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Cooperative Multi-Vehicle Search and Coverage Problem in Uncertain Environments

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***Abstract*— A decentralized approach is proposed to solve a cooperative multi-vehicle search and coverage problem in uncertain environments. Two different types of vehicle are used for search and coverage tasks. The search vehicles have *a priori* probability maps of targets in the environment and they update these maps based on the measurement of their sensors during the search mission. They use a limited look-ahead dynamic programming algorithm to find their own path individually while their objective is to maximize the amount of information gathered by the whole team. The task of service vehicles is to spread out over the environment to optimally cover the terrain. A locational optimization technique is used to assign Voronoi regions to vehicles and the stability of coverage system is guaranteed using LaSalle's invariance principle. The service vehicles modify their configuration using the updated probability maps which are provided by the search vehicles. Simulations show that the proposed approach offers improved performance compared to conventional coverage methods.**

I. INTRODUCTION

Deployment of a group of vehicles over an environment to carry out sensing, surveillance, data collection, or distributed servicing tasks is a principle problem in cooperative multi-vehicles decision making and control. This topic has received considerable attention over the last decade because of technological advances and development of relatively inexpensive communication, computation, and sensing devices [1]-[4].

The cooperative multi-vehicles search and coverage approach is useful for many applications involving distributed sensing and distributed actuation. This framework can be used by groups of vehicles to carry out tasks such as environmental monitoring and clean-up, or search and rescue [5]-[6]. For example, consider a team of Unmanned Aerial Vehicles (UAVs) charged with detecting and extinguishing multiple fires in a partially known environment like a forest. The fire detector UAVs with on-board sensors search the environment to find the centre of fires. Then by using this information, the fire fighter UAVs aggregate in the perimeter of fires. Similarly, consider a group of water-borne vehicles which are in charge of monitoring and cleaning up an oil spill. The monitoring vehicles find the areas where the spill is most severe, while cleaning vehicles distribute themselves over the spill and concentrate their efforts on those most severe areas, without neglecting the areas where the spill is not as severe. In general, any application in which a group of automated mobile agents is required to provide collective sensing and actuation over an environment can be considered as an example of this framework.

In this paper, we consider the case in which some service vehicles deploy to cover an uncertain environment. They are expected to spread out over an environment while aggregating in areas of high service needs. Furthermore, the service vehicles are uncertain about the exact areas of service needs beforehand. In order to decrease the level of uncertainty, the environment is searched by some search vehicles which are equipped with sensors to detect the exact areas of service needs. As mission goes on, the service vehicles use the updated information of search vehicles to change their configuration and cover the environment more efficiently.

In the recent years, search theory has paid attention to the problem of having a team of cooperative searchers. The problem of multi-vehicle search in an uncertain environment has been studied, and a number of approaches have been formulated [1], [4], [8]-[10]. Centralized decision making for a fleet of vehicles is not usually practical due to communication limits, robustness issues, and scalability. Therefore different methods for decentralized decision making are proposed in the literature [4], [8]-[12].

Plenty of research works have been done on the coverage problem as one of the main applications of cooperative control. Normally, the agents move to an optimal configuration to minimize an objective function [13]. The approach is based on Voronoi tessellation and Lloyd algorithm. A decentralized control law is designed for mobile sensors to cover an area partitioned into Voronoi region, in the sense that continually driving the agents toward the centroids of their Voronoi cells [2], [14].

Most of the prior works in the area of Voronoi-based coverage control assume the distribution of sensory information in the environment is required to be known *a priori* by all agents. However, the problem of the online learning of the distribution density function, and estimation of density function using neural networks while moving toward the optimal locations is addressed in [3], [15]

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respectively.

In the aforementioned studies, it is assumed that the density function is measurable and can be measured by each agent at its position. In our work, the density function is still unknown, but it is not directly measurable at each point. Inspired from many real applications, the distribution density function is assumed to be a function of position of some unknown targets which can be detected by appropriate sensors. In order to let service vehicles do their task more efficiently, the task of finding the targets is done by some search vehicles. This idea leads to the covering an uncertain environment more effective and improving the coverage performance.

This paper is organized as follows. In the next section the problem is introduced. Section III deals with the search problem. The probability map updating, dynamic programming formulation and cooperative decision making method are discussed in this section. In section IV, the coverage problem, and derivation of distribution density function based on the probability map is presented. Then the control law for service vehicles is proposed and its stability is proved. Numerical simulation results show the effectiveness of the proposed approach in Section V. Finally, conclusion is provided in Section VI.

II. Problem Statement

This paper addresses the cooperative multi-vehicle search and coverage problem in an uncertain environment. Consider the scenario that some search vehicles are deployed to search and detect some targets in the terrain. There are also some service vehicles that their duty is to spread out over the environment to provide coverage. The search vehicles broadcast their information about the environment to the service vehicles. This information allows the service vehicles to find where in the environment they are most needed and to aggregate in those areas.

For the search problem, the environment is discretized in cells that are described by a probability of target existence. There is an uncertainty region corresponding to each target. Each target is assumed to lie somewhere within its uncertainty region, but its exact position is unknown. Each search vehicle stores a *probability map*, which contains the probability of existence of all targets in any given cell. During the mission, sensors of search vehicles can detect targets in their footprints. The probability map is updated during the mission based on whether or not the target is detected by the sensors. The objective of the cooperative search mission is to maximize the amount of information about the environment.

The objective of service vehicles is to spread out over the area to cover the entire environment. However, in most cases, all points in the environment do not have the same level of importance. We can consider a density function which reflects a measure of relative importance of different points in the environment. The density of each point is a decreasing function of the distance between that point and position of targets. Therefore, closer points to targets have more value and more level of importance in the environment. Since our information about the position of targets improves during the search mission, the density function is changed and gets more accurate as mission goes on.

III. Search Problem

In the search problem, the state of the system consists of the search status and the vehicle status. The search vehicles can communicate with each other so they can form a comprehensive view of the state. The control input, comes from a set of possible assignments such as: turn left, turn right, or go straight.

A. Updating the Probability Map

An initial *probability map* of the environment, which is uncertain and incomplete, is created based on the *a priori* knowledge about the environment. During the mission, the probability map is updated based on whether or not the target is detected by the sensors. We define the events

$E_{x,y}^i$: Target *i* is in the cell (x, y)

D_Ω^i : Target *i* is detected in the region Ω

where Ω is the collection of cells that the vehicle sensor covered during its last step.

1) Target not detected

When the sensor has not detected the target during the last step of the mission, the probability of existence of the target for each cell can be updated as follows, using the Bayes' Rule

$$P\left(E_{x,y}^i \middle| \bar{D}_\Omega^i\right) = \frac{P\left(E_{x,y}^i\right) P\left(\bar{D}_\Omega^i \middle| E_{x,y}^i\right)}{P\left(\bar{D}_\Omega^i\right)}$$

where an overbar on the events represents the complement of the events. The probability of *true positive* and *false positive* measurement of sensors are assumed to be $\gamma \triangleq P(D_\Omega^i|E_\Omega^i)$ and $\varepsilon \triangleq P(D_\Omega^i|\bar{E}_\Omega^i)$ respectively. Therefore, the probability map can be updated as follows

$$P\left(E_{x,y}^i \middle| \bar{D}_\Omega^i\right) = \begin{cases} \frac{P\left(E_{x,y}^i\right)\bar{\gamma}}{P\left(\bar{D}_\Omega^i\right)}, & (x, y) \in \Omega \\ \frac{P\left(E_{x,y}^i\right)\bar{\varepsilon}}{P\left(\bar{D}_\Omega^i\right)}, & (x, y) \notin \Omega \end{cases}$$

In the derivation of second equation, we used the fact that existence of target *i* in a cell outside of coverage region Ω means there is no target inside that region. The probability that target *i* is not detected in the last step of the mission, $P(\bar{D}_\Omega^i)$, can be calculated as follows

$$\begin{aligned} P\left(\bar{D}_\Omega^i\right) &= P\left(\bar{D}_\Omega^i \middle| E_\Omega^i\right) P\left(E_\Omega^i\right) + P\left(\bar{D}_\Omega^i \middle| \bar{E}_\Omega^i\right) P\left(\bar{E}_\Omega^i\right) \\ &= \bar{\gamma}\left(\textstyle\sum_{\forall (x,y)\in\Omega} P\left(E_{\mathrm{x,y}}^i\right)\right) + \bar{\varepsilon}\left(1 - \textstyle\sum_{\forall (x,y)\in\Omega} P\left(E_{\mathrm{x,y}}^i\right)\right) \end{aligned}$$

Therefore, if we define the total probability of finding target *i* in the collection of cells Ω as $N_\Omega^i = \sum_{\forall (x,y)\in\Omega} P\left(E_{\mathrm{x,y}}^i\right)$, the *posterior* probability of existence of target *i* in each cell can be computed as follows

$$P\left(E_{x,y}^{i}\middle|\bar{D}_{\Omega}^{i}\right)=\begin{cases}\frac{P\left(E_{x,y}^{i}\right)\bar{\gamma}}{\bar{\gamma}\,N_{\Omega}^{i}+\bar{\varepsilon}\left(1-N_{\Omega}^{i}\right)}, & (x,y)\in\Omega\\ \frac{P\left(E_{x,y}^{i}\right)\bar{\varepsilon}}{\bar{\gamma}\,N_{\Omega}^{i}+\bar{\varepsilon}\left(1-N_{\Omega}^{i}\right)}, & (x,y)\notin\Omega\end{cases}$$

In the case of informative sensors, it is easy to show that when a target is not detected, the probability of existence of that target in the cells inside the footprint of sensor will be decreased, while the probability of the cells outside the sensor footprint will be increased.

2) Target detected

Using a similar procedure, when the sensor has detected the target, the *posterior* probability of existence of target i in each cell can be computed as follows

$$P\left(E_{x,y}^{i}\middle|D_{\Omega}^{i}\right)=\begin{cases}\frac{P\left(E_{x,y}^{i}\right)\gamma}{\gamma N_{\Omega}^{i}+\varepsilon\left(1-N_{\Omega}^{i}\right)}, & (x,y)\in\Omega\\ \frac{P\left(E_{x,y}^{i}\right)\bar{\varepsilon}}{\gamma N_{\Omega}^{i}+\varepsilon\left(1-N_{\Omega}^{i}\right)}, & (x,y)\notin\Omega\end{cases}$$

B. Dynamic Programming Formulation

The search vehicles must choose a control signal such that it results in the best possible paths, in the sense that the team of search vehicles finds maximum number of targets over the decision process planning horizon. To make the problem tractable and solvable in real-time, Dynamic Programming (DP) approach with a rolling horizon limited look-ahead policy can be utilized [1]. This rolling horizon approximation defines a horizon of time steps T, and then replaces the value of final gain with the gain at T-steps ahead. The optimal decisions can be found by taking the arguments of the maximization of the DP recursion.

The single step gain which is the gain that a search vehicle will receive at one time step (specifically at time step k) can be written as

$$g(x_k,u_k,w_k)=\lambda^k\delta_k\sigma_k \tag{1}$$

where σ_k is the search gain for the vehicle at time step k which is the expected value of the number of targets detected during the mission from time step k to time step $k+1$, δ_k is the probability that the planning vehicle is operational at time k, and $\lambda\ (0\le\lambda<1)$ is the time discount factor. The search gain σ_k can be calculated by adding up the probabilities of existence of targets in all cells that the vehicle covers them during its mission from time step k to time step $k+1$, i.e. $\sum_{\forall i}\sum_{\forall(x,y)\in\Omega_{\mathrm{k}}}P\left(E_{\mathrm{x,y}}^{i}\right)$.

C. Cooperative Decision Making

The objective of search mission is to search the terrain to gain as much information about the environment as possible. To achieve this goal, we use a decentralized method where each vehicle makes a decision about its next action individually. It is desired to obtain localized objective function for each vehicle that aligns with the global objective function. Therefore, when the search vehicles want to make decisions on their next actions, they must simply optimize their own objective functions which also optimize the global objective.

Assume that each vehicle knows the probability of presence of other vehicles in each cell over the future look-ahead horizon. Then the modified search gain of the vehicles can be defined as follows

$$\hat{\sigma}_k=\sum_{\forall i}\sum_{\forall(x,y)\in\Omega_k^j}\left(\rho_{x,y}^{k}\,P\left(E_{x,y}^{i}\right)\right) \tag{2}$$

where Ω_k^j is the collection of cells that the search vehicle j covers during its mission from time step k to time step $k+1$, and $\rho_{\mathrm{x,y}}^{k}$ is a discount factor which is always between zero and one. This discount factor is a decreasing function of the probability that other vehicles also decide to search the cell (x,y) at the next k^{th} step. Now, we can modify the single step gain of vehicles in (1) by replacing σ_k with $\hat{\sigma}_k$.

Evaluating the probability of presence of other vehicles requires each vehicle to expand the planning tree of every other vehicle which is impractical when the number of vehicles or the search horizon increases. To reduce the computational burden, we provide all search vehicles with a look-up table that contains the probability of presence of a search vehicle in each cell at the next k^{th} step, given its current position and heading, i.e. $p(x,y,k|x_0,y_0,\theta_0)$, where (x,y) is the position of vehicle at k-steps ahead and (x_0,y_0,θ_0) is its current position and heading. This table is made off-line and can be produced analytically by using different estimation algorithms or simply through reasonable amount of simulations. It is obvious that since the search vehicles neglect the probability map of targets in the approximation of the next position of other vehicles, the method is not optimal, but it dramatically decreases the computational demand and processing time. This probability, then, can be used to define an appropriate function $\rho_{\mathrm{x,y}}^{k}$ such that the single step gain decreases when the probability of presence of other vehicles increases.

IV. COVERAGE PROBLEM

For the coverage purpose, the environment is denoted by Q which is a convex polytope in $\mathbb{R}^2$ including its interior. An arbitrary point in Q is denoted as q, the position of the i^{th} service vehicle is denoted as p_i, and the set of all service vehicle positions is denoted as $\mathcal{P}=\{p_1,p_2,\dots,p_n\}$. The function $\varphi\colon Q\to\mathbb{R}_+$ is a distribution density function that defines a weight for each point. Thus, the higher the value of $\varphi(q)$ the more attention the group has to pay to q.

Let $V=\{V_1,V_2,\dots,V_n\}$ be the Voronoi partition of Q, for which the service vehicles positions are the generator points. The Voronoi region, V_i, of a given service vehicle is the region of points that are closer to that vehicle than to any other, that is $V_i=\{q\in Q|\ \|q-p_i\|\le\|q-p_j\|,\forall j\ne i\}, i\in\{1,\dots,n\}$, Moreover, two service vehicles V_i and V_j are (Voronoi) neighbors if $V_i\cap V_j\ne 0$.

As a measure of the system performance, the coverage function is defined as:

$$\mathcal{H}(\mathcal{P})=\sum_{i=1}^{n}\int_{V_i}\frac{1}{2}\|q-p_i\|^2\,\varphi(q)dq \tag{3}$$

where it is assumed that the i^{th} service vehicle is responsible for its Voronoi region V_i. Note that the function (3) measures the ability of the coverage provided by the network of service vehicles in Q. Qualitatively, a low value of function (3) corresponds to a good configuration for coverage of the environment Q. Therefore, it is desired to minimize it.

Each Voronoi region has mass M_{V_i}, and centroid C_{V_i} which are defined as:

$$M_{V_i} = \int_{V_i} \varphi(q)dq, \qquad C_{V_i} = \frac{1}{M_{V_i}} \int_{V_i} q\varphi(q)dq \quad (4)$$

respectively. Remarkably, one can show that

$$\frac{\partial \mathcal{H}}{\partial p_i} = -\int_{V_i} (q - p_i)\varphi(q)dq = -M_{V_i}(C_{V_i} - p_i) \quad (5)$$

So the partial derivative of $\mathcal{H}$ with respect to the position of the i^{th} service vehicle only depends on its own position and the position of its Voronoi neighbors. Clearly, the extremum points of $\mathcal{H}$ are those in which every vehicle is at the centroid of its Voronoi region, $p_i = C_{V_i}, \forall i$. The resulting partition of the environment is commonly called a Centroidal Voronoi Tessellation (CVT). More thorough discussions were given in [1].

A. Distribution Density Function

The precise definition of the distribution density function $\varphi(q)$ depends on the desired application. It defines a weight for each point in the environment which is a measure of relative importance of that point. In many applications, there are some critical points and the level of importance of each point in the terrain is inversely proportional to the distance between the point and the critical points. For instance, the critical points can be hotspots in a forest fire or the source of gushing in an oil spill. Let $\varphi(q) = \sum_{i=1}^{n_c} \phi(q, q_c^i)$ where q_c^i is the i^{th} critical point and n_c is the number of critical points. Function $\phi(q, q_c^i)$ is known *a priori* and it has a maximum at the critical point q_c^i. Therefore, knowing the exact location of critical points, we can find the weight of all points ($\varphi(q)$).

In many cases, the location of critical points is not known precisely but it is known that they are lied somewhere inside some uncertainty regions. Knowing the probability distribution of each critical point i in its uncertainty region, $P(q_c^i)$, distribution density function $\varphi(q)$ can be obtained as follows

$$\varphi(q) = \sum_{i=1}^{n_c} \int_{\Lambda_i} \phi(q, q_c^i)\, P(q_c^i) dq_c^i \quad (6)$$

where Λ_i is the uncertainty region of the i^{th} critical point. Indeed, $\int_{\Lambda_i} \phi(q, q_c^i)\, P(q_c^i) dq_c^i$ is the expected value of function $\phi(q, q_c^i)$ with respect to q_c^i.

These critical points are in fact the targets of search problem. Since the search is done in a discrete environment, the probability of all points inside a cell is assumed to be equal. Therefore, (6) can be modified as follows

$$\varphi(q) = \sum_{i=1}^{n_c} \sum_{\forall(x,y)\in\Lambda_i} P(E_{x,y}^i) \int_{q_c^i \in (x,y)} \phi(q, q_c^i)\, dq_c^i$$

B. Distributed Coverage Controller

In this section the coverage control for a group of service vehicles is investigated. These service vehicles are assumed to fly at different levels. Each service vehicle is modeled as a *double-integrator* point mass moving on a two-dimensional (2-D) plane as follows

$$\ddot{p}_i = u_i \quad (7)$$

Equation of motion of a broad class of vehicles can be expressed by a double-integrator dynamic model. In addition, dynamics of many vehicles can be feedback linearized to double integrators.

Consider that the position of the i^{th} service vehicle is denoted by p_i, and C_{V_i} is the center of Voronoi that corresponds to the i^{th} service vehicle. We propose the following position control law for the i^{th} service vehicle:

$$u_i = k_1^i M_{V_i}(C_{V_i} - p_i) - k_2^i \dot{p}_i \quad (8)$$

where k_1^i and k_2^i are the positive gains.

Theorem: Consider a group of n service vehicles whose dynamic models are described in (7). Under control law (8), it is guaranteed that the whole system is asymptotically stable and the planar positions of service vehicles converge to a centroidal Voronoi configuration.

Proof: Consider the Lyapunov function candidate as:

$$\vartheta = K_1\mathcal{H} + \sum_{i=1}^{n} \frac{1}{2} \dot{p}_i^2 \quad (9)$$

where K_1 is the vector of controller gains, and the coverage performance $\mathcal{H}$ is defined in (3). The candidate Lyapunov function ϑ is bounded below by zero since $\mathcal{H}$ is a sum of integrals of strictly positive functions. By substituting (5) and (7) into time derivative of ϑ along the trajectories of systems, and using control input (8), the time derivative of Lyapunov function can be obtained as follows

$$\dot{\vartheta} = \sum_{i=1}^{n} \left[\left(k_1^i M_{V_i} (p_i - C_{V_i})^T + \ddot{p}_i^T \right) \dot{p}_i \right] = \sum_{i=1}^{n} \left(-k_2^i\, \dot{p}_i^2 \right) \quad (10)$$

which is clearly non-positive. Let S be the set of all points in Q where $\dot{\vartheta} = 0$. Due to the convexity of the region Q, one can conclude that each of the Voronoi centroids C_{V_i} lies in the interior of the i^{th} Voronoi region and so in the interior of the region Q. So the vehicles move toward the interior of the region Q and never leave it. Therefore, Q is a positive invariant set for the trajectories of the closed loop system. Since this set is closed and bounded, one can make use of LaSalle's invariance principle to conclude that $p_i = C_{V_i}, \forall i \in N$ is the largest invariant set corresponding to the set of centroidal Voronoi configurations. Therefore, under control law (8), the closed-loop system is asymptotically stable and the planar positions of service vehicles converge to a set of centeroidal Voronoi configuration.

V. Simulation Results

The proposed distributed search and coverage algorithm has been demonstrated via numerical simulations in MATLAB® environment.

The environment used in this simulation is a $1\,km \times 1\,km$ square. Since the search problem has discrete nature, the environment is divided to 10000 cells which make a 100×100 square grid. There exists three targets known to be in the square areas as shown in Fig 1, but their exact positions are unknown. The *a priori* probability of existence of these targets is uniformly distributed in their uncertainty region while their real positions are marked by the * marker.

A group of three fixed-wing search UAVs and ten quad-rotors service UAVs are deployed to search and cover the environment. Each search UAV is equipped with a sensor that can detect targets in its footprint. All three search UAVs start their mission from the south west corner of the terrain, while service UAVs start their mission from their individual bases which are located on the border of the environment as shown in Fig 1. For the purpose of collision avoidance, the UAVs fly in different levels.

At each decision time step, search UAVs must decide to go straight, turn 15 degrees left or turn 15 degrees right. The speed of search UAVs assumed to be constant. In order to execute the simulation in a reasonable amount of time, we set the look-ahead horizon of the DP algorithm to 5 time steps. The model of service UAVs are assumed to be a double integrator and their control law is denoted in (8). Probabilities of *true positive* and *false positive* measurement of sensors are set to be 0.9 and 0.1 respectively.

In this simulation, we used the following Gaussian density function

$$\phi(q, q_c^i) = \frac{1}{\sigma\sqrt{2\pi}}\left(e^{-\frac{(q-q_c^i)^2}{2\sigma^2}}\right)$$

where $\sigma = 70\,m$. The initial and final probability maps and their corresponding distribution density functions are shown in Fig 2.

We consider the scenario in which, at the beginning, service vehicles spread over the terrain based on the imprecise initial distribution density function which is derived from the *a priori* probability maps. The final configuration of planar position and the trajectories of all UAVs are shown in the left of Fig 3-a. The exact distribution density function is also shown in that figure. This distribution is calculated based on the actual position of critical points (targets). The color intensity is proportional to the value of distribution density function at each point. Corresponding distribution density function based on the probability maps is depicted in the right of Fig 3-a. It can be seen from this figure that the configuration of service UAVs in the environment is optimal according to available density function.

Next, search UAVs start their mission to explore the terrain. During the mission, they update the probability maps of targets and transmit these updated maps to the service vehicles on a regular basis. The position and trajectory of all UAVs and the exact distribution density function are shown in the left of Fig 3-b, c, and d for three different time steps. For each figure, the position of service UAVs and the corresponding distribution density function based on the most updated probability maps are shown in the right. It is worth to mention that as search UAVs explore the environment, the probability maps get more precise, and therefore the current distribution density function gets more similar to the exact one. Especially in Fig. 3-d, the density functions in the left and right figures are almost the same. As expected, deployment of search UAVs helps service vehicles to improve their performance to cover the most needed areas. The value of coverage function which is a measure of the system performance is reported in the table 1 for five different times, using the exact density function. It can be seen that the coverage performance is improved about %25 by using this approach.

In order to evaluate the average performance of this approach, different simulations have been done 25 times and the average number of detected targets and the value of coverage function are depicted in Fig 4. The uncertainty regions and actual positions of targets are randomly chosen for each repetition of simulation. As expected, the value of coverage function decreases and the coverage performance improves when the number of detected targets increases.

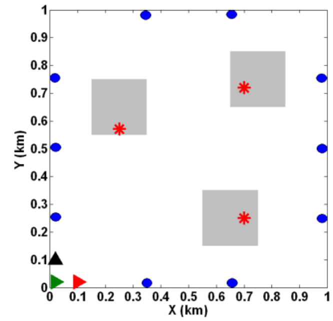

Fig. 1. The problem environment; the grey rectangles are the uncertainty regions of different targets and * denotes the actual position of targets. Search UAVs and service UAVs are shown by ⊳ and o markers respectively.

TABLE 1. THE COVERAGE FUNCTION FOR DIFFERENT TIMES

T (s)	0	120	160	200	240
$\mathcal{H}$	2.4339	0.6771	0.6235	0.5948	0.5443

VI. CONCLUSION

In this paper, the problem of searching and covering an uncertain environment using multi vehicles is presented. A group of vehicles called service vehicle deploys to service the points or areas where they are most needed in the environment based on the Voronoi tessellation. Since the high service areas are not known beforehand, we use a group

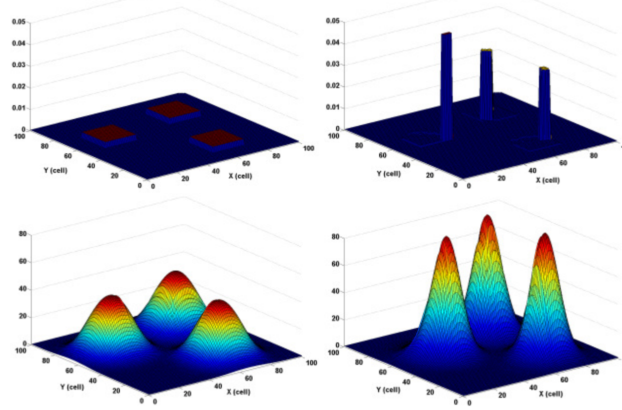

Fig. 2. The initial and final probability maps and their corresponding distribution density functions. Left figures: initial; right figures: final; top figures: probability maps; bottom figures: density functions.

of search vehicles to explore the environment based on the DP method. This approach leads to the covering an uncertain environment more effective and improving the coverage performance. The proposed method has been successfully verified by numerical simulations.

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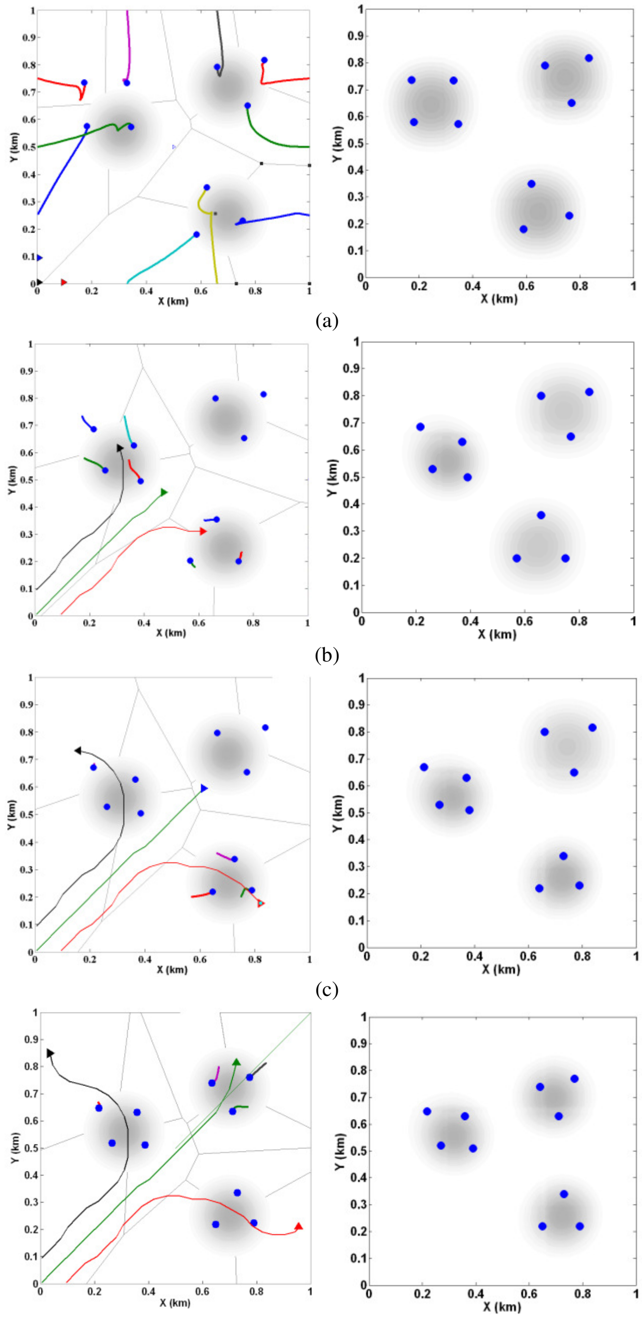

Fig. 3. Left: The configuration and the trajectories of all UAVs and the exact distribution density function. The color intensity is proportional to the value of density function. Search UAVs and service UAVs are shown by ⊳ and ∘ respectively. Right: The configuration of service UAVs and the corresponding distribution density function based on the probability maps. (a) t=120 s; the search has not started yet. All service UAVs are almost in a stationary situation, (b) t=160 s, (c) t=200 s, (d) t= 240 s

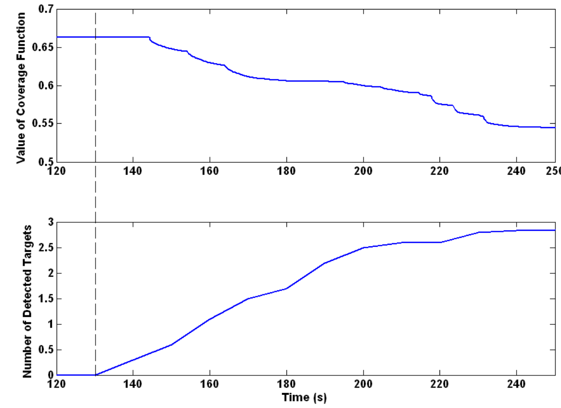

Fig. 4. The average value of coverage function and the average number of detected targets for 25 simulations.

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