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Probabilistic certification of pan-tilt-zoom camera surveillance systems

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Abstract— **In this work a method to evaluate the performance of autonomous patrolling systems is introduced based on stochastic reachability with random sets. We consider set-valued models with stochastic dynamics for multiple pan-tilt-zoom (PTZ) cameras acting as pursuers and a single evader. The problem of maximizing the probability that the evader successfully completes an intrusion objective while avoiding capture by the cameras is considered and posed as a stochastic reach-avoid problem. The solution of the stochastic reach-avoid problem is solved via dynamic programming where the optimal value function is used as a quality indicator of each patrolling strategy. A comparison between multiple patrolling strategies is provided via simulation of a realistic patrolling scenario.**

I. Introduction

The design of algorithms for dynamic patrolling and exploration of an environment is an important subtask in real-time autonomous surveillance [1]. The primary goal of many patrolling tasks is the capture of unknown entities within the environment. While many patrolling algorithms exist in the literature, it is often necessary (and difficult) to choose a patrolling algorithm that best satisfies the objectives of the security task.

The exploration of surveillance objectives of PTZ cameras including patrolling has recently been exploited in various works, e.g. [2]–[8]. In [6], the problem of optimally patrolling a one-dimensional perimeter with a network of cameras was considered resulting in a distributed control strategy based on local asynchronous communication. Optimal camera movement for the objective of minimizing the time necessary to monitor an environment was addressed in [5].In [3] a stochastic MPC approach to optimal patrolling was considered and a target tracking algorithm based on Min-Max and minimum time MPC was proposed. While the main motivation of the prior works was the design of optimal surveillance algorithms, the main motivation of this work is the development of a framework for the systematic analysis of the performance and high-level decisions of surveillance systems employing such algorithms.

In the present work we consider the surveillance task of dynamically patrolling a known environment. We consider the task in the form of a probabilistic pursuit-evasion game [9]–[11] where the camera and evader objects are set-valued and governed by stochastic processes. In contrast to prior works where the focus is the optimality of the autonomous patrolling algorithm given a set of assumptions regarding the dynamical properties of the evader, in this work we consider the problem of maximizing the probability that the evader successfully completes the objective that the security system is designed to prevent. In particular, we assume that the patrolling algorithm is defined for the surveillance system and that the evader has knowledge of this process. Recent results in the theory of stochastic reachability (for discrete-time stochastic hybrid systems (DTSHS)) and random sets [12], [13] are then used to calculate optimal policies for the evader given the objective and policy of the security system. Hence, the worst case performance of the security system (patrolling algorithm) is quantified and can be considered an indicator of the quality of the security system.

To demonstrate the certification framework we consider a surveillance scenario involving a single evader and two cameras. In this scenario the evader objective is to exit the surveilled area before being caught by the cameras. This scenario can be considered equivalent to the task of preventing an intrusion at an office building, or even preventing the escape of an inmate from a prison. For the sake of comparison, three patrolling algorithms for the PTZ cameras are considered: a one-step greedy algorithm [9], a one-step Nash algorithm [11], and a simple random-walk. By solving the dynamic program associated with the stochastic reachability framework considered, the patrolling algorithms are ranked according to their ability to prevent a knowledgeable evader from successfully completing its task.

II. Stochastic Reachability and Random Sets

Here we recall the theory of stochastic reachability for DTSHS [12], [14] and stochastic reachability with random sets [13], [15]. In particular, the results of this section can be found in detail in the work [13].

A DTSHS $\mathcal{H}$ can be described as a Markov control process with state space X, (compact) control space $\mathcal{A}$, and controlled transition probability function Q. Given a Markov control policy $\mu \in \mathcal{M}_m$ (where $\mathcal{M}_m$ denotes the set of all admissible Markov control policies) and initial state $x_0 \in X$, the execution $\{x_k, k = 0, ..., N\}$ is a time inhomogeneous stochastic process defined on the canonical sample space $\Omega = X^{N+1}$, endowed with its product $\sigma-$algebra $\mathcal{B}(\Omega)$ where $\mathcal{B}(\cdot)$ denotes the Borel $\sigma-$algebra. The probability measure $P^{\mu}_{x_0}$ is uniquely defined by the transition kernel

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Q, the Markov policy $\mu \in \mathcal{M}_m$, and the initial condition $x_0 \in X$ (see [16]).

For $k = 0, 1, 2, \ldots, N$, let G_k be a Borel-measurable stochastic kernel on $\mathcal{Y}$ given $\mathcal{Y}$, $G_k : \mathcal{B}(\mathcal{Y}) \times \mathcal{Y} \to [0,1]$, which assigns to each $\xi \in \mathcal{Y}$ a probability measure $G_k(\cdot|\xi)$ on the Borel space $(\mathcal{Y}, \mathcal{B}(\mathcal{Y}))$. That is, let G_k represent a collection of probability measures on $(\mathcal{Y}, \mathcal{B}(\mathcal{Y}))$ parameterized by the elements of $\mathcal{Y}$ and indexed by time k. A discrete-time time-inhomogeneous Markov process $\xi = (\xi_k)_{k \in \mathbb{N}_0}$ taking values in the Borel space $\mathcal{Y}$ is described by the stochastic kernel G_k.

Definition 1: A parameterization of a discrete-time set-valued stochastic process is a discrete-time Markov process $\xi = (\xi_k)_{k \in \mathbb{N}_0}$ with parameter space $\mathcal{Y}$ and transition probability function $G_k : \mathcal{B}(\mathcal{Y}) \times \mathcal{Y} \to [0,1]$ together with a function $\gamma : \mathcal{Y} \to \mathcal{B}(X)$ representing a stochastic (Borel) set-valued evolution on the hybrid state space X (according to the process ξ). Consequently, it holds that there exists a Borel set $\bar{K} \in \mathcal{B}(X \times \mathcal{Y})$ defined

$$\bar{K} = \{(x, \xi) \in X \times \mathcal{Y} | x \in \gamma(\xi)\}.$$

In the spirit of the theory of random closed sets [17], [18], for all $x \in X$, $\xi_{k-1} \in \mathcal{Y}$, and $k \in \mathbb{N}$, we define the following covering function:

$$\begin{aligned} p_{\gamma(\xi_k)}(x) &= P_{\xi_{k-1}}\{x \in \gamma(\xi_k)\} = E_{\xi_{k-1}}[\mathbf{1}_{\gamma(\xi_k)}(x)] \\ &= \int_{\mathcal{Y}} \mathbf{1}_{\bar{K}}(x, \xi_k) G_k(d\xi_k|\xi_{k-1}). \end{aligned}$$

For all $x \in X$ and all $\xi_{k-1} \in \mathcal{Y}$, it follows that the covering function $p_{\gamma(\xi_k)}(x)$ is Borel measurable and bounded between 0 and 1. Now consider the set valued maps $\gamma_1 : \mathcal{Y} \to \mathcal{B}(X)$ and $\gamma_2 : \mathcal{Y} \to \mathcal{B}(X)$ where, for all $\xi \in \mathcal{Y}$, $\gamma_1(\xi) \subseteq \gamma_2(\xi)$. It follows that $p_{\gamma_2(\xi_k) \setminus \gamma_1(\xi_k)}(x) = p_{\gamma_2(\xi_k)}(x) - p_{\gamma_1(\xi_k)}(x)$.

A. Finite Horizon Reach-Avoid

Let $K_k, K_k' \in \mathcal{B}(X)$, with $K_k \subseteq K_k'$ for all $k = 0, 1, \ldots, N$. Our goal is to evaluate the probability that the execution of the Markov control process associated with the Markov policy $\mu \in \mathcal{M}_m$ and the initial condition x_0 will hit K_k before hitting $X \setminus K_k'$ during the time horizon N. Let $\xi = (\xi_k)_{k \in \mathbb{N}_0}$ with stochastic kernel $G_k : \mathcal{B}(\mathcal{Y}) \times \mathcal{Y} \to [0,1]$ together with the functions $\gamma_1 : \mathcal{Y} \to \mathcal{B}(X)$ and $\gamma_2 : \mathcal{Y} \to \mathcal{B}(X)$ be a parameterization of a discrete-time set-valued stochastic process. We assume that the initial set parameter state ξ_0 is known, hence $\gamma_1(\xi_0) = K_0$ and $\gamma_2(\xi_0) = K_0'$ is known, and $\gamma_1(\xi_k) = K_k$ and $\gamma_2(\xi_k) = K_k'$ for $k = 1, \ldots, N$ is an execution of the stochastic set-valued process. The probability that the system initialized at $x_0 \in X$, with control policy $\mu \in \mathcal{M}_m$ and $\xi_0 \in \mathcal{Y}$, reaches K_k while avoiding $X \setminus K_k'$ for all $k = 0, 1, \ldots, N$ is given by

$$\begin{aligned} r^{\mu}_{(x_0,\xi_0)} \;:=\; & P^{\mu}_{(x_0,\xi_0)}\{\exists j \in [0, N] : x_j \in K_j \wedge \\ & \forall i \in [0, j-1] \;\; x_i \in K_i' \setminus K_i\}, \end{aligned}$$

where $\wedge$ denotes the logical AND, and we operate under the assumption that the requirement on i is automatically satisfied when $x_0 \in K_0$; subsequently we will use a similar convention for products, i.e. $\prod_{i=k}^{j}(\cdot) = 1$ if $k > j$. Note that while we assume knowledge of the initial state and initial set parameter set, the consideration of a probabilistic initial condition for each is straightforward.

As in [12], [14], $r^{\mu}_{(x_0,\xi_0)}$ can be expressed as the expectation

$$r^{\mu}_{(x_0,\xi_0)} = E^{\mu}_{(x_0,\xi_0)}\left[\sum_{j=0}^{N}\left(\prod_{i=0}^{j-1}\mathbf{1}_{K_i' \setminus K_i}(x_i)\right)\mathbf{1}_{K_j}(x_j)\right].$$

In this work, as in [13], [15], we assume that the product measure of the parametric process is equal to (or well approximated by) the product measure of time-indexed independent stochastic kernels, i.e. for $N \in \mathbb{N}$

$$\prod_{j=0}^{N} G_j(d\xi_j|\xi_{j-1}) \approx \prod_{j=0}^{N} G_j(d\xi_j).$$

Note that since the initial parameter state ξ_0 of the random set is assumed known, we define $G_0(d\xi_0|\xi_{0-1}) = G_0(d\xi_0) = \delta_{\xi_0}(d\xi_0)$.

For a DTSHS with independent set-valued reach and safe sets ($\gamma_1(\xi_k) \subseteq \gamma_2(\xi_k)$ almost surely), it can be shown that

$$r^{\mu}_{(x_0,\xi_0)} = E^{\mu}_{x_0}\left[\sum_{j=0}^{N}\left(\prod_{i=0}^{j-1} p_{K_i' \setminus K_i}(x_i)\right) p_{K_j}(x_j)\right].$$

The covering functions are defined

$$\begin{aligned} p_{K_i}(x) &= E\left[\mathbf{1}_{\gamma_1(\xi_i)}(x)\right] = \int_{\mathcal{Y}} \mathbf{1}_{\gamma_1(\xi_i)}(x) G_i(d\xi_i), \\ p_{K_i'}(x) &= E\left[\mathbf{1}_{\gamma_2(\xi_i)}(x)\right] = \int_{\mathcal{Y}} \mathbf{1}_{\gamma_2(\xi_i)}(x) G_i(d\xi_i), \\ p_{K_i' \setminus K_i}(x) &= p_{K_i'}(x) - p_{K_i}(x). \end{aligned}$$

Let $\mathcal{F}$ denote the set of functions from X to $\mathbb{R}$ and define the operator $H : X \times \mathcal{A} \times \mathcal{F} \to \mathbb{R}$ as

$$H(x, a, Z) := \int_X Z(y) Q(dy|x, a). \tag{2}$$

The following lemma shows that $r^{\mu}_{(x_0,\xi_0)}$ can be computed via a backwards recursion.

Lemma 3: Fix a Markov policy $\mu = (\mu_0, \mu_1, \ldots \mu_{N-1}) \in \mathcal{M}_m$. The functions $V_k^{\mu} : X \to [0,1]$, $k = 0, 1, \ldots, N-1$ can be computed by the backward recursion:

$$V_k^{\mu}(x) = p_{K_k}(x) + p_{K_k' \setminus K_k}(x) H(x, \mu_k(x), V_{k+1}^{\mu}), \tag{4}$$

initialized with $V_N^{\mu}(x) = p_{K_N}(x)$, $x \in X$.

Definition 5: Let $\mathcal{H}$ be a Markov control process, $\xi = (\xi_k)_{k \in \mathbb{N}_0}$ a parametric stochastic process, $K_k \in \mathcal{B}(X)$, $K_k' \in \mathcal{B}(X)$, with $K_k = \gamma_1(\xi_k)$,$K_k' = \gamma_2(\xi_k)$ and $K_k \subseteq K_k'$ almost surely for all $k = 0, 1, 2, \ldots, N$. A Markov policy μ^* is a maximal reach-avoid policy if and only if $r^{\mu^*}_{(x_0,\xi_0)} = \sup_{\mu \in \mathcal{M}_m} r^{\mu}_{(x_0,\xi_0)}$, for all $x_0 \in X$.

Theorem 6: Define $V_k^* : X \to [0,1]$, $k = 0, 1, \ldots, N$, by the backward recursion:

$$V_k^*(x) = \sup_{a \in \mathcal{A}}\{p_{K_k}(x) + p_{K_k' \setminus K_k}(x) H(x, a, V_{k+1}^*)\} \tag{7}$$

$x \in X$, initialized with $V_N^*(x) = p_{K_N}(x)$, $x \in X$. Then, $V_0^*(x_0) = \sup_{\mu \in \mathcal{M}_m} r^{\mu}_{(x_0,\xi_0)}$, $x_0 \in X$ and $\xi_0 \in \mathcal{Y}$. If $\mu_k^* : X \to \mathcal{A}$, $k \in [0, N-1]$, is such that for all $x \in X$

$$\mu_k^*(x) = \arg\sup_{a \in \mathcal{A}} \{p_{K_k}(x) + p_{K_k' \setminus K_k}(x) H(x, a, V_{k+1}^*)\} \quad (8)$$

then $\mu^* = (\mu_0^*, \mu_1^*, ..., \mu_{N-1}^*)$ is a maximal reach-avoid policy.

III. Probabilistic Certification of Autonomous Patrolling Systems

In this framework the set-valued evader is allowed to move in a spatial frame $X_G \subset \mathbb{R}^2$ (e.g. a prison yard or a building courtyard). We denote by $T \subset X_G$ the target area for the evader, i.e. the area in the spatial frame that the evader would like to reach (e.g. entrance or exit area). The evader is set-valued, denoted by $\mathcal{O} \in \mathcal{B}(X_G)$, where $\mathcal{O}$ is parameterized by a set of parameters $x_e \in X_e$, where X_e is the state space of the evader parameterization. It follows that the evader set and parametrization are constrained to remain in X_G and X_e respectively.

We consider m parametric models for the cameras with state x and state space X, $x \in X$. Naturally, there exists a mapping from the state of the camera parameterization x to the set-valued camera view (defined as the field of view (FOV)) $\mathcal{L}^{(j)}(x) \in \mathcal{B}(X_G)$ in the general spatial frame X_G. In the general case, this function can be defined as a measurable mapping $\mathcal{L}^{(j)} : X \to \mathcal{B}(X_G)$, $j \in \{1, ..., m\}$. We assume that the set-valued pursuers are independent and can intersect. The parametrization of each camera x (*ptz* configuration) is calculated via the patrolling strategy for each $k \in \{0, ..., N\}$.

In the global spatial frame X_G, the set $S = \{x_G \in X_G : x_G \in \cup_j \mathcal{L}^{(j)}\}$ comprises all states that intersect with the set-valued region of one or more pursuers (equivalently the union of the FOVs). S can be seen as the part of the space that the cameras cover at some $k \in \{0, ..., N\}$ for a given x. Likewise, $S_1 = \{x_e \in X_e : \mathcal{O} \cap S = \emptyset\}$ denotes the set of states of the evader that are safe from camera detection. Finally, the set $S_T = \{x_e \in X_e : \mathcal{O} \cap T \neq \emptyset\}$ is the set of evader states $x_e \in X_e$ leading to an intersection between the evader and the target area (i.e. the set of states denoting the success of the evader).

In the surveillance scenario considered above we assume that the evader has prior knowledge of the patrolling strategy. In many cases however, the strategy of the security system contains stochastic components. Thus, even though the evader knows the strategy of the pursuer, in the future the evader (in these cases) may know only that x of each camera $j \in \{1, ..., m\}$ at each time $k \in \{0, ..., N\}$ is distributed according to the probability distribution provided by the patrolling strategy. Thus, at each step in the future, the question is not whether the evader with state x_e will be in the view of the camera, but what the probability is that the evader in state x_e will be in the view of the camera.

It follows that S and S_1 are random sets according to the distribution of the camera and evader parameters. Further, the covering function $p_S(x_G)$ defines the probability that $x_G \in X_G$ is in the view of at least one of the cameras. Similarly, $p_{S_1}(x_e)$ represents the probability that the evader will be free of camera detection. Considering the probabilistic sets detailed above, it is possible to formulate the patrolling surveillance task as a reach-avoid problem by using the stochastic reachability framework of Section II. In particular, consider that the evader objective is to maximize the probability of the evader state x_e reaching S_T at some point during the finite time horizon $k \in \{0, ..., N\}$ while avoiding S_1 for all k. It follows that the reach-avoid problem can be formulated where the set S_1 denotes the safe set and S_T denotes the target set. The optimal control policy is obtained by solving the DP of Theorem 6 and the optimal value function provides the probability (indicator) defining the (worst-case) performance of the patrolling algorithm.

IV. Patrolling Strategies

A. Probabilistic pursuit-evasion game

The probabilistic framework for pursuit-evasion games proposed in [9], [11] is here considered as possible patrolling strategy. In [9] the authors provide a policy to control a set of agents in the pursuit of evaders that do not try to actively avoid detection. Such "greedy" policy directs the pursuers to the locations that maximize the probability of finding the evaders at the next time step. Conversely, in [11], a receding horizon control policy in which the pursuers and the evaders try to respectively maximize and minimize the probability of capture at the next time instant is provided. It is assumed that the evaders have access to pursuers information, i.e., the resulting game is nonzero-sum. The Nash solution to the one-step nonzero-sum game is computed by solving an equivalent zero-sum matrix game.

In our game we consider that pursuers are PTZ cameras. Cameras pan and tilt take place in a two-dimensional grid. Zoom is such that $\mathcal{L}^{(j)}(x)$ has the same area for all x, $j \in 1, 2$. Camera positions $x_1^j, x_2^j \in X, x_1^j \neq x_2^j$, are adjacent if camera j can move from x_1^j to x_2^j (and vice versa) in a sampling time. $\mathcal{A}^{(j)}(x)$ denotes the set of cells adjacent to position x for camera j. Given an initial camera configuration x_0^j at time k, the subset of camera positions to which a pursuer can move at time $k+1$ is given by

$$\mathcal{U}^j(x_0^j) = \{x^j \in X : x^j \in \{x_0^j\} \cup \mathcal{A}^{(j)}(x_0^j)\}$$

We consider the presence of a single evader that can take place in a rectangular two dimensional grid of X_G, see Figure 1. We say that two distinct cells $x_{G_1}, x_{G_2} \in X_G$ are adjacent if they share one side or one corner. The motion of the evader is constrained in that it can remain where it is or move to one of the adjacent cells.

Each pursuer (evader) collects information about X_G at discrete time instants $k \in \mathbb{N}_0$. Each pursuer is capable of determining its current position and sensing all the cells that are contained in $\mathcal{L}(x)$. Each measurement $\mathbf{y}(k)$ is given by $\{\mathbf{v}(k), \mathbf{e}(k)\}$ taking value in a measurement space $\mathcal{Y}$, where $\mathbf{v}(k) = \cup_j \mathcal{L}^{(j)}(x^j(k))$ and $\mathbf{e}(k)$ the cell where the evader is detected. We assume an evader captured if its current cell

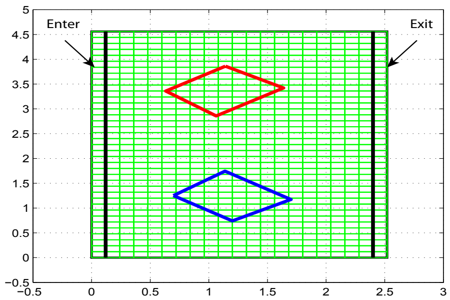


Fig. 1. Rectangular two dimensional grid of X_G

is entirely contained in $\cup_j \mathcal{L}^{(j)}$. We let $\mathbf{Y}_k$ the sequence of measurements $\{\mathbf{y}(1), \cdots, \mathbf{y}(k)\} \subseteq \mathcal{Y}^*$, where $\mathcal{Y}^*$ is the set of all finite sequences of $\mathcal{Y}$.

Let $p_e(x_G, k|\mathbf{Y}(k))$ be the posterior probability of the evader being in cell x_G at time k, given the measurement history $\mathbf{Y}_k$. $p_e(x_G, 0)$ is assumed uniform over X_G. At each k, given $p_e(x_G, k|\mathbf{Y}(k-1))$ and the new measurement $\mathbf{y}(k)$, pursuers recursively estimate $p_e(x_G, k|\mathbf{Y}(k))$ by first computing $p_e(x_G, k|\mathbf{Y}(k)) = \alpha(1-\beta_{in})p_e(x_G, k|\mathbf{Y}(k-1))$ where α is a normalizing constant (see [9] for details) while β_{in} represents the percentage of cell x_G inside $\cup_j \mathcal{L}^{(j)}$. Finally, in order to compute $p_e(x_G, k+1|\mathbf{Y}(k))$, a model for the motion of the evader is needed. Here [9] and [11] are different. In the following we try to give an overview of the evader model used in [9] and [11] and their respective control strategies while, at the same time, make the approaches suitable for PTZ cameras.

1) Evader not avoiding detection [9]: A Markov model for the motion of the evader is assumed. The model is completely determined by the probability $\rho \in [0, 1/8]$ that the evader moves to one of the adjacent cells $\mathcal{A}(x_G)$. The map $p_e(x_G, k+1|\mathbf{Y}(k))$ is given by

$$(1-|\mathcal{A}(x_G)|\rho)p_e(x_G, k|\mathbf{Y}(k))+\rho \sum_{\bar{x}_G \in \mathcal{A}(x_G)} p_e(x_G, k|\mathbf{Y}(k))$$

where $|\mathcal{A}(x_G)|$ indicated the number of adjacent of x_G. We assume that any cell of X_G can be reached by $\cup_j \mathcal{L}^{(j)}$ in finite time $k \leq \delta$. Moreover, given a positive constant $\gamma \leq 1$ and any sequence $\mathbf{Y}(k)$ for which the evader was not captured, we assume that the conditional probability of the evader being at a cell $x_G(k) \in X_G, x_G \notin \cup_j \mathcal{L}^{(j)}(x^j(k))$, satisfies

$$p_e(x_G, k+1|\mathbf{Y}(k)) \geq \gamma p_e(x_G, k|\mathbf{Y}(k-1))$$

Under these assumptions, given δ, we look for the closest (in terms of time steps) cell x_G that satisfies

$$p_e(x_G, k+1|\mathbf{Y}(k)) \geq \gamma^{\delta-m}/n_c$$

with m number of steps s.t. $x_G \in \cup_j \mathcal{L}^{(j)}$ and n_c number of cells of grid X_G. Given m and x_G, the cameras apply a navigation policy that achieves x_G in m steps while maximizing at the same time the probability of capturing the evader at $k+1$, i.e.

$$\max_{x^j \in \mathcal{U}(x^j(k)), \forall j} \sum_{x_G \cup_j \mathcal{L}^{(j)} x^j(k)} p_e(x_G, k+1|\mathbf{Y}(k))$$

2) Evader avoiding detection [11]: Here, at each time instant, the players solve a static game where the pursuers try to maximize the probability of catching the evader at the next time step, while the evader tries to minimize such probability. The game is nonzero-sum because of distinct observations available to cameras and evader ($\mathbf{y}(k)$ and $\mathbf{z}(k)$ respectively). In fact, we assume that, at every time instant k, pursuers sense $\cup_j \mathcal{L}^{(j)}$ while the evader senses its current position x_G plus $\cup_j \mathcal{L}^{(j)}$, i.e. the evader can see what the cameras are looking at. A solution to the game is given by the Nash equilibrium [19] where each player selects an action according to some probability distribution. In [11], it is shown that the determination of a Nash equilibrium for the nonzero-sum static game can be reduced to the determination of a Nash equilibrium for a fictitious zero-sum static game. The stochastic policies associated with the Nash equilibrium can be obtained by solving a linear programming (LP) problem (see [19] for details). In order to make the approach suitable for PTZ cameras we have to consider that pursuers sense a set of cells $\mathcal{L}(x)$ on the X_G grid rather than a single cell.

V. Evaluation scenario

Here we consider a realistic scenario based on common security surveillance objectives. Consider an area patrolled by two cameras where at an unknown time, an evader enters and attempts to exit as fast as possible from the right side of the area (see Figure 1) . As already mentioned, the evader is aware of the probability distribution of the parameters x at any given moment.

With this information, we solve the optimal control problem described in Section II and provide a probability of successfully exiting the room (at a finite number of steps) from any possible initial position of the evader, which includes of course the entrance points. The horizon length is chosen as the maximum length of time an evader has to enter and exit.

The scenario we consider has two PTZ cameras with the state in the camera space given by $x = [\theta, \psi, \zeta]^T$, where θ, ψ and ζ are respectively camera pan, tilt and zoom. In order to compute cameras FOV $\mathcal{L}^{(j)}(x)$, $\forall j \in \{1, ..., m\}$, we first introduce the relation between optical center reference frame (oc) and world reference system (w)

$$\begin{bmatrix} x_w \\ y_w \\ z_w \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ H \end{bmatrix} + R_\theta \left(\begin{bmatrix} D \\ 0 \\ 0 \end{bmatrix} + R_\psi \left(\begin{bmatrix} x_{off} \\ y_{off} \\ z_{off} \end{bmatrix} + \begin{bmatrix} x_{oc} \\ y_{oc} \\ z_{oc} \end{bmatrix} \right) \right)$$

where $p_w = [x_w, y_w, z_w]^T$ are the coordinates of a point p in X_G while $p_{oc} = [x_{oc}, y_{oc}, z_{oc}]^T$ are its coordinates in the optical center reference frame (see Figure 2). $H, D, x_{off}, y_{off}, z_{off}$ are camera-specific parameters and R_ψ,R_θ the relevant rotation matrices. Given a point in optical center reference frame, its position in the image view frame (im) is obtained as follows

$$x_{im} = \lambda(\zeta)\frac{y_{oc}}{x_{oc}} \qquad y_{im} = -\lambda(\zeta)\frac{z_{oc}}{x_{oc}}$$

where $\lambda(\zeta) = \lambda_1 \zeta$ is the focal length for a given level of zoom ζ, while λ_1 is the λ value for $\zeta = 1$. In order to

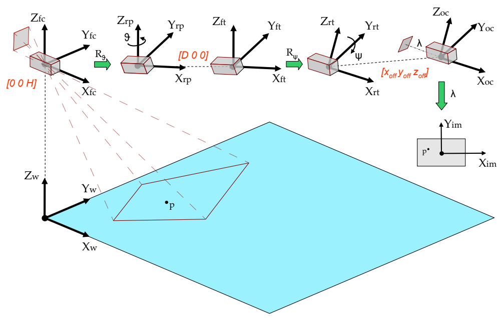


Fig. 2. Pinhole camera model

compute $\mathcal{L}^{(j)}(x)$ (see Figure 2), we first calculate the coordinates of the optical center in X_G i.e. $[\bar{x}_w, \bar{y}_w, \bar{z}_w]^T$.Given $\tilde{x}^i_{im}$, $\tilde{y}^i_{im}$, $i = 1, \cdots, 4$, positions of the vertexes of the FOV in the camera image, we can compute their projection on the ground plane, i.e. $[\tilde{x}^i_w, \tilde{y}^i_w, \tilde{z}^i_w]^T$. Note that the FOV is located at $-\lambda$ on X_{oc} in the optical center reference system.

In order to find the projection of the FOV vertices on the ground plane, i.e. $[\tilde{x}^{iP}_w, \tilde{y}^{iP}_w, \tilde{z}^{iP}_w]^T$, we intersect the line passing through $[\tilde{x}^i_w \; \tilde{y}^i_w, \tilde{z}^i_w]^T$ and $[\bar{x}_w, \bar{y}_w, \bar{z}_w]^T$ with the ground plane $Z_w = 0$.

The dynamics of the camera view over the time horizon $k \in \{0, ..., N\}$, $N \in \mathbb{N}$, are governed by the patrolling strategy. The set-valued evader $\mathcal{O} \in \mathcal{B}(X_G)$, $X_G \subset \mathcal{B}(\mathbb{R}^2)$, is parameterized by its center $[x_e, y_e]^T \in X_e$, according to the relation

$$\mathcal{O} = \{x_G \in X_G : (x_{G,1} - x_e)^2 + (x_{G,2} - y_e)^2 \leq r_e^2\}$$

where $\mathcal{O}$ is a two-dimensional (compact) circle. The movement of the evader in space is completely characterized by the evolution of its parametric center. This is translated via a probabilistic transition kernel Q_e to the possible positions of $[x_{e_{k+1}}, y_{e_{k+1}}]^T$ starting from $[x_{e_k}, y_{e_k}]^T$ with $k \in \{0, ..., N\}$.

VI. Computational Results

Here we consider $m \in \{1, 2\}$ and a finite time horizon of $N = 100$. The choice of the horizon is such as to allow a considerable amount of time for the evader to exit the patrolling area. We evaluate the strategies of Section IV by solving a probabilistic reach-avoid problem with random sets. The probability distribution of the *ptz* parameters along the horizon is estimated via Monte Carlo simulation. In each case, the stochastic reachability problem is solved via dynamic programming based on the computational methods of [20]. The result of the dynamic program is an optimal control policy for the DTSHS that can be applied in open loop (or receding horizon). Intuitively this is the optimal strategy for the evader in order to maximize his probability of successfully exiting the patrolled area.

The calibrated values of cameras set-up have been calculated in an earlier study [21] and are: $H^{(1)} = 2.5$, $H^{(2)} = 2.5$, $D^{(1)} = 0.124$, $D^{(2)} = 0.125$, $x^{(1)}_{off} = 0.1485$, $x^{(2)}_{off} = -0.01$, $y^{(1)}_{off} = -0.0275$, $y^{(2)}_{off} = 0.0437$, $z^{(1)}_{off} = 0.02$, $z^{(2)}_{off} = 0.0917$, $\lambda^{(1)}_1, \lambda^{(2)}_1$ are chosen to keep a constant area for the FOV. The two cameras are positioned at $x_w = 0$, $y_w = 4.55$ meters and $x_w = 0$, $y_w = 0$ meters respectively. The state space bounds for each camera are $x^{(1)}_{1,k} = \in [-\pi/2, 0]$, $x^{(2)}_{1,k} = \in [0, \pi/2]$ and $x^{(1)}_{2,k}, x^{(2)}_{2,k} = \in [\pi/5.5, \pi/2]$. The control inputs for both cameras are bounded according to $u_{1,k} \in [-0.0288, 0.0288]$ and $u_{2,k} \in [-0.0288, 0.0288]$ with units in radians.

The radius of the circular evader is $r_e = 0.06$ meters while the stochastic kernel is a state transition matrix of appropriate dimensions. The planar ground plane, is $X_G = \{[x_w, y_w] : x_w \in [0, 2.52], y_w \in [0, 4.56]\}$ (in meters). For computational purposes, X_G has been discretized with a step of 0.12 meters on both x_w and y_w while the camera state has been discretized by 0.0288 radians.

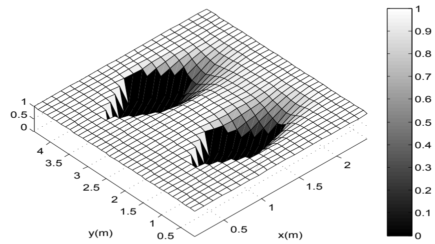
A. Reaching the target set while avoiding the pursuers

Consider the problem of maximizing the probability that the evader at some point during the finite time horizon $k \in \{0, ..., N\}$ reaches the target set while avoiding both cameras $m \in \{1, 2\}$ at each prior time point. Then, for each $k \in \{0, ..., N\}$ we define the sets:

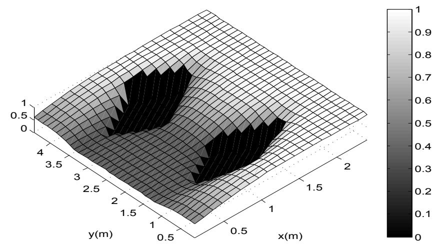
$$S_k = S^{(1)}_k \times S^{(2)}_k = \{x_{e_k} \in X_e : \mathcal{O}(x_{e_k}) \cap T \neq \emptyset\} \times \{x_{e_k} \in X_e : \mathcal{O}(x_{e_k}) \cap \mathcal{L}^{(j)}_k = \emptyset, \;\; \forall j \in \{1, 2\}\}.$$

The target set $K_k = S^{(1)}_k$ comprises all evader states for which there exists an intersection with the exit set T. Hence, $K'_k = S^{(2)}_k$ comprises all states where the evader is not visible to any camera $m \in \{1, 2\}$ at step k. Trivially, $S^{(1)}_k \subseteq S^{(2)}_k$ (Note that this holds always since camera FOVs can never intersect the exit set T). Thus, the patrolling strategy has failed in its objective (equivalently the evader has achieved the target) if there exists a time $k \in \{0, ..., N\}$ such that $x_{e_k} \in S^{(1)}_k$ and $x_{e_j} \in S^{(2)}_j$ for all $j \in \{0, ..., k-1\}$. This is posed as a probabilistic reach-avoid problem where the objective is to maximize the probability of reaching $S^{(1)}_k$ at some point over the time horizon while avoiding leaving $S^{(2)}_k$ at all prior time points. For all $k \in \{0, ..., N\}$, the optimal control policy $\mu^*_k(x_G)$ and the value function $V^*_k(x_e)$ for the patrolling area exit task are computed via dynamic programming, where $V^*_0(x_e)$ indicates the probability that the evader set, initialized at x_e at time $k = 0$, will reach the exit at some point during the time interval while avoiding the union of the camera FOVs at every prior step.

The initial positions of the cameras chosen for the example are such that the respective FOVs are near the center of the search space. For the strategies of Section IV that have randomness associated with the evolution of the camera state x, we have carried out several simulations to obtain the probability density of x over the chosen horizon. In the case of the probabilistic pursuit-evasion approach, this would make no sense as all moves are made with probability 1. Figure 3(a) highlights the fact that the pursuit-evasion approach (which is near deterministic in its evolution) is



(a) Probabilistic pursuit-evasion



(b) One-step Nash

Fig. 3. Value function provided by the solution to the dynamic program for each patrolling strategy

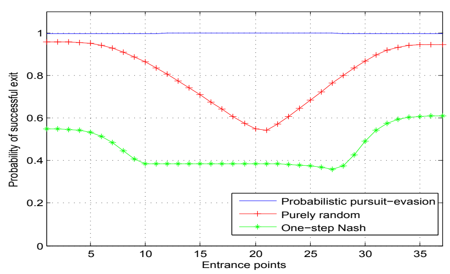


Fig. 4. Probability that the evader successfully exits starting from any one of the enter points (enumerated bottom to top - see Figure 1)

trivial for an evader to avoid if he is aware of the way it is constructed. Thus, for most initial positions $(x(m), y(m))$ of the evader space (including the entrance points) there is a probability near 1 (see Figure 4) that the evader will succeed within N steps. The equivalent value function for the stochastic one-step Nash approach is presented in Figure 3(b). It is more effective since the evader has very low probability of successfully avoiding the camera and reaching the exit area. Figure 4 clearly illustrates this fact by showing the probability of success for the evader initialized at the entrance points. For the sake of completeness, the results of applying a purely random walk as patrolling strategy are also shown in Figure 3(b).

VII. Conclusion

In this work a method of certifying camera-based autonomous patrolling strategies based on the theory stochastic reachability and random sets was presented. For three patrolling strategies, we solved via dynamic programming the problem of maximizing the probability of an evader successfully exiting an area patrolled by active cameras. The systematic method for quantitative analysis of stochastic strategies was shown to provide a valuable indication of patrolling quality under specified objectives. Future work includes utilization of the proposed certification process in order to design more efficient patrolling strategies.

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