

# Improving Industrial MPC Performance with Data-Driven Disturbance Modeling

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**Abstract**—Industrial model predictive control (MPC) usually assumes a step-like disturbance model, which is insufficient when there is model mismatch in the plant or high order disturbances. In this paper, we demonstrate that a disturbance model identified from close-loop data is desirable for dynamic matrix control (DMC). We introduce a subspace based method to obtain such a model. The method estimates Markov parameters of the disturbance model using closed-loop data along with known input-output model information in the DMC controller. Simulation results are given to compare the proposed approach with traditional DMC.

## I. INTRODUCTION

Model predictive control (MPC) is a widely used control technique in process industry. One of the first MPC implementations was developed by Cutler and Ramaker [1] as dynamic matrix control (DMC). The DMC algorithm makes predictions of future outputs using a step response model. The DMC approach was then extended to quadratic DMC (QDMC) [2] to handle constraints in a quadratic programming (QP) problem. Recent survey shows that DMC based MPC products still play an important role in industrial applications [3].

As a model-based control method, MPC relies heavily on the accuracy of the model. Either plant model mismatch or unmodeled disturbances may degrade the control performance. DMC and QDMC uses a step-like disturbance model to achieve offset-free control. Muske and Badgwell [4] proposed a block diagonal disturbance model to include both state and output disturbances. Pannocchia and Rawlings [5] provided a more general disturbance model without special structures. However, the filter gain of the state estimator is predetermined. To avoid the need to specify the disturbances at the input or the output side, the equivalence between different disturbance models are shown in [6]. Recently, Xu et al. [7] proposed an adaptive disturbance modeling method, but it imposes a special structure on the Kalman filter.

Subspace identification (SID) has drawn tremendous interests in the last two decades. Several representative algorithms have been developed, including CVA [8], N4SID [9] and MOESP [10]. The unifying theorem by Van Overschee and

De Moor [11] provides a framework in which these algorithms can be interpreted as a singular value decomposition of a weighted matrix. These algorithms have been widely used for the open-loop systems because of the advantage of simple parametrization. However, due to the correlation between the current input and the past output caused by feedback, the application of most SID methods to the closed-loop data typically gives biased estimation. To address this problem, The SID methods for the closed-loop system were investigated and developed in the last decade. Qin and Ljung developed the closed-loop SID algorithm by innovation estimation [12]. In [13], the pre-estimation of the Markov parameters are proposed to avoid the non-causal projection. The statistical consistency is studied in [14].

Most closed-loop subspace identification methods involve three stages [15]. The first step is to identify a high order ARX model from properly collected input and output data. Providing the observer Markov parameter estimation for high order ARX model, this step by itself is a non-parametric procedure.

Motivated by DMC and subspace estimation of non-parametric models, we propose a new DMC technique with a disturbance model identified by the subspace method using closed loop data. The observer Markov parameters of the plant and disturbance model are obtained. Then, system Markov parameters of the disturbance model is computed recursively. Moreover, we propose a new recursive relationship utilizing the plant model information to better estimate the disturbance model. A more accurate DMC prediction is then made possible by the identified disturbance model. This Scheme can be implemented adaptively as needed.

## II. DYNAMIC MATRIX CONTROL

### A. Plant model in step response form

DMC algorithm uses a step response model of the plant to predict future outputs. Denote  $y_t \in \mathbb{R}^{n_y}$  and  $u_t \in \mathbb{R}^{n_u}$  as the output and input vector of a MIMO plant respectively. The relationship inputs and outputs can be written as [2]

$$y(t) = \sum_{i=1}^N S_i \Delta u(t-i) + y_0 + d(t) \quad (1)$$

where  $y_0$  is the output initial condition,  $d(t)$  denotes unmodelled disturbances  $\Delta u(j) = u(j) - u(j-1)$ ,  $N$  is the number of steps for plant to reach steady-state, and  $S_i$  is the  $i$ th step

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response matrix

$$S_i = \begin{pmatrix} s_{1,1,i} & s_{1,2,i} & \cdots & s_{1,n_u,i} \\ s_{2,1,i} & s_{2,2,i} & \cdots & s_{2,n_u,i} \\ \vdots & \vdots & \ddots & \vdots \\ s_{n_y,1,i} & s_{n_y,2,i} & \cdots & s_{n_y,n_u,i} \end{pmatrix} \quad (2)$$

in which  $s_{j,k,i}$  represents the  $i$ th step response coefficient from the  $k$ th input to the  $j$ th output. The  $k$ -step-ahead prediction can be made by

$$\hat{y}(t+k|t) = y(t) + \sum_{i=1}^k S_i \Delta u(t+k-i) + \hat{d}(t+k|t) \quad (3)$$

where  $\hat{y}(t+k|t)$  and  $\hat{d}(t+k|t)$  denotes the prediction of  $y(t+k)$  and unmodelled disturbances given the information up to time  $t$ , respectively,

The step response model and finite impulse response (FIR) model can be used interchangeably:

$$\begin{pmatrix} S_1 \\ S_2 \\ \vdots \\ S_P \end{pmatrix} = \begin{pmatrix} 1 & & & \\ 1 & 1 & & \\ \vdots & \vdots & \ddots & \\ 1 & & \cdots & 1 \end{pmatrix} \begin{pmatrix} G_1^p \\ G_2^p \\ \vdots \\ G_P^p \end{pmatrix} \quad (4)$$

where  $G_1^p, \dots, G_P^p$  are the impulse response coefficients, also known as system Markov parameters, of the plant model.

### B. DMC predictions and offset-free control

From (3), the predicted outputs can be rewritten as

$$\hat{y}(t+k|t) = \sum_{i=1}^k S_i \Delta u(t+k-i) + \sum_{i=k+1}^P S_i \Delta u(t+k-i) + y_0 + \hat{d}(t+k|t) \quad (5)$$

where  $P$  indicates the prediction horizon. Define

$$y^*(t+k) = y_0 + \sum_{i=k+1}^P S_i \Delta u(t+k-i)$$

which is the effect of past inputs, and stack  $\hat{y}(t+i|t)$ , ( $i = 1, \dots, P$ ) using (5):

$$\begin{pmatrix} \hat{y}(t+1|t) \\ \vdots \\ \hat{y}(t+P|t) \end{pmatrix} = \begin{pmatrix} y^*(t+1) \\ \vdots \\ y^*(t+P) \end{pmatrix} + A_P^M \begin{pmatrix} \Delta u(t) \\ \vdots \\ \Delta u(t+M-1) \end{pmatrix} + \begin{pmatrix} \hat{d}(t+1) \\ \vdots \\ \hat{d}(t+P) \end{pmatrix} \quad (6)$$

where  $M$  is the control horizon and

$$A_P^M = \begin{pmatrix} S_1 & 0 & \cdots & 0 \\ S_2 & S_1 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ S_M & S_{M-1} & \cdots & S_1 \\ \vdots & \vdots & & \vdots \\ S_P & S_{P-1} & \cdots & S_{P-M+1} \\ \vdots & \vdots & & \vdots \\ S_P & S_P & \cdots & S_P \end{pmatrix} \quad (7)$$

is called the dynamic matrix of the system.

DMC assumes a step disturbance model and estimate the step size using the difference between the current measured and estimated output, i.e.,

$$\hat{d}(t+i|t) = y(t) - \hat{y}(t|t-1), \quad i = 1, \dots, P \quad (8)$$

This disturbance model is integrating and is able to achieve offset-free control for DMC.

### C. PLANT MODEL IN STATE-SPACE FORM

We use the state-space form of plant model which will be useful in the derivations later. Assume the plant is described by

$$\begin{cases} x_p(t+1) = A_p x_p(t) + B_p u(t) \\ y_p(t) = C_p x_p(t) \end{cases} \quad (9)$$

which is considered to be deterministic and the disturbance model is written in the following innovation form of Kalman filter:

$$\begin{cases} \hat{x}_d(t+1) = A_d \hat{x}_d(t) + K_d e(t) \\ y_d(t) = C_d \hat{x}_d(t) + e(t) \end{cases} \quad (10)$$

which is considered to be stochastic. When the step-like disturbance model in DMC is assumed,  $A_d = I_{n_y}$ ,  $K_d = I_{n_y}$ , and  $C_d = I_{n_y}$ ; this type of disturbance model corresponds to the case that disturbance is integrated white noise.

By augmenting (9) and (10), we obtain

$$\begin{cases} \hat{x}(t+1) = A_{\text{aug}} \hat{x}(t) + B_{\text{aug}} u(t) + K_{\text{aug}} e(t) \\ y(t) = C_{\text{aug}} \hat{x}(t) + e(t) \end{cases} \quad (11)$$

where

$$\begin{aligned} A_{\text{aug}} &= \begin{pmatrix} A_p & 0 \\ 0 & A_d \end{pmatrix}, & B_{\text{aug}} &= \begin{pmatrix} B_p \\ 0 \end{pmatrix} \\ C_{\text{aug}} &= (C_p \quad C_d), & K_{\text{aug}} &= (0 \quad K_d)^T. \end{aligned} \quad (12)$$

The subscript aug refers to augmented system.

Clearly, the augmented Kalman gain with the structure  $K_{\text{aug}} = (0 \quad K_d)^T$  is insufficient when there exists plant model mismatch or disturbance model mismatch. With the help of closed loop data and subspace identification, it is possible to estimate a complex disturbance model to improve the model prediction and control performance. Instead, a full Kalman gain  $K_{\text{aug}} = (K_p \quad K_d)^T$  will be used. We propose a method to estimate disturbance model from closed-loop data and a revised form of DMC that includes disturbance model.

## III. SUBSPACE IDENTIFICATION OF A DISTURBANCE MODEL

### A. Identification of the observer Markov parameters

In this section, we develop a method for identifying the disturbance model in the closed-loop subspace identification framework. Most closed-loop subspace identification methods involves three stages:

Stage 1: Perform high-order ARX to give observer Markov parameter estimates, which is a non-parametric procedure.

Stage 2: Perform SVD on the Hankel matrix or weighted Hankel matrix, leading to extended observability matrix  $\bar{\Gamma}_f$  and extended controllability  $\bar{L}_p$  estimates, which by themselves are reduced order ( $n$ ) non-parametric models to estimate the state and predict the output.

Stage 3: Perform least squares to estimate  $(\bar{A}, B, C, K)$  from  $\bar{\Gamma}_f$  and  $\bar{L}_p$  leading to state-space parametric models.

The Stage 1 of the closed-loop subspace identification gives the observer Markov parameter estimation in the least-square sense. Collect the sequence of Markov parameters in the following form:

$$\begin{aligned}\bar{G} &= (C_{\text{aug}}\bar{A}_{\text{aug}}^{l-1}B_{\text{aug}} \quad C_{\text{aug}}\bar{A}_{\text{aug}}^{l-2}B_{\text{aug}} \quad \cdots \quad C_{\text{aug}}B_{\text{aug}}) \\ \bar{H} &= (C_{\text{aug}}\bar{A}_{\text{aug}}^{l-1}K \quad C_{\text{aug}}\bar{A}_{\text{aug}}^{l-2}K \quad \cdots \quad C_{\text{aug}}K)\end{aligned}$$

and

$$\begin{aligned}G &= (C_{\text{aug}}A_{\text{aug}}^{l-1}B_{\text{aug}} \quad C_{\text{aug}}A_{\text{aug}}^{l-2}B_{\text{aug}} \quad \cdots \quad C_{\text{aug}}B_{\text{aug}}) \\ H &= (C_{\text{aug}}A_{\text{aug}}^{l-1}K \quad C_{\text{aug}}A_{\text{aug}}^{l-2}K \quad \cdots \quad C_{\text{aug}}K)\end{aligned}$$

where  $l$  denotes the size of the past horizon and  $\bar{A}_{\text{aug}} = A_{\text{aug}} - K_{\text{aug}}C_{\text{aug}}$ . Without loss of generality, we set  $\bar{H}$  to be monic, i.e.,  $H_0 = I_{n_y}$ . The consistent identification of  $G$  and  $H$  is to solve the following equation in by the least-squares,

$$Y_t = C_{\text{aug}}\bar{A}_{\text{aug}}^l X_{t-l} + GU_{t-l|t-1} + HY_{t-l|t-1} + E_t \quad (13)$$

where

$$Y_t = (y(t) \quad \cdots \quad y(t+N-1)), \quad (14)$$

$$E_t = (e(t) \quad \cdots \quad e(t+N-1)), \quad (15)$$

and  $X_{t-l} = (\hat{x}(t-l) \quad \cdots \quad \hat{x}(t+N-l))$  is the sequence of the initial states. We introduce the assumption that  $\bar{A}_{\text{aug}}$  is stable and  $l$  is chosen to be sufficiently large,  $\bar{A}_{\text{aug}}^l \approx 0$ . Then we have the following least squares estimation,

$$\begin{aligned}(\hat{\bar{G}} \quad \hat{\bar{H}}) &= \\ Y_t \begin{pmatrix} U_{t-l|t-1} \\ Y_{t-l|t-1} \end{pmatrix}^T \left[ \begin{pmatrix} U_{t-l|t-1} \\ Y_{t-l|t-1} \end{pmatrix} (U_{t-l|t-1} \quad Y_{t-l|t-1}) \right]^{-1}\end{aligned} \quad (16)$$

where

$$U_{t-l|t-1} = \begin{pmatrix} u(t-l) & u(t-l+1) & \cdots & u(t-l+N-1) \\ \vdots & \vdots & & \vdots \\ u(t-1) & u(t) & \cdots & u(t+N-2) \end{pmatrix}$$

and

$$Y_{t-l|t-1} = \begin{pmatrix} y(t-l) & y(t-l+1) & \cdots & y(t-l+N-1) \\ \vdots & \vdots & & \vdots \\ y(t-1) & y(t) & \cdots & y(t+N-2) \end{pmatrix}.$$

$\hat{\bar{G}}$  and  $\hat{\bar{H}}$  are asymptotically unbiased estimate of the true observer Markov parameters since the innovations term in (13) is uncorrelated to the past input and output data.

## B. System Markov parameters of the disturbance model

In order to apply the estimated system Markov parameters for the disturbance model of MPC, the estimated observer Markov parameters can be converted to the system Markov parameters based on the recursive relationship [16]. Define the estimated observer Markov parameters

$$\hat{\bar{G}} = (\bar{G}_{l-1} \quad \bar{G}_{l-2} \quad \cdots \quad \bar{G}_0), \quad (17)$$

$$\hat{\bar{H}} = (\bar{H}_{l-1} \quad \bar{H}_{l-2} \quad \cdots \quad \bar{H}_0). \quad (18)$$

1) Using  $\hat{\bar{H}}$  to estimate  $H$ : Knowledge of the estimated observer Markov parameters allows one to obtain the estimated open-loop Markov parameters by using the recursion [16]

$$H_i = \bar{H}_i + \sum_{j=1}^{i-1} \bar{H}_j H_{i-j-1}, \quad i = 0, \dots, l-1. \quad (19)$$

2) Using  $G$  and  $\hat{\bar{G}}$  to estimate  $H$ : To better utilize the information of plant model  $G^p$  in DMC, we will show the recursive relationship between  $H_i$ ,  $G$  and  $\hat{\bar{G}}$ .

Equation (5) provides the knowledge of the system Markov parameter of the process model, which are  $G_i^p = C_p A_p^{i-1} B_p$ . Given the structure of the augmented system (12),

$$\begin{aligned}G_i &= C_{\text{aug}} A_{\text{aug}}^{i-1} B_{\text{aug}} \\ &= (C_p \quad C_d) \begin{pmatrix} A_p & 0 \\ 0 & A_d \end{pmatrix}^{i-1} \begin{pmatrix} B_p \\ 0 \end{pmatrix} \\ &= C_p A_p^{i-1} B_p \\ &= G_i^p \quad i = 1, \dots, l-1\end{aligned}$$

which implies that the system Markov parameters of the augmented model equals that of the plant model. Then,  $\bar{H}$  can be determined by

$$G_i = \bar{G}_i + \sum_{j=0}^{i-1} \bar{H}_j G_{i-j-1}, \quad i = 1, \dots, l-1. \quad (20)$$

Therefore,  $H$  can be estimated by using  $\bar{H}$  from (20) along with (19). Note that either method has no approximation; the quality of estimated  $H$  solely depends on how good the estimated observer Markov parameter are.

From either (19) or (20), an FIR model of disturbance for DMC can be built using the estimated system Markov parameters. This procedure is non-parametric and convenient for use in MPC calculations.

## IV. DMC WITH MOVING AVERAGE DISTURBANCE MODEL

In this section, we will discuss how to implement DMC with the information of disturbance model. Although in the last section there is no assumption on  $A_d$  other than that  $\bar{A}_d$  is stable, one may still desire to set  $A_d = I_{n_y}$  because offset-free control can be achieved by augmenting the system state with an integrating disturbance model [17].

It is convenient to differentiate input and output data in order to identify an integrating disturbance model. Converting step response model of the plant to impulse response model, (1) becomes

$$y(t) = \sum_{i=1}^N G_i^p u(t-i) + y_0 + \sum_{i=0}^N H_i e(t-i). \quad (21)$$

Differentiation of (21) gives

$$\Delta y(t) = \sum_{i=1}^N G_i^p \Delta u(t-i) + \sum_{i=0}^N \Delta H_i e(t-i). \quad (22)$$

In this model, the differentiated disturbance model  $\Delta H_i$  is stable with its integrators cancelled by the differentiation. Thus, we can directly identify  $\Delta H_i$  by applying the method introduced in the last section.

In order to calculate predictions, integrating (22), we obtain

$$y(t) = \sum_{i=1}^N S_i \Delta u(t-i) + y_0 + \sum_{i=0}^N H_i e(t-i) + d_0 \quad (23)$$

where  $d_0$  is the initial condition for disturbance. Then,  $k$ -step-ahead prediction can be expressed as

$$\hat{y}(t+k|t) = \sum_{i=1}^k S_i \Delta u(t+k-i) + \sum_{i=k+1}^P S_i \Delta u(t+k-i) + y_0 + \hat{d}(t+k|t) \quad (24)$$

$$\hat{d}(t+k|t) = \sum_{i=0}^N H_i e(t-i) + d_0. \quad (25)$$

The optimal input moves can be optimized by QP problem with the prediction model (24) and (25).

*Remark 1:* Unlike traditional DMC,  $\hat{d}(t+k|t)$  usually does not equal  $\hat{d}(t+1|t)$  for  $k > 1$ , unless there is no model mismatch and disturbances can be modeled as integrated moving average (IMA) (1,1). The reason is that with the mismatch, the Kalman gain for plant model  $K_p$  is nonzero. Thus, the system Markov parameters of disturbance model are

$$\begin{aligned} H_i &= C_{\text{aug}} A_{\text{aug}}^{i-1} K_{\text{aug}} \\ &= (C_p \quad C_d) \begin{pmatrix} A_p & 0 \\ 0 & A_d \end{pmatrix}^{i-1} \begin{pmatrix} K_p \\ K_d \end{pmatrix} \\ &= C_p A_p^{i-1} K_p + C_d K_d \end{aligned} \quad (26)$$

where the second term remains constant while the first term does not. Therefore, it can be inferred that the proposed non-parametric disturbance model is more general than the step-like one used in DMC.

## V. SIMULATION

In the simulation, we use a SISO plant described by the following system matrices.

$$\begin{aligned} A &= \begin{pmatrix} 0.6 & 0.3 \\ 0 & 0.2 \end{pmatrix}, & B &= \begin{pmatrix} 0 \\ 1 \end{pmatrix} \\ C &= (1 \quad 1), & D &= 0. \end{aligned} \quad (27)$$

We generate 10000 data points using an arbitrary MPC in MATLAB. The setpoint is selected to be 0, and a PRBS signal is added to input signal to improve closed-loop system identifiability. The size of past horizon in subspace identification is set as 30.

To compare the proposed control scheme with DMC, we run both controllers with 1000 steps. The prediction and control horizon are chosen as  $P = 15$  and  $M = 4$ . We put zero weights on input and input move and unit weight on output so that output variance can be served as performance metric. The setpoint of reference signal is always 0.

### A. Case I: no mismatch in plant model, subject to IMA(1,1) disturbance

In this case, we assume there is no mismatch of the plant model. The additive output disturbance is generated by

$$d(t) = \frac{1 - 0.3q^{-1}}{1 - q^{-1}} e(t) \quad (28)$$

where  $e(t) \sim N(0, 1)$ . Due to the integrator, the disturbance sequence is non-stationary.

The integrating disturbance model  $A_d = I$  with  $K_d$  is sufficient for (28). Fig. 1 shows the estimated system Markov parameters  $H_i$ . It can be seen that since there is no mismatch,  $H_i$  ( $i = 1, \dots, 30$ ) is flat. The values is quite close to the theoretical one  $H_i = C_d K_d = 0.7$ .

The output variance of traditional DMC is 1.0925, while the output variance of DMC with disturbance model is 0.9989. Since both controllers shares the same model, the difference in control performance is caused by  $K_d$ .

### B. Case II: no plant model mismatch, subject to ARIMA(1,1,1) disturbance

Now consider the disturbance to be more complicated:

$$d(t) = \frac{1 - 0.3q^{-1}}{1 - q^{-1}} \frac{1}{1 + 0.5q^{-1}} e(t) \quad (29)$$

which has an additional first-order filter compared to (28). In this case,  $K_p$  will no longer be zero, and  $H_i$  will be varying in accordance with (26).

Estimated system Markov parameters of disturbance model is demonstrated by Fig. 2. One can observe that unlike Case I, the system Markov parameters do not reach its final value in one step. This is because first term on the right hand side of (26) is nonzero.

The output variance of traditional DMC is 1.8684, whereas the output variance of DMC with disturbance model is 1.4965. If compared to the control results in Case I, DMC with disturbance model has a larger output variance, which is the same as variance of prediction errors when minimum variance control is applied. This shows that reduced order modeling of disturbances tends to yield larger prediction errors.

### C. Case III: with plant model mismatch, subject to ARIMA(1,1,1) disturbance

We further add plant model mismatch to the simulation. Let the model (27) be multiplied by  $1.2 \times (1 - 0.4q^{-1}) / (1 - 0.5q^{-1})$ . It is then converted to step disturbance model and fed into DMC. Then, the system matrices of plant model can be described by

$$A = \begin{pmatrix} 0.6 & 0.3 & 0 \\ 0 & 0.2 & 0.4 \\ 0 & 0 & 0.5 \end{pmatrix}, \quad B = \begin{pmatrix} 0 \\ 1.2 \\ 0.3 \end{pmatrix} \\ C = (1 \quad 1 \quad 0), \quad D = 0.$$

whose order is higher than that of the plant. There is a pole at  $q = 0.5$  in addition to the poles  $q = 0.6$  and  $q = 0.2$  of the plant. The disturbance to be added is still (29).

In this work, our goal is to find the best non-parametric disturbance model that accommodates the existing DMC model. To achieve best control performance, an alternative approach is to substitute both plant and disturbance models from close-loop identification results. We make a comparison of both approaches in this case.

- a) Disturbance model identified by the proposed approach is first shown in Fig. 3. It is observed that due to the model mismatch  $H_i$  oscillates less. This implies that the mismatch has been modeled into the disturbance part. The output variance of traditional DMC is 1.5888, compared to 1.2482 of DMC with disturbance model.
- b) Fig. 4 provides the Markov parameters of the identified plant and disturbance models. The output variance of the process controlled by the MPC using identified process and disturbance models is 1.3950, which is greater than the proposed method only updating disturbance model. After carefully checking the identified model, it is found that the MATLAB n4sid method, which is capable of performing closed-loop subspace identification, has problem in determining the order of augmented system. A second-order is estimated, as one can observe in Fig. 4, while the true augmented system is 5-th order. The reduced-order model identified by traditional SID methods is unable to capture some dynamics in the true model. This problem can be even worse, when the process has considerably higher order than the disturbance model does.

Table I summarizes the control performance of both controllers in all three cases. It can be concluded from the comparison that DMC with non-parametric disturbance model is able to compensate the performance degradation caused by plant model mismatch or high order disturbances.

## VI. CONCLUSIONS

In this paper, we proposed a subspace method for estimating an integrated moving average disturbance model for MPC from closed-loop data. In contrast to the traditional subspace identification method, the proposed approach does not focus on system matrices  $A$ ,  $B$ ,  $C$ , which are known in the MPC model. Instead, we try to utilize closed loop data and the model information in MPC to obtain or update

the disturbance model. Estimation of system matrices and the Kalman gain, which is usually an intermediate step, is then avoided. We further incorporate this non-parametric disturbance model with DMC, which typically uses a (non-parametric) step response plant model. The simulation results demonstrate that DMC with the estimated disturbance model is able to compensate model mismatch and outperforms traditional DMC that uses a step-like disturbance model.

## VII. ACKNOWLEDGEMENTS

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TABLE I  
CONTROL PERFORMANCE OF DIFFERENT DMC SCHEMES

	Output variance		
	Traditional DMC	DMC w/ dist. mdl.	DMC w/ proc. & dist. mdl.
Case I	1.0925	0.9989	N/A
Case II	1.8684	1.4965	N/A
Case III	1.5888	1.2482	1.3950

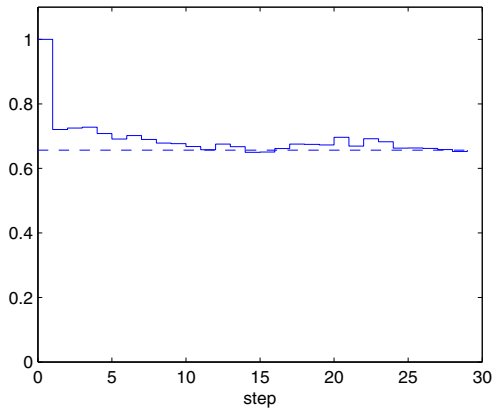


Fig. 1. Identified system Markov parameters of the disturbance model in case I.

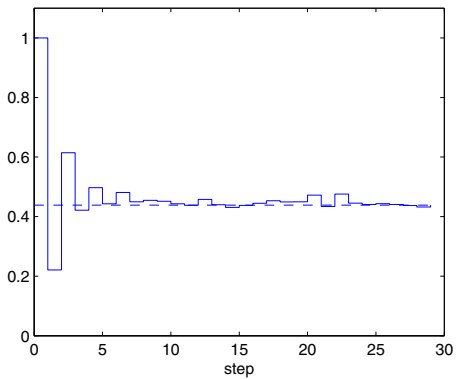


Fig. 2. Identified system Markov parameters of the disturbance model in case II.

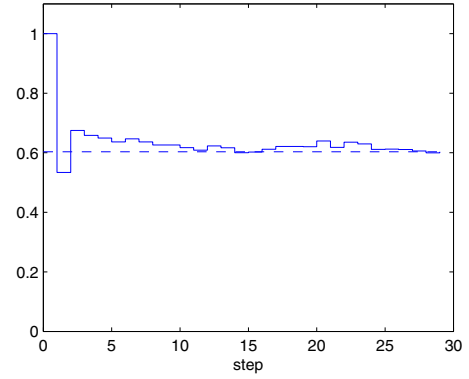


Fig. 3. Identified system Markov parameters of the disturbance model in case III a).

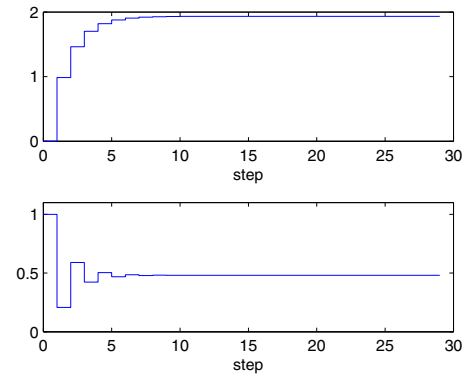


Fig. 4. Identified system Markov parameters of the plant and disturbance model in case III b). The upper one is Markov parameters of the plant model, and the lower one is the Markov parameters of the disturbance model.