $\mathcal{A}$ as follows [8]

$$M_{a.r} = -I_t(\ddot{\alpha_1} + \ddot{\alpha_2}).E_2 \tag{19}$$

The complete expression of the torque vector $\tau = (\tau_1, \tau_2, \tau_3)^T$ acting with respect to the center of mass of the helicopter and expressed in $\mathcal{A}$ is

$$\tau = M_\alpha + M_\beta + Q + M_F + M_{a.r} \tag{20}$$

Finally, replacing the right hand side terms in (20) by their expressions, we obtain

$$\begin{aligned}\tau_1 = & \ l\,(P_1 c_{\alpha_1} - P_2 c_{\alpha_2}) c_\beta + h s_\beta (P_1 - P_2) - (Q_1 s_{\alpha_1} \\ & - Q_2 s_{\alpha_2}) c_\beta + I_r(-\omega_1 \dot{\alpha}_1 c_{\alpha_1} + \omega_2 \dot{\alpha}_2 c_{\alpha_2}) \\ & + I_r \dot{\beta} (\omega_1 s_{\alpha_1} - \omega_2 s_{\alpha_2}) s_\beta \qquad (21)\\ \tau_2 = & \ (Q_1 + Q_2) s_\beta + I_r \dot{\beta}(\omega_1 + \omega_2) c_\beta \\ & + h(P_1 s_{\alpha_1} + P_2 s_{\alpha_2}) c_\beta + M_{a.r} \qquad (22)\\ \tau_3 = & - l(P_1 s_{\alpha_1} - P_2 s_{\alpha_2}) c_\beta - (Q_1 c_{\alpha_1} - Q_2 c_{\alpha_2}) c_\beta \\ & + I_r(\omega_1 \dot{\alpha}_1 s_{\alpha_1} - \omega_2 \dot{\alpha}_2 s_{\alpha_2}) \\ & + I_r \dot{\beta}(\omega_1 c_{\alpha_1} - \omega_2 c_{\alpha_2}) s_\beta \qquad (23)\end{aligned}$$

IV. Model analysis for control design

An examination of the nonlinear model (7) and (21)-(23) reveals that these expressions are very complex and it seems difficult to exploit them directly for the purposes of control design. Indeed, they present high nonlinearities and strong coupling of control inputs. It is thus essential to consider some approximations.

In order to minimize the loss in vertical thrust, the lateral tilt angle β is imposed to be small ($< 15°$). Therefore, we can consider that $\cos\beta \approx 1$ and $\sin\beta \approx \beta$.

The lateral component $(P_1 - P_2)\beta$ of the force vector F in (10) is a parasitic force which occurs only when applying simultaneously lateral tilting and differential rotor speeds. The nominal inputs considered are then defined as

$$u_x = P_1 s_{\alpha_1} + P_2 s_{\alpha_2}\ , \quad u_z = P_1 c_{\alpha_1} + P_2 c_{\alpha_2} \qquad (24)$$

The torque vector τ given by (21)-(23) includes the principal control inputs $(\tau_\phi, \tau_\theta, \tau_\psi)^T$ and parasitic terms which are expected to be of a much smaller magnitude.

Analyzing the expression of τ, we can define the nominal controls associated to the rolling and yawing torques as

$$\tau_\phi = l(P_1 c_{\alpha_1} - P_2 c_{\alpha_2}),\ \ \tau_\psi = -l(P_1 s_{\alpha_1} - P_2 s_{\alpha_2}) \qquad (25)$$

The pitching moment can be expressed in the following form

$$\tau_\theta = \tau_{\theta_\beta} + h u_x + M_{a.r} \qquad (26)$$

From the previous considerations, control vectors can be represented as follows

$$F = (u_x, 0, u_z)^T + \Delta F,\ \tau = (\tau_\phi, \tau_\theta, \tau_\psi)^T + \Delta\tau \qquad (27)$$

where ΔF and $\Delta\tau$ regroup the parasitic terms which are not included in the nominal inputs. These small body forces/moments will allow arbitrary (though bounded) control action, so they are considered as perturbations and set to zero in the control design.

In comparison with other helicopter models (four-rotor [9], conventional helicopter [5]), the birotor model given by (7) and (27) presents some difficulties for control design, due to the coupling of the translational force F and the torque τ through the term u_x. Furthermore, the pitch movement is controlled by two independent inputs (hu_x and τ_{θ_β}). Therefore, the control strategy must take into account this "over-actuation" in order to avoid the cases where the two control inputs, described above, could compensate itself mutually. The synthesized controller have also to be robust against unmodeled dynamics and input uncertainties presented in (27).

Remark 1: It is important to note that the change of variables in (24)-(26) defines a global diffeomorphism. In other words, given $(u_x, u_z, \tau_\phi, \tau_\theta, \tau_\psi)$, it is possible to find the primitive inputs $(P_1, P_2, \beta, \alpha_1, \alpha_2)$ by applying the inverse transformation.

V. Backstepping-based tracking control design

In this section, a control law is derived for the purpose of stabilization and trajectory tracking. This algorithm exploits the procedure of backstepping that we found appropriate for the birotor model.

In order to develop our backstepping procedure, let us rewrite system (7) in the following form

$$\begin{aligned}&\dot{\xi} = v^{\mathcal{I}},\ \ \dot{v}^{\mathcal{I}} = \frac{1}{m} R F - g E_z \\ &\dot{R} = R\,\mathrm{sk}(\Omega),\ \ \dot{\Omega} = J^{-1}\tau - J^{-1}\Omega \times J\Omega\end{aligned} \qquad (28)$$

where $\mathrm{sk}(v)$ is the associated skew-symmetric matrix of a vector v such that $\mathrm{sk}(v)\Omega = v \times \Omega$.
Note that system (28) is not in pure strict-feedback structure, and control vectors F and τ have different relative degree with respect to the position ξ. Another difficulty is the coupling of F and τ via the term u_x (cf. Section IV). All these reasons motivate us to consider a dynamic extension of control vector F by a double integrator.

$$\ddot{F} = \tilde{F} \qquad (29)$$

Thus, both the vector F and its first time derivative $\dot{F}$ become internal variables of a dynamic controller.

Now, it is possible to define a transformation that results in algebraically less complicated equations

$$\begin{aligned}X &:= \frac{1}{m} R F - g E_z \\ Y &:= \dot{X} = \frac{R}{m}(\mathrm{sk}(\Omega) F + \dot{F}) \\ \tau &:= J\tilde{\tau} + \Omega \times J\Omega\end{aligned} \qquad (30)$$

Without loss of generality and recalling (30), we reformat the system (28) to be

$$\begin{aligned}&\dot{\xi} = v^{\mathcal{I}},\ \ \dot{v}^{\mathcal{I}} = X,\ \ \dot{X} = Y \\ &\dot{Y} = \frac{R}{m}(\tilde{F} - \mathrm{sk}(F)\tilde{\tau} + 2\mathrm{sk}(\Omega)\dot{F} + \mathrm{sk}(\Omega)\mathrm{sk}(\Omega)F) := \Gamma\end{aligned} \qquad (31)$$

Now, both the new inputs $\tilde{F}$ and $\tilde{\tau}$ have a relative degree of four with respect to the position ξ, so they can be assigned at the same (last) stage performing also a non-aggressive control for the translation compared to the control of the rotation.

Let $(\hat{\xi}, \hat{\psi}) = (\hat{x}(t), \hat{y}(t), \hat{z}(t), \hat{\psi}(t)) \in \mathbb{R}^4$ be the desired trajectories. Then, the control strategy aims at finding a feedback control $(\tilde{F}, \tilde{\tau})$ such that the tracking error
$\varepsilon := (\xi(t) - \hat{\xi}(t), \psi(t) - \hat{\psi}(t)) \in \mathbb{R}^4$ is asymptotically stable.

The control strategy can be divided into two parts. In the first step, a standard backstepping methodology [7] is applied for system[2] (31). The only difference is the introduction of the yaw error $\varepsilon_3 := \psi - \hat{\psi}$ in the third stage of the backstepping procedure [6], and the definition of a new variable $\nu := \ddot{\psi}$.

The backstepping methodology guarantees exponential convergence of the original tracking error ε to zero, and results in the following control ([6], [7])

$$\begin{aligned}\Gamma &= -5(\xi - \hat{\xi}) - 10(\dot{\xi} - \dot{\hat{\xi}}) - 9(X - \ddot{\hat{\xi}}) \\ &\quad - 4(Y - \hat{\xi}^{(3)}) + \hat{\xi}^{(4)}\end{aligned} \tag{32}$$

$$\nu = -2(\psi - \hat{\psi}) - 2(\dot{\psi} - \dot{\hat{\psi}}) + \ddot{\hat{\psi}} \tag{33}$$

It is important to note that the auxiliary inputs Γ and ν are functions of known signals. The second step is devoted to develop a strategy consisting in expressing $\tilde{F}$ and $\tilde{\tau}$ according to the available variables (Γ, ν).

From (31), we can write

$$\frac{R}{m}(\tilde{F} - \mathrm{sk}(F)\tilde{\tau}) = \Gamma - \frac{R}{m}(2\mathrm{sk}(\Omega)\dot{F} + \mathrm{sk}(\Omega)\mathrm{sk}(\Omega)F) \tag{34}$$

Multiplying this equation by mR^T, we obtain

$$\tilde{F} - \mathrm{sk}(F)\tilde{\tau} = mR^T\Gamma - 2\mathrm{sk}(\Omega)\dot{F} - \mathrm{sk}(\Omega)\mathrm{sk}(\Omega)F := \tilde{\Gamma} \tag{35}$$

The vector $\tilde{\Gamma} \in \mathbb{R}^3$ is an auxiliary variable which is function of known variables.

Concerning the yaw angle control, it is important to verify the possibility of providing the desired control ν. Let us recall the kinematic relationship between the generalized velocities $\dot{\eta} = (\dot{\psi}, \dot{\theta}, \dot{\phi})$ and the angular velocity Ω

$$\dot{\eta} = W^{-1}\Omega \tag{36}$$

where W^{-1} is given by

$$W^{-1}(\eta) = \frac{1}{\cos\theta}\begin{bmatrix} 0 & \sin\phi & \cos\phi \\ 0 & \cos\theta\cos\phi & -\cos\theta\sin\phi \\ \cos\theta & \sin\theta\sin\phi & \sin\theta\cos\phi \end{bmatrix} \tag{37}$$

Differentiating (36), we obtain

$$\ddot{\eta} = \dot{W}^{-1}\Omega + W^{-1}\dot{\Omega} = \dot{W}^{-1}\Omega + W^{-1}\tilde{\tau} \tag{38}$$

From equation (38), $\ddot{\psi}$ expression can be deduced

$$\ddot{\psi} = E_1^T(\dot{W}^{-1}\Omega + W^{-1}\tilde{\tau}) \;\Rightarrow\; E_1^T W^{-1}\tilde{\tau} = \ddot{\psi} - E_1^T\dot{W}^{-1}\Omega$$

with $E_1^T = (1, 0, 0)$ and $\nu = \ddot{\psi}$

By replacing the matrix W^{-1} by its expression in (37) and computing the equation above, one obtains

$$\frac{\cos\phi}{\cos\theta}\tilde{\tau}_\psi + \frac{\sin\phi}{\cos\theta}\tilde{\tau}_\theta = \nu - E_1^T\dot{W}^{-1}\Omega := \tilde{\nu} \tag{39}$$

The new variable $\tilde{\nu} \in \mathbb{R}$ is a function of known parameters. Thus, the control component $\tilde{\tau}_\psi$ can be used for yaw control as long as the following condition is valid

$$(\theta, \phi) \in]\frac{-\pi}{2}, \frac{\pi}{2}[$$

[2] The system (31) is in the form of a chain of four *vector integrators*

Rewriting (35) and (39), we obtain

$$\begin{aligned} &\ddot{u}_x + u_z\tilde{\tau}_\theta = \tilde{\Gamma}_1 \\ &u_x\tilde{\tau}_\psi - u_z\tilde{\tau}_\phi = \tilde{\Gamma}_2 \\ &\ddot{u}_z - u_x\tilde{\tau}_\theta = \tilde{\Gamma}_3 \\ &\frac{\cos\phi}{\cos\theta}\tilde{\tau}_\psi + \frac{\sin\phi}{\cos\theta}\tilde{\tau}_\theta = \tilde{\nu} \end{aligned} \tag{40}$$

Now, it remains to solve system (40) in order to find inputs $(\tilde{F}, \tilde{\tau})$, i.e., $(\ddot{u}_x, \ddot{u}_z, \tilde{\tau}_\phi, \tilde{\tau}_\theta, \tilde{\tau}_\psi)$.

As explained in Section IV, pitch angle is over-actuated, so it is necessary to consider firstly the equation governing this movement

$$\ddot{u}_x + u_z\tilde{\tau}_\theta = \tilde{\Gamma}_1 \tag{41}$$

This equation contains two unknown variables $\ddot{u}_x$ and $\tilde{\tau}_\theta$, so, mathematically it admits an infinite number of solutions. However, some solutions may induce inputs to diverge or oscillate.

Let us choose

$$\tilde{\tau}_\theta = \frac{1}{u_z}(k_1\dot{u}_x + k_2 u_x) \tag{42}$$

where k_1 and k_2 are positive constants and u_z is assumed to be different from zero.
This choice is motivated by the fact that equation (42) represents a stable subsystem with respect to the variable u_x. Moreover, by resolving the differential equation (42) we can show that the two inputs u_x and $\tilde{\tau}_\theta$ are complementary, thereby preventing them from diverging once the control objective is achieved.

From equations (40) and (42), we obtain

$$\ddot{u}_x = \tilde{\Gamma}_1 - (k_1\dot{u}_x + k_2 u_x) \tag{43}$$

Remark 2: Recalling Section II and equation (26), pitch load can be divided according to our wish into τ_{θ_β} and hu_x by a suitable choice of k_1 and k_2. These gains should also be chosen such that subsystem (43) is stable when $\tilde{\Gamma}_1$ converges to zero.

From the third equation of system (40), we deduce

$$\ddot{u}_z = \tilde{\Gamma}_3 + u_x\tilde{\tau}_\theta = \tilde{\Gamma}_3 + \frac{u_x}{u_z}(k_1\dot{u}_x + k_2 u_x) \tag{44}$$

The expression of $\tilde{\tau}_\psi$ is given from (40) by

$$\tilde{\tau}_\psi = \frac{\cos\theta}{\cos\phi}(\tilde{\nu} - \frac{\sin\phi}{\cos\theta}\frac{1}{u_z}(k_1\dot{u}_x + k_2 u_x)) \tag{45}$$

Finally, $\tilde{\tau}_\phi$ is deduced from the second equation of (40)

$$\tilde{\tau}_\phi = \frac{1}{u_z}(u_x\tilde{\tau}_\psi - \tilde{\Gamma}_2), \text{ with } \tilde{\tau}_\psi \text{ is given by (45)} \tag{46}$$

One recalls that all the terms at the left side of equations (42)-(46) are well defined provided that

$$u_z \neq 0,\ \theta \neq \frac{\pi}{2} + k\pi \text{ and } \phi \neq \frac{\pi}{2} + k\pi \text{ with } k \in \mathbb{Z} \tag{47}$$

Using (29) and (30), the original inputs F and τ can be recovered, which of course are themselves functions of all the primitive variables $(P_1, P_2, \beta, \alpha_1, \alpha_2)$ (see (24)-(26)).

VI. SIMULATION RESULTS

In this section, simulation results for two experiments are presented in order to observe the performances of the proposed control law. We considered the case of stabilization and a trajectory tracking problem. The parameters used for the aircraft model are given in the table below. These values are based on the Draganflyer X4 data that we posses in our laboratory.

parameter	value	parameter	value
m	$1\ kg$	I_r	$10^{-3} kg.m^2$
g	$9.81\ m.s^{-1}$	I_t	$10^{-3} kg.m^2$
J	$I_{3.3}$	k_1	0.4
l	$0.2\ m$	k_2	1.8
h	$0.07\ m$		

In the first simulation, the stabilization of the rotorcraft's position is considered. We took the following initial conditions $(x(0), y(0), z(0), \psi(0), \theta(0), \phi(0)) = (1, 1, 0, 0, 0.3, 0.1)$ and $\dot{\xi}(0) = \dot{\eta}(0) = (0, 0, 0)^T$. We also adopted the following choice of initial conditions for the force inputs $u_x(0) = 0$ and $u_z(0) = mg = 9.81$. Thus, the initial force input should compensate the vehicle weight and sustain the rotorcraft.

The desired values of altitude and yaw are fixed at $(\hat{z}, \hat{\psi}) = (3, 0.5)$. The results in Figure 3, obtained by considering the complete model of the birotor, illustrate the performance of the control law developed in this paper. Moreover, the last graph in figure 3 shows that the moments τ_{θ_β} and τ_{θ_α} are complementary for pitch control, and the adverse reaction moment $M_{a.r}$ is compensated by τ_{θ_β} when the later is not desired like at the beginning of tilting. In

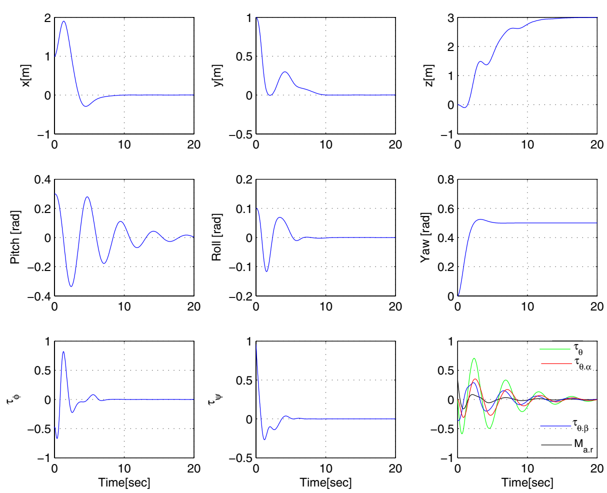
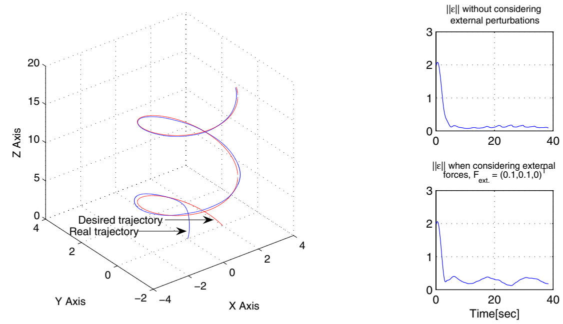


Fig. 3. Stabilization of the birotor dynamics in presence of small body forces/moments

a second part, we ran simulations for a trajectory tracking problem. The desired trajectory was chosen as a helix, i.e., $\hat{\xi} = (2\cos\hat{\psi}, 2\sin\hat{\psi}, 0.5t)^T$ and $\hat{\psi} = 0.6\pi\, t$. The rotorcraft was initially at the position zero, i.e., $\xi(0) = \eta(0) = 0$. The results presented in Figure 4(a) show that the position converges to the desired trajectory after a short transient time with a small error. Robustness against unmodeled inputs and external forces (see Figure 4(b)) is also observed even if a mathematical analysis of the controller robustness is not provided in this paper. However, one can expect some robustness properties of the control law in view of the study developed in [6].



(a) Position tracking

(b) Amplitude of the tracking error ϵ

Fig. 4. Trajectory tracking when considering the complete model

VII. CONCLUSION

The analysis presented here has shown that hover control of a two-propeller VTOL aircraft is possible using two tilting rotors. The pitch stability is increased by combining the opposed lateral tilting with the longitudinal tilting of the two rotors. The resulting oblique tilting appears to be effective and practical. Indeed, the experimental results obtained on prototypes constructed by our team (see Figure 1(b)) are very promising.

In this paper, we have also developed a 6-DOF model of the birotor aircraft and synthesized a nonlinear controller which leads to a satisfactory control. To the authors knowledge, no other control law has been derived for a small tiltrotor-based rotorcraft model.

REFERENCES

[1] G. K. Yamauchi, A. J. Wadcock, and M. R. Derby, "Measured aerodynamic interaction of two tiltrotors," in *AHS 59th Annual Forum*, Phoenix, Arizona, May 2003.

[2] D. Wyatt, "Eagle eye pocket guide," *Bell Helicopter Textron Inc*, Printed in USA, June 2004.

[3] S. M. Barkai, O. Rand, R. J. Peyran, and R. M. Carlson, "Modelling and analysis of tilt-rotor aeromechanical phenomena," in *American Helicopter Society Aeromechanics Specialists Conference*, Bridgeport, October 1995.

[4] G. R. Gress, "Using dual propellers as gyroscopes for tilt-prop hover control," *American Institute of Aeronautics and Astronautics*, 2002.

[5] I. Fantoni and R. Lozano, *Non-linear control for underactuated mechanical systems*, ser. Communications and Control Engineering Series, Springer-Verlag, Ed., 2002.

[6] R. Mahony and T. Hamel, "Robust trajectory tracking for a scale model autonomous helicopter," *Int. J. of Robust and Nonlinear Control*, vol. 14, pp. 1035–1059, 2004.

[7] R. Sepulcre, M. Jankovic, and P. Kokotovic, *Constructive Nonlinear Control*, ser. Communications and Control Engineering Series, Springer-Verlag, Ed., 1997.

[8] G. R. Gress, "A dual-fan vtol aircraft using opposed lateral tilting for pitch control," in *American Helicopter Society 59th Annual Forum*, Phoenix, Arizona, May 2003.

[9] P. Castillo, A. Dzul, and R. Lozano, "Real-time stabilization and tracking of a four rotor mini-rotorcraft," in *Proceedings of 2004 IEEE/RSJ International Conference on Intelligent Robots and Systems*, Sendai, Japan, October 2004.