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Yo-yo Motion Control based on Impulsive Luenberger Observer

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Abstract— **In this paper, a yo-yo motion control called "gravity-pull" is considered with mixing an energy-based stabilizing control and an impulsive Luenberger Observer (ILO). This study aims that this control law is realized without any visual sensor but with force sensors because people can manipulate a yo-yo by use of the force sense at the finger even with eyes closed. Under the assumption that recognizable force is applied to the finger only when a yo-yo arrives at the bottom position, the ILO plays an important role to estimate the state of the yo-yo from the impulsive force because the state as the rotational speed is required for the yo-yo motion control. The precision of a yo-yo physical model affects the estimation accuracy of the ILO, hence a detailed yo-yo model, in which the thickness of the yo-yo's string is considered, is derived. With the estimated state, an energy-based stabilizing control realizes the gravity-pull, the target yo-yo motion.**

I. INTRODUCTION

A yo-yo is an interesting object as a research target, especially in the robotics field. There are many previous studies concerned with a yo-yo, for example, [1], [2], [3], [4]. Žlajpah in [1] focused on the input timing when human manipulates a yo-yo. He found a fundamental rule to keep yo-yo manipulations, that is, human should move his hand upward before a yo-yo reached the bottom position. Based on this observation, Žlajpah succeeded to a yo-yo trick called "gravity-pull" by a manipulator with visual feedback. In the similar direction, Jin and Zackenhouse [2] also realized a yo-yo manipulation with a manipulator by visual feedback. They used a yo-yo's model in one dimensional space to derive their control law. As another example such as model-based control for yo-yo manipulations, Hashimoto and Noritsugu [3] proposed a modeling and model-based control method of robotic yo-yo with visual feedback. On the other hand, Lee, Shim and Bien [4] proposed a soft-oriented control with neuro-fuzzy logic for yo-yo systems.

Basically those studies supposed that some visual feedback was available for yo-yo control. On the other hand, it's the well known fact for human to manipulate a yo-yo even with eyes closed if he has good experience of yo-yo manipulations. We believe this fact might be interpreted as follows: human can manipulate a yo-yo without visual feedback, i.e even with eyes closed, if he knows the dynamics of the yo-yo. Hence, in this paper, we aim to propose a model-based control method for realizing a yo-yo trick called "gravity-pull" by a robot only using force information applied to the yo-yo string. "Gravity-pull" is one of the basic manipulations, and in the trick, human manipulate a yo-yo moving up and down.

One of difficulties toward the goal is caused by the difficulty of force measurement in our case. The recognizable force is detected by a force sensor only when a yo-yo reached the bottom position, and the measured force has an impulsive shape. Human can feel minute force from the string during a yo-yo is falling, and might partially feedback the sensed force to control the yo-yo. However, basically human might control a yo-yo by the estimation of yo-yo motion based on the yo-yo's dynamics, and the estimation might be updated at several distinguishing situations, for example, a yo-yo reaches the bottom position, and is caught by the hand. On the observation, we can conclude the measurement condition in our yo-yo system are basically in alignment with the real human manipulation case. Hence an impulsive Luenberger observer (ILO) is utilized for the estimation of the yo-yo state. The observer state is updated when a yo-yo reaches the bottom position. This condition and situation allow us to use the following information to update the observer state: the position of the yo-yo and the duration from the previous bottom position to the present. Before the yo-yo reaches the bottom position, the yo-yo state is estimated feedforwardly with the yo-yo model. A controller to be proposed can utilized the yo-yo estimated state continuously. In this paper, the energy of the yo-yo is also estimated from the yo-yo estimated state, and an energy-based control is designed to keep the yo-yo trick, "gravity-pull".

As aforementioned, the observer utilizes a yo-yo model to estimate its state, and then a precise yo-yo model is required. Basically a yo-yo can be regarded as a hybrid system whose dynamics changes according to its motion. A modeling process, "Projection Method" proposed by Blajer [5] is utilized to derive the hybrid yo-yo model in this paper. A yo-yo is a toy in which two discs are tied up by a thin axis which is connected a long string. The string is wrapped around the axis, and the axis diameter is when the yo-yo is manipulated. The fluctuation of the axis diameter might influence the estimation accuracy of the observer. Therefore our yo-yo model takes the thickness of the string into account.

This paper is organized as follows; first of all, an equation of motion of a yo-yo in two dimensional space is derived. The projection method [5] is used to derive the equation of motion. In many studies the thickness of the string was disregarded, but the influence of the string's thickness is considered in this study to derive a precise model for the state estimation. The effectiveness of the precise model is illustrated through numerical simulations in which the behav-

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Fig. 1. Photo of the experimental system.

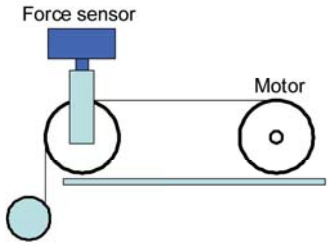

Fig. 2. Schematic figure of the experimental machine.

ior of the derived model is compared with that of an actual yo-yo. Under those conditions, an impulsive Luenberger observer with the derived model is constructed, and the control law based on the estimated energy is also designed. The effectiveness of the total control system is verified by numerical simulations and by an experimental system in Fig. 1 whose schematic figures shown in Fig. 2.

II. YO-YO MODELING

A. Preliminaries

In this section, the equation of motion of the yo-yo is derived. The projection method [5] is used for modeling. The projection method is a modeling method for paying attention to constraints working between system components. This method has a feature that constraint force can be derived comparatively easily. For instance, the impulsive tension of the string generated when a yo-yo reaches the bottom position can be derived from this constraint force. The modeling procedure of the projection method is concisely shown in [5]. Its summary is described as follows:

1) The coordinate system is decided. And the generalized coordinates q_a and the generalization velocity $\dot{q}_a$ of the system are defined.
2) For equations of motion of the unconstrained system, and the generalization mass matrix M and the generalization force h are defined.
3) For the constraint conditions between each element and component, the constraint matrix C that satisfies $C\dot{q}_a = 0$ is derived.
4) $\dot{q}_a$ is decomposed into an independent velocity vector v and others.
5) The complementary orthogonal matrix D that satisfies $\dot{q}_a = Dv$ and $CD = 0$ is derived, and the constrained equation of motion is obtained by substituting $\dot{q}_a = Dv$ into the unconstrained system equations.

The yo-yo is basically regarded as a hybrid system which dynamics changes by the position of the axis and the string. However, the corresponding equation of motion can be derived only by rewriting the constraint conditions in the projection method. The followings describe the concrete modeling process of the yo-yo.

The yo-yo moves along the string. Its motion depends on its rotation, the string's twist and finger tip's position. In this paper, it is assumed that the string's twist can be neglect, therefore, a yo-yo model in two dimensional space is derived. To control it based on its mathematical model, a detailed and precise model that considers the string's diameter and the axis diameter is derived.

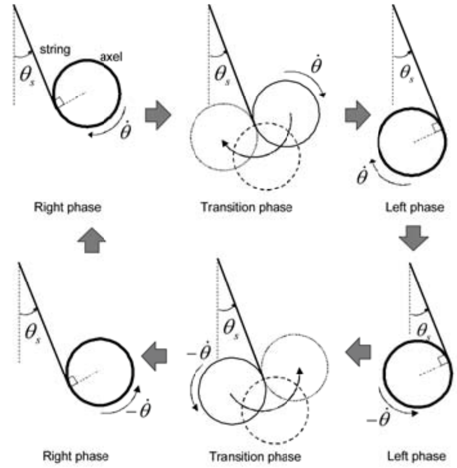

Fig. 3. Phase transition of a yo-yo system.

The 2D yo-yo has four degrees of freedom such as a yo-yo rotational angle θ, a string angle θ_s and the position of the finger tips $x_t,\ y_t$. The phase transition of the position of the string and the axis is shown in Fig. 3. The yo-yo falls down converting the potential energy into the kinetic energy of the rotation. Conversely the yo-yo rises up converting the kinetic energy of the rotation into the potential energy from the bottom position. The yo-yo falls again repeating this process cyclicly.

As a matter of convenience in this paper, the situation that the string exists at the right-hand side of the yo-yo rotational axis is called "right phase", the opposite case is called, "left phase", and the transition phase from the right phase to the left phase is called "transition phase".

B. Modeling

The model figure of the right phase is shown in Fig. 4 and the parameters used are shown in TABLE I. The generalized coordinates q_a, the generalized mass matrix M_a and the generalized force h can be expressed by

$$q_a = [\theta, \theta_s, x_t, y_t, x, y]^T \tag{1}$$

$$M = \mathrm{diag}\,(J,\ J_s(\theta),\ m_t,\ m_t,\ m,\ m) \tag{2}$$

$$h = [-R(\theta)e\dot{\theta},\ -c\dot{\theta}_s,\ F_x,\ F_y - m_t g, 0,\ -mg]^T. \tag{3}$$

F_x and F_y are inputs to the connecting point. The string's moment of inertia $J_s(\theta)$ changes depending on the length of the string. The yo-yo's moment of inertia J also changes according to coiling of the string. However, it is assumed to be able to disregard J because the variation is slight compared with the moment of inertia of the main body of the yo-yo.

From Fig.4, the constraint conditions of the right phase are

$$\begin{cases} x_c = L(\theta)\sin\theta_s + R(\theta)\cos\theta_s + x_t \\ y_c = -L(\theta)\cos\theta_s + R(\theta)\sin\theta_s + y_t. \end{cases} \tag{4}$$

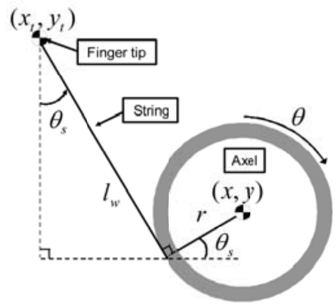

Fig. 4. Schematic figure of yo-yo model (right phase)

TABLE I
PARAMETERS

m, m_t	mass of yo-yo and fingertip
r	minimum radius of axel
r_s	maximum radius of axel
$R(\theta)$	radius of axel
l	maximum length of string
l_w	diameter of string
$L(\theta)$	length of string
$J, J_s(\theta)$	moment of inertia of yo-yo and string
e, c	viscous friction coefficient of yo-yo and string
θ, θ_s	angle of yo-yo and string
x_t, y_t	center of gravity of fingertip
x_c, y_c	center of gravity of yo-yo
F_x, F_y	input to fingertip(x_t, y_t)
θ_l	angle of yo-yo at left phase
θ_t	angle of yo-yo at transition phase

In addition, axis diameter $R(\theta)$ changes by the string's coiling as the constraint condition the string like

$$R(\theta) = r_s - l_w \frac{\theta}{2\pi}, \qquad (r \le R(\theta) \le r_s). \tag{5}$$

r_s is the state when all strings are coiled around the yo-yo. Length $L(\theta)$ of the string changes according to $R(\theta)$. $L(\theta)$ can be shown

$$L(\theta) = r_s\theta - l_w \frac{\theta^2}{4\pi}, \qquad (0 \le L(\theta) \le l), \tag{6}$$

according to thickness l_w of the string and the coiling area.

The constraint condition of Left and Transition Phase can respectively be similarly shown

$$\begin{cases} x_c = L(\theta_l)\sin\theta_s - R(\theta_l)\cos\theta_s + x_t \\ y_c = -L(\theta_l)\cos\theta_s - R(\theta_l)\sin\theta_s + y_t \\ R(\theta_l) = r + l_w \frac{\theta_l}{2\pi} \\ L(\theta_l) = l - \left(r\theta_l - l_w \frac{\theta_l^2}{4\pi}\right) \end{cases} \tag{7}$$

and

$$\begin{cases} x_c = l\sin\theta_s + r\cos(\theta_s - \theta_t) + x_t \\ y_c = -l\cos\theta_s + r\sin(\theta_s - \theta_t) + y_t \\ R = r \end{cases} . \tag{8}$$

where θ_l and θ_t are the angle of rotation from each phase.

From (4)-(8), we derive the constraint matrix C and the orthogonalization matrix D. As the result, the motion equation of the yo-yo can be derived

$$D^T M D\dot{v} + D^T M \dot{D} v = D^T h, \tag{9}$$

where v is tangent speed that equals $[\dot{\theta},\ \dot{\theta}_s,\ \dot{x}_t,\ \dot{y}_t]^T$. Thereafter, to distinguish each phase, subscript "r ", "t" and "l " are used for the right phase, Transition Phase, and left phase , and it is used like D_r, D_t, and D_l. As for $\dot{D}$ and C, the similar manner is applied.

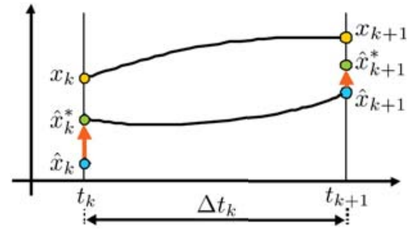

Fig. 5. Schematic figure of estimation in ILO

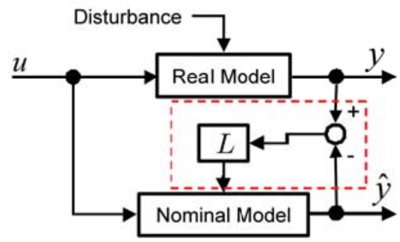

Fig. 6. Block Diagram of ILO

III. OBSERVER DESIGN

A. ILO: Impulsive Luenberger Observer

ILO (Impulsive Luenberger Observers) [7] uses in this study, to estimate the state of the yo-yo. In Fig. 5, Δt_k is a section where information on the yo-yo cannot be observed and in t_k and $t_{k+1}$are time in the bottom position.

ILO is defined by

$$\begin{cases} \dot{\hat{x}}(t) = A(\hat{x})\hat{x}(t) + B(\hat{x})u(t),\ t \notin \{t_k\}_{k=1}^{\infty} \\ \hat{x}^*_{k+1} = \hat{x}_{k+1} - L(\hat{x}_{k+1})\left(\hat{y}_{k+1} - y_{k+1}\right),\ t \in \{t_k\}_{k=1}^{\infty} \end{cases} . \tag{10}$$

(10) is a hybrid system with state estimation at continuous time and the state estimation at discrete time. The state is estimated by using the equation of motion in section Δt_k where information cannot be observed. In bottom position, the error margin is denied by using the equation at discrete time of the observer. The state of the yo-yo is estimated by switching these update equations. Fig.6 shows the block diagram of ILO. In bottom position, by the operation in the dotted line red of the block diagram of Fig.6, the state is updated.

B. ILO Design

The observer gain L of (10) is designed. The error margin in state x_k of a real model and observer's states $\hat{x}^*_k$ is defined as $e_k = x_k - \hat{x}^*_k$. The observer gain is decided so that the absolute value of the eigenvalue of the transition procession of e_k may fit into unit circle.

The equation of motion of the yo-yo with the force sensor is

$$\begin{aligned} \dot{x} &= A(x)x + B(x)u \\ y &= Cx \\ y_q &= Q(y) \end{aligned} , \tag{11}$$

where the function $Q(\cdot)$ means the state observation with the force sensor. $x \in R^{4\times1}$ is the states, $u \in R^{1\times1}$ is the control inputs and $y \in R^{2\times1}$ is the outputs. The function Q is defined as

$$Q(y) = \text{event}(y). \tag{12}$$

$Q(y)$ means yo-yo rotation angle θ can be observed only in bottom position. Moreover, y_q is the outputs in bottom position. The observer is defined as

$$\dot{\hat{x}} = A(\hat{x})\hat{x} + B(\hat{x})u. \tag{13}$$

Control input u is assumed to be given by

$$u = \hat{F}\hat{x}. \tag{14}$$

t_k is the time when the yo-yo reaches the bottom position at past, and t_{k+1} shows current time. The initial state of the plant at t_k is given by $x_0 = x_k$, and the initial state of the holder at t_k is given by $\hat{x}_0 = \hat{x}_k$. The revolutions of the plant and the holder are given by

$$\begin{bmatrix}\dot{x}(\tau)\\ \dot{\hat{x}}(\tau)\end{bmatrix} = \begin{bmatrix}A & B\hat{F}\\ 0 & \hat{A}+\hat{B}\hat{F}\end{bmatrix}\begin{bmatrix}x(\tau)\\ \hat{x}(\tau)\end{bmatrix}, \quad \begin{bmatrix}x(0)\\ \hat{x}(0)\end{bmatrix} = \begin{bmatrix}x_0\\ \hat{x}_0\end{bmatrix}, \tag{15}$$

where $\tau = t - t_k$, $A = A(x)$,$B = B(x)$,$\hat{A} = A(\hat{x})$ and $\hat{B} = B(\hat{x})$. From (13), in $t \approx t_k$, observer's state transition is

$$\hat{x}(\tau) = e^{(\hat{A}+\hat{B}\hat{F})\tau}\hat{x}_0. \tag{16}$$

Therefore, control input u is

$$u = \hat{F}e^{(\hat{A}+\hat{B}\hat{F})\tau}\hat{x}_0. \tag{17}$$

When assuming $A \approx \hat{A}$ and $B \approx \hat{B}$ at time τ, from (16), the state transition is

$$\begin{bmatrix}x(\tau)\\ \hat{x}(\tau)\end{bmatrix} = e^{A_{CL}\tau}\begin{bmatrix}x(0)\\ \hat{x}(0)\end{bmatrix}$$
$$A_{CL} = \begin{bmatrix}\hat{A} & \hat{B}\hat{F}\\ 0 & \hat{A}+\hat{B}\hat{F}\end{bmatrix}.$$

Where

$$A_{CL}^n = \begin{bmatrix}\hat{A}^n & (\hat{A}+\hat{B}\hat{F})^n - \hat{A}^n\\ 0 & (\hat{A}+\hat{B}\hat{F})^n\end{bmatrix},$$

and

$$e^{A_{CL}\tau} = \begin{bmatrix}e^{\hat{A}\tau} & e^{(\hat{A}+\hat{B}\hat{F})\tau} - e^{\hat{A}\tau}\\ 0 & e^{(\hat{A}+\hat{B}\hat{F})\tau}\end{bmatrix}.$$

If $\Delta t_k = t_{k+1} - t_k$, the states are

$$\begin{bmatrix}x_{k+1}\\ \hat{x}_{k+1}\end{bmatrix} = \begin{bmatrix}e^{\hat{A}\Delta t_k} & e^{(\hat{A}+\hat{B}\hat{F})\Delta t_k} - e^{\hat{A}\Delta t_k}\\ 0 & e^{(\hat{A}+\hat{B}\hat{F})\Delta t_k}\end{bmatrix}\begin{bmatrix}x_k\\ \hat{x}_k\end{bmatrix} \tag{18}$$
$$= \begin{bmatrix}\Phi_k & \Psi_k - \Phi_k\\ 0 & \Psi_k\end{bmatrix}\begin{bmatrix}x_k\\ \hat{x}_k\end{bmatrix}, \tag{19}$$

where it is assumed that $\Delta t_k \approx \Delta t_{k+1}$.

The observer update equation is

$$\hat{x}^*_{k+1} = \hat{x}_{k+1} + L_{k+1}(y_{k+1} - C\hat{x}_{k+1}).$$

From (19),

$$\begin{bmatrix}x_{k+1}\\ \hat{x}^*_{k+1}\end{bmatrix} = \begin{bmatrix}I & 0\\ L_{k+1}C & I - L_{k+1}C\end{bmatrix}\begin{bmatrix}x_{k+1}\\ \hat{x}_{k+1}\end{bmatrix}$$
$$= \begin{bmatrix}\Phi_k & \Psi_k - \Phi_k\\ L_{k+1}C\Phi_k & -L_{k+1}C\Phi_k + \Psi_k\end{bmatrix}\begin{bmatrix}x_k\\ \hat{x}^*_k\end{bmatrix}.$$

From $e_k = x_k - \hat{x}^*_k$,

$$\begin{bmatrix}x_{k+1}\\ e_{k+1}\end{bmatrix} = \begin{bmatrix}\Psi_k & \Phi_k - \Psi_k\\ 0 & \Phi_k - L_{k+1}C\Phi_k\end{bmatrix}\begin{bmatrix}x_k\\ e_k\end{bmatrix}.$$

Therefore, $e_{k+1} = (\Phi_k - L_{k+1}C\Phi)e_k$. If $L_{k+1}$is chosen that the eigenvalue of $\Phi_k - L_{k+1}C\Phi$ fits into unit circle, $e_k \to 0$.

$\Delta t_k \approx 2.0$ [sec] because Δt_k is time from bottom position to next bottom position when the yo-yo manipulating It is necessary to derive Φ_k by dividing Δt_k, because accuracy cannot be kept when Δt_k is large. From $e_{k+1} = (e^{\hat{A}\Delta t_k} - L_{k+1}Ce^{\hat{A}\Delta t_k})e_k$, $\tilde{A}_k = (e^{\hat{A}\Delta t_k})^T$,$\tilde{B}_k = (Ce^{\hat{A}\Delta t_k})^T$. From the duality, we consider

$$\xi_{i+n} = \tilde{A}_k\xi_i + \tilde{B}_k\mu_k. \tag{20}$$

Gain L_{k+1}that puts the eigenvalue of (20) into unit cycle is derived. Where $u_k = L_{k+1}e_k$. The number of partitions of Δt_k is assumed to be n, and it is assumed that $\xi_i = \xi_k$, and $\xi_{i+n} = \xi_{k+1}$.

$$\begin{aligned}
\xi_{i+1} &= \tilde{A}_i\xi_i + \tilde{B}_i\mu_k\\
&= \tilde{A}^1\xi_i + \tilde{B}^1\mu_k\\
\xi_{i+2} &= \tilde{A}_{i+1}\xi_{i+1} + \tilde{B}_{i+1}\mu_k\\
&= \tilde{A}_{i+1}(\tilde{A}^1\xi_i + \tilde{B}^1\mu_k) + \tilde{B}_{i+1}\mu_k\\
&= \tilde{A}_{i+1}\tilde{A}^1\xi_i + (\tilde{A}_{i+1}\tilde{B}^1 + \tilde{B}_{i+1})\mu_k\\
&= \tilde{A}^2\xi_i + \tilde{B}^2\mu_k.
\end{aligned}$$

To similar

$$\begin{aligned}
\xi_{i+n} &= \tilde{A}_{i+n-1}\xi_{i+n-1} + \tilde{B}_{i+n-1}\mu_k\\
&= \tilde{A}_{i+n-1}(\tilde{A}^{n-1}\xi_i + \tilde{B}^{n-1}\mu_k) + \tilde{B}_{i+n-1}\mu_k\\
&= \tilde{A}_{i+n-1}\tilde{A}^{n-1}\xi_i + (\tilde{A}_{i+n-1}\tilde{B}^{n-1} + \tilde{B}_{i+n-1})\mu_k\\
&= \tilde{A}^n\xi_i + \tilde{B}^n\mu_k.
\end{aligned} \tag{21}$$

And $e^{\hat{A}\Delta t_k} = (\tilde{A}^n)^T$,$Ce^{\hat{A}\Delta t_k} = (\tilde{B}^n)^T$. From (21), observer gain L_{k+1} is derived.

IV. ENERGY-BASED CONTROL DESIGN

The conditions precedent in the control design is as follows.

1) Gravity-pull that is one of the yo-yo manipulations is achieved.
2) The control system is constructed based on force at time in bottom position, without visual information.
3) From experimental result of Žlajpah, the control input is added when the yo-yo falls.

From the condition 2, yo-yo's states is observed in bottom position. Therefore, a continuous feedback control cannot be used. We basically think about the energy control of the yo-yo based on the numerical model. The estimation of an internal state of the yo-yo is updated in bottom position. From the condition 3, Gravity-pull is achieved by adding the input when the yo-yo falls. Gravity-pull is to achieve a certain limit cycle. The control target comes to keep a targeted value the amplitude in the yo-yo orbit in the meaning.

We think about function V that shows the energy status of the yo-yo.

$$V = \frac{1}{2}(E - E_d)^2 \quad , \qquad \dot{V} = (E - E_d)\dot{E}$$

E assumes the kinetic energy of the yo-yo, and is

$$E = \frac{1}{2}J\dot{\theta}_v^2 \quad , \qquad \dot{E} = J\dot{\theta}_v\ddot{\theta}_v.$$

Subscript v means the variable of the internal model. Moreover, when Gravity-pull is achieved, E_d is assumed to be a kinetic energy of the ideal of the yo-yo. However, when each Phase is switched, V is discontinuous. We think about energy function V_r and V_l of each Phase. Do not consider Transition Phase because it is not possible to control.

First of all, we think about right phase. From (9), the equation of motion of $\ddot{\theta}_v$ at right phase is

$$\ddot{\theta}_v = \frac{R_v\left(mF_y - (m+m_t)e\dot{\theta}_v + \frac{L_w m m_t \dot{\theta}_v^2}{2\pi}\right)}{J(m+m_t) + \frac{L_w^2 m}{4\pi^2}(m+m_t) + m m_t R_v^2}. \tag{22}$$

From (22), the energy function in right phase is

$$\dot{V}_r = (E - E_d)J\dot{\theta}_v \frac{R_v\left(mF_y - (m+m_t)e\dot{\theta}_v + \frac{L_w m m_t \dot{\theta}_v^2}{2\pi}\right)}{J(m+m_t) + \frac{L_w^2 m}{4\pi^2}(m+m_t) + m m_t R_v^2}.$$

Clearly, $V_r > 0$ and $V_r = 0$ is filled in $E = E_d$. $\dot{V}_r$ is changed by input F_y to the finger tip. Then, the input that fills $\dot{V}_r < 0$ is requested. From experimental result of Žlajpah, the control input is added when the yo-yo falls. At this time, $\dot{\theta}_v > 0$,$R_v > 0$. Then,

$$(E - E_d)\left(mF_y - (m+m_t)e\dot{\theta}_v + \frac{L_w m m_t \dot{\theta}_v^2}{2\pi}\right) < 0.$$

As for the input, energy is not decreased. If $E - E_d \geq 0$, $F_y = 0$. If $E - E_d < 0$,

$$F_y > \frac{m+m_t}{m}e\dot{\theta}_v - \frac{L_w m_t \dot{\theta}_v^2}{2\pi}. \tag{23}$$

$\dot{V}_r < 0$ is filled by input F_y that fills (23). In similar for left phase, control input is

$$F_y > -\frac{m+m_t}{m}e\dot{\theta}_v - \frac{L_w m_t \dot{\theta}_v^2}{2\pi}. \tag{24}$$

Section T where the input to the yo-yo is added has been limited while the yo-yo is falling. Thus, it is necessary to achieve $V = 0$ in the section. From (23), control input F_y of right phase is

$$F_y = \alpha\left(\frac{m+m_t}{m}e\dot{\theta}_v - \frac{L_w m_t \dot{\theta}_v^2}{2\pi}\right),$$

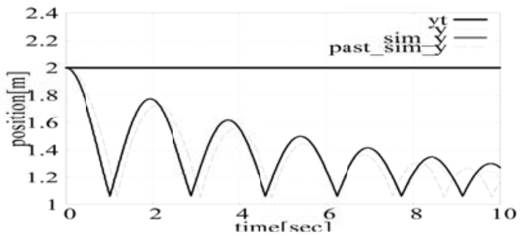

Fig. 7. Trajectories of a real yo-yo and its precise model

where $1 \leq \alpha < \infty$. To decide the value of α, set $\xi = \{\xi_1, \xi_2, \cdots, \xi_k, \cdots\}$ where ξ_k means the energy function V when the yo-yo arrives at the bottom. If this set ξ settles to 0 in $k \to \infty$, the gravity-pull is achieved. α that fills this is assumed to be

$$\alpha = \alpha_0 + \sum_{k=1}^{n}\left(K_p\Delta\xi_k + K_d\frac{\Delta\xi_k - \Delta\xi_{k-1}}{\Delta t} + K_i\sum_{j=1}^{k}\Delta\xi_j\right), \tag{25}$$

where $\Delta\xi_k = \xi_k - \xi_*$, and ξ_* is a targeted value of energy function V. Moreover, time in ξ_k with t_k and $\Delta t = t_k - t_{k-1}$. α_0, K_p, K_d, and K_i are adjustable parameters, and positive constants. From (25), this is decided to fill $\Delta\xi_k \to 0$ with $k \to \infty$. As for left phase, it is similar.

V. EXPERIMENTAL VERIFICATION

A. Model Verification

The behavior of the derived equation of motion is compared with the actual measurement value and the effectiveness of the model is confirmed. The actual yo-yo behavior of comparison was measured as follows. The condition that the point of the string was fixed to the stand, and the input from the finger tip did not join was imposed. The marker was installed in the disc of the yo-yo, and eight cameras were set up to enclose the yo-yo, and measured. The frame rate was 60[fps].

Each parameter is shown in TABLE II. The comparative result is shown in Fig. 7. `y` is behavior of an actual yo-yo, `sim_y` is behavior of the model derived in this paper, `past_sim_y` is behavior of the model to disregard the axis diameter change and `yt` is height of the finger tip. From Fig.7, the simulation result of the model to disregard the diameter of the string is not corresponding to the behavior of an actual yo-yo. The behavior of a simulation of the derived model and an actual yo-yo must be almost corresponding. However, the error margin is caused in the amplitude as the rotational speed of the yo-yo lowers. This is because the movement of three dimensions due to the twist of the string etc. becomes remarkable. The gyro effect works to maintain the rotation axis of the yo-yo when the rotational speed of the yo-yo is fast. If the gyro effect weakens as the rotational speed lowers, the situation in which the twist of the string and the interference with the string and the disc occur easily. Therefore, The error margin has been caused in behavior of the model and actual behavior. On the other hand, Gravity-pull is difficult in the low-speed rotation region where the twist of the string occurs. Therefore, the proposed

TABLE II

SIMULATION PARAMETERS

m	52.0[g]	l_w	1.0[mm]
m_t	52.0[mg]	J	0.00006[kgm^2]
r	10.0[mm]	e	0.00065[Ns/rad]
l	93.64[mm]	c	0.1[Ns/rad]

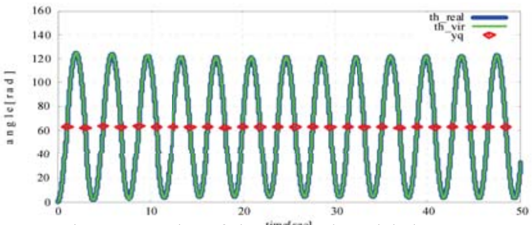

Fig. 8. Angles of the real and modeled yo-yo.

model to whom behavior in the state with the rotational speed is corresponding is useful enough. The effectiveness of the proposed model with behavior that was nearer an actual yo-yo than the past model in considering the diameter of the string was confirmed. From here onwards, to estimate the behavior of a yo-yo actual also as for the observer that constructed, it has enough accuracy.

To verify the effectiveness of the control system, the experiment machine that showed in figure 1 was produced. The composition of the experiment machine in which force in a perpendicular lower direction was able to be measured with the force sensor was an observer to adjust.

B. Control experiments

The gravity-pull motion with the designed control laws is verified . The identified model parameter was shown in TABLE II, and each gain was tuned with $\alpha_0 = 30.0$, $K_p = 30.0$, $K_i = 0.1$, and $K_d = 5.0$. The targeted energy value E_d was chosen as the energy of the yo-yo in bottom position at the first time. The state of the mathematical internal model is updated every time when the yo-yo reaches the bottom position by using the ILO observer that is constructed with Section III.

The results are shown in Figs. 8–10. Fig. 8 shows the yo-yo's angle θ, Fig.9 shows the position of finger tip and the center of gravity of yo-yo, and Fig. 10 is energy function V. `_real` is the state of the real plant, and `_vir` is the state of the internal model. It found that the amplitude of the yo-yo's angle θ was kept a constant value from Fig. 8, the yo-yo moved periodically from Fig. 9, and the gravity-pull was realized. Moreover, the energy function V settled finally to 0 from Fig.10, and the control target was achieved. The influence due to modeling error and some disturbance considered, $\dot{V} < 0$ is never secured, however, the control objective was achieved in this case by some robustness. The total stability should be investigated as our future works, but, this verification shows the effectiveness of the proposed method.

VI. CONCLUSIONS

In this paper, an energy-based stabilizing control for a yo-yo was proposed to realize a yo-yo trick called "gravity-pull" without any visual sensor but with a force sensor. This study

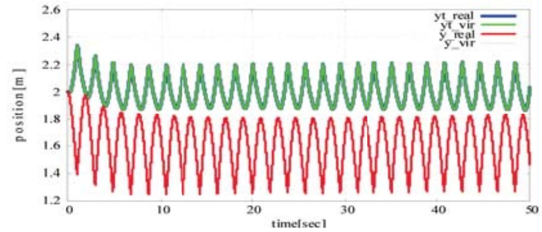

Fig. 9. The finger tip position (y_t) and the center of gravity of yo-yo (y)

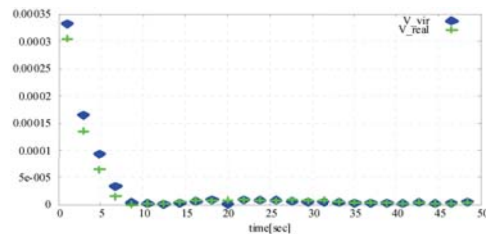

Fig. 10. yo-yo's energy transition

was regarded as a trial to mimic a human skillful operation in that human manipulated a yo-yo by use only of the knowledge of the yo-yo's dynamics and the force sense at the finger where the yo-yo string was tied, i.e. with eyes closed. An ILO (Impulsive Luenberger Observer) was introduced to estimate the yo-yo state from the impulsive force arising when the yo-yo reached the bottom position. The ILO required a precise yo-yo model to achieve good estimation, and there a two-dimensional yo-yo model, in which the fluctuation of the axis diameter caused by rolling up the yo-yo's string, was derived by projection method. The effectiveness of the proposed method was verified trough some numerical simulations with modeling errors and measurement disturbances, and by an experimental system.

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