2011 50th IEEE Conference on Decision and Control and
European Control Conference (CDC-ECC)
Orlando, FL, USA, December 12-15, 2011

Discrete-Time Gain-Scheduled Output-Feedback Controllers Exploiting Inexact Scheduling Parameters via Parameter-Dependent Lyapunov Functions

Masayuki Sato

Abstract— **This paper addresses the design problem of Gain-Scheduled Output-Feedback (GSOF) controllers for discrete-time Linear Parameter-Varying (LPV) systems via Parameter-Dependent Lyapunov Functions (PDLFs). In our problem setting, it is supposed that scheduling parameters are provided with some uncertainties. In this practical problem setting, we give a sufficient condition for designing GSOF controllers exploiting the uncertain scheduling parameters which minimize the induced l_2-norm in terms of Parameter-Dependent Bilinear Matrix Inequalities (PDBMIs). Two algorithms for solving the PDBMIs, i.e. line search and iterative algorithms, are shown. Under the ideal situation, i.e. exact values of scheduling parameters are available, our method then recovers the conventional design method via PDLFs.**

I. INTRODUCTION

It is widely known that Gain-Scheduling (GS) strategy is a promising method to address the changes of plant dynamics, and this strategy has widely been applied to many real systems, e.g. aircraft, missile [1], etc. Thus, a lot of papers tackling the design problem of GS Output-Feedback (GSOF) controllers for Linear Parameter-Varying (LPV) systems have been reported, e.g. [2]–[7] and references therein. Most of those papers suppose that scheduling parameters are exactly available; however this assumption does not always hold true in real systems. In other words, the scheduling parameters are usually provided with some uncertainties. Several researchers have already tackled this issue and some design methods of GSOF controllers exploiting uncertain scheduling parameters have successfully been proposed [8]–[11].

Most of the foregoing papers address the design problem of GSOF controllers for continuous-time LPV systems. This consequently means that the designed GSOF controllers are also continuous-time systems. On the other hand, digital computers are widely used for controlling real systems. Thus, if the designed GSOF controllers are continuous-time, it is required to discretize the GSOF controllers in some manner. On this issue, a good paper has already been published [12], in which trapezoidal approximation has been proposed. In the approximation, it is supposed that $I - \frac{T_s}{2} A_c(\theta)$, where T_s and $A_c(\theta)$ respectively denote the sampling time and the continuous-time state-space matrix of the designed gain-scheduled controller, is non-singular. One of the simplest methods to satisfy this condition is to impose the poles of the controller matrix to lie in some specific region. However, additional requirements for controllers generally increase conservatism. Thus, designing discrete-time GSOF controllers directly for discrete-time LPV systems is another promising method from a viewpoint of controller implementation. The author has already proposed a design method along this approach [13], in which Parameter-inDependent Lyapunov Functions (PiDLFs) are used.

It is widely known that Parameter-Dependent Lyapunov Functions (PDLFs) produce less conservative controllers than using PiDLFs when scheduling parameters vary with bounded rates. Thus, PDLFs are recommended for designing GSOF controllers, as in [3]–[5], [7], [9].

Considering these, this paper tackles the design problem of discrete-time GSOF controllers for discrete-time LPV systems via PDLFs under the condition that only inexact scheduling parameters are available. Using the design method in [14], which is an extension of [15] to LPV systems, we propose a design method of GSOF controllers exploiting uncertain scheduling parameters via PDLFs.

In this note, we use the following notations: $\mathsf{He}\{X\}$ is a shorthand notation for $X + X^T$, $\mathbf{I}_n$, $\mathbf{I}$ and $\mathbf{0}$ respectively denote an $n \times n$ dimensional identity matrix, an identity matrix and a zero matrix of appropriate dimensions, $\mathbb{Z}_+$ denotes the set of non-negative integers, $\mathbb{R}^{n\times m}$ and $\mathbb{S}^n$ respectively denote sets of $n \times m$ dimensional real matrices and $n \times n$ dimensional symmetric real matrices, $*$ denotes an abbreviated off-diagonal block in a symmetric matrix, and $\mathsf{diag}(X_1, \cdots, X_k)$ denotes a block-diagonal matrix composed of $X_1, \cdots, X_k$.

II. PRELIMINARIES

A. System Definitions

We consider the following discrete-time LPV generalized plant $G(\theta)$ with l independent scalar parameters $\theta(k) = [\theta_1(k) \ \cdots \ \theta_l(k)]^T$:

$$G(\theta) : \begin{cases} x(k+1) = A(\theta(k))x(k) + B_1(\theta(k))w(k) \\ \qquad\qquad\quad + B_2 u(k) \\ z(k) \quad = C_1(\theta(k))x(k) + D_{11}(\theta(k))w(k) \\ \qquad\qquad\quad + D_{12}(\theta(k))u(k) \\ y(k) \quad = C_2 x(k) + D_{21}(\theta(k))w(k) \end{cases}, \tag{1}$$

where $x(k) \in \mathbb{R}^n$, $w(k) \in \mathbb{R}^{n_w}$, $u(k) \in \mathbb{R}^{n_u}$, $z(k) \in \mathbb{R}^{n_z}$ and $y(k) \in \mathbb{R}^{n_y}$ are respectively the state with $x(0) = \mathbf{0}$, the

This work was supported by the Ministry of Education, Culture, Sports, Science and Technology of Japan under Grant-in-Aid for Young Scientists (B) No. 23760398.

M. Sato is with the Institute of Space Technology and Aeronautics, Japan Aerospace Exploration Agency, Mitaka, Tokyo 181-0015, Japan sato.masayuki@jaxa.jp

978-1-61284-799-3/11/$26.00 ©2011 IEEE

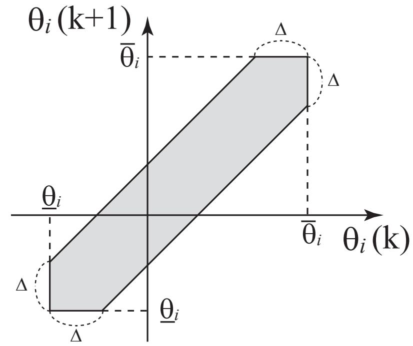

Fig. 1. Example of admissible region Λ_θ (Δ denotes the maximum deviation of θ for one sampling step, and $\overline{\theta}_i$ and $\underline{\theta}_i$ respectively denote the maximum and minimum values of θ_i)

disturbance input, the control input, the performance output and the measurement output at step k with $k \in \mathbb{Z}_+$. The parameters θ_i, which represent changes of plant dynamics, are supposed to be time-varying (possibly time-invariant).

As indicated in (1), the matrices B_2 and C_2 are set to be constant. This assumption slightly restricts the applicability of the exposed method hereafter; however, if strictly proper Linear Time-Invariant (LTI) filters are applied to the original signals $u(k)$ and $y(k)$ then this assumption is satisfied, similarly to a continuous-time case [2].

The state-space matrices in (1) are supposed to have compatible dimensions and to be polynomial with respect to $\theta_i(k)$. The parameters $\theta_i(k)$ are supposed to lie in a hyper-rectangle Ω_θ which is known in advance: $\theta(k) \in \Omega_\theta,\ \forall k \in \mathbb{Z}_+$. Similarly to [7], the parameter deviation for one sampling step, i.e. $\theta(k+1) - \theta(k)$, is also supposed to be bounded. The admissible region for $(\theta(k), \theta(k+1))$ is supposed to be given as a polytope and is denoted by Λ_θ. (See also Fig. 1.)

For the LPV system $G(\theta)$ in (1), we consider a parameter-dependent full-order GSOF controller exploiting the available scheduling parameters. The scheduling parameters are supposed to be available with some uncertainties; that is, the i-th scheduling parameter at step k is provided as $\theta_i(k)+\delta_i(k)$ with an associated uncertainty $\delta_i(k)$. The vector $\delta(k) = [\delta_1(k) \cdots \delta_l(k)]^T$ represents the uncertainties in the provided scheduling parameters. It is supposed that the uncertainties δ_i are independent from each other. The GSOF controller $K(\theta+\delta)$ to be designed is defined as follows:

$$K(\theta+\delta): \begin{cases} x_K(k+1) = A_K(\theta(k)+\delta(k))x_K(k) \\ \qquad\qquad\quad + B_K(\theta(k)+\delta(k))y(k) \\ \quad u(k) \quad = C_K(\theta(k)+\delta(k))x_K(k) \\ \qquad\qquad\quad + D_K(\theta(k)+\delta(k))y(k) \end{cases}, \quad (2)$$

where $x_K(k) \in \mathbb{R}^n$ denotes the state at step k with $x_K(0) = \mathbf{0}$, and matrices $A_K(\theta(k)+\delta(k))$, etc. are appropriately dimensioned parameter-dependent matrices to be designed.

The uncertainties $\delta_i(k)$ are supposed to lie in a hyper-rectangle Ω_δ which is known in advance: $\delta(k) \in \Omega_\delta,\ \forall k \in \mathbb{Z}_+$.

The closed-loop system $G_{cl}(\theta, \theta+\delta)$ comprising $G(\theta)$ and $K(\theta+\delta)$ is given as follows:

$$G_{cl}(\theta, \theta+\delta): \begin{cases} x_{cl}(k+1) = A_{cl}(\theta(k), \theta(k)+\delta(k))x_{cl}(k) \\ \qquad\qquad\quad + B_{cl}(\theta(k), \theta(k)+\delta(k))w(k) \\ \quad z(k) \quad = C_{cl}(\theta(k), \theta(k)+\delta(k))x_{cl}(k) \\ \qquad\qquad\quad + D_{cl}(\theta(k), \theta(k)+\delta(k))w(k) \end{cases}, \quad (3)$$

where $x_{cl}(k) = \left[x(k)^T\ x_K(k)^T\right]^T$, and

$$\begin{aligned} & A_{cl}(\theta(k), \theta(k)+\delta(k)) \\ &= \begin{bmatrix} A(\theta(k)) + B_2 D_K(\theta(k)+\delta(k))C_2 & B_2 C_K(\theta(k)+\delta(k)) \\ B_K(\theta(k)+\delta(k))C_2 & A_K(\theta(k)+\delta(k)) \end{bmatrix}, \\ & B_{cl}(\theta(k), \theta(k)+\delta(k)) \\ &= \begin{bmatrix} B_1(\theta(k)) + B_2 D_K(\theta(k)+\delta(k))D_{21}(\theta(k)) \\ B_K(\theta(k)+\delta(k))D_{21}(\theta(k)) \end{bmatrix}, \\ & C_{cl}(\theta(k), \theta(k)+\delta(k)) \\ &= \big[C_1(\theta(k)) + D_{12}(\theta(k))D_K(\theta(k)+\delta(k))C_2 \\ & \qquad\qquad D_{12}(\theta(k))C_K(\theta(k)+\delta(k))\big], \\ & D_{cl}(\theta(k), \theta(k)+\delta(k)) \\ &= D_{11}(\theta(k)) + D_{12}(\theta(k))D_K(\theta(k)+\delta(k))D_{21}(\theta(k)). \end{aligned}$$

B. Problem Definition

We are now ready to define our problem.

Problem 1: Suppose that the scheduling parameters $\theta_i(k)$ are provided to the controller as $\theta_i(k) + \delta_i(k)$ with uncertainties $\delta_i(k)$. For a given positive number γ_∞, find a controller $K(\theta+\delta)$ which stabilizes $G_{cl}(\theta, \theta+\delta)$ and satisfies (4) for all combinations of admissible trajectories $(\theta(k), \theta(k+1)) \in \Lambda_\theta$ and uncertainties $\delta(k) \in \Omega_\delta$.

$$\sup_{w \in l_2, w \neq 0} \|z\|_2 \,/\, \|w\|_2 < \gamma_\infty \quad (4)$$

C. Basic Lemmas

Hereafter the step index for $\theta(k)$, i.e. k, is omitted if it is obvious, and $\theta(k+1)$ is denoted by θ^+. Similarly, the index for $\delta(k)$ is also omitted if it is obvious.

Now let us consider the case in which $\delta = \mathbf{0}$ holds. Suppose that some full-order controller as in (2) but δ being set as $\mathbf{0}$, $K(\theta)$, is given. Then, the following lemma holds.

Lemma 1: [7] For a given positive number γ_∞, if there exists a parameter-dependent matrix $X_{cl}(\theta) \in \mathbb{S}^{2n}$ such that (5) holds for all pairs $(\theta, \theta^+) \in \Lambda_\theta$, then the closed-loop system $G_{cl}(\theta)$ is asymptotically stable and satisfies (4) for all admissible trajectories $(\theta, \theta^+) \in \Lambda_\theta$.

$$\begin{bmatrix} X_{cl}(\theta) & * & * & \mathbf{0} \\ A_{cl}(\theta)X_{cl}(\theta) & X_{cl}(\theta^+) & \mathbf{0} & B_{cl}(\theta) \\ C_{cl}(\theta)X_{cl}(\theta) & \mathbf{0} & \gamma_\infty \mathbf{I}_{n_z} & D_{cl}(\theta) \\ \mathbf{0} & * & * & \gamma_\infty \mathbf{I}_{n_w} \end{bmatrix} > 0 \quad (5)$$

Applying the well-known Finsler's lemma, one gets the following lemma which is equivalent to Lemma 1, similarly to an LTI case [15].

Lemma 2: [14] For a given positive number γ_∞, if there exist parameter-dependent matrices $X_{cl}(\theta) \in \mathbb{S}^{2n}$ and $\mathcal{G}(\theta) \in \mathbb{R}^{2n\times 2n}$ such that (6) holds for all pairs $(\theta, \theta^+) \in \Lambda_\theta$, then the closed-loop system $G_{cl}(\theta)$ is asymptotically stable and satisfies (4) for all admissible trajectories $(\theta, \theta^+) \in \Lambda_\theta$.

$$\begin{bmatrix} \mathsf{He}\{\mathcal{G}(\theta)\} - X_{cl}(\theta) & * & * & \mathbf{0} \\ A_{cl}(\theta)\mathcal{G}(\theta) & X_{cl}(\theta^+) & \mathbf{0} & B_{cl}(\theta) \\ C_{cl}(\theta)\mathcal{G}(\theta) & \mathbf{0} & \gamma_\infty \mathbf{I}_{n_z} & D_{cl}(\theta) \\ \mathbf{0} & * & * & \gamma_\infty \mathbf{I}_{n_w} \end{bmatrix} > 0 \quad (6)$$

Next let us consider the case in which there exist uncertainties in the available scheduling parameters, i.e. $\delta \neq \mathbf{0}$ holds. Suppose that some full-order controller $K(\theta+\delta)$ defined in (2) is given. Considering that the controller $K(\theta+\delta)$ has additional parameters δ_i, then the following lemma is directly derived from Lemma 2.

Lemma 3: For a given positive number γ_∞, if there exist parameter-dependent matrices $X_{cl}(\theta) \in \mathbb{S}^{2n}$ and $\mathcal{G}(\theta) \in \mathbb{R}^{2n\times 2n}$ such that (7) holds for all triplets $((\theta, \theta^+), \delta) \in \Lambda_\theta \times \Omega_\delta$, then the closed-loop system $G_{cl}(\theta, \theta+\delta)$ is asymptotically stable and satisfies (4) for all combinations of admissible trajectories $(\theta, \theta^+) \in \Lambda_\theta$ and uncertainties $\delta \in \Omega_\delta$.

$$\begin{bmatrix} \mathsf{He}\{\mathcal{G}(\theta)\} - X_{cl}(\theta) & * & * & \mathbf{0} \\ A_{cl}(\theta,\theta+\delta)\mathcal{G}(\theta) & X_{cl}(\theta^+) & \mathbf{0} & B_{cl}(\theta,\theta+\delta) \\ C_{cl}(\theta,\theta+\delta)\mathcal{G}(\theta) & \mathbf{0} & \gamma_\infty \mathbf{I}_{n_z} & D_{cl}(\theta,\theta+\delta) \\ \mathbf{0} & * & * & \gamma_\infty \mathbf{I}_{n_w} \end{bmatrix} > 0 \quad (7)$$

In the next section, we propose our design method for Problem 1 using Lemma 3. However, we need to cope with the gap between $\theta+\delta$ and θ for constructing the state-space matrices of $K(\theta+\delta)$. To do that, a technical lemma is used. It is a variation on the celebrated property $\mathsf{He}\{X^T Y\} \leq X^T \mathcal{E} X + Y^T \mathcal{E}^{-1} Y,\ \forall \mathcal{E} = \mathcal{E}^T > 0$ (see [16]). Combined to a Schur complement, the result is used to approximate in a linear fashion some products of decision matrices.

Lemma 4: Suppose that a symmetric matrix Υ_0 and matrices Υ_1, Υ_2 with compatible dimensions are given. If one of the following two inequalities holds for some positive definite matrix $\mathcal{E}$ with compatible dimensions:

$$\begin{bmatrix} \Upsilon_0 & * \\ \left[\Upsilon_1\ \mathcal{E}\Upsilon_2\right] & \mathcal{E} \end{bmatrix} > 0 \quad (8)$$

$$\begin{bmatrix} \Upsilon_0 & * \\ \mathsf{diag}\left(\Upsilon_1, \mathcal{E}\Upsilon_2\right) & \mathsf{diag}\left(\mathcal{E}, \mathcal{E}\right) \end{bmatrix} > 0 \quad (9)$$

then $\Upsilon_0 - \begin{bmatrix} \mathbf{0} & * \\ \Upsilon_2^T \Upsilon_1 & \mathbf{0} \end{bmatrix} > 0$ holds.

Proof: Applying a Schur complement to (8) gives

$$\Upsilon_0 - \begin{bmatrix} \mathbf{0} & * \\ \Upsilon_2^T \Upsilon_1 & \mathbf{0} \end{bmatrix} > \begin{bmatrix} \Upsilon_1^T \mathcal{E}^{-1} \Upsilon_1 & \mathbf{0} \\ \mathbf{0} & \Upsilon_2^T \mathcal{E} \Upsilon_2 \end{bmatrix} \geq 0.$$

Thus, it proves the lemma for (8). Similarly,

$$\Upsilon_0 - \begin{bmatrix} \mathbf{0} & * \\ \Upsilon_2^T \Upsilon_1 & \mathbf{0} \end{bmatrix} > \begin{bmatrix} \Upsilon_1^T \\ -\Upsilon_2^T \mathcal{E} \end{bmatrix} \mathcal{E}^{-1} \left[\Upsilon_1\ -\mathcal{E}\Upsilon_2\right] \geq 0$$

is obtained from (9). Thus, it proves the lemma for (9). ■

III. MAIN RESULTS

As our method is based on the formulation in [14], we briefly review the design method in [14] below[1].

A candidate of the Lyapunov functions in Lemma 2 is set as $x_{cl}^T X_{cl}(\theta)^{-1} x_{cl}$ using a parameter-dependent matrix $X_{cl}(\theta) \in \mathbb{S}^{2n}$ satisfying $X_{cl}(\theta) > 0,\ \forall \theta \in \Omega_\theta$. Matrix $\mathcal{G}(\theta)$ is now set to be constant $\mathcal{G}$ with some conservatism being admitted. Then, matrix $\mathcal{G}$ and its inverse are set as $\begin{bmatrix} \mathcal{X} & Z_1 \\ \mathcal{U} & Z_2 \end{bmatrix}$ and $\begin{bmatrix} \mathcal{Y}^T & Z_3 \\ \mathcal{V}^T & Z_4 \end{bmatrix}$ respectively. Using the change-of-variables $\mathcal{K}(\theta) = \begin{bmatrix} \mathcal{V} & \mathcal{Y}B_2 \\ \mathbf{0} & \mathbf{I}_{n_u} \end{bmatrix} \bar{K}(\theta) \begin{bmatrix} \mathcal{U} & \mathbf{0} \\ C_2\mathcal{X} & \mathbf{I}_{n_y} \end{bmatrix} + \begin{bmatrix} \mathcal{Y}A(\theta)\mathcal{X} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} \end{bmatrix}$, where $\bar{K}(\theta) = \begin{bmatrix} A_K(\theta) & B_K(\theta) \\ C_K(\theta) & D_K(\theta) \end{bmatrix}$, a design method of GSOF controllers has been proposed.

Remark 1: If matrix $\mathcal{G}$ is set to be parameter-dependent $\mathcal{G}(\theta)$, then the designed GSOF controller depends on the current scheduling parameters $\theta_i(k)$ as well as future ones $\theta_i(k+1)$ [14]. Considering this property, this paper adopts constant $\mathcal{G}$ even though some conservatism is introduced.

A. Proposed Method

We propose the following theorem for Problem 1. (Some equations are given at the top of the next page.)

Theorem 1: For a given positive number γ_∞, suppose that there exist parameter-dependent symmetric matrices $\mathcal{P}(\theta), \mathcal{H}(\theta+\delta), \mathcal{E}(\theta,\delta) \in \mathbb{S}^n$, parameter-dependent matrices $\mathcal{J}(\theta) \in \mathbb{R}^{n\times n}$, $\mathcal{K}(\theta+\delta) \in \mathbb{R}^{(n+n_u)\times(n+n_y)}$, and constant matrices $\mathcal{X}, \mathcal{S}, \mathcal{Y} \in \mathbb{R}^{n\times n}$ such that (10) holds.

$$\begin{bmatrix} \Upsilon_\infty(\theta, \theta^+, \theta+\delta) & * \\ \left[\Upsilon(\theta,\delta)\ \mathbf{0}\ \mathbf{0}\right] & \Gamma(\theta,\delta) \end{bmatrix} > 0,\ \forall \left((\theta, \theta^+), \delta\right) \in \Lambda_\theta \times \Omega_\delta, \quad (10)$$

where $\Upsilon_\infty(\theta, \theta^+, \theta+\delta)$ is defined in (11) using matrices in (12), and $(\Upsilon(\theta,\delta), \Gamma(\theta,\delta))$ is any pair chosen from (13). Then, the controller $K(\theta+\delta)$ whose state-space matrices $\bar{K}(\theta+\delta)$ are given in (14), in which $A(\theta+\delta)$ denotes the matrix $A(\theta)$ in (1) with $\theta+\delta$ instead of θ and matrices $U, V \in \mathbb{R}^{n\times n}$ are non-singular matrices satisfying $VU + \mathcal{Y}\mathcal{X} = \mathcal{S}$, makes the closed-loop system $G_{cl}(\theta, \theta+\delta)$ asymptotically stable and satisfies (4) for all combinations of admissible trajectories $(\theta, \theta^+) \in \Lambda_\theta$ and uncertainties $\delta \in \Omega_\delta$.

Proof: Lemma 4 is applied to (10), then one gets the following inequality:

$$\Upsilon_\infty(\theta, \theta^+, \theta+\delta) + \begin{bmatrix} \begin{bmatrix} \mathbf{0} & \mathbf{0}\ \mathbf{0}\ * \\ \mathbf{0} & \mathbf{0}\ \mathbf{0}\ \mathbf{0} \\ \mathbf{0} & \mathbf{0}\ \mathbf{0}\ \mathbf{0} \\ \mathcal{Y}\left(A(\theta) - A(\theta+\delta)\right)\mathcal{X} & \mathbf{0}\ \mathbf{0}\ \mathbf{0} \end{bmatrix} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} \end{bmatrix} > 0. \quad (15)$$

[1] [14] considers the design problem with the direct term $D_K(\theta)$ being set to zero. Thus, a slight extension is conducted.

$$\Upsilon_\infty(\theta,\theta^+,\theta+\delta) = \begin{bmatrix} -\begin{bmatrix} \mathcal{P}(\theta) & \mathcal{J}(\theta) \\ * & \mathcal{H}(\theta) \end{bmatrix} + \mathsf{He}\left\{\begin{bmatrix} \mathcal{X} & \mathbf{I}_n \\ \mathcal{S} & \mathcal{Y} \end{bmatrix}\right\} & * & * & \mathbf{0} \\ \Upsilon_A(\theta,\theta+\delta) & \begin{bmatrix} \mathcal{P}(\theta^+) & \mathcal{J}(\theta^+) \\ * & \mathcal{H}(\theta^+) \end{bmatrix} & \mathbf{0} & \Upsilon_B(\theta,\theta+\delta) \\ \Upsilon_C(\theta,\theta+\delta) & \mathbf{0} & \gamma_\infty \mathbf{I}_{n_z} & \Upsilon_D(\theta,\theta+\delta) \\ \mathbf{0} & * & * & \gamma_\infty \mathbf{I}_{n_w} \end{bmatrix} \tag{11}$$

$$\left[\begin{array}{c|c} \Upsilon_A(\theta,\theta+\delta) & \Upsilon_B(\theta,\theta+\delta) \\ \hline \Upsilon_C(\theta,\theta+\delta) & \Upsilon_D(\theta,\theta+\delta) \end{array}\right] = \left[\begin{array}{cc|c} A(\theta)\mathcal{X} & A(\theta) & B_1(\theta) \\ \mathbf{0} & \mathcal{Y}A(\theta) & \mathcal{Y}B_1(\theta) \\ \hline C_1(\theta)\mathcal{X} & C_1(\theta) & D_{11}(\theta) \end{array}\right] + \left[\begin{array}{cc} \mathbf{0} & B_2 \\ \mathbf{I}_n & \mathbf{0} \\ \hline \mathbf{0} & D_{12}(\theta) \end{array}\right] \mathcal{K}(\theta+\delta) \left[\begin{array}{cc|c} \mathbf{I}_n & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & C_2 & D_{21}(\theta) \end{array}\right] \tag{12}$$

	$\Upsilon(\theta,\delta)$	$\Gamma(\theta,\delta)$
(a)	$\begin{bmatrix} \mathcal{E}(\theta,\delta)\mathcal{X} & \mathbf{0} & \mathbf{0} & (A(\theta+\delta)-A(\theta))^T \mathcal{Y}^T \end{bmatrix}$	$\mathcal{E}(\theta,\delta)$
(b)	$\begin{bmatrix} (A(\theta+\delta)-A(\theta))\mathcal{X} & \mathbf{0} & \mathbf{0} & \mathcal{E}(\theta,\delta)\mathcal{Y}^T \end{bmatrix}$	$\mathcal{E}(\theta,\delta)$
(c)	$\begin{bmatrix} \begin{bmatrix} \mathcal{E}(\theta,\delta)\mathcal{X} \\ \mathbf{0} \end{bmatrix} & \mathbf{0} & \mathbf{0} & \begin{bmatrix} \mathbf{0} \\ (A(\theta+\delta)-A(\theta))^T \mathcal{Y}^T \end{bmatrix} \end{bmatrix}$	$\mathsf{diag}(\mathcal{E}(\theta,\delta),\mathcal{E}(\theta,\delta))$
(d)	$\begin{bmatrix} \begin{bmatrix} (A(\theta+\delta)-A(\theta))\mathcal{X} \\ \mathbf{0} \end{bmatrix} & \mathbf{0} & \mathbf{0} & \begin{bmatrix} \mathbf{0} \\ \mathcal{E}(\theta,\delta)\mathcal{Y}^T \end{bmatrix} \end{bmatrix}$	$\mathsf{diag}(\mathcal{E}(\theta,\delta),\mathcal{E}(\theta,\delta))$

(13)

$$\bar{K}(\theta+\delta) := \begin{bmatrix} A_K(\theta+\delta) & B_K(\theta+\delta) \\ C_K(\theta+\delta) & D_K(\theta+\delta) \end{bmatrix} = \begin{bmatrix} V & \mathcal{Y}B_2 \\ \mathbf{0} & \mathbf{I}_{n_u} \end{bmatrix}^{-1} \left(\mathcal{K}(\theta+\delta) - \begin{bmatrix} \mathcal{Y}A(\theta+\delta)\mathcal{X} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} \end{bmatrix}\right) \begin{bmatrix} U & \mathbf{0} \\ C_2\mathcal{X} & \mathbf{I}_{n_y} \end{bmatrix}^{-1} \tag{14}$$

From (14), the matrix $\mathcal{K}(\theta+\delta)$ is represented as follows:

$$\mathcal{K}(\theta+\delta) = \begin{bmatrix} V & \mathcal{Y}B_2 \\ \mathbf{0} & \mathbf{I}_{n_u} \end{bmatrix} \bar{K}(\theta+\delta) \begin{bmatrix} U & \mathbf{0} \\ C_2\mathcal{X} & \mathbf{I}_{n_y} \end{bmatrix} + \mathsf{diag}\left(\mathcal{Y}A(\theta+\delta)\mathcal{X}, \mathbf{0}\right).$$

After substituting this matrix into (15), one gets

$$\mathcal{T}^T \left(\text{left hand side of (7)}\right) \mathcal{T} > 0, \tag{16}$$

where $\mathcal{T} = \mathsf{diag}\left(T, T, \mathbf{I}_{n_z}, \mathbf{I}_{n_w}\right)$ with $T = \begin{bmatrix} \mathbf{I}_n & \mathcal{Y}^T \\ \mathbf{0} & V^T \end{bmatrix}$, in a similar way to [14]. As long as matrix T is nonsingular, (16) implies that (7) holds with $X_{cl}(\theta+\delta) = T^{-T} \begin{bmatrix} P(\theta) & J(\theta) \\ * & H(\theta) \end{bmatrix} T^{-1}$ and $\mathcal{G}(\theta) = \mathcal{G} = T^{-T} \begin{bmatrix} \mathcal{X} & \mathbf{I}_n \\ \mathcal{S} & \mathcal{Y} \end{bmatrix} T^{-1}$. The nonsingularity of T is equivalent to that of V, which is always possible with slight perturbations for decision matrices $\mathcal{X}, \mathcal{Y}, \mathcal{S}$ if necessary. ■

Remark 2: Theorem 1 has four different formulations for the same assertion. At the current stage, it is not sure which is the best with respect to conservatism. However, the formulations using (a) and (b) are always better than the others with respect to the numerical complexity of the controller design process, because the row numbers of those LMIs are smaller than the others by n.

Remark 3: Theorem 1 introduces additional terms to address the gap between $A(\theta)$ and $A(\theta+\delta)$. If only either B_2 or C_2 is parameter-dependent, then we can obtain similar formulations exposed in Theorem 1. However, the term addressing the gap of those matrices is further introduced. This increases conservatism and numerical complexity. Thus, matrices B_2 and C_2 are supposed to be constant in this paper.

If we minimize γ_∞ for Theorem 1, then we can obtain optimal GSOF controllers.

B. Recovery of Conventional Design Method

Here, we claim the following: (i) When exact scheduling parameters are available our method recovers the method in [14] in which PDLFs are used but exact scheduling parameters are supposed to be available, and (ii) our method encompasses the method in [13] in which uncertainties on scheduling parameters are supposed but PiDLFs are used.

To do that, we first present a method in [14]. The following lemma is slightly extended from the result in [14].

Lemma 5: Suppose that $\delta = \mathbf{0}$ and $\Omega_\delta = \{\mathbf{0}\}$ hold. For a given positive number γ_∞, suppose that there exist parameter-dependent symmetric matrices $\mathcal{P}(\theta), \mathcal{H}(\theta) \in \mathbb{S}^n$, parameter-dependent matrices $\mathcal{J}(\theta) \in \mathbb{R}^{n\times n}, \mathcal{K}(\theta) \in \mathbb{R}^{(n+n_u)\times(n+n_y)}$, and constant matrices $\mathcal{X}, \mathcal{S}, \mathcal{Y} \in \mathbb{R}^{n\times n}$ such that $\Upsilon_\infty(\theta,\theta^+) > 0$, $\forall(\theta,\theta^+) \in \Lambda_\theta$, where $\Upsilon_\infty(\theta,\theta^+)$ is the same as in (11) but with δ being set to be $\mathbf{0}$, holds. Then, the controller $K(\theta)$ whose state-space matrices are given in (14) but with $\delta = \mathbf{0}$ makes the closed-loop system $G_{cl}(\theta)$ asymptotically stable and satisfies (4) for all admissible trajectories $(\theta,\theta^+) \in \Lambda_\theta$.

We make the following assertion.

Theorem 2: Let any performance level γ_∞, the following two propositions are equivalent:

-1- A solution $(\mathcal{P}(\theta), \mathcal{H}(\theta), \mathcal{J}(\theta), \mathcal{K}(\theta), \mathcal{X}, \mathcal{S}, \mathcal{Y})$ exists to the condition of Lemma 5.

-2- A solution $(\mathcal{P}(\theta), \mathcal{H}(\theta), \mathcal{J}(\theta), \mathcal{K}(\theta), \mathcal{X}, \mathcal{S}, \mathcal{Y}, \mathcal{E}(\theta))$, i.e. with uncertainties δ being set to zero, exists to the condition of Theorem 1.

Proof: It is obvious that if the condition of Theorem 1 holds then the condition of Lemma 5 holds with the same solution $(\mathcal{P}(\theta), \mathcal{H}(\theta), \mathcal{J}(\theta), \mathcal{K}(\theta), \mathcal{X}, \mathcal{S}, \mathcal{Y})$ of Theorem 1.

Let us next prove that *-1-* implies *-2-*. The condition $\Upsilon_\infty(\theta,\theta^+) > 0$ holds for all $(\theta,\theta^+) \in \Lambda_\theta$, i.e. on a compact set. Thus, there exists a sufficiently small positive scalar ε such that for all $(\theta,\theta^+) \in \Lambda_\theta$

$$\Upsilon_\infty(\theta,\theta^+) > \mathsf{diag}(\varepsilon\mathcal{X}^T\mathcal{X},\ \mathbf{0},\ \mathbf{0},\ \mathbf{0},\ \mathbf{0},\ \mathbf{0}). \tag{17}$$

Take $\mathcal{E}(\theta) = \varepsilon\mathbf{I}_n$, the right-hand term of (17) can be factorized either as $\begin{bmatrix} \varepsilon\mathcal{X} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} \end{bmatrix}^T (\varepsilon\mathbf{I}_n)^{-1} \begin{bmatrix} \varepsilon\mathcal{X} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} \end{bmatrix}$ or as $\begin{bmatrix} \varepsilon\mathcal{X} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} \end{bmatrix}^T \begin{bmatrix} \varepsilon\mathbf{I}_n & \mathbf{0} \\ \mathbf{0} & \varepsilon\mathbf{I}_n \end{bmatrix}^{-1} \begin{bmatrix} \varepsilon\mathcal{X} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} \end{bmatrix}$. A Schur complement applied to (17) with either these factorizations gives (10) for the two possible choices, i.e. (a) and (c), of the pair $(\Upsilon(\theta), \Gamma(\theta))$ in (13).

Similar discussion can be done for choices (b) and (d) of the pair $(\Upsilon(\theta), \Gamma(\theta))$ but starting from the existence of $\varepsilon > 0$ such that for all $(\theta, \theta^+) \in \Lambda_\theta$

$$\Upsilon_\infty(\theta, \theta^+) > \mathsf{diag}(\mathbf{0},\ \mathbf{0},\ \mathbf{0},\ \varepsilon\mathcal{Y}\mathcal{Y}^T,\ \mathbf{0},\ \mathbf{0}), \tag{18}$$

and choosing $\mathcal{E}(\theta) = \varepsilon\mathbf{I}_n$. ■

We next show an extended version of the method in [13] for using $\mathcal{E}(\theta, \delta)$.

Lemma 6: For a given positive number γ_∞, suppose that there exist a parameter-dependent symmetric matrix $\mathcal{E}(\theta, \delta) \in \mathbb{S}^n$, constant symmetric matrices $\mathcal{X}_c, \mathcal{Z}_c \in \mathbb{S}^n$, and a parameter-dependent matrix $\mathcal{K}(\theta + \delta) \in \mathbb{R}^{(n+n_u)\times(n+n_y)}$ such that (19) holds.

$$\begin{bmatrix} \Upsilon^c_\infty(\theta, \theta+\delta) & * \\ \left[\Upsilon^c(\theta, \delta)\, \mathbf{0}\, \mathbf{0}\right] & \Gamma^c(\theta, \delta) \end{bmatrix} > 0,\ \forall(\theta, \delta) \in \Omega_\theta \times \Omega_\delta, \tag{19}$$

where $\Upsilon^c_\infty(\theta, \theta + \delta)$ is defined in (20) using matrices in (12) with $\mathcal{X}$ and $\mathcal{Y}$ being respectively replaced by $\mathcal{X}_c$ and $\mathcal{Z}_c$, and $(\Upsilon^c(\theta, \delta), \Gamma^c(\theta, \delta))$ is any pair chosen from (13) with $\mathcal{X}$ and $\mathcal{Y}$ being respectively replaced by $\mathcal{X}_c$ and $\mathcal{Z}_c$. Then, the controller $K(\theta + \delta)$ whose state-space matrices $\bar{K}(\theta + \delta)$ are given in (14), in which matrices U and V are respectively set as $\mathcal{X}_c - \mathcal{Z}_c^{-1}$ and $-\mathcal{Z}_c$, makes the closed-loop system $G_{cl}(\theta, \theta + \delta)$ asymptotically stable and satisfies (4) for all combinations of admissible trajectories $\theta \in \Omega_\theta$ and uncertainties $\delta \in \Omega_\delta$.

$$\Upsilon^c_\infty(\theta, \theta+\delta) = \begin{bmatrix} \begin{bmatrix} \mathcal{X}_c & \mathbf{I}_n \\ * & \mathcal{Z}_c \end{bmatrix} & * & * & \mathbf{0} \\ \Upsilon_A(\theta, \theta+\delta) \begin{bmatrix} \mathcal{X}_c & \mathbf{I}_n \\ * & \mathcal{Z}_c \end{bmatrix} & \mathbf{0} & \Upsilon_B(\theta, \theta+\delta) \\ \Upsilon_C(\theta, \theta+\delta) & \mathbf{0} & \gamma_\infty\mathbf{I}_{n_z} & \Upsilon_D(\theta, \theta+\delta) \\ \mathbf{0} & * & * & \gamma_\infty\mathbf{I}_{n_w} \end{bmatrix} \tag{20}$$

We make the following assertion.

Theorem 3: Suppose that GSOF controllers can be designed for $G(\theta)$ by using Theorem 1 and Lemma 6, and that the optimally minimized values of γ_∞ are respectively obtained as γ_∞ and γ^c_∞. Then, $\gamma_\infty \le \gamma^c_\infty$ holds.

Proof: Set the decision matrices in Theorem 1 as follows: $\mathcal{P}(\theta) = \mathcal{P}(\theta^+) = \mathcal{X} = \mathcal{X}_c$, $\mathcal{H}(\theta) = \mathcal{H}(\theta^+) = \mathcal{Y} = \mathcal{Z}_c$, $\mathcal{J}(\theta) = \mathcal{J}(\theta^+) = \mathcal{S}^T = \mathbf{I}_n$, then one gets (19) from (10). Thus, the inequality holds. ■

C. Algorithm for Solving PDBMIs

Note that inequality (10) is PDBMI and becomes PDLMI with *a priori* defined $\mathcal{E}(\theta, \delta)$. Considering this property, we show the following simple method for solving (10).

[Algorithm I (line search algorithm)]: Minimize γ_∞ under (10) with single line search parameter ε ($\mathcal{E}(\theta, \delta) = \varepsilon\mathbf{I}_n$).

This algorithm is very simple; however, the numerical complexity may be huge for fine gridding of ε. In addition, restricting $\mathcal{E}(\theta, \delta)$ to be $\varepsilon\mathbf{I}_n$ introduces conservatism. To reduce conservatism, the following algorithm is proposed.

[Algorithm II (iterative algorithm)]:

- Step 0 Set $i = 0$, $\gamma_{\infty,i} = \infty$, and $\mathcal{E}(\theta, \delta) = \mathcal{E}_0(\theta, \delta) = \varepsilon_0\mathbf{I}_n$ with some given positive scalar ε_0, e.g. 1.
- Step i.1 Set $i = i+1$. Minimize γ_∞ under (10) with fixed $\mathcal{E}_{i-1}(\theta, \delta)$, and set $\mathcal{X}_{i-1}$ be the optimum of $\mathcal{X}$ ((a) or (c) is chosen for (13)) or $\mathcal{Y}_{i-1}$ be the optimum of $\mathcal{Y}$ ((b) or (d) is chosen for (13)).
- Step i.2 Minimize γ_∞ under (10) with fixed $\mathcal{X}_{i-1}$ or $\mathcal{Y}_{i-1}$, and set $\mathcal{E}_i(\theta, \delta)$ and $\gamma_{\infty,i}$ be the optima of $\mathcal{E}(\theta, \delta)$ and γ_∞, respectively.
- Step i.3 If $\gamma_{\infty,i-1} - \gamma_{\infty,i}$ is below some predefined threshold ρ, then stop the iteration. Otherwise, return to Step i.1.

Although this algorithm does not always converge to the global optima, conservatism reduction is expected compared to Algorithm I as the structural constraint for $\mathcal{E}(\theta, \delta)$ is relaxed.

Those algorithms require solving PDLMIs. In contrast to solving PDBMIs, one can easily solve them by using Sum-Of-Squares techniques, e.g. [17] and references therein, Slack Variable (SV) approach [18], etc.

Remark 4: Suppose that the state-space matrices of LPV system (1) are affine with respect to parameters, and matrices $D_{12}(\theta)$ and $D_{21}(\theta)$ are constant. Under these assumptions, if parameter-dependent decision matrices in Theorem 1 are also set to be affine with respect to θ_i and/or $\theta_i + \delta_i$, then the inequality (10) becomes parametrically affine. Thus, we only have to check the feasibility of the inequality at all vertices of associated parameters with neither SOS relaxations nor SV approaches being applied.

IV. NUMERICAL EXAMPLE

Let us consider a discrete-time LPV system with the following state-space matrices:

$$\left[\begin{array}{c|c|c} A(\theta) & B_1(\theta) & B_2 \\ \hline C_1(\theta) & D_{11}(\theta) & D_{12}(\theta) \\ \hline C_2 & D_{21}(\theta) & \end{array}\right] = \left[\begin{array}{c|c|c} \mu\begin{pmatrix} 1-\theta & 0 & -2+\theta \\ 2-\theta & -1 & 1-\theta \\ -1+\theta & 1-3\theta & -\theta \end{pmatrix} & \begin{matrix} 0 \\ 1-\theta \\ \theta \end{matrix} & \begin{matrix} 1 \\ 0 \\ 0 \end{matrix} \\ \hline \begin{matrix} 1 & 1 & 1 \end{matrix} & 0 & 0 \\ \hline \begin{matrix} 1 & 0 & 0 \end{matrix} & 1 & \end{array}\right],$$

where $\mu = 0.4525$ and the scheduling parameter θ is bounded as $0 \le \theta \le 1$. This example is borrowed from [19] with a slight revision.

Hereafter, we set all parameter-dependent matrices to be affine with respect to associated parameters among θ, $\theta + \delta$ and δ, then all inequalities are affine with respect to the parameters and are solved at all vertices of the parameters. (See Remark 4.)

The bound of the deviation from $\theta(k)$ to $\theta(k + 1)$, i.e. $|\theta(k+1) - \theta(k)|$, is set as Δ. We consider 4 cases for Δ, i.e. $\Delta = 0, 0.01, 0.1$ and 1.0, and 3 cases for the uncertainty on scheduling parameter θ, i.e. $|\delta| \le \xi$ with $\xi = 0, 0.01$ and 0.2. When ξ is set as 0.5, we couldn't design any stabilizing controllers by using our method.

We design GSOF controllers using Theorem 1 with iterative algorithm, Lemma 5, and Lemma 6 with line search algorithm with YALMIP [20] and SeDuMi [21]. The results are shown in Tables I $\sim$ III, where "∞" denotes that no

TABLE I

OPTIMAL γ_∞ VIA THEOREM 1 WITH ITERATIVE ALGORITHM

(13)	ξ	$\Delta = 0$	$\Delta = 0.01$	$\Delta = 0.1$	$\Delta = 1.0$
(a)	0	2.485	2.486	2.503	2.642
	0.01	2.490	2.491	2.511	2.907
	0.2	5.293	5.377	6.241	∞
(b)	0	2.484	2.486	2.503	2.617
	0.01	2.488	2.490	2.509	2.907
	0.2	5.298	5.381	6.241	∞
(c)	0	2.486	2.486	2.503	2.762
	0.01	2.489	2.490	2.510	2.829
	0.2	4.325	4.371	4.887	20.347
(d)	0	2.484	2.486	2.503	2.617
	0.01	2.487	2.489	2.509	2.824
	0.2	4.325	4.367	4.863	19.652

TABLE II

OPTIMAL γ_∞ VIA LEMMA 5

$\Delta = 0$	$\Delta = 0.01$	$\Delta = 0.1$	$\Delta = 1.0$
2.484	2.486	2.503	2.616

TABLE III

OPTIMAL γ_∞ VIA LEMMA 6 WITH LINE SEARCH ALGORITHM

ξ	(a)	(b)	(c)	(d)
0	2.616	2.617	2.616	2.617
0.01	3.148	3.720	3.053	3.485
0.2	∞	∞	∞	∞

stabilizing controllers can be designed. When Theorem 1 was applied, the threshold ρ for stopping criterion of the iterations was set as 10^{-3}. When Lemma 6 was applied, line search for ε ($\mathcal{E}(\theta, \delta) = \varepsilon \mathbf{I}_n$) was conducted with 10 points linearly gridded over a logarithmic scale in $[10^{-10},\ 10^5]$. For reference, we designed a GSOF controller using PiDLFs via the method in [2]. The optimized γ_∞ is obtained as 2.616, which is exactly the same as the results by Lemma 5 with $\Delta = 1.0$, and almost the same as the results by Lemma 6 with $\xi = 0$.

The values of γ_∞ in Tables I for $\xi = 0$ are almost the same as those in Table II, which indicates that Theorem 2 holds. The values of γ_∞ in Tables I apart from the case with $\Delta = 1.0$ and $\xi = 0$ are all less than the corresponding ones in Table III, which confirms that Theorem 3 holds. Thus, our method produces GSOF controllers which are robust against the uncertainties on scheduling parameters with conservatism being reduced by using PDLFs compared to using PiDLFs.

V. CONCLUSIONS

This paper tackles the design problem of discrete-time Gain-Scheduled Output-Feedback (GSOF) controllers for discrete-time Linear Parameter-Varying (LPV) systems under the condition that only inexact scheduling parameters are available. We propose a design method for our problem in terms of Parameter-Dependent Bilinear Matrix Inequalities (PDBMIs) using Parameter-Dependent Lyapunov Functions (PDLFs). We also show the following: (i) When the uncertainties in the provided scheduling parameters vanish, our method recovers the conventional method via PDLFs assuming that exact scheduling parameters are available, and (ii) our method encompasses the conventional method via Parameter-inDependent Lyapunov Functions (PiDLFs) but assuming that inexact scheduling parameters are available. A simple numerical example illustrates those properties and demonstrates the effectiveness of our method.

REFERENCES

[1] J. S. Shamma and J. R. Cloutier, "Gain-Scheduled Missile Autopilot Design Using Linear Parameter Varying Transformations," *AIAA J. of Guidance, Control, and Dynamics*, Vol. 16, No. 2, pp. 256–263, 1993.

[2] P. Apkarian, P. Gahinet, and G. Becker, "Self-Scheduled H_∞ Control of Linear Parameter-Varying Systems: a Design Example," *Automatica*, Vol. 31, No. 9, pp. 1251–1261, 1995.

[3] F. Wu, X. H. Yang, A. Packard, and G. Becker, "Induced L_2-norm Control for LPV Systems with Bounded Parameter Variation Rates," *Int. J. Robust and Nonlinear Control*, Vol. 6, Nos. 9-10, pp. 983–998, 1996.

[4] C. Scherer, "Mixed H_2/H_∞ Control for Time-Varying and Linear Parametrically-Varying Systems," *Int. J. Robust and Nonlinear Control*, Vol. 6, 1996, pp. 929-952.

[5] P. Apkarian and R. J. Adams, "Advanced Gain-Scheduling Techniques for Uncertain Systems," *IEEE Trans. Control Systems Technology*, Vol. 6, No. 1, 1998, pp. 21-32.

[6] E. Prempain and I. Postlethwaite, "L_2 and H_2 Performance Analysis and Gain-Scheduling Synthesis for Parameter-Dependent Systems," *Automatica*, Vol. 44, pp. 2081–2089, 2008.

[7] R. A. Borges, R. C. L. F. Oliveira, C. T. Abdallah, and P. L. D. Peres, "A BMI Approach for H_∞ Gain Scheduling of Discrete Time-Varying Systems, *Int. J. Robust and Nonlinear Control*, Vol. 20, No. 11, pp. 1255–1268, 2010.

[8] A. Ohara, Y. Yamaguchi, and T. Morito, "LPV Modeling and Gain Scheduled Control of Re-entry Vehicle in Approach and Landing Phase," AIAA Guidance, Navigation, and Control Conference, *AIAA 2001-4038*, 2001.

[9] I. Masubuchi, J. Kato, M. Saeki, and A. Ohara, "Gain-Scheduled Controller Design Based on Descriptor Representation of LPV Systems: Application to Flight Vehicle Control," *Proc. CDC*, pp. 815–820, 2004.

[10] J. Daafouz, J. Bernussou, and J. C. Geromel, "On Inexact LPV Control Design of Continuous-Time Polytopic Systems," *IEEE Trans. Automatic Control*, Vol. 53, No. 7, pp. 1674–1678, 2008.

[11] M. Sato, "Gain-Scheduled Output-Feedback Controllers Using Inexactly Measured Scheduling Parameters," *Proc. CDC*, pp. 3174–3180, 2010.

[12] P. Apkarian, "On the Discretization of LMI-Synthesized Linear Parameter-Varying Controllers," *Automatica*, Vol. 33, No. 4, pp. 655–661, 1997.

[13] M. Sato, "Discrete-Time Gain-Scheduled Output-Feedback Controllers Exploiting Inexact Scheduling Parameters," *Proc. MSC*, 2011 (to appear).

[14] J. de Caigny, J. F. Camino, R. C. L. F. Oliveira, P. L. D. Peres, and J. Swevers, "Gain-Scheduled Dynamic Output Feedback Control for Discrete-Time LPV Systems," *Int. J. Robust and Nonlinear Control*, (to appear).

[15] M. C. de Oliveira, J. C. Geromel, and J. Bernussou, "Extended H_2 and H_∞ Norm Characterizations and Controller Parametrizations for Discrete-Time Systems," *Int. J. Control*, Vol. 75, No. 9, pp. 666–679, 2002.

[16] Xie, L. and de Souza, C. E. (1992). Robust H_∞ Control for Linear Systems with Norm-Bounded Time-Varying Uncertainty. IEEE Trans. on Automatic Control, 37(8):1188–1191.

[17] C. W. Scherer and C. W. J. Hol, "Matrix Sums-Of-Squares Relaxations for Robust Semi-Definite Programs," *Mathematical Programming Ser. B*, Vol. 107, No. 1-2, pp. 189–211, 2006.

[18] D. Peaucelle and M. Sato, "LMI Tests for Positive Definite Polynomials: Slack Variable Approach," *IEEE Trans. Automatic Control*, Vol. 54, No. 4, pp. 886–891, 2009.

[19] J. De Caigny, J. F. Camino, R. C. L. F. Oliveira, P. L. D. Peres, and J. Swevers, "Gain-Scheduled H_∞-Control of Discrete-Time Polytopic Time-Varying Systems," *Proc. CDC*, pp. 3872–3877, 2008.

[20] J. Löfberg, "YALMIP: A Toolbox for Modeling and Optimization in MATLAB," *Proc. the CACSD Conference*, Taipei, Taiwan, 2004, `http://control.ee.ethz.ch/~joloef/yalmip.php`.

[21] J. S. Sturm, "Using SeDuMi 1.02, a MATLAB Toolbox for Optimization Over Symmetric Cones," *Optimization Methods and Software*, Vols. 11-12, pp. 625–653, 1999.