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A Markovian Jump Guaranteed Cost Congestion Control Strategy for Mobile Networks Subject to Differentiated Services Traffic

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Abstract— **In this paper, a novel congestion control strategy for mobile networks with differentiated services traffic is proposed. The switching or changes in the network topology is modeled by a Markovian process. By utilizing the guaranteed cost control approach, a maximum bound on the jump quadratic cost function is guaranteed for each traffic class with respect to its specific QoS requirements. The proposed Markovian jump guaranteed cost congestion controller (MJ-GCC) is shown to be robust to the *unknown* and *time-varying* network delays and the non-stationary network topologies. Numerical simulation results are presented to illustrate the effectiveness and capabilities of our proposed MJ-GCC strategies.**

I. INTRODUCTION

The congestion control problem is of paramount importance in communication networks. In particular, when the network nodes are mobile the neighboring set of each node becomes time-varying, which results in a switching network topologies. This fact does necessitate the design and utilization of more robust and effective congestion control algorithms. On the other hand, the traffic flows in a network belong to various classes due to the diverse range of applications. Specifically, the Internet Engineering Task Force (IETF) has proposed the Differentiated Services (Diff-Serv) architecture [1] to deliver aggregated quality of service (QoS) in IP networks. In the Diff-Serv architecture the traffic is aggregated into different classes of flows and the bandwidth allocation and the packet dropping rules are applied to the traffic classes according to their QoS requirements and specifications.

Recently, several new congestion control schemes for Diff-Serv networks have been developed by using sliding mode control [2], robust adaptive control [3], and switching control [4], [5]. However, the nature of discontinuities of the sliding mode controller may result and introduce unavoidable and undesirable oscillations in the closed-loop system [6], and therefore reduce the effectiveness of the developed congestion control solutions. The approach in [3] is designed for *only* a *cascade network* and the unknown and time-varying delays are *not* considered in the design of the congestion control scheme. On the other hand, the approach in [5] needs to regulate the traffic compression gains among the network nodes for guaranteeing stability. However, in some cases this regulation may lead to conservative results and low quality of service.

The goal of this paper is to improve the performance of the switching congestion control approach developed in [5] by utilizing the guaranteed cost control approach [7], [8]. The changes and switching of the network topologies is modeled by a Markov chain and the dynamics of the mobile network is represented by a nonlinear time-delay system with Markovian jump parameters. The transmission, processing, and propagation delays are considered as unknown and time-varying parameters in the dynamic model. The bandwidth, buffer size, and the transmission constraints in the communication network are considered as state and input constraints of the system model. The guaranteed cost control approach is then applied for synthesis and design of congestion control strategies for the differentiated services traffic in mobile networks.

The organization of this paper is as follows. In Section 2, the dynamical models of the mobile network is presented. In Section 3, our proposed Markovian jump guaranteed cost congestion control (MJ-GCC) strategy is presented. The stability conditions incorporating all the physical constraints are derived in Section 4. Simulations results are presented in Section 5 and the conclusions are stated in Section 6.

II. PROBLEM FORMULATION

A. Dynamical Model of Diff-Serv Networks

In this paper, we assume that the dynamics of a queue is governed by an M/M/1. The M/M/1 queue is a single-server queue model where the packets arrives according to a Poisson process, and the queuing service time is exponentially distributed.

The M/M/1 queuing system can be applied to describe a wide variety of queuing models as found in systems with a very large number of independent customers/nodes that can be approximated as a Poisson process. Given an M/M/1 queue the dynamics of a single node can be expressed as follows [3], [9]

$$\dot{x}_i(t) = -\mu_i \frac{x_i(t)}{1+x_i(t)} C_i(t) + \lambda_i(t) \tag{1}$$

where $x_i(t)$ is the queuing length, $C_i(t)$ is the link capacity, $\lambda_i(t)$ is the average rate of incoming traffic, and $1/\mu_i$ is the average length of the packets being transmitted in the network.

Consider now a general network with n nodes. In a large scale network the input traffic to each node can consist of two parts, namely: (1) the external traffic $\lambda_i(t)$ which in principle could represent the traffic that is being sent from nodes of other clusters (defined as groups of nodes not belonging to the nearest neighboring set $\wp_i$) as well as disturbances or environmental stimuli, and (2) the internal traffic $\lambda_j(t-\tau_{ji}(t))$ which is the delayed input traffic from all the neighboring nodes within a given cluster.

Therefore, by using the representation (1), the fluid flow model corresponding to each node is governed by

$$\dot{x}_i(t) = -f(x_i(t))C_i(t) + \lambda_i(t) + \sum_{j\in\wp_i(\alpha_t)} \lambda_j(t-\tau_{ji}(t))g_{ji} \quad (2)$$

$$\lambda_j(t-\tau_{ji}(t)) = f(x_j(t-\tau_{ji}(t)))C_j(t-\tau_{ji}(t)) \quad (3)$$

where $f(x_i(t)) = \mu_i x_i(t)/(1+x_i(t))$, $\wp_i$ is the set of the nearest neighboring nodes associated with the node i, $g_{ji}(t)$ is the traffic compression gain from node j to node i, $\tau_{ji}(t)$ is the time-varying delay between node j and node i, and α_t is a Markov chain that represents the rule for changes and switching in the neighboring sets.

The Markov chain α_t is defined on a complete probability space $\{\Omega, \mathscr{F}, P\}$ that takes values in a finite space $\mathscr{S} = \{1,...,M\}$ which describes the switching between different modes, and whose evolution is governed by the following probability transitions

$$P[\alpha_{t+\Delta} = k \mid \alpha_t = l] = \begin{cases} \pi_{kl}\Delta + o(\Delta), & k \neq l; \\ 1+\pi_{kk}\Delta + o(\Delta), & k = l. \end{cases} \quad (4)$$

where $\pi_{kl} \geq 0$ is the transition rate from mode k to mode l, $\pi_{kk} = -\sum_{k=1,k\neq l}^{M} \pi_{kl}$, and $o(\Delta)$ is a function satisfying $\lim_{\Delta\to 0} \frac{o(\Delta)}{\Delta} = 0$. In this work the modes $1,...,M$ correspond to the topologies that are possible in the network due to the nodes mobility.

Any communication network is characterized by a number of physical resources constraints. A typical set of *physical constraints* corresponding to the network are now specified as follows

$$0 < x_i(t) \leq x_{buffer,i} \qquad 0 \leq C_i(t) \leq C_{server,i} \quad (5)$$

where $x_{buffer,i}$ is the buffer size and $C_{server,i}$ is the link capacity of node i.

On the other hand, the instantaneous traffic transmission rate and its rate of change at each node should satisfy

$$\lambda_i(t) \leq \lambda_i^{max} \leq C_{server,i} \qquad \dot{\lambda}_i(t) \in \mathscr{L}_\infty \quad (6)$$

Finally, the following two assumptions are made in this work

Assumption 1: The time-varying and unknown *delays $\tau_{ji}(t)$ are upper bounded and the maximum upper bound is a known constant, that is*

$$0 \leq \tau_{ji}(t) \leq h_{ji} \quad \textit{with} \quad h = max\{h_{ji}\} \quad (7)$$

Assumption 2: The external incoming traffic to each node is L_2 norm bounded, that is

$$\int_0^\infty \|\lambda_i(t)\|^2 dt \leq \gamma_i, \qquad \gamma_i > 0 \quad (8)$$

B. Guaranteed Cost Control

The guaranteed cost control approach was first introduced in [10], which is an extension to the classical LQR regulation problem for linear systems with parametric uncertainties. The conceptual objective of the GCC is to design a feedback controller such that for all admissible uncertainties the closed-loop system is asymptotically stable and an upper bound on the corresponding cost function is guaranteed [12].

In this paper, the transmission, the processing, and the propagation delays in the mobile network are considered as unknown and time-varying variables in the dynamical system model (2). The guaranteed cost control problem for system (2) is then defined as follows.

Definition 1: [8] *For the Markovian jump time-delay system (2)-(3), the following jump quadratic cost function is defined*

$$J_i \;=\; E\{\int_0^\infty [x_i^T(t)Q_i(\alpha_t)x_i(t) + u_i^T(t)R_i(\alpha_t)u_i(t)]dt\} \quad (9)$$

where $x_i(t)$ is the state, $u_i(t)$ is the control input, $Q_i(\alpha_t)$ and $R_i(\alpha_t)$ are positive definite matrices. Provided there exists a control law $u_i^*(t)$ and a positive scalar J_i^* such that the closed-loop system is stochastically stable and the cost function J_i satisfies

$$J_i \leq J_i^*$$

then J_i^* is the stochastic guaranteed cost *of the system (2)-(3) and $u_i^*(t)$ is the* stochastic guaranteed cost controller *of the system (2)-(3).*

III. Proposed Guaranteed Cost Congestion Control Strategy

In this paper, we consider three kinds of traffic, namely the *premium* (denoted by "p"), *ordinary* (denoted by "r"), and the *best-effort* according to the definitions proposed by IETF [1]. The dynamic queuing models of the mobile network (2)-(3) are valid for each traffic class. The control objective of the premium and the ordinary traffic classes are to maintain their queuing lengthes as close as possible to the corresponding reference values, such that the QoS specifications such as the queuing delay and packet loss rate can be ensured indirectly.

Therefore, based on the dynamical queuing model (2)-(3), the congestion control strategy for the premium traffic is to allocate the output capacity $C_{pi}(t)$ such that the queuing length of the premium traffic is as close as possible to its reference value. On the other hand, the strategy for the ordinary traffic is to simultaneously regulate the incoming flow rate $\lambda_{ri}(t)$ and allocate the capacity $C_{ri}(t)$ such that its queuing length is as close as possible to its reference value. Finally, for the best-effort traffic, no explicit active control is designed in this paper since this traffic does not have any QoS requirements.

A. Premium Traffic Control Strategy

The control input for the premium traffic is the link capacity, that is $u_{pi}(t) = C_{pi}(t)$. Based on the nonlinear system model (2)-(3), the following feedback linearization scheme is first applied

$$u_{pi} = f^{-1}(x_{pi}(t))\bar{u}_{pi} \qquad z_{pi}(t) = x_{pi}(t) - x_{pi}^{ref}$$

where $\bar{u}_{pi}(t)$ denotes a state feedback controller, $z_{pi}(t)$ denotes the new state of the transformed linear system, and x_{pi}^{ref} denotes the reference queuing length at node i.

Thus the nonlinear dynamical model (2)-(3) is transformed into the following equivalent linear one

$$\dot{z}_{pi}(t) = -\bar{u}_{pi}(t) + \lambda_{pi}(t) + \sum_{j\in\wp_i(\alpha_t)} \bar{u}_{pj}(t-\tau_{ji}(t))g_{ji}^{p}(t) \quad (10)$$

Due to the presence of the unknown external incoming traffic $\lambda_{pi}(t)$, the state feedback controller $\bar{u}_{pi}(t)$ is now selected as follows

$$\begin{aligned}\bar{u}_{pi}(t) &= K_{i1}(\alpha_t)z_{pi}(t) + K_{i2}(\alpha_t)\hat{\lambda}_{pi}(t)\\ &= K_i(\alpha_t)\bar{z}_{pi}(t) \qquad (11)\end{aligned}$$

where $\bar{z}_{pi}(t) = [z_{pi}(t) \;\; \hat{\lambda}_{pi}(t)]^T$.

The variable $\hat{\lambda}_{pi}(t)$ is an estimate of $\lambda_{pi}(t)$ and is updated according to the parameter projection method [13]

$$\dot{\hat{\lambda}}_{pi}(t) = \begin{cases} \delta_{pi}(\alpha_t)z_{pi}(t) - \beta_{pi}(\alpha_t)\hat{\lambda}_{pi}(t), & \text{if } 0 \le \hat{\lambda}_{pi}(t) \le \lambda_{pi}^{max} \text{ or} \\ & \hat{\lambda}_{pi}(t) = 0,\ z_{pi}(t) \ge 0 \text{ or} \\ & \hat{\lambda}_{pi}(t) = \lambda_{pi}^{max},\ z_{pi}(t) \le 0 \\ -\beta_{pi}(\alpha_t)\hat{\lambda}_{pi}(t), & \text{otherwise} \end{cases} \quad (12)$$

where $\delta_{pi}(\alpha_t) > 0$ and $\beta_{pi}(\alpha_t) > 0$ are design parameters.

Therefore, the dynamical system (10) in the new coordinates $\bar{z}_{pi}(t)$ can be written as

$$\begin{aligned}\dot{\bar{z}}_{pi}(t) &= A_{i0}^{k}(\alpha_t)\bar{z}_{pi}(t) + B_{i0}\bar{u}_{pi}(t) + B_{\lambda_i}\lambda_{pi}(t) \qquad (13)\\ &\quad + \sum_{j\in\wp_i(\alpha_t)} B_j\bar{u}_j(t-\tau_{ji}(t))\end{aligned}$$

$$\bar{z}_{pi}(t) = \varphi_i(t); \;\; \varphi_i(t) \in [-h, 0]; \quad k \in \aleph, \aleph = 1, 2;$$
$$\alpha_t \in \mathscr{S}, \mathscr{S} = \{1, ..., M\}$$

where $A_{i0}^{k}(\alpha_t)$, B_{i0}, B_j, and B_{λ_i}, are the system matrices that are defined according to

$$A_{i0}^{1}(\alpha_t) = \begin{bmatrix} 0 & 0 \\ \delta_{pi}(\alpha_t) & -\beta_{pi}(\alpha_t) \end{bmatrix}; \;\; B_{i0} = \begin{bmatrix} -1 \\ 0 \end{bmatrix}$$

$$A_{i0}^{2}(\alpha_t) = \begin{bmatrix} 0 & 0 \\ 0 & -\beta_{pi}(\alpha_t) \end{bmatrix}; \;\; B_j = \begin{bmatrix} g_{ji}^{p} \\ 0 \end{bmatrix}; \;\; B_{\lambda_i} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

The following jump quadratic cost function is now considered for the premium traffic

$$J_{pi} = E\{\int_0^\infty (\bar{z}_{pi}^T(t)Q_i(\alpha_t)\bar{z}_{pi}(t) + \bar{u}_{pi}^T(t)R_i(\alpha_t)\bar{u}_{pi})dt\} \quad (14)$$

where $Q_i(\alpha_t)$ and $R_i(\alpha_t)$ are given positive definite matrices for each mode.

The following lemma is presented to show that the state feedback controller $\bar{u}_{pi}(t) = K_i(\alpha_t)\bar{z}_{pi}(t)$ is a stochastic guaranteed cost control law [8] for the system (13).

Lemma 1: Given the cost function (14) and under Assumption 2, if there exist symmetric positive definite matrices $\Lambda_{i1}^T(\alpha_t)$, $\bar{X}_{ik}(\alpha_t)$, $\bar{V}_{ii}(\alpha_t)$, $\bar{T}_i(\alpha_t)$, and matrices U_i, $N_i(\alpha_t)$, Λ_{i3}^T, and $\bar{S}_i(\alpha_t)$ for $k = 1, 2$, $i = 1, ..., n$, and $\alpha_t \in \mathscr{S} = \{1, \ldots, M\}$ such that the LMI condition $\Omega_{ik}(\alpha_t) < 0$ is satisfied with $\Omega_{ik}(\alpha_t)$ given by

$$\begin{bmatrix} \bar{X}_{ik}(\alpha_t) & h^2(\bar{V}_{ik}^T(\alpha_t) + \bar{T}_i(\alpha_t))B_{ji} + B_{ji} & \Lambda_{i1}^T(\alpha_t) \\ * & h^2 B_{ji}^T(U_i + N_i(\alpha_t))B_{ji} & 0 \\ * & * & -\Lambda_{i3}^T - (1-h)\bar{S}_i(\alpha_t) \end{bmatrix}$$

then the controller $\bar{u}_{pi}(t) = K_{pi}(\alpha_t)\bar{z}_{pi}(t)$ is the stochastic guaranteed cost controller of system (13), and the decentralized control gains are given by $K_{pi}(\alpha_t) = B_{i0}^{+}T_i(\alpha_t)\Lambda_{i1}^{-1}(\alpha_t)$ ("+" denotes the Moore-Penrose inverse [14]).

Proof: Consider the following stochastic Lyapunov-Krasovskii functional candidate

$$\begin{aligned} V_i(\bar{z}_{pi}(t), \alpha_t) &= V_{i1} + V_{i2} + V_{i3} + V_{i4} \qquad (15)\\ V_{i1} &= \bar{z}_{pi}(t)^T P_i(\alpha_t)\bar{z}_{pi}(t)\\ V_{i2} &= \int_{t-h}^{t} \bar{z}_{pi}^T(s)S_i(\alpha_t)\bar{z}_{pi}(s)ds\\ V_{i3} &= h\int_{-h}^{0}\int_{t+\theta}^{t} \dot{\bar{z}}_{pi}^T(s)U_i\dot{\bar{z}}_{pi}(s)dsd\theta\\ V_{i4} &= \int_{-h}^{0}\int_{t+\theta}^{t} \bar{z}_{pi}^T(s)S_i(\alpha_t)\bar{z}_{pi}(s)dsd\theta \end{aligned}$$

where $P_i(\alpha_t)$, $S_i(\alpha_t)$, and U_i are positive definite matrices. Then, for each $\alpha_t = l \in \mathscr{S}$, the infinitesimal generator [11] $\mathscr{L}V_i$ can be obtained as

$$\begin{aligned} \mathscr{L}V_{i1} &= 2\bar{z}_{pi}^T(t)P_i(\alpha_t)[A_{ic}^k(\alpha_t)\bar{z}_{pi}(t) + \sum_{j\in\wp_i(\alpha_t)} B_jK_{pj}(\alpha_t)\bar{z}_{pj}(t-\tau_{ji}(t))]\\ &\quad + \bar{z}_{pi}^T(t)\sum_{l=1}^{M}\pi_{\alpha_t l}P_i(l)\bar{z}_{pi}(t) + 2\bar{z}_{pi}^T(t)P_i(\alpha_t)B_{\lambda_i}\lambda_{pi}(t)\\ \mathscr{L}V_{i2} &= \bar{z}_{pi}^T(t)S_i(\alpha_t)\bar{z}_{pi}(t) - (1-h)\bar{z}_{pi}^T(t-h)S_i(\alpha_t)\bar{z}_{pi}(t-h)\\ &\quad + \int_{t-h}^{t}\bar{z}_{pi}^T(s)\sum_{l=1}^{M}\pi_{\alpha_t l}S_i(l)\bar{z}_{pi}(s)ds\\ \mathscr{L}V_{i3} &= h^2[A_{ic}^k(\alpha_t)\bar{z}_{pi}(t) + \sum_{j\in\wp_i(\alpha_t)} B_jK_{pj}(\alpha_t)\bar{z}_{pj}(t-\tau_{ji}(t))\\ &\quad + B_{\lambda_i}\lambda_{pi}(t)]^T U_i[A_{ic}^k(\alpha_t)\bar{z}_{pi}(t) + B_{\lambda_i}\lambda_{pi}(t)\\ &\quad + \sum_{j\in\wp_i(\alpha_t)} B_jK_{pj}(\alpha_t)\bar{z}_{pj}(t-\tau_{ji}(t))] - h\int_{t-h}^{t}\dot{\bar{z}}_{pi}^T(s)U_i\dot{\bar{z}}_{pi}(t)ds\\ \mathscr{L}V_{i4} &= h\bar{z}_{pi}^T(t)S_i(\alpha_t)\bar{z}_{pi}(t) - \int_{t-h}^{t}\bar{z}_{pi}^T(s)\sum_{l=1}^{M}\pi_{\alpha_t l}S_i(l)\bar{z}_{pi}(s)ds \end{aligned}$$

where $k = 1, 2$.

Let us define $B_{ji} = vec\{B_j\}$, $K_{ji}(\alpha_t) = diag\{K_{pj}(\alpha_t)\}$, and $\bar{Z}_{pj}(t-\tau) = vec\{\bar{z}_{pj}^T(t-\tau_{ji}(t))\}$, then one can obtain

$$\mathscr{L}V_i \;\; \le \;\; \eta_i^T(t,\tau,h)W_{ik}(\alpha_t)\eta_i(t,\tau,h) + \lambda_{pi}^T(t)\Psi_i(\alpha_t)\lambda_{pi}(t) \quad (16)$$

where $\eta_i(t,\tau,h) = [\bar{z}_{pi}^T(t) \;\bar{Z}_{pj}^T(t-\tau)\; \bar{z}_{pi}^T(t-h)]^T$, M_i and N_i are positive definite matrices, $Y_i(\alpha_t) = h^2A_{ic}^k(\alpha_t)U_i + P_i(\alpha_t)$, and the matrix W_{ik} and the variable Ψ_i are given by

$$\begin{aligned} W_{ik}(\alpha_t) &= \begin{bmatrix} w_{ik}^1(\alpha_t) & w_{ik}^2(\alpha_t) & U_i \\ * & w_i^3(\alpha_t) & 0 \\ * & * & -U_i - (1-h)S_i(\alpha_t) \end{bmatrix} \qquad (17)\\ \Psi_i(\alpha_t) &= B_{\lambda_i}(h^2U_i + Y_i^T(\alpha_t)M_i^{-1}(\alpha_t)Y_i(\alpha_t) + h^2N_i^{-1}(\alpha_t))B_{\lambda_i}\\ w_{ik}^1(\alpha_t) &= (2P_i(\alpha_t) + h^2(A_{ic}^k)^T(\alpha_t)U_i)A_{ic}^k(\alpha_t)\\ &\quad + \sum_{l=1}^{M}\pi_{\alpha_t l}P_i(l) + (1+h)S_i(\alpha_t) - U_i + M_i(\alpha_t)\\ w_{ik}^2(\alpha_t) &= (h^2(A_{ic}^k)^T(\alpha_t)U_i + P_i(\alpha_t))B_{ji}K_{ji}(\alpha_t)\\ w_i^3(\alpha_t) &= h^2K_{ji}^T(\alpha_t)B_{ji}^T(U_i + N_i(\alpha_t))B_{ji}K_{ji}(\alpha_t) \end{aligned}$$

Let us furthermore define $\bar{W}_{ik}(\alpha_t) = W_{ik}(\alpha_t) + Q_i(\alpha_t) + K_{pi}^T(\alpha_t)R_i(\alpha_t)K_{pi}(\alpha_t)$, $\Lambda_{i1}(\alpha_t) = P_i^{-1}(\alpha_t)$, $\Lambda_{i2}(\alpha_t) = K_{ji}^{-1}(\alpha_t)$, $\Lambda_{i3} = U_i^{-1}$, and $\Lambda_i(\alpha_t) = diag\{\Lambda_{i1}(\alpha_t), \Lambda_{i2}(\alpha_t), \Lambda_{i3}\}$, then by pre and post multiplying the matrix $\bar{W}_{ik}(\alpha_t)$ with Λ_i^T and Λ_i, respectively, the following matrix is obtained

$$\begin{aligned} &\Lambda_i^T(\alpha_t)\bar{W}_{ik}(\alpha_t)\Lambda_i(\alpha_t)\\ &\quad = \begin{bmatrix} \Lambda_{i1}^T(\alpha_t)X_{ik}(\alpha_t)\Lambda_{i1}(\alpha_t) & h^2\Lambda_{i1}^T(\alpha_t)(A_{ic}^k)^T(\alpha_t)U_iB_{ji} + B_{ji} \\ * & h^2B_{ji}^T(U_i + N_i(\alpha_t))B_{ji} \\ * & * \end{bmatrix} \end{aligned}$$

$$
\begin{matrix} \Lambda_{i1}^T(\alpha_t) \\ 0 \\ -\Lambda_{i3}^T - (1-h)\Lambda_{i3}^T S_i(\alpha_t)\Lambda_{i3} \end{matrix} \Bigg]
$$

$$
= \begin{bmatrix} \bar{X}_{ik}(\alpha_t) & h^2(\bar{V}_{ik}^T + \bar{T}_i)B_{ji} + B_{ji} & \Lambda_{i1}^T(\alpha_t) \\ * & h^2 B_{ji}^T(U_i + N_i(\alpha_t))B_{ji} & 0 \\ * & * & -\Lambda_{i3}^T - (1-h)\bar{S}_i(\alpha_t) \end{bmatrix}
$$

where

$$
\begin{aligned}
\bar{X}_{ik}(\alpha_t) &= V_{ik}(\alpha_t) + V_{ik}^T(\alpha_t) + T_i + T_i^T + h^2\bar{U}_i(\alpha_t) \\
&\quad + (1 + h + \sum_{l=1}^{M} \pi_{\alpha_t l})\Lambda_{i1}^T(\alpha_t) + \bar{Q}_i(\alpha_t) + \bar{R}_i(\alpha_t)
\end{aligned}
$$

$$
\begin{array}{llll}
A_{i0}^k = V_{ik}\Lambda_{i1}^{-1}; & \bar{V}_{ik}^T = V_{ik}^T U_i; & \bar{T}_i = T_i^T U_i; & S_i = P_i \\
B_{i0}K_{pi} = T_i\Lambda_{i1}^{-1}; & M_i = U_i; & \bar{U}_i = V_{ik}^T U_i V_{ik}; & \bar{S}_i = \Lambda_{i3}^T S_i \Lambda_{i3}^T \\
\bar{Q}_i = \Lambda_{i1}^T Q_i \Lambda_{i1}; & \bar{R}_i = \Lambda_{i1}^T K_{pi}^T R_i K_{pi} \Lambda_{i1} & &
\end{array}
$$

Therefore, if $\Omega_{ik}(\alpha_t) < 0$, one gets $\bar{W}_{ik}(\alpha_t) < 0$, and hence $W_{ik}(\alpha_t) < 0$. Now, according to (16), one get

$$
\begin{aligned}
\mathscr{L}V_i &\le \eta_i^T(t,\tau,h)[\bar{W}_{ik}(\alpha_t) - Q_i(\alpha_t) - K_{pi}^T(\alpha_t)R_i(\alpha_t)K_{pi}(\alpha_t)]\eta_i(t,\tau,h) \\
&\quad + \lambda_{pi}^T(t)\Psi_i(\alpha_t)\lambda_{pi}(t) \\
&\le -\bar{z}_{pi}^T(t)(Q_i(\alpha_t) + K_{pi}^T(\alpha_t)R_i(\alpha_t)K_{pi}(\alpha_t))\bar{z}_{pi}(t) \qquad (18) \\
&\quad + \lambda_{pi}^T(t)\Psi_i(\alpha_t)\lambda_{pi}(t)
\end{aligned}
$$

Therefore, for any $\bar{z}_{pi}(t)$ that satisfies

$$
\|\bar{z}_{pi}(t)\|^2 \ge \frac{\lambda_{max}\{\Psi_i(\alpha_t)\}}{\lambda_{min}(Q_i(\alpha_t) + K_{pi}^T(\alpha_t)R_i(\alpha_t)K_{pi}(\alpha_t))}\|\lambda_{pi}(t)\|^2 \qquad (19)
$$

we will have $\mathscr{L}V_i < 0$, where λ_{max} and λ_{min} denote the maximum and minimum eigenvalues of the corresponding matrices. Consequently, the system (13) is now stochastically ultimately bounded. It should be noted that since $\Psi_i(\alpha_t)$ is a scaler, hence $\lambda_{max}\{\Psi_i(\alpha_t)\} = max\{\Psi_i(\alpha_t)\}$.

Furthermore, from (14) and (18) we have

$$
\begin{aligned}
J_{pi} &\le E\{\int_0^\infty (-\mathscr{L}V_i + \lambda_{pi}^T(t)\Psi_i(\alpha_t)\lambda_{pi}(t))dt\} \\
&= V_i(\bar{z}_{pi}(0),0,r_0) - \lim_{t\to\infty} V_i(\bar{z}_{pi}(t),t,\alpha_t) + E\{\int_0^\infty \Psi_i(\alpha_t)\lambda_{pi}^2(t)dt\} \\
&\le V_i(\bar{z}_{pi}(0),0,r_0) - \bar{z}_{pi}^T(\infty)P_i(r_\infty)\bar{z}_{pi}(\infty) + \gamma_i max(\Psi_i(\alpha_t)) \qquad (20)
\end{aligned}
$$

Therefore, the upper bound of the cost function J_{pi} is given by (since $\bar{z}_{pi}(\infty) \ge 0$)

$$
J_{pi} \quad < \quad V_i(\bar{z}_{pi}(0),0,r_0) + \gamma_i max(\Psi_i(\alpha_t)) = J_{pi}^* \qquad (21)
$$

Therefore, the closed-loop system performance cost incurred by delays is guaranteed to be less than the scalar J_{pi}^*. According to the Definition 1, the controller $\bar{u}_{pi}(t) = K_{pi}(\alpha_t)\bar{z}_{pi}(t)$ is the stochastic guaranteed cost controller of system (13).

Furthermore, by solving the LMI conditions $\Omega_{ik}(\alpha_t) < 0$, one can obtain the control gains $K_{pi}(\alpha_t)$ and the system matrices as $K_{pi}(\alpha_t) = B_{i0}^+ T_i(\alpha_t)\Lambda_{i1}^{-1}(\alpha_t)$ and $A_{i0}^k(\alpha_t) = V_{ik}(\alpha_t)\Lambda_{i1}^{-1}(\alpha_t)$, where "+" denotes the Moore-Penrose inverse. This completes the proof of Lemma 1. ■

B. Ordinary Traffic Control Strategy

Since the incoming traffic of the ordinary traffic $\lambda_{ri}(t)$ is measurable and available for control, the control inputs for the ordinary traffic are the link capacity and the incoming traffic, namely $u_{ri}^1(t) = C_{ri}(t)$ and $u_{ri}^2(t) = \lambda_{ri}(t)$. Similar to the premium traffic, we first apply the following feedback linearization scheme to the open-loop system (2)-(3), namely

$$
z_{ri}(t) = x_{ri}(t) - x_{ri}^{ref} \quad \text{and} \quad u_{ri}(t) = F^{-1}(x_{ri},t)\bar{u}_{ri}(t)
$$

where $u_{ri}(t) = [u_{ri}^1(t), u_{ri}^2(t)]^T$, $\bar{u}_{ri}(t) = [\bar{u}_{ri}^1(t), \bar{u}_{ri}^2(t)]^T$, and $F(x_{ri}(t)) = diag\{f(x_{ri}(t)), 1\}$.

The resulting dynamical model (2)-(3) with respect to the ordinary traffic becomes

$$
\dot{z}_{ri}(t) \quad = \quad B_{i0}\bar{u}_{ri}(t) + \sum_{j\in\wp_i(\alpha_t)} B_j\bar{u}_{rj}(t - \tau_{ji}(t)) \qquad (22)
$$

where $B_{i0} = \begin{bmatrix} -1 & 1 \end{bmatrix}$ and $B_j = \begin{bmatrix} g_{ji}^r & 0 \end{bmatrix}$ are the system matrices. The performance cost function for the ordinary traffic is selected as

$$
J_{ri} = E\{\int_0^\infty (z_{ri}^T(t)Q_i(\alpha_t)z_{ri}(t) + \bar{u}_{ri}^T(t)R_i(\alpha_t)\bar{u}_{ri}(t))dt\} \qquad (23)
$$

where $Q_i(\alpha_t)$ and $R_i(\alpha_t)$ are given positive definite matrices.

The following lemma shows that the state feedback controller $\bar{u}_{ri}(t) = K_{ri}(\alpha_t)z_{ri}(t)$ is a stochastic guaranteed cost control law [8] for the system (22).

Lemma 2: Given the cost function (23) and under Assumption 2, if there exist symmetric positive definite matrices $\Lambda_{i1}^T(\alpha_t)$, $\bar{X}_i(\alpha_t)$, $\bar{V}_{ii}(\alpha_t)$, $\bar{T}_i(\alpha_t)$, and matrices U_i, $N_i(\alpha_t)$, Λ_{i3}^T, and $\bar{S}_i(\alpha_t)$ for $i = 1,....n$, and $\alpha_t \in \mathscr{S} = \{1,\ldots,M\}$ such that the LMI condition $\Omega_i(\alpha_t) < 0$ is satisfied and where $\Omega_i(\alpha_t)$ is given by

$$
\begin{bmatrix} \bar{X}_{ik}(\alpha_t) & h^2(\bar{V}_{ik}^T(\alpha_t) + \bar{T}_i(\alpha_t))B_{ji} + B_{ji} & \Lambda_{i1}^T(\alpha_t) \\ * & h^2 B_{ji}^T U_i B_{ji} & 0 \\ * & * & -\Lambda_{i3}^T - (1-h)\bar{S}_i(\alpha_t) \end{bmatrix} \qquad (24)
$$

then the controller $\bar{u}_{ri}(t) = K_{ri}(\alpha_t)z_{ri}(t)$ is the stochastic guaranteed cost controller of system (22), and the decentralized control gains are given by $K_{ri}(\alpha_t) = B_{i0}^+ T_i(\alpha_t)\Lambda_{i1}^{-1}(\alpha_t)$ ("+" denotes the the Moore-Penrose inverse).

Proof: Consider the following stochastic Lyapunov-Krasovskii functional candidate

$$
\begin{aligned}
V_i(z_{ri}(t),\alpha_t) &= V_{i1} + V_{i2} + V_{i3} + V_{i4} \qquad (25) \\
V_{i1} &= z_{ri}(t)^T P_i(\alpha_t)z_{ri}(t) \\
V_{i2} &= \int_{t-h}^{t} z_{ri}^T(s)S_i(\alpha_t)z_{ri}(s)ds \\
V_{i3} &= h\int_{-h}^{0}\int_{t+\theta}^{t} \dot{z}_{ri}^T(s)U_i\dot{z}_{ri}(s)dsd\theta \\
V_{i4} &= \int_{-h}^{0}\int_{t+\theta}^{t} z_{ri}^T(s)S_i(\alpha_t)z_{ri}(s)dsd\theta
\end{aligned}
$$

where $P_i(\alpha_t)$, $S_i(\alpha_t)$, U_i are positive definite matrices with appropriate dimensions. Then, for each $\alpha_t = l \in \mathscr{S}$ we have

$$
\begin{aligned}
\mathscr{L}V_i &\le z_{ri}^T(t)(2P_i(\alpha_t)B_{i0}K_{ri}(\alpha_t) + \sum_{l=1}^{M}\pi_{\alpha_t l}P_i(l) + (1+h)S_i(\alpha_t))z_{ri}(t) \\
&\quad + h^2 z_{ri}^T(t)((B_{i0}K_{ri})^T(\alpha_t)U_iB_{i0}K_{ri}(\alpha_t) - U_i)z_{ri}(t) \\
&\quad + 2z_{ri}^T(t)(h^2(B_{i0}K_{ri}(\alpha_t))^T U_i + P_i(\alpha_t)) \sum_{j\in\wp_i(\alpha_t)} B_jK_{rj}(\alpha_t)z_{rj}(t - \tau_{ji}(t)) \\
&\quad + h^2[\sum_{j\in\wp_i(\alpha_t)} B_jK_{rj}(\alpha_t)z_{rj}(t-\tau_{ji}(t))]^T U_i[\sum_{j\in\wp_i(\alpha_t)} B_jK_{rj}(\alpha_t)z_{rj}(t-\tau_{ji}(t))] \\
&\quad - z_{ri}^T(t-h)(U_i + (1-h)S_i(\alpha_t))z_{ri}(t-h)
\end{aligned}
$$

By defining $B_{ji} = vec\{B_j\}$, $K_{ji}(\alpha_t) = diag\{K_{rj}(\alpha_t)\}$, $Z_{rj}(t - \tau) = vec\{z_{rj}^T(t - \tau_{ji}(t))\}$, we obtain

$$
\mathscr{L}V_i \quad \le \quad \eta_i^T(t,\tau,h)W_i(\alpha_t)\eta_i(t,\tau,h) \qquad (26)
$$

where $\eta_i(t,\tau,h) = [z_{ri}^T(t)\ \bar{Z}_{rj}^T(t-\tau)\ z_{ri}^T(t-h)]^T$. The matrix

$W_i(\alpha_t)$ is defined by

$$W_i(\alpha_t) = \begin{bmatrix} w_i^1(\alpha_t) & w_i^2(\alpha_t) & U_i \\ * & w_i^3(\alpha_t) & 0 \\ * & * & -U_i-(1-h)S_i(\alpha_t) \end{bmatrix}$$
$$w_i^1(\alpha_t) = (2P_i(\alpha_t)+h^2(B_{i0}K_{ri})^T(\alpha_t)U_i)B_{i0}K_{ri}(\alpha_t) + \sum_{l=1}^{M}\pi_{\alpha_t l}P_i(l)+(1+h)S_i(\alpha_t)-U_i$$
$$w_i^2(\alpha_t) = (h^2(B_{i0}K_{ri})^T(\alpha_t)U_i+P_i(\alpha_t))B_{ji}K_{ji}(\alpha_t)$$
$$w_i^3(\alpha_t) = h^2K_{ji}^T(\alpha_t)B_{ji}^TU_iB_{ji}K_{ji}(\alpha_t)$$

Therefore, if $W_i(\alpha_t) < 0$, then we will have $\mathscr{L}V_i < 0$ and the system (22) is stochastically stable.

However, since the matrix inequality $\bar{W}_i(\alpha_t)$ is not linear with respect to the control gain K_{ri} let us define

$$\bar{W}_i(\alpha_t) = W_i(\alpha_t)+Q_i(\alpha_t)+K_{pi}^T(\alpha_t)R_i(\alpha_t)K_{pi}(\alpha_t)$$
$$B_{i0}K_{ri}(\alpha_t) = T_i\Lambda_{i1}^{-1}(\alpha_t); \qquad \bar{T}_i = T_i^TU_i$$
$$S_i(\alpha_t) = P_i(\alpha_t); \qquad M_i(\alpha_t) = U_i$$
$$\bar{Q}_i(\alpha_t) = \Lambda_{i1}^T(\alpha_t)Q_i(\alpha_t)\Lambda_{i1}(\alpha_t); \qquad \bar{S}_i(\alpha_t) = \Lambda_{i3}^TS_i\Lambda_{i3}^T$$
$$\bar{R}_i(\alpha_t) = \Lambda_{i1}^T(\alpha_t)K_{ri}^T(\alpha_t)R_i(\alpha_t)K_{ri}(\alpha_t)\Lambda_{i1}(\alpha_t)$$

By pre pre and post multiplying the matrix $\bar{W}_{ik}(\alpha_t)$ with Λ_i^T and Λ_i, respectively, the following matrix is obtained

$$\begin{aligned}\Omega_i(\alpha_t) &= \Lambda_i^T(\alpha_t)\bar{W}_i(\alpha_t)\Lambda_i(\alpha_t) \\ &= \begin{bmatrix} \Lambda_{i1}^T(\alpha_t)X_{ik}(\alpha_t)\Lambda_{i1}(\alpha_t) & h^2\Lambda_{i1}^T(\alpha_t)(B_{i0}K_{ri})^T(\alpha_t)U_iB_{ji}+B_{ji} & \Lambda_{i1}^T(\alpha_t) \\ * & h^2B_{ji}^T(U_i+N_i(\alpha_t))B_{ji} & 0 \\ * & * & -\Lambda_{i3}^T-(1-h)\Lambda_{i3}^TS_i(\alpha_t)\Lambda_{i3} \end{bmatrix} \\ &= \begin{bmatrix} \bar{X}_i(\alpha_t) & h^2\bar{T}_iB_{ji}+B_{ji} & \Lambda_{i1}^T(\alpha_t) \\ * & h^2B_{ji}^TU_iB_{ji} & 0 \\ * & * & -\Lambda_{i3}^T-(1-h)\bar{S}_i(\alpha_t) \end{bmatrix}\end{aligned} \tag{27}$$

where
$$\bar{X}_i(\alpha_t) = T_i+T_i^T+(1+h+\sum_{l=1}^{M}\pi_{\alpha_t l})\Lambda_{i1}^T(\alpha_t)+\bar{Q}_i(\alpha_t)+\bar{R}_i(\alpha_t)$$
$$B_{i0}K_{ri}(\alpha_t) = T_i\Lambda_{i1}^{-1}(\alpha_t); \qquad \bar{T}_i = T_i^TU_i$$
$$S_i(\alpha_t) = P_i(\alpha_t); \qquad M_i(\alpha_t) = U_i$$
$$\bar{Q}_i(\alpha_t) = \Lambda_{i1}^T(\alpha_t)Q_i(\alpha_t)\Lambda_{i1}(\alpha_t); \qquad \bar{S}_i(\alpha_t) = \Lambda_{i3}^TS_i\Lambda_{i3}^T$$
$$\bar{R}_i(\alpha_t) = \Lambda_{i1}^T(\alpha_t)K_{ri}^T(\alpha_t)R_i(\alpha_t)K_{ri}(\alpha_t)\Lambda_{i1}(\alpha_t)$$

Therefore, if $\Omega_i(\alpha_t) < 0$, one will also have $\bar{W}_i < 0$. By solving the LMI conditions $\Omega_i(\alpha_t) < 0$, one can then obtain the control gains $K_{ri}(\alpha_t) = B_{i0}^+T_i(\alpha_t)\Lambda_{i1}^{-1}(\alpha_t)$, where "+" is the Moore-Penrose inverse.

Furthermore, according to (26), the following inequality also holds for $\mathscr{L}V_i$, namely

$$\begin{aligned}\mathscr{L}V_i &\le \eta_i^T(t,\tau,h)[\bar{W}_{ik}(\alpha_t)-Q_i(\alpha_t)-K_{pi}^T(\alpha_t)R_i(\alpha_t)K_{pi}(\alpha_t)]\eta_i(t,\tau,h) \\ &\le -z_{ri}^T(t)(Q_i(\alpha_t)+K_{pi}^T(\alpha_t)R_i(\alpha_t)K_{pi}(\alpha_t))z_{ri}(t)\end{aligned} \tag{28}$$

Consequently, in view of (23) and (28) one can obtain

$$\begin{aligned}J_{ri} &\le -E\{\int_0^\infty(\mathscr{L}V_i(z_{ri}(t),\alpha_t))dt\} \\ &= V_i(z_{ri}(0),0,r_0)-\lim_{t\to\infty}V_i(z_{ri}(t),t,\alpha_t) \\ &\le V_i(z_{ri}(0),0,r_0) = J_{ri}^*\end{aligned} \tag{29}$$

Therefore, the performance cost of the closed-loop system is guaranteed for any admissible time-varying delays satisfying Assumption 1.

According to the Definition 1, the state feedback controller $\bar{u}_{ri}(t)$ is therefore the stochastic guaranteed cost controller of system (22) and the scalar J_{ri}^* is the stochastic guaranteed cost of (22). This completes the proof of Lemma 2. ■

IV. Stability Conditions Incorporating the Network Physical Constraints

In this section, the network physical constraints (5)-(6) are transformed into mode-dependent LMI conditions. These complementary LMIs, together with the stability conditions provided in Lemmas 1 and 2 will be taken into account for determining a complete solution to the guaranteed cost congestion control problem.

A. Mode-Dependent Physical Constraints of the Premium Traffic

The state constraints for system (13) can be expressed as follows

$$\bar{z}_{pi}^{min} \le \bar{z}_{pi}(t) \le \bar{z}_{pi}^{max} \tag{30}$$

where $\bar{z}_{pi}^{min} = [-x_{pi}^{ref}\ \ 0]^T$ and $\bar{z}_{pi}^{max} = [x_{pi}^{buffer}-x_{pi}^{ref}, \lambda_{pi}^{max}]^T$ denote the minimum and the maximum bounds of the new state. By squaring (30) one will have

$$\bar{z}_{pi}^T(t)\bar{z}_{pi}(t) \le \|\bar{z}_{pi}^{max}\|^2 \tag{31}$$

Consider the following ellipsoid for a given parameter $\varepsilon_{1i} > 0$

$$\mathbb{F}_i(\alpha_t) = \{\bar{z}_{pi}(t)|\bar{z}_{pi}^T\Lambda_{i1}^{-1}(\alpha_t)\bar{z}_{pi} \le \varepsilon_{1i}\} \tag{32}$$

According to the definitions of the Lyapunov functional V_i in (15), since $\Lambda_{i1}^{-1}(\alpha_t) = P_i(\alpha_t)$, we get

$$\bar{z}_{pi}^T\Lambda_{i1}^{-1}(\alpha_t)\bar{z}_{pi} \le V_i(\bar{z}_{pi}(t),\alpha_t) \tag{33}$$

By integrating (18), from 0 to t and considering that $V_i(\bar{z}_{pi}(0),r_0) = 0$, we get

$$\begin{aligned}V_i &\le -\int_0^t\bar{z}_{pi}^T(t)(Q_i(\alpha_t)+K_{pi}^T(\alpha_t)R_i(\alpha_t)K_{pi}(\alpha_t))\bar{z}_{pi}(t)dt \\ &\quad +\int_0^t\lambda_{pi}^T(t)\Psi_i(\alpha_t)\lambda_{pi}(t)dt \\ &< \int_0^\infty\lambda_{pi}^T(t)\Psi_i(\alpha_t)\lambda_{pi}(t)dt \\ &< \gamma_i max(\Psi_i(\alpha_t))\end{aligned} \tag{34}$$

Therefore, the state $\bar{z}_{pi}(t)$ will belong to the set $\mathbb{F}_i(\alpha_t)$ for all the modes α_t if

$$\gamma_i max(\Psi_i(\alpha_t)) \le \varepsilon_{1i} \tag{35}$$

Consequently, the right-hand side of the state constraint (30) is satisfied if

$$\varepsilon_{1i}/||\bar{z}_{pi}^{max}||^2 \le \Lambda_{i1}^{-1}(\alpha_t) \tag{36}$$

By applying the Schur complement to (36), the right-hand side of the state constraint (30) will hold if the following LMI conditions are satisfied

$$\Omega_{c1i}^p(\alpha_t) \triangleq \gamma_i max\{\Psi_i(\alpha_t)\} \le \varepsilon_{1i} \tag{37}$$

$$\Omega_{c2i}^p(\alpha_t) \triangleq \begin{bmatrix} \Lambda_{i1}(\alpha_t) & \Lambda_{i1}^T(\alpha_t) \\ \Lambda_{i1}(\alpha_t) & \|\bar{z}_{pi}^{max}\|^2/\varepsilon_{1i} \end{bmatrix} \ge 0 \tag{38}$$

On the other hand, the left-hand side of the state constraint (30) can be rewritten as $\bar{z}_{pi}(t)-\bar{z}_{pi}^{min} \ge 0$.

According to the definition of non-negative systems [15], if the above system is non-negative, then the left-hand side of the state constraint (30) holds. By selecting the matrix

$\Lambda_{i1}(\alpha_t)$ as a diagonal positive definite matrix, the non-negative condition of the closed-loop system matrices can be expressed as follows

$$\begin{aligned}
\Omega^p_{c3i}(\alpha_t) &\triangleq (T_i(\alpha_t))_{ij} \geq 0 \qquad (39)\\
V_{ik}(\alpha_t) &= \begin{bmatrix} V^1_{ik}(\alpha_t) & V^2_{ik}(\alpha_t) \\ V^3_{ik}(\alpha_t) & V^4_{ik}(\alpha_t) \end{bmatrix}\\
V^1_{i1}(\alpha_t) &= V^1_{i2}(\alpha_t) = 0\\
V^2_{i1}(\alpha_t) &= V^2_{i2}(\alpha_t) = 0\\
V^3_{i2}(\alpha_t) &= 0\\
V^3_{i1}(\alpha_t) &> 0 \text{ and is diagonal}\\
V^4_{i1}(\alpha_t) &= V^4_{i2}(\alpha_t) < 0 \text{ and is diagonal}
\end{aligned}$$

The input constraint of the system (13) can be expressed as

$$0 \leq \bar{u}_{pi}(t) \leq C_{server,i}(\alpha_t) \qquad (40)$$

Noting that $\bar{u}_{pi}(t) = B^+_{i0}T_i(\alpha_t)\Lambda^{-1}_{i1}(\alpha_t)\bar{z}_{pi}(t)$, hence the input constraint (40) becomes

$$0 \leq B^+_{i0}T_i(\alpha_t)\Lambda^{-1}_{i1}(\alpha_t)\bar{z}_{pi}(t) \leq C_{server,i}(\alpha_t) \qquad (41)$$

Consider the ellipsoid (32), so that the right-hand side of the input constraint is satisfied if

$$(B^+_{i0}T_i(\alpha_t)\Lambda^{-1}_{i1}(\alpha_t))^T(\varepsilon_{i1}/C^2_{server,i}(\alpha_t))B^+_{i0}T_i(\alpha_t)\Lambda^{-1}_{i1}(\alpha_t) \leq \Lambda^{-1}_{i1}(\alpha_t) \qquad (42)$$

The above condition can be transformed into the following LMI condition

$$\Omega^p_{c4i}(\alpha_t) \triangleq \begin{bmatrix} I & K^T_i(\alpha_t) \\ K_i(\alpha_t) & (C^2_{server,i}(\alpha_t)/\varepsilon_{1i})\Lambda_{i1}(\alpha_t) \end{bmatrix} \geq 0 \qquad (43)$$

The non-negative constraint of the input is satisfied if the control gain $(K_{pi}(\alpha_t))_{ij} > 0$. Hence, by using $K_{pi}(\alpha_t) = B^+_{i0}T_i(\alpha_t)\Lambda^{-1}_{i1}(\alpha_t)$ and noting that $\Lambda^{-1}_{i1}(\alpha_t)$ is set to be a diagonal positive definite matrix, then B_{i0} is negative definite.

The left-hand side of the input constraint can be transformed into the following LMI condition

$$\Omega^p_{c5i}(\alpha_t) \triangleq (T_i(\alpha_t))_{ij} \leq 0 \qquad (44)$$

Therefore, the above results and the LMI conditions that are given in Lemma 1 can all be summarized into the following theorem.

Theorem 1: The decentralized Markovian jump guaranteed cost congestion controller (MJ-GCC) for the premium traffic in a mobile network is determined by $\bar{u}_{pi} = K_{pi}(\alpha_t)\bar{z}_{pi}$, if the mode-dependent LMI conditions given in Lemma 1 subject to the positive definite diagonal matrix $\Lambda^{-1}_{i1}(\alpha_t)$ and the mode-dependent LMI conditions of $\Omega^p_{c1i}(\alpha_t)$ to $\Omega^p_{c5i}(\alpha_t)$ for $i = 1,\ldots n$, $\alpha_t \in \mathscr{S} = \{1,\ldots,M\}$, as given in (37), (38), (39), (43), and (44), respectively, are all satisfied.

Proof: Follows along the same lines as the derivations in Lemma 1 and the analysis of the physical constraints. These details are omitted due to the space limitations. ■

B. Mode-Dependent Physical Constraints of the Ordinary Traffic

The physical constraints for the ordinary traffic in a mobile network are listed as

$$z^{min}_{ri} \leq z_r(t) \leq z^{max}_{ri}; \qquad 0 \leq \bar{u}_{ri}(t) \leq c_{ri}(\alpha_t)$$

where $z^{maz}_{ri} = x^{buffer}_{ri} - x^{ref}_{ri}$ and $z^{min}_{ri} = -x^{ref}_{ri}$.

To avoid any confusion, in the remainder of this section we use the notations Λ_{pi1} and Λ_{ri1} to denote the Lyapunov matrix Λ_{i1} that is used in Lemmas 1 and 2, for the premium and the ordinary traffic, respectively, and the following analysis of the physical constraints can be obtained.

For the state constraints, consider the following ellipsoid for a given parameter $\varepsilon_{i2} > 0$, namely

$$\mathbb{S}_i = \{z^T_{ri}(\tilde{P}_{ri})^{-1}(\alpha_t)z_{ri} < \varepsilon_{i2}\} \qquad (45)$$

From the definition of the Lyapunov function given in (25) and the stability conditions given in Lemma 2, we will have

$$z^T_r(t)\Lambda^{-1}_{ri1}z_r(t) \leq V_i(z_{ri}(t), \alpha_t) \qquad (46)$$

Now, by integrating (28) on both sides from 0 to t and considering $V(z_{ri}(0), r_0) = 0$, we will have

$$V_i \leq -\int_0^t z^T_{ri}(t)(Q_i(\alpha_t) + K^T_{pi}(\alpha_t)R_i(\alpha_t)K_{pi}(\alpha_t))z_{ri}(t)dt < 0 \qquad (47)$$

Therefore, $z_{ri}(t)$ belongs to the set $\mathbb{S}_i$ for all $t > 0$. Consequently, the right-hand side of the state constraints can be expressed according to the following LMI condition

$$\Omega^r_{c1i}(\alpha_t) \triangleq \begin{bmatrix} \Lambda_{ri1}(\alpha_t) & \Lambda^T_{ri1}(\alpha_t) \\ \Lambda_{ri1}(\alpha_t) & (z^{max}_{ri})^2/\varepsilon_{i2} \end{bmatrix} \geq 0 \qquad (48)$$

On the other hand, the left-hand side of the state constraints can be considered by the following non-negative constraint

$$z_{ri}(t) - z^{min}_{ri} \geq 0 \qquad (49)$$

Following along the similar lines as those in deriving the LMI conditions for the physical constraints of the premium traffic, and noting that the matrix Λ_{ri1} is set to be diagonal and positive definite, and given that $B_{i0} < 0$, the non-negative constraint of the state can be expressed by the following LMI conditions

$$\Omega^r_{c2i}(\alpha_t) \triangleq (T_i(\alpha_t))_{ij} \leq 0 \qquad (50)$$

For the constraints on the input $\bar{u}_{ri}$, by taking into account that $\bar{u}_{ri}(t) = K_{ri}(\alpha_t)z_{ri}(t)$, it can be stated that

$$0 \leq B^+_{i0}T_i(\alpha_t)\Lambda^{-1}_{ri1}(\alpha_t)z_{ri}(t) \leq c_{ri}(\alpha_t) \qquad (51)$$

Note that $c_{ri}(\alpha_t) = C_{server,i}(\alpha_t) - K_{pi}(\alpha_t)\bar{z}_{pi}(t)$, where $K_{pi}(\alpha_t)$ is the control gain of the premium traffic controller. Consequently, the input constraints of the ordinary traffic can be expressed as follows

$$0 \leq K_{ri}(\alpha_t)z_{ri}(t) \leq C_{server,i}(\alpha_t) - K_{pi}(\alpha_t)\bar{z}_{pi}(t) \qquad (52)$$

From the right-hand side of (52) one can have

$$K_{ri}(\alpha_t)z_{ri}(t) + K_{pi}(\alpha_t)\bar{z}_{pi}(t) \leq C_{server,i}(\alpha_t) \qquad (53)$$

By squaring (53) we obtain

$$\begin{bmatrix} z_{ri}(t) \\ z_{pi}(t) \end{bmatrix}^T \begin{bmatrix} K^T_{ri} \\ K^T_{pi} \end{bmatrix} \begin{bmatrix} K_{ri} & K_{pi} \end{bmatrix} \begin{bmatrix} z_{ri}(t) \\ z_{pi}(t) \end{bmatrix} \leq \|C_{server,i}(\alpha_t)\|^2 \qquad (54)$$

Therefore, by considering the ellipsoid $\mathbb{F}_i$ and the set $\mathbb{S}_i$, the right-hand side of the input constraints will be satisfied if the following LMI conditions hold

$$\Omega^r_{c3i}(\alpha_t) \triangleq \gamma_i max\{\Psi_i(\alpha_t)\} \leq \varepsilon_{i1} \qquad (55)$$

$$\Omega^r_{c4i}(\alpha_t) \triangleq \begin{bmatrix} I & K_{ri}(\alpha_t) & K_{pi}(\alpha_t) \\ K^T_{ri}(\alpha_t) & \frac{C^2_{server,i}(\alpha_t)}{\varepsilon_{i1}+\varepsilon_{i2}}\Lambda_{ri1}(\alpha_t) & 0 \\ K^T_{pi}(\alpha_t) & 0 & \frac{C^2_{server,i}(\alpha_t)}{\varepsilon_{i1}+\varepsilon_{i2}}\Lambda_{pi1}(\alpha_t) \end{bmatrix} \geq 0 \qquad (56)$$

The model-dependent LMI conditions derived above together with the stability conditions obtained in Lemma 2 can be summarized according to the following theorem.

Theorem 2: A decentralized Markovian jump guaranteed cost congestion controller (MJ-GCC) for the dynamical queuing system for the ordinary traffic in each node i is obtained provided that the mode-dependent conditions that are given in Lemma 2 are satisfied, subject to the mode-dependent LMIs $\Omega^r_{c1i}(\alpha_t)$ to $\Omega^r_{c4i}(\alpha_t)$ that are governed by equations (48), (50), (55), and (56), respectively.

Proof: The proof follows along the same lines as those given in Lemma 2 and the derivations and analysis for the physical constraints that are given in this subsection. ■

V. Simulation Results

The simulation results presented in this section are intended to demonstrate the effectiveness and capabilities of our proposed decentralized Markovian jump guaranteed cost congestion (MJ-GCC) strategy to mobile Diff-Serv networks.

A. Performance Metrics

In our simulations we denote the link between nodes by a connectivity parameter $a_{ij}(\alpha_t)$ which is defined as

$$a_{ij}(\alpha_t) = \begin{cases} 1, & \text{if nodes } i \text{ and } j \text{ are connected} \\ 0, & \text{otherwise} \end{cases} \tag{57}$$

where α_t represents the changes in the network topology.

The packet loss rate (PLR) for the premium traffic in the mobile network is defined as

$$PLR_{pi}(t) = \frac{P^p_{bi} + P^p_{ci}}{\lambda_{pi}(t) + \sum_{j\in\wp_i} \lambda^p_{ji}(t) g^p_{ji}(t) a_{ji}(\alpha_t)} \tag{58}$$

$$P_{bi}(t) = max\{0, \lambda_{pi}(t) + \sum_{j\in\wp_i} \lambda^p_{ji}(t) g^p_{ji}(t) a_{ji}(\alpha_t) - (x_{buffer,i} - x_{pi}(t))\} \tag{59}$$

$$P_{ci}(t) = \sum_{k\in\wp_i} \lambda^p_{ik}(t) g^p_{ik}(t)(1 - a_{ik}(\alpha_t)) \tag{60}$$

where P_{bi} is the packet loss induced by the buffer overflow and P_{ci} is the packet loss due to the network topology changes. The PLR for the ordinary traffic in the mobile network is then defined according to

$$PLR_{ri}(t) = \frac{P^r_{bi}(t) + P^r_{fi}(t) + P^r_{ci}(t)}{\lambda_{ri}(t) + \sum_{j\in\wp_i} \lambda^r_{ji}(t) g^r_{ji}(t) a_{ji}(\alpha_t)} \tag{61}$$

$$P^r_{bi}(t) = max\{0, \lambda_{ri}(t) + \sum_{j\in\wp_i} \lambda^r_{ji}(t) g^r_{ji}(t) a_{ji}(\alpha_t) - (x_{buffer,i} - x_{ri}(t))\} \tag{62}$$

$$P^r_{fi}(t) = \lambda^a_{ri}(t) - \lambda_{ri}(t) \tag{63}$$

$$P^r_{ci}(t) = \sum_{k\in\wp_i} \lambda^r_{ik}(t) g^r_{ik}(t)(1 - a_{ik}(\alpha_t)) \tag{64}$$

where P^r_{bi} is the packet loss due to the buffer overflow, P^r_{fi} is the packet loss due to the inadequate flow rate regulation, and $P^r_{ci}(t)$ is the packet loss due to disconnection.

Moreover, the average queuing delay of a mobile network is defined as

$$E\{T^i_q\} = \frac{E\{x_i(t)\}}{E\{\lambda_i(t)\} + \sum_{j\in\wp_i} E\{\lambda_{ji}(t) g_{ji}(t) a_{ji}(\alpha_t)\}} \tag{65}$$

where $E\{T^i_q\}$ is the average queuing delay and $x_i(t)$ is the present queuing state.

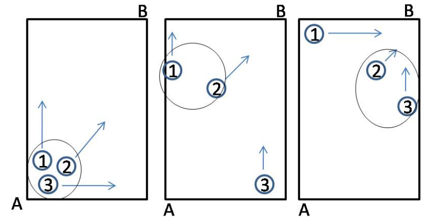

Fig. 1. The schematic of the network topologies and configurations for three "typical" modes corresponding to a mobile network.

B. Simulation Scenario

Consider a network with 3 nodes that are supposed to explore a rectangular planner area from location A to location B as shown in Fig. 1. The first node moves towards north first and then towards east, the second node moves towards northeast directly, and the third node moves towards east and then towards north. It is assumed that the network is fully connected at the start. The capacity of each link is considered to be 10 Mbps, and the maximum buffer size is 5 Mbits. The simulation time duration is selected to be 30s. A total of 5 switching modes are defined based on the possible network topologies. In other words we consider the following network modes $M_1 = \{1,2,3\}$, $M_2 = \{1,2\}$, $\{3\}$, $M_3 = \{1\}$, $\{2,3\}$, $M_4 = \{1,3\}$, $\{2\}$, and $M_5 = \{1\}$, $\{2\}$, $\{3\}$.

The heterogeneous delays among the network nodes are simulated by a *randomly generated* signals as $h \sim N(\mu, \sigma^2)$, and $\tau = min\{0, max\{h_{max}, h\}\}$, where $\mu = 10$ ms is the mean value of delay, $\sigma^2 = 5$ ms is the standard deviation, and $h_{max} = 20$ is the maximum value of transmission delay.

The transition probabilities π_{kl} among different modes are taken to be random and the following transition probability matrix is considered for the Markovian jump model of the switchings in the network topologies, namely

$$\Pi = \begin{bmatrix} \pi_{11} & \cdots & \pi_{15} \\ \vdots & \ddots & \vdots \\ \pi_{51} & \cdots & \pi_{55} \end{bmatrix} = \begin{bmatrix} 0.2 & 0.1 & 0.3 & 0.3 & 0.1 \\ 0.1 & 0.4 & 0.2 & 0.15 & 0.15 \\ 0.15 & 0.15 & 0.6 & 0.05 & 0.05 \\ 0.2 & 0.6 & 0.05 & 0.1 & 0.05 \\ 0.2 & 0.2 & 0.2 & 0.2 & 0.2 \end{bmatrix}$$

C. Performance Analysis

A comparative study of our proposed MJ-GCC strategy with two other congestion control strategies in the literature, namely the Integrated Dynamic Congestion Control (IDCC) [16] and the Markovian Jump Switching Congestion Control (MJ-SCC) [5], are summarized in this section. The IDCC method is designed for a cascade network of ATM switches with fixed topology, and has shown effective performance by the authors in [16]. On the other hand, the MJ-SCC is an extension of IDCC to the mobile environment and is designed for a fully connected network topology. In this paper, we select these two methods for comparison since they are both developed based on analytical fluid flow model. Especially, the IDCC method is considered as a bench-mark congestion control strategy in the control community.

As shown in Tables I and II, one can observe that the packet loss rate of the ordinary traffic by utilizing the MJ-GCC strategy is greatly decreased. The reason is that in the

TABLE I

PACKET LOSS RATE

Premium	IDCC [16]	MJ-SCC [5]	MJ-GCC
Node 1	99.33%	0.029%	0.012%
Node 2	96.15%	0.034%	0.032%
Node 3	93.59%	0.017%	0.011%
Ordinary	IDCC [16]	MJ-SCC [5]	MJ-GCC
Node1	89.51%	9.62%	2.61%
Node 2	96.50%	9.80%	1.32%
Node 3	98.85%	9.94%	1.41%

TABLE II

AVERAGE QUEUING DELAY

Premium	IDCC [16]	MJ-SCC [5]	MJ-GCC
Node 1	∞	52.7 ms	42.70ms
Node 2	∞	47.2 ms	44.80ms
Node 3	∞	25.6 ms	23.70ms
Ordinary	IDCC [16]	MJ-SCC [5]	MJ-GCC
Node 1	∞	570.1 ms	65.41ms
Node 2	∞	406.3 ms	48.22ms
Node 3	∞	205.3 ms	26.10ms

switching congestion control (SCC) strategy [5], one needs to regulate the traffic compression gains in order to guarantee that the network is working in the safe operating range (to satisfy the physical constraints). However, in the guaranteed cost congestion control (GCC) strategy, the physical constraints are expressed as a set of complementary LMIs that affect the control parameters. Therefore a higher traffic compression gains are possible, which in turn results in a lower packet loss rate.

On the other hand, Table III presents the buffer characteristics of each node for both the premium and the ordinary traffic services based on different levels of the time-delays having maximum bounds of $h = \{20;\ 40;\ 80\}$ ms.

By inspecting the above numerical results one can observe that as the level of the delay increases our proposed decentralized MJ-GCC approach can still maintain a robust performance on the packet loss rate and the average queuing delay, despite the changes in dynamical network topologies. Indeed, the packet loss rate in the network remains less than 0.1% for the premium traffic and less than 6% for the ordinary traffic. The average queuing delay for the premium traffic remains less than 53 ms and for the ordinary traffic remains less than 70 ms.

VI. CONCLUSIONS

In this paper, the congestion control problem of mobile Diff-Serv networks is considered. By utilizing the guaranteed cost control theory, a novel decentralized Markovian jump guaranteed cost congestion control (MJ-GCC) algorithm is developed for the premium and the ordinary traffic in the presence of changing and switching network topologies as well as time-varying and unknown delays. The proposed MJ-GCC strategy is shown to be capable of stabilizing the buffer queues and maintaining the robustness of the system with respect to the admissible time-varying delays.

TABLE III

THE QUEUING PERFORMANCE BY UTILIZING THE DECENTRALIZED MJ-GCC APPROACH HAVING DIFFERENT DELAY LEVELS.

PLR	Node 1		Node 2		Node 3	
h	P	O	P	O	P	O
20 ms	0.012%	2.61%	0.032%	1.32%	0.011%	1.41%
40 ms	0.012%	2.91%	0.034%	4.66%	0.013%	2.12%
80 ms	0.099%	3.42%	0.037%	5.95%	0.034%	5.57%
Delay	Node 1		Node 2		Node 3	
h	P	O	P	O	P	O
20 ms	42.70 ms	65.41 ms	44.80 ms	48.22 ms	23.70 ms	26.10 ms
40 ms	51.30 ms	66.56 ms	44.90 ms	49.64 ms	24.80 ms	27.87 ms
80 ms	52.60 ms	67.03 ms	47.70 ms	50.12 ms	25.80 ms	30.48 ms

Furthermore, the mode-dependent physical constraints of the mobile network are guaranteed by satisfying a set of complementary LMIs. The simulation results and numerical comparisons show that the performance of our proposed MJ-GCC algorithm has great superiority when compared to the other available methods in the literature.

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