2011 50th IEEE Conference on Decision and Control and
European Control Conference (CDC-ECC)
Orlando, FL, USA, December 12-15, 2011

A Distributed Guaranteed Cost Congestion Control Strategy for Mobile Networks with Differentiated Services Traffic

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Abstract— **In this paper, a novel distributed guaranteed cost congestion control (DGCC) strategy for mobile networks with differentiated services traffic is developed. The switching or changes in the network topologies is modeled by a Markovian process. By incorporating communication capability among controllers, the distributed congestion controller is shown to be in fact equivalent to a local state feedback control plus a nearest neighboring controllers' adjustment with proportional gains. Numerical simulation results are presented to illustrate the effectiveness and capabilities of our proposed DGCC strategies.**

I. INTRODUCTION

Congestion control problem is one of the main challenges in the mobile networks, which in general deals with the problem of flow rate control and network resource allocation. Standard congestion control schemes have shown poor performances in wireless mobile networks [1], [2], [3] and [4]. Furthermore, networks with differentiated services traffic require highly adaptive congestion control algorithms. Also, the nodes mobility, bandwidth limitation and the power constrains of network nodes make these developments more challenging.

Recently, several decentralized congestion control schemes have been developed by using the sliding mode control [8], the switching control [10], [12] and the guaranteed cost control [13] techniques. Although decentralized congestion control approaches are economical in implementation and have been shown effective in the above works, nevertheless performance of a decentralized controller is generally conservative due to the fact that the decisions are only based on local information. Therefore, new congestion control approaches which respect the underlying interconnections, adopts a distributed architecture and scales well to large networks are highly in demand.

The objective of this paper is to improve on the congestion control strategy in [13] by incorporating the possibility of communications among the controllers and to develop a *distributed congestion control* strategy for mobile networks with differentiated services traffic.

The proposed distributed congestion controller is shown to be in fact equivalent to a local state feedback control that is embedded with a nearest neighboring controllers' adjustment mechanism. The resulting guaranteed cost control problem is then cast as a quadratic regulation problem of a time-delay system with free parameters (gains) that need to be selected. Therefore, the decisions of each controller is based on the local information of each node and the adjusting information from the nearest neighboring controllers.

II. PROBLEM FORMULATION

A. Dynamical Model of Diff-Serv Networks

In this paper, we assume that the dynamics of a queue is governed by an M/M/1. The resulting queuing system can be applied to describe a wide variety of queuing models as found in systems with a very large number of independent customers/nodes that can be approximated as a Poisson process.

Given an M/M/1 queue the dynamics of a single node can be expressed as follows [9], [17], [18]

$$\dot{x}_i(t) = -\mu_i \frac{x_i(t)}{1+x_i(t)} C_i(t) + \lambda_i(t) \tag{1}$$

where $x_i(t)$ is the queuing length, $C_i(t)$ is the link capacity, $\lambda_i(t)$ is the average rate of incoming traffic, and $1/\mu_i$ is the average length of the packets being transmitted in the network.

Consider a general network with n nodes. In a large scale network the input traffic to each node can consist of two parts, namely: (1) the external traffic $\lambda_i(t)$ which in principle could represent the traffic that is being sent from nodes of other clusters (defined as groups of nodes not belonging to the nearest neighboring set $\wp_i$) as well as disturbances or environmental stimuli, and (2) the internal traffic $\lambda_j(t-\tau_{ji}(t))$ which is the delayed input traffic from all the neighboring nodes within a given cluster.

Therefore, by using the representation (1), the fluid flow model corresponding to each node is governed by

$$\dot{x}_i(t) = -f(x_i(t))C_i(t) + \lambda_i(t) + \sum_{j\in\wp_i(\alpha_t)} \lambda_j(t-\tau_{ji}(t))g_{ji} \tag{2}$$

$$\lambda_j(t-\tau_{ji}(t)) = f(x_j(t-\tau_{ji}(t)))C_j(t-\tau_{ji}(t)) \tag{3}$$

where $f(x_i(t)) = \mu_i x_i(t)/(1+x_i(t))$, $\wp_i$ is the set of the nearest neighboring nodes associated with the node i, $g_{ji}(t)$ is the traffic compression gain from node j to node i, $\tau_{ji}(t)$ is the time-varying delay between node j and node i, and α_t is a Markov chain that represents the rule for changes and switching in the neighboring sets.

The Markov chain α_t is defined on a complete probability space $\{\Omega, \mathscr{F}, P\}$ that takes values in a finite space $\mathscr{S} = \{1,...,M\}$ which describes the switching between different modes, and whose evolution is governed by the following probability transitions

$$P[\alpha_{t+\Delta} = k \mid \alpha_t = l] = \begin{cases} \pi_{kl}\Delta + o(\Delta), & k \neq l; \\ 1 + \pi_{kk}\Delta + o(\Delta), & k = l. \end{cases} \quad (4)$$

where $\pi_{kl} \geq 0$ is the transition rate from mode k to mode l, $\pi_{kk} = -\sum_{k=1,k\neq l}^{M} \pi_{kk}$, and $o(\Delta)$ is a function satisfying $\lim_{\Delta\to 0} \frac{o(\Delta)}{\Delta} = 0$. In this work the modes $1,\ldots,M$ correspond to the topologies that are possible in the network due to the nodes mobility.

Any communication network is characterized by a number of physical resources constraints. A typical set of *physical constraints* corresponding to the network are now specified as follows

$$0 < x_i(t) \leq x_{buffer,i} \qquad 0 \leq C_i(t) \leq C_{server,i} \quad (5)$$

where $x_{buffer,i}$ is the buffer size and $C_{server,i}$ is the link capacity of node i.

On the other hand, the instantaneous traffic transmission rate and its rate of change at each node should satisfy

$$\lambda_i(t) \leq \lambda_i^{max} \leq C_{server,i} \qquad \dot{\lambda}_i(t) \in \mathscr{L}_\infty \quad (6)$$

Finally, the following two assumptions are made in this work

Assumption 1: The time-varying and unknown *delays $\tau_{ji}(t)$ are upper bounded and the maximum upper bound is a known constant, that is*

$$0 \leq \tau_{ji}(t) \leq h_{ji} \quad \text{with} \quad h = max\{h_{ji}\} \quad (7)$$

Assumption 2: The external incoming traffic to each node is L_2 norm bounded, that is

$$\int_0^\infty \|\lambda_i(t)\|^2 dt \leq \gamma_i, \qquad \gamma_i > 0 \quad (8)$$

B. Guaranteed Cost Control

The guaranteed cost control approach was first introduced in [19], which is an extension to the classical LQR regulation problem for linear systems with parametric uncertainties. The conceptual objective of the GCC is to design a feedback controller such that for all admissible uncertainties the closed-loop system is asymptotically stable and an upper bound on the corresponding cost function is guaranteed [16], [20], [21].

In this paper, the transmission, the processing, and the propagation delays in the mobile network are considered as unknown and time-varying variables in the dynamical system model (2). The guaranteed cost control problem of system (2) is then defined as follows.

Definition 1: [15] *For the Markovian jump time-delay system (2)-(3), the following jump quadratic cost function is defined*

$$J_i = E\{\int_0^\infty [x_i^T(t)Q_i(\alpha_t)x_i(t) + u_i^T(t)R_i(\alpha_t)u_i(t)]dt\} \quad (9)$$

where $x_i(t)$ is the state, $u_i(t)$ is the control input, and $Q_i(\alpha_t)$ and $R_i(\alpha_t)$ are positive definite matrices. Provided there exist a control law $u_i^*(t)$ and a positive scalar J_i^* such that the closed-loop system is stochastically stable and the cost function J_i satisfies

$$J_i \leq J_i^*$$

then J_i^* is the stochastic guaranteed cost *of the system (2)-(3) and $u_i^*(t)$ is the* stochastic guaranteed cost controller *of the system (2)-(3).*

III. Distributed Guaranteed Cost Congestion Control Strategy

In this paper, we consider three kinds of traffic, namely the *premium* (denoted by ”p”), the *ordinary* (denoted by ”r”), and the *best-effort* according to the definitions proposed by IETF [5]. The dynamic queuing models of the mobile network (2)-(3) are valid for each traffic class.

According to the dynamical queuing model (2)-(3), the congestion control strategy for the premium traffic is to allocate the output capacity $C_{pi}(t)$ such that the queuing length of the premium traffic is as close as possible to its reference value. On the other hand, the strategy for the ordinary traffic is to simultaneously regulate the incoming flow rate $\lambda_{ri}(t)$ and allocate the capacity $C_{ri}(t)$ such that its queuing length is as close as possible to its reference value. Finally, for the best-effort traffic, no explicit active control is designed in this paper since this traffic does not have any QoS requirements.

A. Premium Traffic Control Strategy

The control input for the premium traffic is the link capacity, that is $u_{pi}(t) = C_{pi}(t)$. Based on the nonlinear system model (2)-(3), the following feedback linearization scheme is first applied

$$u_{pi} = f^{-1}(x_{pi}(t))\bar{u}_{pi} \qquad z_{pi}(t) = x_{pi}(t) - x_{pi}^{ref}$$

where $\bar{u}_{pi}(t)$ denotes a state feedback controller, $z_{pi}(t)$ denotes the new state of the transformed linear system, and x_{pi}^{ref} denotes the reference queuing length at node i.

The nonlinear dynamical model (2)-(3) is transformed into the following equivalent linear one

$$\dot{z}_{pi}(t) = -\bar{u}_{pi}(t) + \lambda_{pi}(t) + \sum_{j\in\wp_i(\alpha_t)} \bar{u}_{pj}(t - \tau_{ji}(t))g_p^{ji}(t) \quad (10)$$

The distributed congestion controller of the premium traffic is selected as

$$\bar{u}_{pi}(t) = K_{pi}(\alpha_t)\bar{z}_{pi}(t) + W_{ji}(\alpha_t)K_{ji}(\alpha_t)\bar{Z}_{pj}(t) \quad (11)$$

where $\bar{z}_{pi} = [z_{pi}(t) \;\; \hat{\lambda}_{pi}(t)]^T$, $\bar{Z}_{pj}(t-\tau) = vec\{\bar{z}_{pj}^T(t-\tau_{ji}(t))\}$, and $K_{ji}(\alpha_t) = diag\{K_{pj}(\alpha_t)\}$.

The signal $\hat{\lambda}_{pi}(t)$ is an adaptive estimator used to estimate the unknown external incoming premium traffic $\lambda_{pi}(t)$ and compensates for its effect via feedback which is defined as follows

$$\dot{\hat{\lambda}}_{pi}(t) = \begin{cases} \delta_{pi}(\alpha_t)z_{pi}(t) - \beta_{pi}(\alpha_t)\hat{\lambda}_{pi}(t), & \text{if } 0 \leq \hat{\lambda}_{pi}(t) \leq \lambda_{pi}^{max} \text{ or} \\ & \hat{\lambda}_{pi}(t) = 0,\ z_{pi}(t) \geq 0 \text{ or} \\ & \hat{\lambda}_{pi}(t) = \lambda_{pi}^{max},\ z_{pi}(t) \leq 0 \\ -\beta_{pi}(\alpha_t)\hat{\lambda}_{pi}(t), & \text{otherwise} \end{cases} \quad (12)$$

Therefore, the closed-loop system of the premium traffic (10) after applying the distributed controller (11) becomes

$$\begin{aligned}\dot{\bar{z}}_{pi}(t) &= A_{ic}^{k}(\alpha_t)\bar{z}_{pi}(t) + \sum_{j\in\wp_i(\alpha_t)} B_{i0} w_{ji}^{p}(\alpha_t) K_{pj}(\alpha_t)\bar{z}_{pj}(t) \\ &+ \sum_{j\in\wp_i(\alpha_t)} B_j K_{pj}(\alpha_t)\bar{z}_{pj}(t-\tau_{ji}(t)) \\ &+ \sum_{\substack{j\in\wp_i(\alpha_t)\\ k\in\wp_j(\alpha_t)}} B_j w_{kj}^{p} K_{pk}(\alpha_t)\bar{z}_{pk}(t-\tau_{ji}(t)) + B_{\lambda_i}\lambda_{pi}(t)\end{aligned} \tag{13}$$

where $A_{ic}^{k}(\alpha_t) = A_{i0}^{k}(\alpha_t) + B_{i0}K_{pi}(\alpha_t)$, $A_{j0}^{k}(\alpha_t)$, B_{i0}, B_j, and B_{λ_i}, for $i,j = 1,...,n$ are the system matrices that are defined as follows

$$A_{i0}^{1}(\alpha_t) = \begin{bmatrix} 0 & 0 \\ \delta_{pi}(\alpha_t) & -\beta_{pi}(\alpha_t)\end{bmatrix} \qquad B_{i0} = \begin{bmatrix} -1 \\ 0 \end{bmatrix}$$
$$A_{i0}^{2}(\alpha_t) = \begin{bmatrix} 0 & 0 \\ 0 & -\beta_{pi}(\alpha_t)\end{bmatrix} \quad B_j = \begin{bmatrix} g_{ji}^{p} \\ 0 \end{bmatrix} \quad B_{\lambda_i} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

In view of the closed-loop system (13), the control objective of the distributed guaranteed cost congestion control (DGCC) problem is to actually select the local state feedback control gains $K_{pi}(\alpha_t)$ and the adjusting weights of the neighboring controllers $W_{ji}(\alpha_t)$, such that the system (13) is stable and the following jump quadratic cost function is bounded

$$J_{pi} = E\{\int_0^{\infty}(\bar{z}_{pi}^{T}(t)Q_i(\alpha_t)\bar{z}_{pi}(t) + \bar{u}_{pi}^{T}R_i(\alpha_t)\bar{u}_{pi}(t))dt\} \tag{14}$$

where $Q_i(\alpha_t)$ and $R_i(\alpha_t)$ are positive definite matrices with respect to each mode $\alpha_t \in \mathscr{S} = \{1,...,M\}$.

Lemma 1: Given the cost function (14), if there exist symmetric positive definite matrices $\Lambda_{i1}^{T}(\alpha_t)$, $\bar{X}_{ik}(\alpha_t)$, $\bar{V}_{ii}(\alpha_t)$, $\bar{T}_i(\alpha_t)$, and matrices U_i, $N_i(\alpha_t)$, Λ_{i3}^{T}, and $\bar{S}_i(\alpha_t)$ for $k = 1,2$, $i = 1,....n$, and $\alpha_t \in \mathscr{S} = \{1,\dots,M\}$ such that the following LMI conditions are satisfied

$$\bar{\Omega}_{ik} = \begin{bmatrix} Y_{ik} & B_{i0}W_{ji} + h^2\bar{W}_{ji} & I + h^2(\bar{V}_{ik}+\bar{T}_i) & I + h^2(\bar{V}_{ik}+\bar{T}_i) \\ * & h^2\tilde{W}_{ji} + \tilde{R}_i & h^2\hat{W}_{ji} & h^2\hat{W}_{ji} \\ * & * & h^2U_i & h^2U_i \\ * & * & * & h^2U_i \end{bmatrix} < 0$$

$$Y_{ik} = 2(V_{ik}+T_i) + \sum_{k=1}^{M}\pi_{\alpha_t k}\Lambda_{i1} + h^2\tilde{V}_{ik}(\alpha_t) + (1+h)\bar{S}_i - \bar{U}_i + \bar{Q}_i + \bar{R}_i$$

then the distributed controller (11) is a stochastic guaranteed cost controller of the system (10), and the state feedback control gain is given by $K_{pi}(\alpha_t) = B_{i0}^{+}T_i(\alpha_t)\Lambda_{i1}^{-1}(\alpha_t)$, where "+" denotes the Moore-Penrose inverse.

Proof: Consider the following stochastic Lyapunov-Krasovskii functional candidate

$$\begin{aligned}&V_i(\bar{z}_{pi}(t),\alpha_t) = V_{i1} + V_{i2} + V_{i3} + V_{i4} \\ &V_{i1} = \bar{z}_{pi}(t)^{T}P_i(\alpha_t)\bar{z}_{pi}(t) \\ &V_{i2} = \int_{t-h}^{t}\bar{z}_{pi}^{T}(s)S_i(\alpha_t)\bar{z}_{pi}(s)ds \\ &V_{i3} = h\int_{-h}^{0}\int_{t+\theta}^{t}\dot{\bar{z}}_{pi}^{T}(s)U_i\dot{\bar{z}}_{pi}(s)dsd\theta \\ &V_{i4} = \int_{-h}^{0}\int_{t+\theta}^{t}\bar{z}_{pi}^{T}(s)S_i(\alpha_t)\bar{z}_{pi}(s)dsd\theta\end{aligned} \tag{15}$$

and $P_i(\alpha_t)$, $S_i(\alpha_t)$, U_i are positive definite matrices with appropriate dimensions. For each mode $\alpha_t = k \in \mathscr{S}$, the infinitesimal generator [11] of the Lyapunov function can be derived as follows

$$\begin{aligned}\mathscr{L}V_{i1} &= 2\bar{z}_{pi}^{T}(t)P_i(\alpha_t)[A_{ic}^{k}(\alpha_t)\bar{z}_{pi}(t) + \sum_{j\in\wp_i(\alpha_t)}B_jK_{pj}(\alpha_t)\bar{z}_{pj}(t-\tau_{ji}(t)) \\ &+ \sum_{j\in\wp_i(\alpha_t)}B_{i0}w_{ji}^{p}(\alpha_t)K_{pj}(\alpha_t)\bar{z}_{pj}(t) \\ &+ \sum_{\substack{j\in\wp_i(\alpha_t)\\ k\in\wp_j(\alpha_t)}}B_jw_{kj}^{p}K_{pk}(\alpha_t)\bar{z}_{pk}(t-\tau_{ji}(t))] \\ &+ \bar{z}_{pi}^{T}(t)\sum_{k=1}^{M}\pi_{\alpha_t k}P_i(k)\bar{z}_{pi}(t) + 2\bar{z}_{pi}^{T}(t)P_i(\alpha_t)B_{\lambda_i}\lambda_{pi}(t) \\ \mathscr{L}V_{i2} &= \bar{z}_{pi}^{T}(t)S_i(\alpha_t)\bar{z}_{pi}(t) - (1-h)\bar{z}_{pi}^{T}(t-h)S_i(\alpha_t)\bar{z}_{pi}(t-h) \\ &+ \int_{t-h}^{t}\bar{z}_{pi}^{T}(s)\sum_{k=1}^{M}\pi_{\alpha_t k}S_i(k)\bar{z}_{pi}(s)ds \\ \mathscr{L}V_{i3} &= h^2[A_{ic}^{k}(\alpha_t)\bar{z}_{pi}(t) + \sum_{j\in\wp_i(\alpha_t)}B_jK_{pj}(\alpha_t)\bar{z}_{pj}(t-\tau_{ji}(t)) \\ &+ B_{\lambda_i}\lambda_{pi}(t) + \sum_{j\in\wp_i(\alpha_t)}B_{i0}w_{ji}^{p}(\alpha_t)K_{pj}(\alpha_t)\bar{z}_{pj}(t) \\ &+ \sum_{\substack{j\in\wp_i(\alpha_t)\\ k\in\wp_j(\alpha_t)}}B_jw_{kj}^{p}K_{pk}(\alpha_t)\bar{z}_{pk}(t-\tau_{ji}(t))]^{T}U_i \\ &[A_{ic}^{k}(\alpha_t)\bar{z}_{pi}(t) + \sum_{j\in\wp_i(\alpha_t)}B_jK_{pj}(\alpha_t)\bar{z}_{pj}(t-\tau_{ji}(t)) \\ &+ B_{\lambda_i}\lambda_{pi}(t) + \sum_{j\in\wp_i(\alpha_t)}B_{i0}w_{ji}^{p}(\alpha_t)K_{pj}(\alpha_t)\bar{z}_{pj}(t) \\ &+ \sum_{\substack{j\in\wp_i(\alpha_t)\\ k\in\wp_j(\alpha_t)}}B_jw_{kj}^{p}K_{pk}(\alpha_t)\bar{z}_{pk}(t-\tau_{ji}(t))] - h\int_{t-h}^{t}\dot{\bar{z}}_{pi}^{T}(s)U_i\dot{\bar{z}}_{pi}(t)ds \\ \mathscr{L}V_{i4} &= h\bar{z}_{pi}^{T}(t)S_i(\alpha_t)\bar{z}_{pi}(t) - \int_{t-h}^{t}\bar{z}_{pi}^{T}(s)\sum_{k=1}^{M}\pi_{\alpha_t k}S_i(k)\bar{z}_{pi}(s)ds\end{aligned}$$

Let us define $\bar{Z}_{pk}(t-\tau) = vec\{\bar{z}_{pk}^{T}(t-\tau_{ji}(t))\} \quad k \in \wp_j(\alpha_t)$, Then by adding up $\mathscr{L}V_{i1}$ to $\mathscr{L}V_{i4}$ one can obtain

$$\begin{aligned}\mathscr{L}V_i &\le \eta_i^{T}(t,\tau,h)\Sigma_{ik}(\alpha_t)\eta_i(t,\tau,h) + \eta_i^{T}(t,\tau,h)\Theta_{ik}(\alpha_t)B_{\lambda_i}\lambda_{pi}(t) \\ &+ h^2\lambda_{pi}^{T}(t)B_{\lambda_i}^{T}U_iB_{\lambda_i}\lambda_{pi}(t) \\ &\le \eta_i^{T}(t,\tau,h)[\Sigma_{ik}(\alpha_t) + M_{ik}(\alpha_t)]\eta_i(t,\tau,h) \\ &+ \lambda_{pi}^{T}(t)B_{\lambda_i}^{T}[\Theta_{ik}^{T}M_{ik}^{-1}\Theta_{ik}(\alpha_t) + h^2U_i]B_{\lambda_i}\lambda_{pi}(t) \\ &= \eta_i^{T}(t,\tau,h)W_{ik}(\alpha_t)\eta_i(t,\tau,h) + \lambda_{pi}^{T}(t)\Psi_{ik}\lambda_{pi}(t)\end{aligned} \tag{16}$$

where $\eta_i(t,\tau,h) = [\bar{z}_{pi}^{T}(t)\ \bar{Z}_{pj}^{T}(t)\ \bar{Z}_{pj}^{T}(t-\tau)\ \bar{Z}_{pk}^{T}(t-\tau)\ \bar{z}_{pi}^{T}(t-h)]^{T}$; $M_{ik}(\alpha_t)$ is a positive definite matrix, and the matrices Σ_{ik} and Θ_{ik} are defined as

$$\Sigma_{ik}(\alpha_t) = \begin{bmatrix} \sigma_1 & \sigma_2 & [P_i(\alpha_t) + h^2(A_{ic}^{k}(\alpha_t))^{T}U_i]B_{ji}K_{ji}(\alpha_t) & \sigma_3 & U_i \\ * & \sigma_4 & h^2K_{ji}^{T}(\alpha_t)W_{ji}^{T}(\alpha_t)B_{i0}^{T}U_iB_{ji}K_{ji}(\alpha_t) & \sigma_5 & 0 \\ * & * & h^2K_{ji}^{T}(\alpha_t)B_{ji}^{T}U_iB_{ji}K_{ji}(\alpha_t) & \sigma_6 & 0 \\ * & * & * & \sigma_7 & 0 \\ * & * & * & * & -U_i-(1-h)S_i(\alpha_t) \end{bmatrix} \tag{17}$$

$$\begin{aligned}&W_{ik}(\alpha_t) = \Sigma_{ik}(\alpha_t) + M_{ik}(\alpha_t) \\ &\Psi_i(\alpha_t) = B_{\lambda_i}^{T}[\Theta_{ik}^{T}M_{ik}^{-1}\Theta_{ik}(\alpha_t) + h^2U_i]B_{\lambda_i} \\ &\Theta_{ik}(\alpha_t) = \begin{bmatrix}\theta_1 & \theta_2 & \theta_3 & \theta_4 & 0\end{bmatrix}^{T} \\ &\theta_1 = 2P_i(\alpha_t) + 2h^2(A_{ic}^{k}(\alpha_t))^{T}U_i; \quad \theta_2 = 2h^2K_{ji}^{T}(\alpha_t)W_{ji}^{T}(\alpha_t)B_{i0}^{T}U_i \\ &\theta_3 = 2h^2K_{ji}^{T}(\alpha_t)B_{ji}^{T}U_i; \quad \theta_4 = 2h^2K_{kj}^{T}(\alpha_t)W_{kj}^{T}(\alpha_t)B_{kj}^{T}U_i\end{aligned}$$

Let us define $\bar{W}_{ik}(\alpha_t) = W_{ik}(\alpha_t) + diag\{Q_i(\alpha_t) + K_{pi}^{T}(\alpha_t)R_i(\alpha_t)K_{pi}(\alpha_t)\ \ K_{ji}^{T}W_{ji}^{T}R_iW_{ji}K_{ji}\ \ 0\ \ 0\ \ 0\}$, and the following matrices

$$\Lambda_{i1}(\alpha_t) = P_i^{-1}(\alpha_t); \qquad \Lambda_{i2}(\alpha_t) = K_{ji}^{+}(\alpha_t)$$
$$\Lambda_{i3}(\alpha_t) = [B_{ji}K_{ji}(rt)]^{-1}; \quad \Lambda_{i4}(\alpha_t) = [B_{kj}W_{kj}(\alpha_t)K_{ji}(\alpha_t)]^{-1}$$
$$\Lambda_{i5}(\alpha_t) = 0; \qquad \Lambda_i(\alpha_t) = diag\{\Lambda_{ij}(\alpha_t)\}$$

By pre and post multiplying the matrix $\bar{W}_{ik}(\alpha_t)$ with Λ_i^T and Λ_i, respectively, we obtain

$$\Omega_{ik}(\alpha_t) = \Lambda_i^T(\alpha_t)\bar{W}_{ik}(\alpha_t)\Lambda_i(\alpha_t) = \begin{bmatrix} \bar{\Omega}_{ik} & 0 \\ 0 & 0 \end{bmatrix} \quad (18)$$

where

$$\bar{\Omega}_{ik} = \begin{bmatrix} X_{ik} & B_{i0}W_{ji} + h^2A_{ic}^kU_iB_{i0}W_{ji} & I + h^2A_{ic}^kU_i & I + h^2A_{ic}^kU_i \\ * & h^2W_{ji}^TB_{i0}^TU_iB_{i0}W_{ji} + W_{ji}^TR_iW_{ji} & h^2W_{ji}^TB_{i0}^TU_i & h^2W_{ji}^TB_{i0}^TU_i \\ * & * & h^2U_i & h^2U_i \\ * & * & * & h^2U_i \end{bmatrix} \quad (19)$$

$$X_{ik} = 2A_{ic}^k\Lambda_{i1} + \sum_{k=1}^{M}\pi_{\alpha_t k}\Lambda_{i1} + h^2\Lambda_{i1}^T(A_{ic}^k)^TU_iA_{ic}^k\Lambda_{i1} + (1+h)\Lambda_{i1}^TS_i\Lambda_{i1} - \Lambda_{i1}^TU_i\Lambda_{i1} + \Lambda_{i1}^T(Q_i + K_{pi}^TR_iK_{pi})\Lambda_{i1}$$

Therefore, if we further define

$$A_{i0}^k(\alpha_t) = V_{ik}(\alpha_t)\Lambda_{i1}^{-1}(\alpha_t) \qquad B_{i0}K_{pi}(\alpha_t) = T_i(\alpha_t)\Lambda_{i1}^{-1}(\alpha_t)$$
$$\bar{V}_{ik}(\alpha_t) = V_{ik}^T(\alpha_t)U_i; \qquad \bar{T}_i^T(\alpha_t) = T_i^T(\alpha_t)U_i$$
$$\bar{R}_i = \Lambda_{i1}^TK_{pi}^TR_iK_{pi}\Lambda_{i1}; \qquad \tilde{R}_i = W_{ji}^TR_iW_{ji}$$
$$\bar{W}_{ji} = A_{ic}^kU_iB_{i0}W_{ji}; \qquad \hat{W}_{ji} = W_{ji}^TB_{i0}^TU_i$$
$$\tilde{W}_{ji} = \hat{W}_{ji}B_{i0}W_{ji}; \qquad \bar{S}_i = \Lambda_{i1}^TS_i\Lambda_{i1}$$
$$\bar{Q}_i = \Lambda_{i1}^TQ_i\Lambda_{i1}; \qquad \bar{U}_i = \Lambda_{i1}^TU_i\Lambda_{i1}$$
$$\tilde{V}_{ik}(\alpha_t) = (\bar{V}_{ik}(\alpha_t) + \bar{T}_i^T(\alpha_t))(V_{ik}(\alpha_t) + T_i(\alpha_t))$$

the matrix $\bar{\Omega}_{ik}(\alpha_t)$ becomes

$$\bar{\Omega}_{ik} = \begin{bmatrix} Y_{ik} & B_{i0}W_{ji} + h^2\bar{W}_{ji} & I + h^2(\bar{V}_{ik} + \bar{T}_i) & I + h^2(\bar{V}_{ik} + \bar{T}_i) \\ * & h^2\tilde{W}_{ji} + \tilde{R}_i & h^2\hat{W}_{ji} & h^2\hat{W}_{ji} \\ * & * & h^2U_i & h^2U_i \\ * & * & * & h^2U_i \end{bmatrix}$$

$$Y_{ik} = 2(V_{ik} + T_i) + \sum_{k=1}^{M}\pi_{\alpha_t k}\Lambda_{i1} + h^2\tilde{V}_{ik}(\alpha_t) + (1+h)\bar{S}_i - \bar{U}_i + \bar{Q}_i + \bar{R}_i$$

Therefore, if $\bar{\Omega}_{ik}(\alpha_t) < 0$, one will have $\Omega_{ik} < 0$, and hence $\bar{W}_{ik} < 0$. Furthermore, by solving the LMI conditions $\bar{\Omega}_{ik}(\alpha_t) < 0$, the weight matrix $W_{ji}(\alpha_t)$ can be obtained directly. The state feedback control gain $K_{pi}(\alpha_t)$ and the system matrix A_{i0}^k can be expressed as $K_{pi}(\alpha_t) = B_{i0}^{+}T_i(\alpha_t)\Lambda_{i1}^{-1}(\alpha_t)$ and $A_{i0}^k(\alpha_t) = V_{ik}(\alpha_t)\Lambda_{i1}^{-1}(\alpha_t)$.

Furthermore, since $\bar{W}_{ik}(\alpha_t) < 0$, one will have $W_{ik}(\alpha_t) < 0$. From (16), it then follows that

$$\mathscr{L}V_i \leq -\bar{z}_{pi}^T(t)(Q_i(\alpha_t) + K_{pi}^T(\alpha_t)R_i(\alpha_t)K_{pi}(\alpha_t))\bar{z}_{pi}(t) - \bar{Z}_{pj}^TK_{ji}^TW_{ji}^TR_iW_{ji}K_{ji}\bar{Z}_{pj}(t) + \lambda_{pi}^T(t)\Psi_i(\alpha_t)\lambda_{pi}(t) \quad (20)$$

Therefore, for any $[\bar{z}_{pi}(t) \;\; \bar{Z}_{pj}(t)]$ that satisfies

$$\begin{bmatrix} \bar{z}_{pi}^T(t) \\ \bar{Z}_{pj}^T(t) \end{bmatrix}^T \mathbb{C}_{ik} \begin{bmatrix} \bar{z}_{pi}(t) \\ \bar{Z}_{pj}(t) \end{bmatrix} \geq \Psi_i(\alpha_t)\lambda_{pi}^2(t)$$

$$\mathbb{C}_{ik} = \begin{bmatrix} Q_i(\alpha_t) + K_{pi}^T(\alpha_t)R_i(\alpha_t)K_{pi}(\alpha_t) & 0 \\ 0 & K_{ji}^TW_{ji}^TR_iW_{ji}K_{ji} \end{bmatrix}$$

one will have $\mathscr{L}V_i < 0$.

Therefore, the system (13) is stochastically ultimately bounded and the ultimate bounded region is given by

$$\|\bar{z}_{pi}(t)\|^2 + \|\bar{Z}_{pj}(t)\|^2 \geq \frac{max(\Psi_i(\alpha_t))}{\lambda_{min}(\mathbb{C}_{ik})}\lambda_{pi}^2(t) \quad (21)$$

On the other hand, in review of (20), we have

$$J_{pi} \leq E\{\int_0^\infty(-\mathscr{L}V_i + \lambda_{pi}^T(t)\Psi_i(\alpha_t)\lambda_{pi}(t))dt\}$$
$$= V_i(\bar{z}_{pi}(0), 0, r_0) - \lim_{t\to\infty} V_i(\bar{z}_{pi}(t), t, \alpha_t) + E\{\int_0^\infty \Psi_i(\alpha_t)\lambda_{pi}^2(t)dt\}$$
$$\leq V_i(\bar{z}_{pi}(0), 0, r_0) - \bar{z}_{pi}^T(\infty)P_i(r_\infty)\bar{z}_{pi}(\infty) + \gamma_i max(\Psi_i(\alpha_t))$$

Therefore, the upper bound of the cost function J_{pi} is (since $\bar{z}_{pi}(\infty) = 0$)

$$J_{pi} < V_i(\bar{z}_{pi}(0), 0, r_0) + \gamma_i max(\Psi_i(\alpha_t)) = J_{pi}^* \quad (22)$$

Therefore, the distributed control law (11) is a guaranteed cost controller for the system (13). This completes the proof of Lemma 1. ■

B. Ordinary Traffic Control Strategy

Since the incoming ordinary traffic $\lambda_{ri}(t)$ is measurable and available for control, the control inputs for the ordinary traffic are the link capacity and the incoming traffic, that are $u_{ri}^1(t) = C_{ri}(t)$ and $u_{ri}^2(t) = \lambda_{ri}(t)$. Similar to the premium traffic, we first apply the following feedback linearization scheme to the open-loop system (2)-(3), namely

$$z_{ri}(t) = x_{ri}(t) - x_{ri}^{ref} \quad \text{and} \quad u_{ri}(t) = F^{-1}(x_{ri}, t)\bar{u}_{ri}(t)$$

where $u_{ri}(t) = [u_{ri}^1(t), u_{ri}^2(t)]^T$, $\bar{u}_{ri}(t) = [\bar{u}_{ri}^1(t), \bar{u}_{ri}^2(t)]^T$, and $F(x_{ri}(t)) = diag\{f(x_{ri}(t)), 1\}$.

The resulting dynamical model (2)-(3) with respect to the ordinary traffic becomes

$$\dot{z}_{ri}(t) = B_{i0}\bar{u}_{ri}(t) + \sum_{j\in\wp_i(\alpha_t)} B_{ij}\bar{u}_{rj}(t - \tau_{ji}(t)) \quad (23)$$

where $B_{i0} = \begin{bmatrix} -1 & 1 \end{bmatrix}$ and $B_{ij} = \begin{bmatrix} g_r^{ji} & 0 \end{bmatrix}$ are the system matrices. The performance cost function for the ordinary traffic is selected as

$$J_{ri} = E\{\int_0^\infty(z_{ri}^T(t)Q_i(\alpha_t)z_{ri}(t) + \bar{u}_{ri}^T(t)R_i(\alpha_t)\bar{u}_{ri}(t))dt\} \quad (24)$$

where $Q_i(\alpha_t)$ and $R_i(\alpha_t)$ are given positive definite matrices.

Based on the decentralized model (23), the distributed congestion controller for the ordinary traffic is selected as

$$\bar{u}_{ri}(t) = K_{ri}(\alpha_t)z_{ri}(t) + W_{ji}(\alpha_t)K_{ji}(\alpha_t)Z_{rj}(t) \quad (25)$$

where $W_{ji}(\alpha_t) = vec\{w_{ji}^r(\alpha_t)\}$ and $K_{ji}(\alpha_t) = diag\{K_{ri}(\alpha_t)\}$.

Therefore, the control objective for the ordinary traffic is to select the distributed control gain $K_{ri}(\alpha_t)$ and the distributed weight matrix W_{ji} such that the system (23) is stochastically stable and the following cost function is upper bounded:

$$J_{ri} = E\{\int_0^\infty(z_{ri}^T(t)Q_i(\alpha_t)z_{ri}(t) + \bar{u}_{ri}^T(t)R_i(\alpha_t)\bar{u}_{ri}(t))dt\} \quad (26)$$

where $Q_i(\alpha_t)$ and $R_i(\alpha_t)$ are given positive definite matrices.

Lemma 2: Given the cost function (26), the distributed controller (25) is the stochastic guaranteed cost controller for the system (23), if there exist symmetric positive definite matrices $\Lambda_{i1}(\alpha_t)$, $\bar{S}_i(\alpha_t)$, U_i, $\bar{U}_i$, $\bar{Q}_i(\alpha_t)$, $\bar{R}_i(\alpha_t)$, positive definite matrices $T_i(\alpha_t)$, $\tilde{T}_i(\alpha_t)$, $\bar{W}_{ji}(\alpha_t)$, $\hat{W}_{ji}(\alpha_t)$, and $\tilde{W}_{ji}(\alpha_t)$, for $i = 1, ..., n$, $\alpha_t \in \mathscr{S} = \{1, ...M\}$, such that the following LMI conditions are satisfied

$$\bar{\Sigma}_{ik}(\alpha_t) = \begin{bmatrix} Y_{ik} & B_{i0}W_{ji} + h^2\bar{W}_{ji} & I + h^2\bar{T}_i & I + h^2\bar{T}_i \\ * & h^2\tilde{W}_{ji} & h^2\hat{W}_{ji} & h^2\hat{W}_{ji} \\ * & * & h^2U_i & h^2U_i \\ * & * & * & h^2U_i \end{bmatrix} < 0$$

$$Y_{ik} = 2T_i(\alpha_t) + \sum_{k=1}^{M}\pi_{\alpha_t k}\Lambda_{i1} + h^2\tilde{T}_i(\alpha_t) + (1+h)\bar{S}_i - \bar{U}_i + \bar{Q}_i + \bar{R}_i$$

and the distributed control gain is given by $K_{ri}(\alpha_t) = B_{i0}^{+}T_i(\alpha_t)\Lambda_{i1}^{-1}(\alpha_t)$.

Proof: Consider the following stochastic Lyapunov-Krasovskii functional candidate

$$V_i(\bar{z}_{ri}(t),\alpha_t) = V_{i1}+V_{i2}+V_{i3}+V_{i4} \quad (27)$$
$$V_{i1} = \bar{z}_{ri}(t)^T P_i(\alpha_t) z_{ri}(t)$$
$$V_{i2} = \int_{t-h}^{t} z_{ri}^T(s) S_i(\alpha_t) z_{ri}(s) ds$$
$$V_{i3} = h\int_{-h}^{0}\int_{t+\theta}^{t} \dot{z}_{ri}^T(s) U_i \dot{\bar{z}}_{ri}(s) ds d\theta$$
$$V_{i4} = \int_{-h}^{0}\int_{t+\theta}^{t} z_{ri}^T(s) S_i(\alpha_t) z_{ri}(s) ds d\theta$$

and $P_i(\alpha_t)$, $S_i(\alpha_t)$, U_i are positive definite matrices with appropriate dimensions. For each mode $\alpha_t = k \in \mathscr{S}$, the infinitesimal generator of the Lyapunov function can then be derived as follows

$$\begin{aligned}
\mathscr{L}V_{i1} &= \lim_{\Delta\to 0^+}\frac{1}{\Delta}\{E[V_{i1}(z_{ri}(t+\Delta),\alpha_{t+\delta},t+\Delta)|z_{ri}(t),\alpha_t=k] \\
&\quad -V_{i1}(z_{ri}(t),k,t)\} \\
&= 2z_{ri}^T(t)P_i(\alpha_t)\dot{\bar{z}}_{ri}(t)+\sum_{k=1}^{M}\pi_{\alpha_t k} z_{ri}^T(t)P_i(k)z_{ri}(t) \\
&= 2z_{ri}^T(t)P_i(\alpha_t)[B_{i0}K_{ri}(\alpha_t)z_{ri}(t)+\sum_{j\in\wp_i(\alpha_t)} B_jK_{rj}(\alpha_t)z_{rj}(t-\tau_{ji}(t)) \\
&\quad +\sum_{j\in\wp_i(\alpha_t)} B_{i0}w_{ji}^p(\alpha_t)K_{rj}(\alpha_t)z_{rj}(t) \\
&\quad +\sum_{\substack{j\in\wp_i(\alpha_t)\\ k\in\wp_j(\alpha_t)}} B_jw_{kj}^pK_{rk}(\alpha_t)z_{rk}(t-\tau_{ji}(t))]+z_{ri}^T(t)\sum_{k=1}^{M}\pi_{\alpha_t k}P_i(k)z_{ri}(t) \\
\mathscr{L}V_{i2} &= \int_{t-h}^{t} 2z_{ri}^T(s)S_i(\alpha_t)\dot{\bar{z}}_{pi}(s)ds+\int_{t-h}^{t} z_{ri}^T(s)\sum_{k=1}^{M}\pi_{\alpha_t k}S_i(k)z_{ri}(s)ds \\
&= z_{ri}^T(t)S_i(\alpha_t)z_{ri}(t)-(1-h)z_{ri}^T(t-h)S_i(\alpha_t)z_{ri}(t-h) \\
&\quad +\int_{t-h}^{t} z_{ri}^T(s)\sum_{k=1}^{M}\pi_{\alpha_t k}S_i(k)z_{ri}(s)ds \\
\mathscr{L}V_{i3} &= h^2\dot{z}_{ri}^T(t)U_i\dot{z}_{ri}(t)-h\int_{t-h}^{t}\dot{z}_{ri}^T(s)U_i\dot{\bar{z}}_{ri}(s)ds \\
&= h^2[B_{i0}K_{ri}(\alpha_t)z_{ri}(t)+\sum_{j\in\wp_i(\alpha_t)} B_jK_{rj}(\alpha_t)z_{rj}(t-\tau_{ji}(t)) \\
&\quad +\sum_{j\in\wp_i(\alpha_t)} B_{i0}w_{ji}^p(\alpha_t)K_{rj}(\alpha_t)z_{rj}(t) \\
&\quad +\sum_{\substack{j\in\wp_i(\alpha_t)\\ k\in\wp_j(\alpha_t)}} B_jw_{kj}^pK_{rk}(\alpha_t)z_{rk}(t-\tau_{ji}(t))]^TU_i \\
&\quad \times[B_{i0}K_{ri}(\alpha_t)z_{ri}(t)+\sum_{j\in\wp_i(\alpha_t)} B_jK_{rj}(\alpha_t)z_{rj}(t-\tau_{ji}(t)) \\
&\quad +\sum_{j\in\wp_i(\alpha_t)} B_{i0}w_{ji}^p(\alpha_t)K_{rj}(\alpha_t)z_{rj}(t) \\
&\quad +\sum_{\substack{j\in\wp_i(\alpha_t)\\ k\in\wp_j(\alpha_t)}} B_jw_{kj}^pK_{rk}(\alpha_t)z_{rk}(t-\tau_{ji}(t))]-h\int_{t-h}^{t}\dot{z}_{ri}^T(s)U_i\dot{\bar{z}}_{ri}(t)ds \\
\mathscr{L}V_{i4} &= hz_{ri}^T(t)S_i(\alpha_t)z_{ri}(t)-\int_{t-h}^{t} z_{ri}^T(s)\sum_{k=1}^{M}\pi_{\alpha_t k}S_i(k)z_{ri}(s)ds
\end{aligned}$$

By defining $Z_{rk}(t-\tau) = vec\{\bar{z}_{rk}^T(t-\tau_{ji}(t))\} \quad k\in\wp_j(\alpha_t)$ and $B_{ji}K_{ji}(\alpha_t)z_{rj}(t-\tau) = \sum_{j\in\wp_i(\alpha_t)} B_jK_{rj}(\alpha_t)z_{rj}(t-\tau_{ji}(t))$, and adding up $\mathscr{L}V_{i1}$ to $\mathscr{L}V_{i4}$, one gets

$$\mathscr{L}V_i \leq \eta_i^T(t,\tau,h)\Sigma_i(\alpha_t)\eta_i(t,\tau,h) \quad (28)$$

where $\eta_i(t,\tau,h) = [z_{ri}^T(t)\ \bar{Z}_{rj}^T(t)\ \bar{Z}_{rj}^T(t-\tau)\ \bar{Z}_{rk}^T(t-\tau)\ z_{ri}^T(t-h)]^T$, and Σ_i is given by

$$\Sigma_i(\alpha_t) = \begin{bmatrix} \sigma_1 & \sigma_2 & [P_i(\alpha_t)+h^2(B_{i0}K_{ri}(\alpha_t))^TU_i]B_{ji}K_{ji}(\alpha_t) & \sigma_3 & U_i \\ * & \sigma_4 & h^2K_{ji}^T(\alpha_t)W_{ji}^T(\alpha_t)B_{i0}^TU_iB_{ji}K_{ji}(\alpha_t) & \sigma_5 & 0 \\ * & * & h^2K_{ji}^T(\alpha_t)B_{ji}^TU_iB_{ji}K_{ji}(\alpha_t) & \sigma_6 & 0 \\ * & * & * & \sigma_7 & 0 \\ * & * & * & * & -U_i-(1-h)S_i(\alpha_t) \end{bmatrix} \quad (29)$$

Let us define

$$\bar{\Sigma}_{ik}(\alpha_t) = \Sigma_i(\alpha_t)+\begin{bmatrix} Q_i+K_i^TR_iK_i & 0 \\ 0 & K_{ji}^TW_{ji}^TR_iW_{ji}K_{ji} \end{bmatrix} < 0 \quad (30)$$

and the matrices

$$\begin{aligned}
&\Lambda_{i1}(\alpha_t) = P_i^{-1}(\alpha_t); && \Lambda_{i2}(\alpha_t) = K_{ji}^{+}(\alpha_t) \\
&\Lambda_{i3}(\alpha_t) = [B_{ji}K_{ji}(rt)]^{-1}; && \Lambda_{i4}(\alpha_t) = [B_{kj}W_{kj}(\alpha_t)K_{ji}(\alpha_t)]^{-1} \\
&\Lambda_{i5}(\alpha_t) = 0; && \Lambda_i(\alpha_t) = diag\{\Lambda_{ij}(\alpha_t)\}
\end{aligned}$$

By pre and post multiplying the matrix $\Sigma_i(\alpha_t)$ with Λ_i^T and Λ_i, respectively, one will obtain

$$\bar{\Sigma}_{ik}(\alpha_t) = \Lambda_i^T(\alpha_t)\Sigma_i(\alpha_t)\Lambda_i(\alpha_t) = \begin{bmatrix} \tilde{\Sigma}_{ik} & 0 \\ 0 & 0 \end{bmatrix} \quad (31)$$

where:

$$\tilde{\Sigma}_{ik} = \begin{bmatrix} X_{ik} & B_{i0}W_{ji}+h^2B_{i0}K_{ri}U_iB_{i0}W_{ji} & I+h^2B_{i0}K_{ri}U_i & I+h^2B_{i0}K_{ri}U_i \\ * & h^2W_{ji}^TB_{i0}^TU_iB_{i0}W_{ji}+W_{ji}^TR_iW_{ji} & h^2W_{ji}^TB_{i0}^TU_i & h^2W_{ji}^TB_{i0}^TU_i \\ * & * & h^2U_i & h^2U_i \\ * & * & * & h^2U_i \end{bmatrix} \quad (32)$$

$$\begin{aligned}
X_{ik} &= 2B_{i0}K_{ri}\Lambda_{i1}+\sum_{k=1}^{M}\pi_{\alpha_t k}\Lambda_{i1}+h^2\Lambda_{i1}^T(B_{i0}K_{ri})^TU_iB_{i0}K_{ri}\Lambda_{i1} \\
&\quad +(1+h)\Lambda_{i1}^TS_i\Lambda_{i1}-\Lambda_{i1}^TU_i\Lambda_{i1}+\Lambda_{i1}^T(Q_i+K_{pi}^TR_iK_{pi})\Lambda_{i1}
\end{aligned}$$

Therefore, if we define

$$\begin{aligned}
&B_{i0}K_{ri}(\alpha_t) = T_i(\alpha_t)\Lambda_{i1}^{-1}(\alpha_t); && \bar{T}_i^T(\alpha_t) = T_i^T(\alpha_t)U_i \\
&\bar{W}_{ji} = B_{i0}K_{ri}U_iB_{i0}W_{ji}; && \hat{W}_{ji} = W_{ji}^TB_{i0}^TU_i \\
&\tilde{W}_{ji} = \hat{W}_{ji}B_{i0}W_{ji}; && \bar{R}_i = \Lambda_{i1}^TK_{pi}^TR_iK_{pi}\Lambda_{i1} \\
&\bar{S}_i = \Lambda_{i1}^TS_i\Lambda_{i1}; && \bar{U}_i = \Lambda_{i1}^TU_i\Lambda_{i1} \\
&\bar{Q}_i = \Lambda_{i1}^TQ_i\Lambda_{i1}; && \tilde{T}_i(\alpha_t) = \bar{T}_i^T(\alpha_t)T_i(\alpha_t))
\end{aligned}$$

then the matrix $\tilde{\Sigma}_{ik}(\alpha_t)$ becomes

$$\tilde{\Sigma}_i(\alpha_t) = \begin{bmatrix} Y_{ik} & B_{i0}W_{ji}+h^2\bar{W}_{ji} & I+h^2\bar{T}_i & I+h^2\bar{T}_i \\ * & h^2\tilde{W}_{ji} & h^2\hat{W}_{ji} & h^2\hat{W}_{ji} \\ * & * & h^2U_i & h^2U_i \\ * & * & * & h^2U_i \end{bmatrix}$$

$$Y_{ik} = 2T_i+\sum_{k=1}^{M}\pi_{\alpha_t k}\Lambda_{i1}+h^2\tilde{T}_i(\alpha_t)+(1+h)\bar{S}_i-\bar{U}_i+\bar{Q}_i+\bar{R}_i$$

Therefore, if $\tilde{\Sigma}_i(\alpha_t) < 0$, one will have $\Sigma_i(\alpha_t) < 0$, and the system (23) is stochastically stable. By solving the LMI conditions $\bar{\Omega}_{ik}(\alpha_t) < 0$, and the weight matrix $\tilde{\Sigma}_{ik}(\alpha_t) < 0$, one can obtain $K_{ri}(\alpha_t) = B_{i0}^{+}T_i(\alpha_t)\Lambda_{i1}^{-1}(\alpha_t)$.

Moreover, in view of (28) and (29) we also have

$$\mathscr{L}V_i \leq -z_{ri}^T[Q_i+K_{ri}^TR_iK_{ri}]z_{ri}-Z_{rj}^T(t)K_{ji}^TW_{ji}^TR_iW_{ji}K_{ji}Z_{rj}(t)) < 0 \quad (33)$$

Consequently, one obtains

$$\begin{aligned}
J_{ri} &= E\{\int_0^{\infty}(z_{ri}^T[Q_i+K_{ri}^TR_iK_{ri}]z_{ri}+Z_{rj}^T(t)K_{ji}^TW_{ji}^TR_iW_{ji}K_{ji}Z_{rj}(t))dt\} \\
&\leq -E\int_0^{\infty}\mathscr{L}V_i dt \\
&= V(z_{ri}(0),0,r_0) = J_{ri}^{*} \quad (34)
\end{aligned}$$

Therefore, according to Definition 1, the scalar J_{ri}^{*} is the stochastic guaranteed cost of system (23). This completes the proof of Lemma 2. ■

IV. Stability Conditions Incorporating the Network Physical Constraints

In this section, the network physical constraints (5)-(6) are transformed into LMI mode-dependent conditions. These complementary LMIs, together with the stability conditions provided in Lemmas 1 and 2 will be taken into account for determining a complete solution to the guaranteed cost congestion control problem.

A. Mode-Dependent Physical Constraints of the Premium Traffic

The state constraints for system (10) can be expressed as follows

$$\bar{z}_{pi}^{min} \leq \bar{z}_{pi}(t) \leq \bar{z}_{pi}^{max} \tag{35}$$

where $\bar{z}_{pi}^{min} = [-x_{pi}^{ref} \quad 0]^T$ and $\bar{z}_{pi}^{max} = [x_{pi}^{buffer} - x_{pi}^{ref}, \lambda_{pi}^{max}]^T$ denote the minimum and the maximum bounds of the new state. By squaring (35) one will have

$$\bar{z}_{pi}^T(t)\bar{z}_{pi}(t) \leq \|\bar{z}_{pi}^{max}\|^2 \tag{36}$$

Consider the following ellipsoid for a given parameter $\varepsilon_{1i} > 0$

$$\mathbb{F}_i(\alpha_t) = \{\bar{z}_{pi}(t)|\bar{z}_{pi}^T\Lambda_{i1}^{-1}(\alpha_t)\bar{z}_{pi} \leq \varepsilon_{1i}\} \tag{37}$$

According to the definitions of the Lyapunov functional V_i in (15), since $\Lambda_{i1}^{-1}(\alpha_t) = P_i(\alpha_t)$, we have

$$\bar{z}_{pi}^T\Lambda_{i1}^{-1}(\alpha_t)\bar{z}_{pi} \leq V_i(\bar{z}_{pi}(t), \alpha_t) \tag{38}$$

By integrating (20), from 0 to t and considering that $V_i(\bar{z}_{pi}(0), r_0) = 0$, we get

$$\begin{aligned} V_i &\leq -\int_0^t \bar{z}_{pi}^T(t)(Q_i(\alpha_t) + K_{pi}^T(\alpha_t)R_i(\alpha_t)K_{pi}(\alpha_t))\bar{z}_{pi}(t)dt \\ &\quad + \int_0^t \lambda_{pi}^T(t)\Psi_i(\alpha_t)\lambda_{pi}(t)dt \\ &< \int_0^\infty \lambda_{pi}^T(t)\Psi_i(\alpha_t)\lambda_{pi}(t)dt \\ &< \gamma_i max(\Psi_i(\alpha_t)) \end{aligned} \tag{39}$$

Therefore, the state $\bar{z}_{pi}(t)$ will belong to the set $\mathbb{F}_i(\alpha_t)$ for all the modes α_t if $\gamma_i max(\Psi_i(\alpha_t)) \leq \varepsilon_{1i}$. Consequently, the right hand side of the state constraint (35) is satisfied if $\varepsilon_{1i}/(\bar{z}_{pi}^{max})^2 \leq \Lambda_{i1}^{-1}(\alpha_t)$.

By applying the Schur complement, the right-hand side of the state constraint (35) will hold if the following LMI conditions are satisfied

$$\Omega_{c1i}^p(\alpha_t) \triangleq \gamma_i max\{\Psi_i(\alpha_t)\} \leq \varepsilon_{1i} \tag{40}$$

$$\Omega_{c2i}^p(\alpha_t) \triangleq \begin{bmatrix} \Lambda_{i1}(\alpha_t) & \Lambda_{i1}^T(\alpha_t) \\ \Lambda_{i1}(\alpha_t) & \|\bar{z}_{pi}^{max}\|^2/\varepsilon_{1i} \end{bmatrix} \geq 0 \tag{41}$$

On the other hand, the left-hand side of the state constraint (35) can be rewritten as $\bar{z}_{pi}(t) - \bar{z}_{pi}^{min} \geq 0$.

According to the definition of non-negative systems [22], if the above system is non-negative, then the left-hand side of the state constraint (35) holds. By selecting the matrix $\Lambda_{i1}(\alpha_t)$ as a diagonal positive definite matrix, the non-negative condition of the closed-loop system matrices can be expressed as follows

$$\begin{aligned} \Omega_{c3i}^p(\alpha_t) &\triangleq (T_i(\alpha_t))_{ij} \geq 0 \\ V_{ik}(\alpha_t) &= \begin{bmatrix} V_{ik}^1(\alpha_t) & V_{ik}^2(\alpha_t) \\ V_{ik}^3(\alpha_t) & V_{ik}^4(\alpha_t) \end{bmatrix} \\ V_{i1}^1(\alpha_t) &= V_{i2}^1(\alpha_t) = 0 \\ V_{i1}^2(\alpha_t) &= V_{i2}^2(\alpha_t) = 0 \\ V_{i2}^3(\alpha_t) &= 0 \\ V_{i1}^3(\alpha_t) &> 0 \text{ and is diagonal} \\ V_{i1}^4(\alpha_t) &= V_{i2}^4(\alpha_t) < 0 \text{ and is diagonal} \end{aligned} \tag{42}$$

The input constraint of the system (10) is expressed as

$$0 \leq \bar{u}_{pi}(t) \leq C_{server,i}(\alpha_t) \tag{43}$$

Noting that $\bar{u}_{pi}(t) = B_{i0}^+T_i(\alpha_t)\Lambda_{i1}^{-1}(\alpha_t)\bar{z}_{pi}(t)$, hence the input constraint (43) becomes

$$0 \leq B_{i0}^+T_i(\alpha_t)\Lambda_{i1}^{-1}(\alpha_t)\bar{z}_{pi}(t) \leq C_{server,i}(\alpha_t) \tag{44}$$

Consider the ellipsoid (37), so that the right-hand side of the input constraint will be satisfied if

$$(B_{i0}^+T_i(\alpha_t)\Lambda_{i1}^{-1}(\alpha_t))^T(\varepsilon_{i1}/C_{server,i}^2(\alpha_t))B_{i0}^+T_i(\alpha_t)\Lambda_{i1}^{-1}(\alpha_t) \leq \Lambda_{i1}^{-1}(\alpha_t) \tag{45}$$

The above condition can be transformed into the following LMI condition

$$\Omega_{c4i}^p(\alpha_t) \triangleq \begin{bmatrix} I & K_i^T(\alpha_t) \\ K_i(\alpha_t) & (C_{server,i}^2(\alpha_t)/\varepsilon_{1i})\Lambda_{i1}(\alpha_t) \end{bmatrix} \geq 0 \tag{46}$$

The non-negative constraint of the input will be satisfied if the control gain $(K_{pi}(\alpha_t))_{ij} > 0$. Hence, by using $K_{pi}(\alpha_t) = B_{i0}^+T_i(\alpha_t)\Lambda_{i1}^{-1}(\alpha_t)$ and noting that $\Lambda_{i1}^{-1}(\alpha_t)$ is set to be a diagonal positive definite matrix, then B_{i0} is negative definite.

The left-hand side of the input constraint can be transformed into the following LMI condition

$$\Omega_{c5i}^p(\alpha_t) \triangleq (T_i(\alpha_t))_{ij} \leq 0 \tag{47}$$

Therefore, the above results and the LMI conditions given in Lemma 1 can be summarized into the following theorem.

Theorem 1: A distributed guaranteed cost congestion controller (DGCC) for the premium traffic in a mobile network is designed according to

$$\bar{u}_{pi} = K_{pi}(\alpha_t)\bar{z}_{pi} + W_{ji}(\alpha_t)K_{ji}(\alpha_t)\bar{Z}_{pj}(t) \tag{48}$$

if the LMI conditions that are given in Lemma 1 subject to the positive definite diagonal matrix $\Lambda_{i1}^{-1}(\alpha_t)$ and the LMI conditions $\Omega_{c1i}^p(\alpha_t)$ to $\Omega_{c5i}^p(\alpha_t)$ for $i = 1, \ldots n$, $\alpha_t \in \mathscr{S} = \{1, \ldots, M\}$, as given in (40), (41), (42), (46), and (47), respectively, are all satisfied.

Proof: Follows along the same lines as in the derivations for Lemma 1 and the analysis of the physical constraints. These details are omitted due to the space limitations. ■

B. Mode-Dependent Physical Constraints of the Ordinary Traffic

The physical constraints for the ordinary traffic in a mobile network are listed as

$$z_{ri}^{min} \leq z_r(t) \leq z_{ri}^{max}; \quad 0 \leq \bar{u}_{ri}(t) \leq c_{ri}(\alpha_t)$$

where $z_{ri}^{maz} = x_{ri}^{buffer} - x_{ri}^{ref}$ and $z_{ri}^{min} = -x_{ri}^{ref}$.

To avoid any confusion, in the remainder of this section we use the notations Λ_{pi1} and Λ_{ri1} to denote the Lyapunov matrix Λ_{i1} that is used in Lemmas 1 and 2, for the premium and the ordinary traffic, respectively, and the following analysis of the physical constraints can be obtained.

For the state constraints, consider the following ellipsoid for a given parameter $\varepsilon_{i2} > 0$, namely

$$\mathbb{S}_i = \{z_{ri}^T(\tilde{P}_{ri})^{-1}(\alpha_t)z_{ri} < \varepsilon_{i2}\} \quad (49)$$

From the definition of the Lyapunov function given in (27) and the stability conditions given in Lemma 2, we will have

$$z_r^T(t)\Lambda_{ri1}^{-1}z_r(t) \leq V_i(z_{ri}(t), \alpha_t) \quad (50)$$

Now, by integrating (33) on both sides from 0 to t and considering $V(z_{ri}(0), r_0) = 0$, we will have

$$V_i \leq -\int_0^t z_{ri}^T(t)(Q_i(\alpha_t) + K_{pi}^T(\alpha_t)R_i(\alpha_t)K_{pi}(\alpha_t))z_{ri}(t)dt < 0 \quad (51)$$

Therefore, $z_{ri}(t)$ belongs to the set $\mathbb{S}_i$ for all $t > 0$. Consequently, the right-hand side of the state constraints can be expressed according to the following LMI condition

$$\Omega_{c1i}^r(\alpha_t) \triangleq \begin{bmatrix} \Lambda_{ri1}(\alpha_t) & \Lambda_{ri1}^T(\alpha_t) \\ \Lambda_{ri1}(\alpha_t) & (z_{ri}^{max})^2/\varepsilon_{i2} \end{bmatrix} \geq 0 \quad (52)$$

On the other hand, the left-hand side of the state constraints can be considered by the following non-negative constraint

$$z_{ri}(t) - z_{ri}^{min} \geq 0 \quad (53)$$

Following along the similar lines as those in deriving the LMI conditions for the physical constraints of the premium traffic, and noting that the matrix Λ_{ri1} is set to be diagonal and positive definite, and given that $B_{i0} < 0$, the non-negative constraint of the state can be expressed by the following LMI conditions

$$\Omega_{c2i}^r(\alpha_t) \triangleq (T_i(\alpha_t))_{ij} \leq 0 \quad (54)$$

For the constraints on the input $\bar{u}_{ri}$, by taking into account that $\bar{u}_{ri}(t) = K_{ri}(\alpha_t)z_{ri}(t)$, it can be stated that

$$0 \leq B_{i0}^+T_i(\alpha_t)\Lambda_{ri1}^{-1}(\alpha_t)z_{ri}(t) \leq c_{ri}(\alpha_t) \quad (55)$$

Note that $c_{ri}(\alpha_t) = C_{server,i}(\alpha_t) - K_{pi}(\alpha_t)\bar{z}_{pi}(t)$, where $K_{pi}(\alpha_t)$ is the control gain of the premium traffic controller. Consequently, the input constraints of the ordinary traffic can be expressed as follows

$$0 \leq K_{ri}(\alpha_t)z_{ri}(t) \leq C_{server,i}(\alpha_t) - K_{pi}(\alpha_t)\bar{z}_{pi}(t) \quad (56)$$

From the right-hand side of (56) one can have

$$K_{ri}(\alpha_t)z_{ri}(t) + K_{pi}(\alpha_t)\bar{z}_{pi}(t) \leq C_{server,i}(\alpha_t) \quad (57)$$

By squaring (57) we obtain

$$\begin{bmatrix} z_{ri}(t) \\ z_{pi}(t) \end{bmatrix}^T \begin{bmatrix} K_{ri}^T \\ K_{pi}^T \end{bmatrix} \begin{bmatrix} K_{ri} & K_{pi} \end{bmatrix} \begin{bmatrix} z_{ri}(t) \\ z_{pi}(t) \end{bmatrix} \leq \|C_{server,i}(\alpha_t)\|^2 \quad (58)$$

Therefore, by considering the ellipsoid $\mathbb{F}_i$ and the set $\mathbb{S}_i$, the right-hand side of the input constraints will be satisfied if the following LMI conditions hold

$$\Omega_{c3i}^r(\alpha_t) \triangleq \gamma_i max\{\Psi_i(\alpha_t)\} \leq \varepsilon_{i1} \quad (59)$$

$$\Omega_{c4i}^r(\alpha_t) \triangleq \begin{bmatrix} I & K_{ri}(\alpha_t) & K_{pi}(\alpha_t) \\ K_{ri}^T(\alpha_t) & \frac{C_{server,i}^2(\alpha_t)}{\varepsilon_{i1}+\varepsilon_{i2}}\Lambda_{ri1}(\alpha_t) & 0 \\ K_{pi}^T(\alpha_t) & 0 & \frac{C_{server,i}^2(\alpha_t)}{\varepsilon_{i1}+\varepsilon_{i2}}\Lambda_{pi1}(\alpha_t) \end{bmatrix} \geq 0 \quad (60)$$

The model-dependent LMI conditions derived above together with the stability conditions obtained in Lemma 2 can be summarized according to the following theorem.

Theorem 2: A distributed guaranteed cost congestion controller (DGCC) for the ordinary traffic in a mobile network is designed if the conditions given in Lemma 2 is satisfied subject to the LMIs Ω_{c1i}^r to Ω_{c4i}^r that are governed by equations (52), (54), (59), and (60), respectively.

Proof: The proof follows along the same lines as those given in Lemma 2 and the derivations and analysis for the physical constraints that are given in this subsection. ■

V. Simulation Results

The simulation results presented in this section are intended to demonstrate the effectiveness and capabilities of our proposed decentralized Markovian jump guaranteed cost congestion (MJ-GCC) strategy to mobile Diff-Serv networks.

A. Performance Metrics

In the simulations of this paper, one denotes the link between nodes by a connectivity parameter $a_{ij}(\alpha_t)$ which is defined as

$$a_{ij}(\alpha_t) = \begin{cases} 1, & \text{if nodes } i \text{ and } j \text{ are connected} \\ 0, & \text{otherwise} \end{cases} \quad (61)$$

where α_t represents the changes of network topology.

Now, let the packet loss rate (PLR) for the premium traffic in the mobile network be defined as $PLR_{pi}(t) = \frac{P_{bi}+P_{ci}}{\lambda_{pi}(t)+\sum_{j\in\wp_i}\lambda_{ji}(t)g_{ji}(t)a_{ji}(\alpha_t)}$, $P_{bi}(t) = max\{0, \lambda_{pi}(t) + \sum_{j\in\wp_i}\lambda_{ji}(t)g_{ji}(t)a_{ji}(\alpha_t) - (x_{buffer,i} - x_{pi}(t))\}$, and $P_{ci}(t) = \sum_{k\in\wp_i}\lambda_{ik}(t)g_{ik}(t)(1 - a_{ik}(\alpha_t))$, where P_{bi} is the packet loss induced by the buffer overflow and P_{ci} is the packet loss due to the network topology changes. The PLR for the ordinary traffic in the mobile network is similarly defined.

On the other hand, the average queuing delay of a mobile network can be extended to the mobile network as

$$E\{T_q^i\} = \frac{E\{x_i(t)\}}{E\{\lambda_i(t)\} + \sum_{j\in\wp_i} E\{\lambda_{ji}(t)g_{ji}(t)a_{ji}(\alpha_t)\}} \quad (62)$$

where $E\{T_q^i\}$ is the average queuing delay and $x_i(t)$ is the present queuing state.

The network we consider consists of three clusters where each cluster has five nodes. In each cluster, one of the five nodes act as the decision maker and the other four nodes act as sensors. Only the decision makers can communicate with each other to share the information among the three clusters. This network configuration is quite general and can be found in many applications such as sensor/actuator networks, cooperative team of unmanned vehicles [6]-[7], and high speed Ethernet networks. The physical constraints of the network are set to $C_{server,i} = 10$ Mb, $x_{buffer,i} = 5$ Mb, for $i = 1, \ldots, 15$. Each source node generates a premium random traffic with a mean packet size of 512 bytes and pace the packets into the network every 10ms. The premium traffic is assumed to be bounded such that $\lambda_{pi}^{max} = 0.8$ Mbps.

The comparative results of the buffer characteristics in node 11 by utilizing the proposed DGCC strategy, the decentralized GCC [13], and an equivalent centralized guaranteed cost congestion control strategy (the details are not provided due to space limitations) are summarized in Tables I and II.

TABLE I

THE PREMIUM TRAFFIC PERFORMANCE OF THE NODE 11

h_{max} = 80 ms	Decentralized GCC [13]	Centralized GCC	Distributed GCC
PLR	0	0	0
Queuing Delay	26.0 ms	24.5 ms	24.6 ms
Mean Error	3.83%	2.87%	2.93%
Settling Time	0.09s	0.11s	0.09s
Max cost J_p^*	5.05×10^{20}	2.94×10^{20}	3.27×10^{20}
Num of LMIs	21	8	18
Max dimension of LMIs	10×10	18×18	10×10

TABLE II

THE ORDINARY TRAFFIC PERFORMANCE OF THE NODE 11.

h_{max} = 80 ms	Decentralized GCC [13]	Centralized GCC	Distributed GCC
PLR	0.98%	0.36%	0.78%
Queuing Delay	23.55ms	23.14ms	23.16ms
Mean Error	0.68%	0.43%	0.65%
Settling Time	0.09s	0.24s	0.11s
Max cost J_r^*	3.01×10^{20}	1.45×10^{20}	2.04×10^{20}
Num of LMIs	18	7	15
Max dimension of LMIs	7×7	18×18	10×10

As can be inspected from these numerical results, one can readily observe that:

- The DGCC strategy can obtain a more accurate control result (mean error) than the decentralized GCC algorithm, and respond faster (settling time) than the centralized GCC. This is due to the fact that by incorporating the adjustments from the nearest neighboring nodes, the coupling effects of the neighboring states are considered explicitly. Therefore, by properly selecting the distributed weights, the DGCC approach can obtain a more effective control than the decentralized one. On the other hand, the DGCC controller is implemented at each node and updated only based on the local information. Therefore, the buffer response of the DGCC is faster than the centralized one, and
- The upper bound of the guaranteed cost by utilizing the DGCC approach is between the decentralized and the centralized GCC approaches. However, the number of LMIs and the maximum dimension of the LMIs by utilizing the DGCC approach is similar to the decentralized one. These results confirm again that the DGCC approach is also scalable to large scale networks.

VI. CONCLUSIONS

In this paper, a distributed guaranteed cost congestion control (DGCC) strategy for a mobile network with Diff-Serv traffic is proposed. By taking the advantages of the Markovian jump and the guaranteed cost control principles, the proposed DGCC algorithm is shown to be in fact equivalent to a local state feedback control plus a nearest neighboring controllers that are adjusted with proportional gains. The resulting congestion control problem is then cast as a quadratic regulation problem of a time-delay system with free parameters (gains) that need to be selected. The analytical results are confirmed through a number of simulation studies. The comparative results demonstrate that the DGCC strategy significantly enhances the scalability of the centralized algorithm and improves the performance when compared to the other congestion control approaches in the literature.

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