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Formation Control with Velocity Assignment for Second-order Multi-agent Systems with Heterogeneous Time-delays

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***Abstract*— This paper deals with a formation control problem for second-order multi-agent systems with heterogeneous time-delays. We aim to control multiple agents so as to form a desired formation pattern and to move with an assigned velocity in an assigned direction. The assigned velocity and direction are transmitted through long-distance communication from human operators as external signals. Some of the agents, called leaders, can receive the external signals. As shown in this paper, in multi-agent systems with heterogeneous time-delays, there remain formation and velocity errors because of the asymmetry of the time-delays. We investigate how large errors arise in these systems, and propose the best selection of leaders so as to minimize the velocity error. Finally, some numerical results demonstrate the validity of the proposed analyses.**

I. INTRODUCTION

Recently, the autonomous navigation of multi-agents, e.g. autonomous robots and unmanned air vehicles (UAVs), has been an area of significant interest. Thus, formation control is the most essential for navigation in order to guide the agents in a desired formation pattern. Fax and Murray [1] have derived the necessary and sufficient condition to achieve an assigned formation for multi-agent systems. These agents can communicate with each other through a network such as wireless LAN. Based on the local information transmitted through the network communication, the group of agents can achieve an assigned formation.

In many papers concerning formation control, e. g. [1], [2], deploying the agents in a desired formation pattern is dealt with as the control objective. In practice, the formation control is available to redeploy the agents in a desired direction. The command of the redeployment is transmitted via long-distance communication from human operators as external signals including a desired direction and velocity. The expense to equip all agents with long-distance communication units to receive the external signals will be huge in large-scale multi-agent systems. Therefore, the number of the agents, called *leaders*, which can receive the external signals is limited. The rest of the agents do not receive the command signals, but can move according to network-connected agents. As a result, the group of agents are expected to move in the assigned direction at the assigned velocity.

On the other hand, time-delays of local information are caused by radio disturbance on networks. As this paper will reveal, these time-delays possibly interfere with the desired formation and/or desired velocity in the case that not all the agents can receive the external signals. We are naturally interested in how accurately the agents form the formation and move at the desired velocity in the presence of time-delays. However, there are few papers which investigate the accuracy of the formation in connection with time-delays.

This paper deals with a formation control problem with constant velocity assignment for second-order multi-agent systems with heterogeneous time-delays. The first contribution of this paper is to derive the formation and velocity errors in terms of the network structures of the time-delayed multi-agent systems. This result suggests that not only the network structures but also the assignment of the leaders affect the accuracy of the formation and velocity. The second contribution of this paper is to propose the best selection of leaders so as to minimize the velocity error. Using the proposed method, we can decide which agents should be leaders to improve the control performance.

For single-order multi-agent systems, Ren *et al.*[3] have investigated the situation where the external signals can be received by all agents. Ghabchelee *et al.*[4] have dealt with the same situation for multi-agent systems with time-delayed networks. As for second-order systems, Ren [5], [6] has considered a tracking problem for multi-agent systems without time-delays based on consensus, where time-varying reference velocities are considered. J. Hu, Y. Hong [7], H. Su and X. Wang [8] have dealt with multi-agent systems including uniform time-delays. These existing papers have mainly dealt with the case where all the agents can receive the external signals; namely all of them are leaders. On the other hand, this paper deals with the case where only some of the agents are leaders, and investigates which agents should be leaders to reduce undesirable errors.

The following notations are used in this paper: I_n and $O_n \in \mathbb{R}^{n\times n}$ are the identity and zero matrices, respectively. 1_n and $0_n \in \mathbb{R}^n$ are the n dimensional vectors all of whose entries are 1 and 0, respectively. $E_{ij} \in \mathbb{R}^{n\times n}$ is the matrix whose (i, j) entry is 1 and the other entries are 0. Let $\|\cdot\|$ be the Euclidean norm for a vector. Let $|\cdot|$ be the number of the elements for a countable set.

II. SECOND-ORDER MULTI-AGENT SYSTEM

Consider the multi-agent system consisting of n agents which are governed by the second-order dynamics

$$\ddot{x}_i(t) = u_i(t),\ i \in \mathcal{N} \tag{1}$$

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where $\mathcal{N} = \{1, 2, \cdots, n\}$ is the set of the agent numbers, and $x_i(t),\ u_i(t) \in \mathbb{R}$ are the position and input of agent i, respectively. Although this paper deals with the one-dimensional space for simplicity, it is easy to extend the following results to higher-dimensional spaces.

Each agent in this system communicates with the others over a network, where the agent can measure the relative positions and velocities from the network-connected agents. The network structure is described by the coupling gains

$$a_{ij} \begin{cases} > 0, & \text{if agents } i \text{ and } j \text{ can communicate} \\ = 0, & \text{if they cannot communicate.} \end{cases}$$

Note that this network is directed, thus $a_{ij} \neq a_{ji}$ in general. We assume that this network has a globally reachable agent [9]. The agents can compare their positions and velocities with the network-connected agents' ones. Then, the group of agents are expected to form a desired formation by the relative information via network communication.

Information through this network includes time-delays in practice. We consider two types of time-delays: *communication-delay* and *computation-delay*. Let $\tau_{ij}^c \geq 0$ be the communication-delay, which occurs when agent i receives information from agent j via network communication. Let $\tau_{ij}^s \geq 0$ be the computation-delay, which occurs when agent i compares his position and velocity with agent j's ones by using agent i's poor computational ability. In general, these time-delays are heterogeneous, that is, $\tau_{ij}^c \neq \tau_{kl}^c$, $\tau_{ij}^s \neq \tau_{kl}^s$ and $\tau_{ij}^s \neq \tau_{kl}^c$ for i, j, k and $l \in \mathcal{N}$.

Some matrices are defined in order to describe the network structure of the multi-agent system in question. Let Laplacian $L \in \mathbb{R}^{n\times n}$ be

$$[L]_{ij} = \begin{cases} \sum_{k=1}^{n} a_{ik}, & \text{if } j = i \\ -a_{ij}, & \text{otherwise} \end{cases} \tag{2}$$

where $[L]_{ij}$ is the (i,j) entry of L. Note that all of the row-sums of Laplacian L are 0, that is, $L1_n = 0_n$, which means that one of the eigenvalues of L is zero. Let $p \in \mathbb{R}^{1\times n}$ be the left eigenvector of L corresponding to the zero eigenvalue, whose sum of elements is 1, that is, $pL = 0_n^\top$ and $p1_n = 1$. Note that the row vector p is non-negative, namely, $p_i \geq 0$ for all $i \in \mathcal{N}$, where p_i is the i-th element of p [10]. The delay-Laplacian $L_\tau \in \mathbb{R}^{n\times n}$ describes the time-delayed network as follows:

$$[L_\tau]_{ij} = \begin{cases} \sum_{k=1}^{n} \tau_{ik}^s a_{ik}, & \text{if } j = i \\ -\tau_{ij}^c a_{ij}, & \text{otherwise} \end{cases}, \tag{3}$$

which is weighted with the time-delays τ_{ik}^s and τ_{ik}^c compared with Laplacian L in (2). This matrix has been introduced by the authors to express time-delays according to network structures [11].

For this system, command signals are transmitted from human operators. Only some of the agents are equipped with long-distance communication devices, and can receive these signals. Such agents are called *leaders*, which might be plural or might be singular. The leaders guide the other agents, *followers*. The followers do not receive the command signal, but can move according to network-connected agents. As a

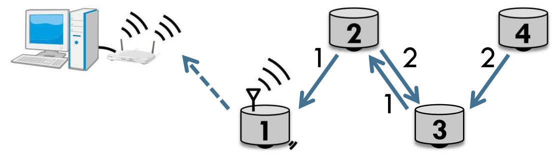

Fig. 1. Network structure of a multi-agent system

result, the group of agents, both leaders and followers, are expected to move at a desired velocity which the leaders receive via the long-distance communication. The set of leaders is given by $\mathcal{L} \subset \mathcal{N}$. Whether agent i can receive or not is expressed by the receiving gains such that

$$b_i \begin{cases} > 0, & i \in \mathcal{L} \text{ (agent } i \text{ receives command)} \\ = 0, & i \notin \mathcal{L} \text{ (agent } i \text{ cannot receive command).} \end{cases}$$

Let the receiving matrix be defined as

$$B = \mathrm{diag}(b_1, b_2, \cdots, b_n) \in \mathbb{R}^{n\times n}. \tag{4}$$

Consider an example of the multi-agent systems.

Example 1: Fig. 1 depicts a networked multi-agent system consisting of 4 agents, numbered from 1 to 4. The arrows between the agents represent communication paths between the agents. The numbers beside the arrows denote the computation-delays τ_{ij}^s and communication-delays τ_{ij}^c. The dashed arrow means that agent 1 can receive a command signal from a human operator; thus the set of leaders is given by $\mathcal{L} = \{1\}$. Let the coupling gains a_{ij} and receiving gains b_i be 1 if there is corresponding communication; otherwise 0. Then, the parameters a_{ij}, b_i, τ_{ij}^s and τ_{ij}^c are given by

$$a_{21} = 1,\ a_{23} = 1,\ a_{32} = 1,\ a_{43} = 1,\ b_1 = 1$$
$$\tau_{21}^s = \tau_{21}^c = \tau_{32}^s = \tau_{32}^c = 1,\ \tau_{23}^s = \tau_{23}^c = \tau_{43}^s = \tau_{43}^c = 2$$

and the rest are 0. Then, the corresponding Laplacian, delay-Laplacian and receiving matrix are given by

$$L = \begin{bmatrix} 0 & 0 & 0 & 0 \\ -1 & 2 & -1 & 0 \\ 0 & -1 & 1 & 0 \\ 0 & 0 & -1 & 1 \end{bmatrix},\ L_\tau = \begin{bmatrix} 0 & 0 & 0 & 0 \\ -1 & 3 & -2 & 0 \\ 0 & -1 & 1 & 0 \\ 0 & 0 & -2 & 2 \end{bmatrix}$$

and $B = \mathrm{diag}(1, 0, 0, 0)$ from (2), (3) and (4).

III. Problem Setting

The control objective for the multi-agent system (1) is that the group of agents achieve a desired formation and move at an assigned velocity. Fig. 2 illustrates an example of a formation pattern of 4 agents. The assigned formation is given by the constant value $x_i^* \in \mathbb{R}$, which is the desired relative distance of agent i from a certain base point $c(t) \in \mathbb{R}$. Note that the base point $c(t)$ cannot be predetermined because all agents do not obtain the information on their

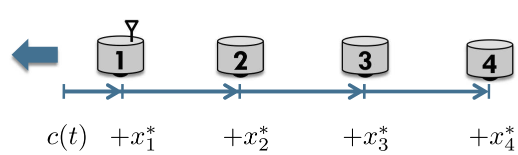

Fig. 2. Example of a formation pattern

absolute positions. Instead, we can assign the formation pattern through the relative positions x_i^*. Let $\nu^* \in \mathbb{R}$ be the desired velocity of the group of agents. Then, our control objective is represented as

$$\begin{cases} \lim_{t\to\infty}(\dot{x}_i(t) - \nu^*) = 0 \\ \lim_{t\to\infty}\{x_i(t) - (c(t) + x_i^*)\} = 0 \end{cases} \quad (5)$$

for a time-varying function $c(t)$. This is called *formation control with velocity assignment*. Note that the second equation of (5) is reduced to the condition of the relative positions as

$$\lim_{t\to\infty}(x_i(t) - x_j(t)) = l_{ij}^*,\ \forall(i,j) \in \mathcal{N} \times \mathcal{N} \quad (6)$$

for the desired relative distance $l_{ij}^* = x_i^* - x_j^*$ between agents i and j. (6) is equivalent to the consensus condition with the bias l_{ij}^*.

In order to achieve (5), we consider following controller, which is a time-delayed version of the conventional formation controller introduced by Ren [6].

$$u_i(t) = -\sum_{j=1}^{n} a_{ij}\{((x_i(t-\tau_{ij}^s) - x_j(t-\tau_{ij}^c)) - l_{ij}^*) \\ +k_v(\dot{x}_i(t-\tau_{ij}^s) - \dot{x}_j(t-\tau_{ij}^c))\} - b_i(\dot{x}_i(t-\tau_{ij}^s) - \nu^*) \quad (7)$$

Here, k_v is a damping gain common among the agents. The first/second term in the summation is the error feedback between the relative position/velocity from agent j, which is available when $a_{ij} > 0$. The final term represents the feedback of the error between the absolute velocity and the desired velocity. This feedback is available when $b_i > 0$, that is, agent i is a leader. These feedbacks includes the computation-delay τ_{ij}^s at the agent i's own state $q_i(t)$ and the communication-delay τ_{ij}^c at the agent j's state $q_j(t)$. Note that the controller (7) does not use the information on the absolute positions. Thus, all agents do not require GPS tracking equipment.

Our question is whether the formation control (5) with velocity assignment is realized, or not. If the formation control is not perfectly realized, we are interested in how accurately the agents achieve the desired formation and move at the assigned velocity. In order to estimate these accuracies, we introduce the formation error F_e and the velocity error V_e. The formation error F_e is defined as the sum of the squared steady errors of the agents' positions as follows:

$$F_e := \lim_{t\to\infty}\min_{c(t)\in\mathbb{R}}\sqrt{\sum_{i=1}^{n}\{x_i(t) - (c(t) + x_i^*)\}^2}, \quad (8)$$

where the base point $c(t)$ is regarded as a point with which the formation pattern is the most similar to the desired formation pattern. The velocity error F_v is defined as the sum of the squared steady errors of the agents' velocities:

$$V_e := \lim_{t\to\infty}\sqrt{\sum_{i=1}^{n}(\dot{x}_i(t) - \nu^*)^2}. \quad (9)$$

The first topic of this paper is to investigate how accurately the formation control (5) is achieved by calculating the formation and velocity errors F_e and V_e.

Problem 1: Consider the multi-agent system (1) with the network where the coupling and receiving gains are given by $a_{ij},\ b_i \geq 0$, respectively. Let $\tau_{ij}^s,\ \tau_{ij}^c \geq 0$ be the computation-delays and communication-delays. For this system and controller (7), derive the formation and velocity errors F_e, V_e defined by (8) and (9), respectively.

Naturally, we expect that the group of agents to move at the desired velocity as accurately as they can so as to reach a goal on time. In order to satisfy this expectation, we minimize the velocity error by appropriately assigning leaders. Now, the number $|\mathcal{L}|$ of leaders is limited to m due to cost controlling. Moreover, the gain b_i should be smaller than the upper limit $\bar{b}_i$ from the viewpoint of the S/N ratio. These assumptions are summarized as follows:

$$b_i \in [0, \bar{b}_i] \text{ for } i \in \mathcal{L}, \quad b_i = 0 \text{ for } i \notin \mathcal{L}. \quad (10)$$

The second topic of this paper is given as follows:

Problem 2: Consider the same setting to Problem 1 and the additional condition of the upper limit $\bar{b}_i > 0$ of the receiving gain b_i. Then, determine the set $\mathcal{L} \subset \mathcal{N}$ of leaders such that $|\mathcal{L}| = m$, and design the receiving gains b_i satisfying (10) so as to minimize the velocity error V_e defined in (9), that is

$$\min_{\substack{\mathcal{L}\subset\mathcal{N} \\ |\mathcal{L}|=m}} \min_{\substack{b_i\leq\bar{b}_i, i\in\mathcal{L} \\ b_i=0, i\notin\mathcal{L}}} V_e. \quad (11)$$

IV. Estimation of formation and velocity errors

In this section, we consider Problem 1. Our strategy to calculate the formation and velocity errors F_e and V_e is to reduce the formation control problem (5) into a consensus problem of a transformed system. Then, many results on the consensus problem are available.

The expressions in (5) suggest that the base point $c(t)$ finally moves as a time-varying point $(\nu^* t + \alpha)$ where ν^* is the assigned velocity and α is a bias of the position. Thus, we expect that the position $x_i(t)$ will converge to the formation pattern x_i^* plus this constantly moving point, namely, the time-varying point $(x_i^* + \nu t + \alpha)$. However, as shown later, this expectation is not correct due to the effect of the time-delays of the controller (7). Let x_i^e and ν^e be the errors of agent i's position and velocity from the expected point $(x_i^* + \nu t + \alpha)$, respectively. Then, the new variable including the errors x_i^e and ν^e is introduced as

$$\hat{x}_i(t) = x_i(t) - \{(\nu^* + \nu^e)t + \alpha + (x_i^* + x_i^e)\}, \quad (12)$$

and their collection with respect to all the agents is given by

$$\hat{x}(t) = x(t) - (x^* + x^e) - \{(\nu^* + \nu^e)t + \alpha\}1_n \quad (13)$$

where $\hat{x}(t) = [\hat{x}_1(t), \hat{x}_2(t), \cdots, \hat{x}_n(t)]^\top$, $x(t) = [x_1(t), x_2(t), \cdots, x_n(t)]^\top$, $x^* = [x_1^*, x_2^*, \cdots, x_n^*]^\top$ and $x^e = [x_1^e, x_2^e, \cdots, x_n^e]^\top \in \mathbb{R}^n$. If the new variable $\hat{x}(t)$ is appropriately chosen, the formation and velocity errors F_e and V_e can be represented by the errors x_i^e and ν^e as follows.

Lemma 1: Assume that the system of $\hat{x}(t)$ defined by (13) achieves consensus with zero velocities, that is

$$\lim_{t\to\infty}(\hat{x}_i(t) - \hat{x}_j(t)) = 0,\ \lim_{t\to\infty}\dot{\hat{x}}_i(t) = 0,\ \forall i,j \in \mathcal{N},$$

for a constant vector $x^e \in \mathbb{R}^n$ and a constant value $\nu^e \in \mathbb{R}$. Then, the formation and velocity errors F_e and V_e in (8) and (9) are represented as

$$F_e = \|(I_n - C)x^e\|,\ V_e = \sqrt{n}|\nu^e| \tag{14}$$

where $C = 1_n 1_n^\top / n$.

The proof is omitted due to the limitation of space.

From (1), the dynamics of the variable $\hat{x}(t)$ is given as

$$\begin{aligned}\ddot{\hat{x}}(t) \ =\ & -\sum_{i,j\in\mathcal{N}} a_{ij}(E_{ii}\hat{x}(t-\tau_{ij}^s) - E_{ij}\hat{x}(t-\tau_{ij}^c)) \\ & -k_v \sum_{i,j\in\mathcal{N}} a_{ij}(E_{ii}\dot{\hat{x}}(t-\tau_{ij}^s) - E_{ij}\dot{\hat{x}}(t-\tau_{ij}^c)) \\ & -\sum_{i\in\mathcal{N}} b_i E_{ii}\dot{\hat{x}}(t-\tau_{ij}^s) + d,\end{aligned} \tag{15}$$

where the vector $d \in \mathbb{R}^n$ is

$$d := \nu^* L_\tau 1_n - Lx^e + (L_\tau - B)1_n \nu^e. \tag{16}$$

Thus, the vector d works on the system (15) as a disturbance. This term can be canceled with an appropriate choice of x_i^e and ν^e as follows.

Lemma 2: The constant vector d in (16) is 0_n by choosing the constants $x^e \in \mathbb{R}^n$ and $\nu^e \in \mathbb{R}$ as

$$x^e = \nu^*[U,\ 0_n][LU,\ (B-L_\tau)1_n]^{-1}L_\tau 1_n + 1_n\gamma \tag{17}$$

$$\nu^e = \nu^* \frac{pL_\tau 1_n}{p(B-L_\tau)1_n} \tag{18}$$

for any constant $\gamma \in \mathbb{R}$ and matrix $U \in \mathbb{R}^{n\times(n-1)}$ satisfying $\mathrm{rank}[LU,\ (B-L_\tau)1_n] = n$.

The proof is omitted due to the limitation of space.

From Lemma 2, the system (15) of the new variable $\hat{x}(t)$ is not affected by the disturbance d for x^e and ν^e in (17) and (18). Then, the time-delayed system (15) does not include any disturbances, and can achieve consensus with zero velocities under some assumptions. Then, the formation and velocity errors F_e and V_e are derived from Lemmas 1 and 2.

Theorem 1: Consider the multi-agent system given by Problem 1 with the time-delayed controller (7). Assume that the time-delayed system (15) achieves consensus with zero velocities for $d = 0$. Then, the formation and velocity errors of the original multi-agent system (1) defined in (8) and (9) are given by

$$F_e = \left\|\nu^*[(I_n - C)U\ 0_n][LU\ (B-L_\tau)1_n]^{-1}L_\tau 1_n\right\| \tag{19}$$

$$V_e = \sqrt{n}\left|\nu^* \frac{pL_\tau 1_n}{p(B-L_\tau)1_n}\right|, \tag{20}$$

where $U \in \mathbb{R}^{n\times(n-1)}$ is a matrix such that $\mathrm{rank}[LU\ (B - L_\tau)1_n] = n$.

Proof: Let $\hat{x}(t)$ be the variable defined by (13) with the constant vector x^e and value ν^e given by (17) and (18). Then, Lemma 2 guarantees that the constant vector d in (16) is zero. From the assumption of this theorem, the system of $\hat{x}(t)$ achieves consensus with zero velocities. Then, Lemma 1 guarantees that the formation and velocity errors F_e and V_e are given by (14). Thus, by replacing x^e and ν^e with (17) and (18) in the errors (14), the resultant errors (19) and (20) are derived. ■

Remark 1: Our stance in this paper is to investigate the formation and velocity errors in the original multi-agent system (1) under assumption that the time-delayed system (15) achieves consensus. See other papers for the consensus problems of multi-agent systems with/without time-delays [10], [12], [13].

V. LEADER SELECTION PROBLEM

In this section, we consider Problem 2, and propose a way to select leaders from the viewpoint of minimizing the velocity error (9). The velocity error V_e calculated as (20) can be minimized by maximizing $pB1_n = \sum_{i=1}^n p_i b_i$. Thus, the minimizing problem (11) is reduced to how to choose m agents as leaders whose values of $p_i b_i$ are larger than the others. From this viewpoint, the following theorem summarizes the ways to select leaders and to design their receiving gains.

Theorem 2: Consider Problem 2. The velocity error V_e in (9) is minimized in terms of (11) for the set $\mathcal{L} \subset \mathcal{N}$ of leaders such that $|\mathcal{L}| = m$ and

$$\min_{i\in\mathcal{L}} p_i\bar{b}_i \geq \max_{i\notin\mathcal{L}} p_i\bar{b}_i, \tag{21}$$

and the receiving gains

$$b_i = \bar{b}_i,\ i \in \mathcal{L}, \quad b_i = 0,\ i \notin \mathcal{L}. \tag{22}$$

Proof: From the calculated velocity error (20), the problem (11) is equivalent to maximizing $pB1_n = \sum_{i=1}^n p_i b_i$. Thus, we consider the following problem instead of (11).

$$Q := \max_{\substack{\mathcal{L}\subset\mathcal{N} \\ |\mathcal{L}|=m}} \max_{\substack{b_i\leq\bar{b}_i, i\in\mathcal{L} \\ b_i=0, i\notin\mathcal{L}}} \sum_{i=1}^n p_i b_i \tag{23}$$

Let $\mathcal{L}_* \subset \mathcal{N}$ be the set of m agents, which satisfies (21) for $\mathcal{L} = \mathcal{L}_*$. Let $\mathcal{L} \subset \mathcal{N}$ be a set of m agents, which is not necessarily $\mathcal{L}_*$. Note that the receiving gains b_i have to satisfy the conditions in (10). Then, the following equations hold for the set $\mathcal{L}$:

$$\sum_{i=1}^n p_i b_i = \sum_{i\in\mathcal{L}} p_i b_i \leq \sum_{i\in\mathcal{L}} p_i \bar{b}_i, \tag{24}$$

where we use the property $p_i \geq 0$ of the left eigenvector p of Laplacian L. Define the following sets:

$$\mathcal{N}_1 = \mathcal{L}_* \cap \mathcal{L},\ \mathcal{N}_2 = \mathcal{L}_* \cap \mathcal{L}^c,\ \mathcal{N}_3 = \mathcal{L}_*^c \cap \mathcal{L},$$

where the superscript c denotes the complimentary set. Note that $\mathcal{L}_* = \mathcal{N}_1 \cup \mathcal{N}_2$ and $\mathcal{L} = \mathcal{N}_1 \cup \mathcal{N}_3$, and that $|\mathcal{N}_2| = |\mathcal{N}_3|$ because $|\mathcal{L}_*| = |\mathcal{L}| = m$. From the inequality (21) for $\mathcal{L} = \mathcal{L}_*$ and the fact that $\mathcal{N}_2 \subset \mathcal{L}_*$ and $\mathcal{N}_3 \subset \mathcal{L}_*^c$, the inequality $p_i\bar{b}_i \geq p_j\bar{b}_j$ holds for any pairs $(i,j) \in \mathcal{N}_2 \times \mathcal{N}_3$. Then, the following expressions hold:

$$\begin{aligned}\sum_{i\in\mathcal{L}} p_i\bar{b}_i \ &= \ \sum_{i\in\mathcal{N}_1} p_i\bar{b}_i + \sum_{i\in\mathcal{N}_3} p_i\bar{b}_i \\ &\leq \ \sum_{i\in\mathcal{N}_1} p_i\bar{b}_i + \sum_{i\in\mathcal{N}_2} p_i\bar{b}_i = \sum_{i\in\mathcal{L}_*} p_i\bar{b}_i.\end{aligned} \tag{25}$$

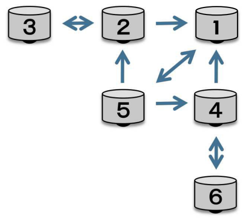

Fig. 3. Network for simulations

Note that $\mathcal{L}$ can be any subset of $\mathcal{N}$ which has m elements. Thus, from (24) and (25), the value (23) satisfies

$$Q \leq \max_{\substack{\mathcal{L}\subset\mathcal{N}\\ |\mathcal{L}|=m}} \sum_{i\in\mathcal{L}} p_i \bar{b}_i \leq \sum_{i\in\mathcal{L}_*} p_i \bar{b}_i.$$

The equations hold in these expressions if $\mathcal{L} = \mathcal{L}_*$ in the middle of the expressions. Thus, $Q = \sum_{i\in\mathcal{L}_*} p_i \bar{b}_i$ is the solution. The proof is completed. ■

Note that the value p_i represents agent i's power of propagating information to the group of agents through the network represented by Laplacian L. This propagation is amplified via the receiving gain b_i. Thus, as shown in Theorem 2, it is reasonable that the value p_i and the upper limit $\bar{b}_i$ of the receiving gain hold the keys to minimizing the velocity error V_e. (21) suggests how to select leaders according to the network structure through the left eigenvector p of Laplacian L. Note that (21) does not depend on the time-delays τ_{ij}^s and τ_{ij}^c.

VI. Numerical examples

This section demonstrates the validity of the proposed methods with simulation results. Consider the network depicted in Fig. 3. The coupling gains a_{ij} of connected agents take the value 1, and the rest of a_{ij} take 0. Then, the Laplacian is given by

$$L = \begin{bmatrix} 1 & 0 & 0 & 0 & -1 & 0 \\ -1 & 2 & -1 & 0 & 0 & 0 \\ 0 & -1 & 1 & 0 & 0 & 0 \\ -1 & 0 & 0 & 2 & 0 & -1 \\ -1 & -1 & 0 & -1 & 3 & 0 \\ 0 & 0 & 0 & -1 & 0 & 1 \end{bmatrix}$$

from (2). We consider two cases according to time-delays. The first case is that the computation-delays and the communication-delays are uniform such as $\tau_{ij}^s = \tau_{ij}^c = 0.15$. The second case is that the time-delays are non-uniform; the communication-delays $\tau_{ij}^c = 0.3$, and no computation-delays $\tau_{ij}^s = 0$. Then, from (3), the delay-Laplacians in the two cases are described as $L_\tau = 0.15L$ and

$$L_\tau = -0.3 \begin{bmatrix} 0 & 0 & 0 & 0 & 1 & 0 \\ 1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 1 \\ 1 & 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \end{bmatrix}, \qquad (26)$$

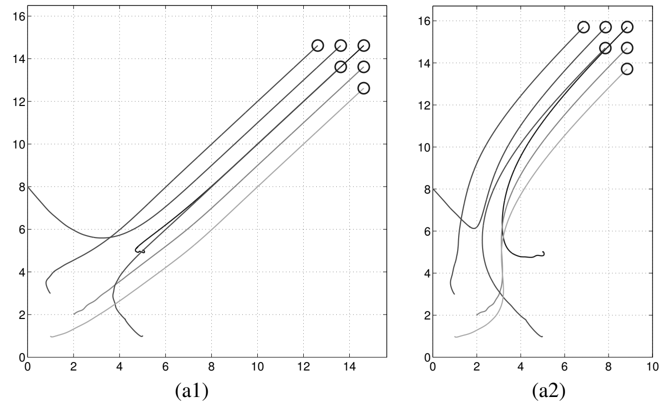

Fig. 4. Trajectories of the agents on the x-y plane with uniform time-delays: In (a1) and (a2), agents 1 and 2 are assigned as leaders, respectively.

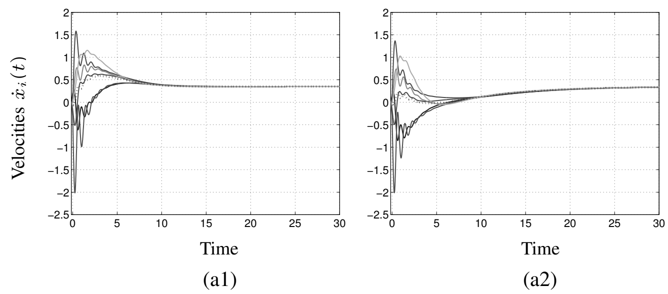

Fig. 5. Velocities of the agents with uniform time-delays.

respectively. The damping gain k_v is given by 2. We will assign one leader ($m = 1$) from the 6 agents with the upper limit $\bar{b}_i = 1$ of the receiving gain b_i.

The setting of simulations is as follows: We consider the 2-dimensional space for clear pictures on simulation results, but actually two dimensions are individually dealt with. The desired formation pattern is assigned as

$$x_1^* = (0,0),\ x_2^* = (-1,0),\ x_3^* = (-2,0)$$
$$x_4^* = (0,-1),\ x_5^* = (-1,-1),\ x_6^* = (0,-2)$$

on the x-y plane. Let the assigned velocity be $\nu^* = (0.35, 0.35)$, whose elements represent the velocity along x and y axes, respectively. The initial positions of the agents are given by

$$x_1(0) = (5,5),\ x_2(0) = (0,8),\ x_3(0) = (5,0)$$
$$x_4(0) = (2,2),\ x_5(0) = (4,6),\ x_6(0) = (1,1).$$

Case 1 ($\tau_{ij}^s = \tau_{ij}^c = 0.15$): the delay-Laplacian satisfies $L_\tau 1_n = 0.3L1_n = 0_n$. Thus, Theorem 1 guarantees that the formation and velocity errors are $F_e = V_e = 0$ from (19) and (20) for any receiving matrix B. Therefore, the formation control (5) with velocity assignment is achieved regardless of the choice of the leader. We verify this analysis result by two simulations: either of agents 1 and 2 is the leader, and $b_i = 1$ for the leader and the rest of b_i are zero. Figs. 4 depict the trajectories of the agents on the x-y plane from time $t = 0$ to 30. Figures (a1) and (a2) refer to the situations where

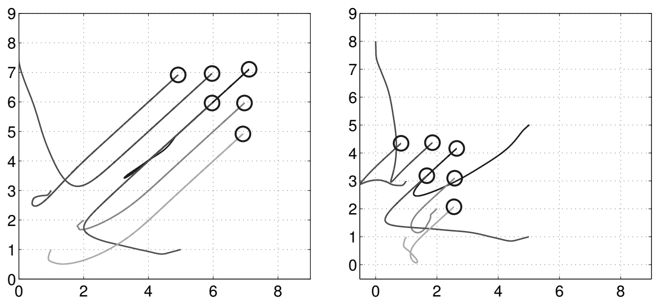

Fig. 6. Trajectories of the agents on the x-y plane with non-uniform time-delays: In (a1) and (a2), agents 1 and 2 are assigned as leaders, respectively.

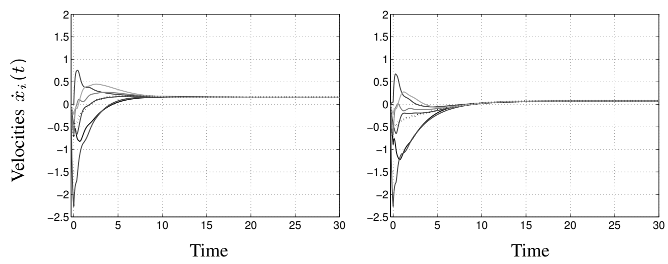

Fig. 7. Velocities of the agents with non-uniform time-delays.

agents 1 and 2 are the leaders, respectively. The circle marks represent the positions of the agents at $t = 30$. In both the simulations, the agents gather and form the desired formation pattern assigned by x_i^*. Figs. 5 (a1) and (a2) depict the time-plots of the velocities of the agents along the x axis for agent 1 or 2 as the leader. It is observed that the velocities of all the agents converge to the desired velocity 0.35 in both the simulations. These results show that the formation control (5) with velocity assignment is achieved even if any agent is the leader. Thus, the validity of the analysis result in Theorem 1 is verified.

Case 2 ($\tau_{ij}^c = 0.3$, $\tau_{ij}^s = 0$): for the delay-Laplacian (26), $L_\tau 1_n = -0.3[1, 2, 1, 2, 3, 1]^\top \neq 0_n$ is calculated; thus, the formation and velocity errors will occur according to Theorem 1. Now, we assign one agent to the leader in order to minimize the velocity error V_e from the leader selection method proposed in Theorem 2. Note that the left eigenvector p of Laplacian L is given by

$$p = [0.38,\ 0.13,\ 0.13,\ 0.13,\ 0.13,\ 0.13].$$

Among the elements of p, $p_1 = 0.38$ is the largest. Thus, the leader's condition (21) holds for $\mathcal{L} = \{1\}$ because the upper limits of the receiving gains are given as $\bar{b}_i = 1$ for all the agents. Then, agent 1 should be the leader in order to minimize the velocity error V_e. The receiving gains are designed as $b_1 = 1$ and $b_i = 0$ for the rest from (22). We confirm this result by two simulations: either of agent 1 or 2 is the leader, $b_i = 1$ for the leader, and the rest of b_i are zero. Figs. 6 (a1) and (a2) show the trajectories of the agents on the x-y plane from time $t = 0$ to 30. In both the simulations, the agents gather, but do not perfectly form the desired formation. Figs. 7 (a1) and (a2) depict the time-plots of the velocities of the agents along the x axis. It is observed that the velocities of all the agents converge to 0.16 and 0.076, respectively. Although the desired velocity 0.35 is not achieved in both the cases, the convergent velocities in the former case are closer to the desired velocity. Thus, agent 1 is the best leader to minimize the velocity error, which illustrates the validity of Theorem 2.

VII. Conclusion

This paper dealt with a formation control problem with velocity assignment for multi-agent systems with heterogeneous time-delays. We revealed that heterogeneous time-delays possibly cause formation and velocity errors, and derived the formation and velocity errors according to network structures of time-delayed multi-agent systems. Moreover, we proposed a leader selection method in order to improve the performance with respect to the velocity error. The main result showed that the best leaders can be selected independently of the time-delays of networks.

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