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A Fading Memory Data-Driven Algorithm for Controller Switching

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***Abstract*— A data-driven controller switching algorithm used for adaptive control is investigated. A new cost-detectable cost function based on fading memory data is constructed so as to reduce the influence of older data. A new controller switching algorithm is designed to guarantee that switching stops and that the closed-loop system is stable. Theoretical analyses and simulations are presented to show that, when the plant changes slowly or infrequently, the new algorithm can detect instability and switch to a stabilizing controller sooner and more smoothly, once the currently active controller becomes destabilizing for the new plant dynamics. It is also shown that this algorithm can be used to attenuate the Dehghani-Anderson-Lanzon phenomenon.**

***Index Terms*— adaptive control, unfalsified, data-driven, fading memory, cost-detectability**

I. INTRODUCTION

In most theoretical studies of adaptive control, it is assumed that the structure of the plant is known. However, in practice, it is difficult to determine this structure. Even when the plant's structure is known, it is still difficult to identify its parameters. These difficulties motivate the study of logic-based switching control [1] and data-driven control approaches, such as iterative learning control (ILC) [2], virtual reference feedback tuning (VRFT) [3], and iterative feedback tuning (IFT) [4].

Unfalsified adaptive control (UAC) [5] is an important real-time approach for data-driven control. In UAC, a supervisor manages the closed-loop system based on measured data. If the active controller does not meet performance requirements such as stability, the supervisor will switch it off and try another candidate controller. With a cost-detectable cost function, the supervisor will find a controller satisfying the performance requirements and keep it in the closed-loop, if there exists such a controller. Although plant models are important for populating this set of candidate controllers a priori, once the system is online, the supervisor only relies on data to make switching decisions. In this way, UAC can be robust against plant model mismatch and uncertainty.

In recent years, there have been numerous papers investigating the features and limitations of UAC [6], [7], [8], [9], [10], [11], [12], [13], [14], [15]. In most of these papers, timeworn data carry the same weight as current data when making switching decisions. This is problematic in some cases, because a basic motivation for adaptive control is to deal with systems where the plant varies slowly or infrequently over time. Such time varying systems may become unstable after being stable for a long time. In these cases, using old data may lessen the ability of the supervisor to detect the destabilizing controller quickly, which can be dangerous in practice. To overcome the influence of old data, a natural idea is to use a method based on fading memory data [11], [16], which would let old data have less of an effect on current switching decisions. With fading memory data, the adaptive supervisor would pay more attention to recent data and detect instability earlier. For these reasons, we investigate the issue of stability when using fading memory data.

In this paper, the fading memory controller switching algorithm for UAC is studied. First, we design a cost function, in which fading memory data are used. Then, we prove the cost-detectability of the cost function and controller set pair, which guarantees that the closed-loop system will be stable if switching stops. Next, a controller switching algorithm is designed. We present a theorem and prove that, with this algorithm, switching will stop if there is at least one stabilizing controller in the candidate controller set. With these results, UAC guarantees the stability of the closed-loop system with fading memory data.

The algorithm in this paper performs better than existing UAC approaches when used with slowly or infrequently varying plants. Once the system becomes unstable, the supervisor will detect the instability and switch controllers sooner, because very old data, which no longer reflects current plant behavior, will not continue to incorrectly unfalsify the active controller. This can be essential for maintaining the stability of adaptive systems when plant changes are large and unexpected, which may occur as a result of unanticipated equipment failure for example.

Furthermore, the algorithm in this paper can attenuate the Dehghani-Anderson-Lanzon (DAL) phenomenon presented in [17]. In [17], examples illustrate that using the ϵ-hysteresis algorithm [18] without fading memory data might cause the supervisor to repeatedly insert a destabilizing controller in the loop. As a result, the magnitudes of the control and output signals might increase to an unacceptable level. Our new algorithm exponentially increases the cost level at which controllers are falsified so as to reduce the chance of switching to a destabilizing controller, thereby preventing the magnitudes of the control and output signals from becoming

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too large.

This paper is organized as follows. In Section II, the problem is formulated. In Section III, a background of UAC is given. Section IV contains our main results, including the construction of our fading memory cost function, the design of our controller switching algorithm, and a discussion of the advantages and disadvantages of the algorithm. In Section V, simulations illustrating the effectiveness of the new algorithm are presented. Section VI gives the proof of the convergence theorem of the algorithm.

II. Problem Formulation

In this paper, we consider discrete-time signals and systems. $\mathbb{N}$ denotes the set of nonnegative integers, $\mathbb{R}_+$ denotes the set of nonnegative real numbers, $\emptyset$ denotes the empty set, and the set $X \backslash Y = \{x : x \in \text{set } X,\ x \notin \text{set } Y\}$. For a real-valued signal y defined on $\mathbb{N}$, its truncation is defined as

$$y_\tau(t) = \begin{cases} y(t) & , \quad 0 \le t \le \tau \\ 0 & , \quad \text{otherwise.} \end{cases}$$

Given y_τ, its $\mathcal{L}_\infty$-norm is

$$\|y_\tau\|_\infty = \max_{t \in \{0,1,2,\cdots,\tau\}} |y(t)|.$$

If $\|y_\tau\|_\infty < \infty$ for each $0 \le \tau < \infty$, then $y \in \mathcal{L}_{\infty e}(\mathbb{N})$. If $\exists k < \infty$ such that $\|y_\tau\|_\infty \le k,\ \forall \tau \ge 0$, then $y \in \mathcal{L}_\infty(\mathbb{N})$.

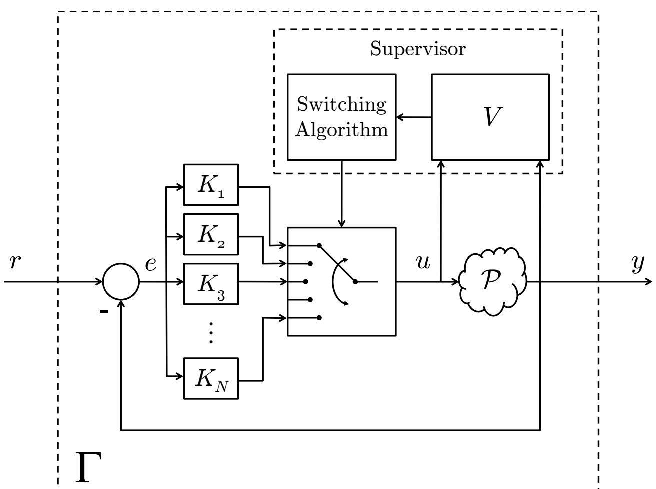

Fig. 1. The switching adaptive control system.

Problem 1. (UAC.) Suppose we have the system Γ shown in Figure 1, in which there is an unknown $\mathcal{L}_{\infty e}(\mathbb{N}) \to \mathcal{L}_{\infty e}(\mathbb{N})$ plant $\mathcal{P}$, a finite candidate controller set $\mathbb{K} = \{K_1, K_2, \ldots, K_N\}$, and a supervisor. The signals $r(t)$, $y(t)$, and $u(t)$ are the reference signal, output signal, and control signal, respectively. Let $d(t) = \begin{bmatrix} u(t) & y(t) \end{bmatrix}'$ denote a data pair, and let $\mathbb{D}$ and $\mathbb{D}_\tau$ denote the space of all possible data d and d_τ respectively.

At each instant, only one candidate controller is active. The active controller at time t is denoted $\hat{K}(t)$. That is, $\hat{K}(t) = K_i \in \mathbb{K}$ if controller K_i is active at time t. Let $\Gamma(\hat{K}(t), \mathcal{P})$ denote the closed-loop system. Thus, if K_i is the only active controller for all time, then the system is $\Gamma(K_i, \mathcal{P})$.

The supervisor monitors the performance of the system using collected data and switches controllers when necessary. The supervisor consists of a cost function $V(K, d_t, t)$ used to order candidate controllers and a controller switching algorithm that uses the cost function to make switching decisions. We focus on stability as our performance goal. The problem at hand is to design a data-driven mechanism to guarantee that the switching adaptive control system $\Gamma(\hat{K}(t), \mathcal{P})$ is stable. Here, for the mechanism to be "data-driven," neither the cost function $V(K, d_t, t)$ nor the switching algorithm may make any prior assumptions about the plant $\mathcal{P}$. □

In this paper, the concepts of stability and feasibility are defined as follows.

Definition 1. (Stability.) Consider the system T in Figure 2 with input r, control signal u, and output y. System T is *stable* if there exist constants $\alpha_1, \alpha_2, \beta_1, \beta_2 \ge 0$ such that, for each input $r \in \mathcal{L}_{\infty e}(\mathbb{N})$ and each $\tau \ge 0$, we have

$$\begin{aligned} \|u_\tau\|_\infty &\le \beta_1 \|r_\tau\|_\infty + \alpha_1 \\ \|y_\tau\|_\infty &\le \beta_2 \|r_\tau\|_\infty + \alpha_2. \end{aligned}$$

Otherwise, T is *unstable*. □

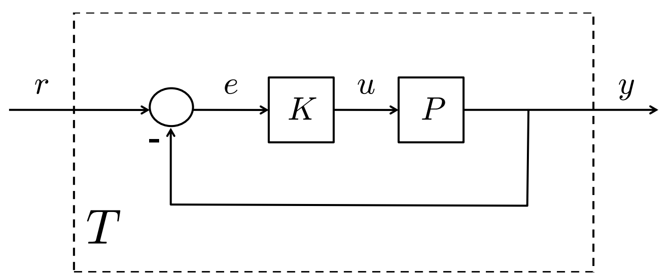

Fig. 2. The SISO system.

Definition 2. (Feasibility.) Problem 1 is *feasible* if there exists at least one controller in the candidate controller set $\mathbb{K}$ that stabilizes the closed-loop system. If $\Gamma(K, \mathcal{P})$ is stable, then the controller K is a *feasible controller*. □

In this paper, we make the following assumption.

Assumption A1. (Feasibility.) Problem 1 is feasible. □

III. Basic Concepts of Unfalsified Adaptive Control

Stability is a property that must hold for all input-output data pairs. However, in a single experiment, only a single data pair is obtained. As such, it is necessary to introduce the concept of unfalsification, defined as follows.

Definition 3. (Unfalsification of Stability.) Consider the system T in Figure 2. Suppose we perform an experiment with the input data r_1, and we collect the output data $d_1 = \begin{bmatrix} u_1 & y_1 \end{bmatrix}'$. The stability of T is *unfalsified* by the data pair (r_1, d_1) if there exist constants $\alpha_1, \alpha_2, \beta_1, \beta_2 \ge 0$ such that, for each $\tau \ge 0$, we have

$$\begin{aligned} \|(u_1)_\tau\|_\infty &\le \beta_1 \|(r_1)_\tau\|_\infty + \alpha_1 \\ \|(y_1)_\tau\|_\infty &\le \beta_2 \|(r_1)_\tau\|_\infty + \alpha_2. \end{aligned}$$

Otherwise, the stability of T is *falsified* by (r_1, d_1). □

By Definitions 1 and 3, system T in Figure 2 is stable if and only if its stability is unfalsified by each input-output data pair. That is, T is definitely unstable if its stability is falsified by data from an experiment. However, if data from the experiment unfalsify the stability of T, we cannot conclude that the system is stable.

A major concern in adaptive control is the analysis of the performance of nonactive controllers. For this reason, UAC makes use of the following definition.

Definition 4. (Fictitious Reference Signal (FRS).) Given the data $d = \begin{bmatrix} u & y \end{bmatrix}'$, $\tau \geq 0$, and a candidate controller K, a *fictitious reference signal* (FRS) $\tilde{r}(K, d_\tau)$ is a hypothetical signal that would have reproduced exactly the measured data d_τ had the controller K been in the loop for the time period over which the data d_τ was collected. For brevity, we denote $\tilde{r}_K = \tilde{r}(K, d_\tau)$ and $\tilde{r}_i = \tilde{r}(K_i, d_\tau)$. □

Definition 5. (CLI, SCLI, and FRSG.) The candidate controller K is *causally left invertible* (CLI) if for each $\tau \geq 0$ and d_τ, the fictitious reference signal $\tilde{r}(K, d_\tau)$ is unique. For each such K, we denote by $\mathfrak{K}_{\text{CLI}}$ the induced causal map $d_\tau \mapsto \tilde{r}$. We call $\mathfrak{K}_{\text{CLI}}$ the *fictitious reference signal generator* (FRSG) for controller K. When $\mathfrak{K}_{\text{CLI}}$ satisfies the condition that, for every two data pairs d_1, d_2, there exist constants $\alpha, \beta \geq 0$ such that

$$\|(\mathfrak{K}_{\text{CLI}} d_1 - \mathfrak{K}_{\text{CLI}} d_2)_\tau\|_\infty \leq \beta \|(d_1 - d_2)_\tau\|_\infty + \alpha,$$

K is *stable causally left invertible* (SCLI). □

With Definitions 3 and 4, we can falsify or unfalsify the stability of system $\Gamma(K_i, \mathcal{P})$ for each $K_i \in \mathbb{K}$; that is, when $\hat{K}(t) = K_i,\ \forall t$. However, in practice, it is more important to decide whether or not to switch off the currently active controller. If the input-output data show that the active controller is likely to be destabilizing, it is essential to take it offline as soon as possible. That is, we need some approach to falsify active controllers in finite time, even though stability itself cannot be falsified in finite time.

Definition 6. (Cost Function.) $V(K, d_t, t)$ is a *cost function* if it is a mapping $V : \mathbb{K} \times \mathbb{D}_t \times \mathbb{N} \to \mathbb{R}_+$. □

Definition 7. (Falsification at a Level.) Given the pair $(V, \mathbb{K})$ and a scalar $\gamma \geq 0$, a controller $K_i \in \mathbb{K}$ is *falsified* at time t at cost level γ by past measurement data d_t if $V(K_i, d_t, t) > \gamma$. Otherwise it is *unfalsified* at time t at cost level γ by d_t. □

Another key concept of UAC is cost-detectability, which establishes the relation between a cost function and the stability of system $\Gamma(\hat{K}(t), \mathcal{P})$. It is defined as follows.

Definition 8. (Cost-Detectability.) Consider the switching adaptive control system Γ shown in Figure 1. Suppose $\hat{K}(t) \in \mathbb{K},\ \forall t$. The cost function and controller set pair $(V, \mathbb{K})$ is *cost-detectable* if for every $\hat{K}(t)$ with at most finitely many switches, the following two statements are equivalent.

1. $V(K_f, d_t, t)$ is bounded as $t \to \infty$, where K_f is the final controller.
2. The stability of the system $\Gamma(\hat{K}(t), \mathcal{P})$ is unfalsified by the data pair (r, d_t).

□

We now give sufficient conditions for cost-detectability.

Lemma 9. (Cost-Detectability.) If for every $K_i \in \mathbb{K}$ and $d \in \mathcal{L}_{\infty e}(\mathbb{N})$, $V(K_i, d_t, t)$ is bounded for each $t \geq 0$ if and only if stability is unfalsified by the input-output data pair $(\tilde{r}(K_i, d_t), d_t)$, then a sufficient condition for the pair $(V, \mathbb{K})$ to be cost-detectable is that all $K_i \in \mathbb{K}$ be SCLI. If all $K_i \in \mathbb{K}$ are LTI, then it is also a necessary condition. □

Proof Since $d \in \mathcal{L}_{\infty e}(\mathbb{N})$, we have that $\tilde{r}(K_i, d_t) \in \mathcal{L}_{\infty e}(\mathbb{N})$ exists for each SCLI controller $K_i \in \mathbb{K}$. The remainder of the proof is identical to that of Theorem 1 of [6] after substitution of continuous-time data with discrete-time data and the $\mathcal{L}_2$-norm with the $\mathcal{L}_\infty$-norm. □

IV. Main Results

We use the following two assumptions in this paper.

Assumption A2. (Reference Signal.) $r \in \mathcal{L}_\infty(\mathbb{N})$. □

Assumption A3. (SCLI.) Each $K_i \in \mathbb{K}$ is SCLI. □

A. The Fading Memory Cost Function

We now introduce the fading memory cost function for Problem 1. For each $K_i \in \mathbb{K}$, the FRS $\tilde{r}_i$ is generated by the following equation.

$$\tilde{r}_i(t) = \begin{cases} r(t) & , \quad K_i = \hat{K}(t) \\ y(t) + K_i^{-1} u(t) & , \quad \text{otherwise.} \end{cases} \tag{1}$$

For the signal $y \in \mathcal{L}_{\infty e}(\mathbb{N})$, let the fading memory functional be

$$F_\eta(y_t, t) = \sum_{\tau=0}^{t} y^2(\tau) \eta^{t-\tau}, \tag{2}$$

where $\eta < 1$ is the fading memory parameter. We use the cost function defined as

$$V(K_i, d_t, t) = \frac{F_\eta((\tilde{r}_i - y)_t, t) + F_\eta(u_t, t)}{F_\eta((\tilde{r}_i)_t, t) + c}, \tag{3}$$

where c is a constant.

Theorem 1. (Cost-Detectability with a Bounded FRS.) Suppose Assumptions 1-3 hold. If $\forall K_i \in \mathbb{K}$, the FRS $\tilde{r}_i \in \mathcal{L}_\infty(\mathbb{N})$, then the pair $(V, \mathbb{K})$ is cost-detectable. □

Proof Since $\tilde{r}_i \in \mathcal{L}_\infty(\mathbb{N})$, it is easy to verify that for each $K_i \in \mathbb{K}$, the cost function (3) is bounded for all $t \geq 0$ if and only if the stability of $\Gamma(K_i, \mathcal{P})$ is unfalsified by the data $(\tilde{r}_i, d)$. Because all controllers are SCLI, by Lemma 9, the pair $(V, \mathbb{K})$ is cost-detectable. □

B. The Controller Switching Algorithm

We now present the following controller switching algorithm. In this algorithm, t is the time variable; $b, \theta_0 > 0$ and $\lambda > 1$ are constants; and θ, γ, Q, and $B_{K_i}(t)$ are variables. Here, b is an upper bound of r. That is, $b \in \{b : |r(t)| \leq b,\ \forall t \in \mathbb{N}\}$. Since $r \in \mathcal{L}_\infty(\mathbb{N})$, such a b exists. θ_0 and λ are used to generate θ, and θ is used to generate the cost level γ used to falsify the active controller. Q is the set of currently available candidate controllers, and $K_0 \in \mathbb{K}$ is the initial controller. For each controller K_i, define $B_{K_i}(t)$ as

$$B_{K_i}(t) = \max\{\|(\tilde{r}_i)_t\|_\infty, b\}.$$

Algorithm I. (The Controller Switching Algorithm.)

1. Initialization. Set $\theta_0 > 0$, $\lambda > 1$, $\theta \leftarrow \theta_0$, $\gamma \leftarrow \theta b$, $t \leftarrow 0$, $Q \leftarrow \mathbb{K}$, and $\hat{K}(0) \leftarrow K_0$.
2. Set $t \leftarrow t+1$. Collect data $r(t)$, $u(t)$, and $y(t)$. $\forall K_i \in \mathbb{K}$, update $\tilde{r}_i(t)$, calculate $V(K_i, d_t, t)$, and calculate $B_{K_i}(t)$.
3. If $V(\hat{K}(t-1), d_t, t) \leq \gamma$, then $\hat{K}(t) \leftarrow \hat{K}(t-1)$. Otherwise,
   a. $Q \leftarrow Q \backslash \hat{K}(t-1)$.
   b. If $Q = \emptyset$, then $Q \leftarrow \mathbb{K}$.
   c. $\hat{K}(t) \leftarrow \operatorname*{argmin}_{K_i \in \mathbb{K}} V(K_i, d_t, t)$. $\theta \leftarrow \theta\lambda$.
   d. $\begin{cases} \text{If} \quad : & \theta(B_{\hat{K}(t)}(t))^2 < \gamma, \\ \text{Then:} & \gamma \leftarrow \lambda\gamma. \\ \text{Else :} & \gamma \leftarrow \theta(B_{\hat{K}(t)}(t))^2. \end{cases}$
4. Go to Step 2.

□

Algorithm I continues indefinitely. In Step 2, the supervisor collects data at time t and calculates the value of the cost function for each controller. In Step 3, if the active controller is falsified by the current cost level γ, it will be switched off and moved out of the currently available controller set Q. If the set Q is empty, then it will be reset to $\mathbb{K}$. The candidate controller with the minimal cost function value will be switch on, and the cost level γ will be increased.

We now give sufficient conditions for convergence.

Theorem 2. (Convergence of Algorithm I.) Suppose Assumptions 1-3 hold. Then, with the FRSG given by (1), the fading memory cost function given by (3), and Algorithm I, the switching adaptive control system $\Gamma(\hat{K}(t), \mathcal{P})$ shown in Figure 1 is stable. □

Proof See Section VI. □

C. Discussion

Like our previous work on UAC in [6], [7], in Algorithm I, we do not make any assumptions on the plant $\mathcal{P}$. It may be nonlinear, time-delayed, or time varying. It may also be a discretization of a continuous-time plant. Thus, Algorithm I can be widely used, and Theorem 2 guarantees that, for each reference signal $r \in \mathcal{L}_\infty(\mathbb{N})$, the closed-loop system $\Gamma(\hat{K}(t), \mathcal{P})$ will be stable with Algorithm I. After at most a finite number of switches, the supervisor will find a controller with which the stability of $\Gamma(\hat{K}(t), \mathcal{P})$ will be unfalsified. That is, $\Gamma(\hat{K}(t), \mathcal{P})$ will be stable.

There are three main modification of Algorithm I over the ϵ-hysteresis algorithm used in [6], [7]. First, as can be seen from Definitions 1 and 3, stability and unfalsification are defined with the $\mathcal{L}_\infty$-norm instead of the $\mathcal{L}_2$-norm. Second, in the cost function (3), fading memory data are used. Third, the logical structure of Algorithm I is different from that of the ϵ-hysteresis algorithm. These modifications have the following advantages.

Advantage 1. (Lower Transient.) When used with a slowly or infrequently time varying plant, which Theorem 2 permits, Algorithm I may lower the large transient caused by the change in the plant. Suppose that, with the currently active controller $K_c \in \mathbb{K}$, the closed-loop system has been stable for a long time but has now become unstable due to a change in the plant. If the unstable modes are excited by the reference signal, then $\|d_t\|_\infty$ will increase rapidly. As such, the stability of $\Gamma(K_c(t), \mathcal{P})$ will be quickly falsified at cost level γ by the data pair $(\tilde{r}(K_c, d_t), d_t)$. To try to stabilize the closed-loop system, the supervisor will switch K_c off, switch on another controller, and increase γ. With Theorem 2, we know that, if there exists at least one controller that stabilizes the plant after the change in plant dynamics, and if the plant varies slowly or infrequently enough so that Algorithm I has sufficient time to find a stabilizing controller, then Algorithm I will find such a controller and keep it online after at most a finite number of switches. That is, after a brief transient, the closed-loop system will become stable again.

We note that, before the supervisor switches to the new controller and stops switching thereafter, there must be an impulse. The reason for the impulse lies in the active destabilizing controller, and the height of the impulse depends on the cost function. To falsify the active destabilizing controller K_c, the value of the cost function $V(K_c, d_t, t)$ must greater than the current value of γ. For the same γ, different cost functions require different peak values of $\|d_t\|_\infty$. We compare the cost function (3) with the cost function used in [6], [17], which is modified for the discrete-time case and reproduced as follows.

$$V'(K_i, d_t, t) \quad = \quad \max_{\tau \leq t} \frac{W((\tilde{r}_i - y)_\tau, \tau) + W(u_\tau, \tau)}{W((\tilde{r}_i)_\tau, \tau) + c}, \tag{4}$$

where

$$W(y_\tau, \tau) \quad = \quad \sum_{k=0}^{\tau} y^2(k), \tag{5}$$

and c is a small positive number.

When the reference signal is bounded, the denominator of (3) remains bounded as $t \to \infty$, whereas the denominator of (4) might increase unboundedly. That is, if the current controller suddenly becomes destabilizing after the system has been stable for a long time, to falsify it at a given γ with (3), the minimum required value of $\|d_t\|_\infty$ is bounded, whereas the minimum required value of (4) increases as the time during which the system has been previously stable increases. This is dangerous, because it can produce a large transient. Using fading memory reduces this danger. □

Advantage 2. (Attenuating the DAL Phenomenon.) Algorithm I can attenuate the Dehghani-Anderson-Lanzon (DAL) phenomenon addressed in [17]. This phenomenon occurs when the ϵ-hysteresis algorithm is used with (4). In this case, the supervisor may repeatedly insert a destabilizing controller in the loop, thus causing $\|u_t\|_\infty$ to increase to a very large value before switching stops. The reason is that, when the hysteresis constant ϵ is small, there are too many chances for every controller to be switched on. Once a destabilizing controller is switched on, $\|u_t\|_\infty$ and/or $\|y_t\|_\infty$ will increase until the controller is falsified and switched off. However, once it is switched on again, $\|u_t\|_\infty$ and/or $\|y_t\|_\infty$ will reach an even higher value.

In Algorithm I, exponentially increasing the cost level γ reduces the chances for a destabilizing controller to be inserted into the loop. Thus, γ quickly reaches a level high enough that feasible controllers will not be falsified. Then, after at most $(2N-2)$ switches, a feasible controller will be switched on, whereupon switching will stop. This is explained in more detail in Section VI, which contains the proof of Theorem 2. □

Algorithm I also has a notable disadvantage. Since, all N controllers might be switched on after a reset of the set of available controllers Q, Algorithm I might be inefficient when the number of candidate controllers is large.

V. Simulations

In this section, we present some numerical examples to demonstrate the performance of Algorithm I.

Example 1. (Finding a Feasible Controller.) Suppose that the time varying plant in Figure 1 is as follows.

$$P(z)=\begin{cases}\frac{0.1}{z-1.1} & , \quad 0\le t\le T\\ \frac{-0.1}{z-1.1} & , \quad t>T.\end{cases} \tag{6}$$

That is, at time T, $P(z)$ changes from $P_1(z)=\frac{0.1}{z-1.1}$ to $P_2(z)=\frac{-0.1}{z-1.1}$. Let the controller set be $\mathbb{K}=\{K_1=2\ ,\ K_2=-2\}$. Note that $\Gamma(K_1,P_1)$ and $\Gamma(K_2,P_2)$ are stable, but $\Gamma(K_1,P_2)$ and $\Gamma(K_2,P_1)$ are unstable. Let the reference input $r(t)$ be the unit step signal, the initial state of the plant be zero, and the initial controller be K_1.

We use the cost function given by (3), the plant $P(z)$ given by (6) with $T=200$, and Algorithm I with $\eta=0.9$, $c=3$, $\theta_0=2$, and $\lambda=1.2$. The Matlab® simulation results are shown in Figure 3, which display, from top to bottom, the plant index, the output y, the control input u, and the controller index. For $t<T$, Algorithm I switches to controller K_1 for the last time after only two switches. After the plant changes at $t=T$, Algorithm I quickly switches to controller K_2 and keeps it active for the remaining time.

As a comparison, we perform a simulation using the cost function (4), which does not have fading memory, as was the case in [17], [6]. For this simulation, we set $c=0.01$ and use the ϵ-hysteresis algorithm with $\epsilon=0.2$. The Matlab® simulation results are shown in Figure 4. The results seem similar to the case with fading memory shown in Figure 3. The controller switches two times fewer than the simulation with fading memory, while the peak values of y and u after the plant changes are higher. □

Example 2. (Insensitivity of Transient Peaks due to Different Plant Switch Times.) We consider the same plant $P(z)$ given in Example 1, but with the plant switch times $T\in\{1000,2000,4000,8000\}$. The results of the simulations with these changes are shown in Figure 5. Because of page limitations, only the sections of the graphs representing $t\in[T,T+100]$ are shown. The graphs in the top row use the fading memory cost function, and the graphs in the bottom row use a cost function without fading memory. From left to right, the graphs in each row correspond to the case when $T=1000$, 2000, 4000, and 8000 respectively.

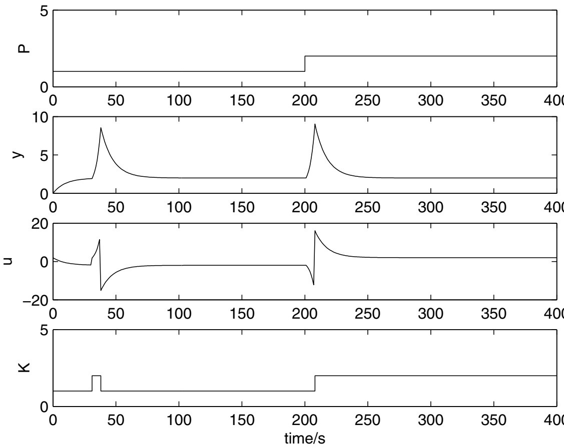

Fig. 3. Simulation results of Algorithm I with fading memory data.

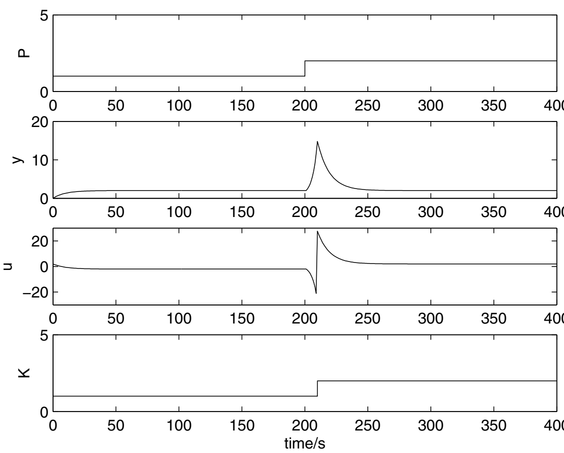

Fig. 4. Simulation results of the ϵ-hysteresis algorithm without fading memory.

As shown by Figure 5, as T increases, the peaks in the graphs in the top row do not increase much, while the peaks in the graphs in the bottom row increase noticeably. The reason is that, when old data and new data are given the same weight, the denominator of the cost function given by (4) will increase as T increases. That is, to falsify the current destabilizing controller at the same cost level γ, the value of $\|d_t\|_\infty$ must be larger whenever T is larger. □

Example 3. (Lessening the DAL Phenomenon.) Consider the plant $P(z)=\frac{e^{0.01}-1}{z-e^{0.01}}$ and the controller set $\mathbb{K}=\{K_1=2\ ,\ K_2=0.5\}$. This is the discretized version of the example used in [17] with sampling interval $h=0.01$. Let the reference signal be $r(t)=\sin(0.01t)$ and the initial state be zero. Simulation results of using Algorithm I and the ϵ-hysteresis algorithm are shown in Figure 6. The graphs on the left show simulation results using Algorithm I with the cost function given by (3) with $\eta=0.9$, $c=3$, $\theta_0=2$,

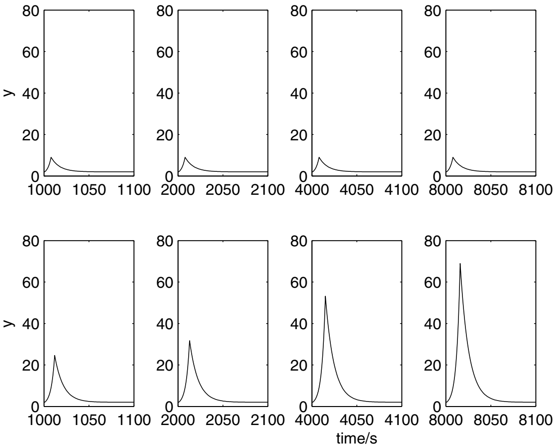

Fig. 5. A comparison of the simulation results of using a cost function with and without fading memory.

and $\lambda = 1.1$. The graphs on the right show simulation results using the ϵ-hysteresis algorithm with $\epsilon = 0.2$ and the cost function given by (4) with $c = 0.01$. Because of page limitations, only the controller index and u are shown.

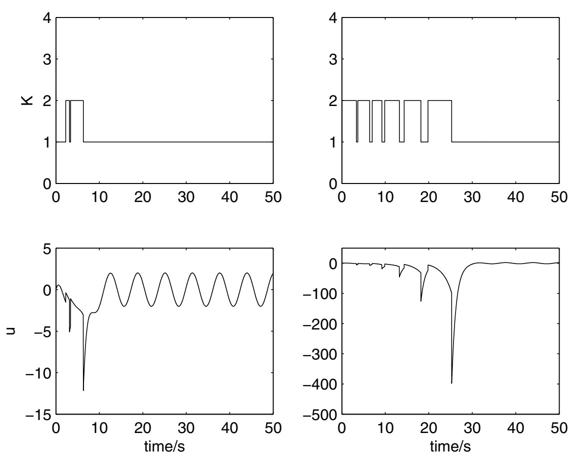

Fig. 6. Simulation results showing the DAL phenomenon when using Algorithm I with the cost function (3) *(left)* and the ϵ-hysteresis algorithm with the cost function (4) *(right)*.

From the graphs on the right in Figure 6, it is easy to witness the DAL phenomenon. The destabilizing controller K_2 is inserted in the loop many times before the supervisor settles on the stabilizing controller K_1 at $t = 25.28$s. As a result, $\max\{|u(t)|\} \approx 400$. However, as can be seen from the graphs on the left, using fading memory decreases both the time in which K_2 is in the loop as well as the value of $\max\{|u(t)|\}$. $\square$

VI. Proof of Theorem 2

Proof We shall prove that, for each reference signal $r \in \mathcal{L}_\infty(\mathbb{N})$, stability will be unfalsified.

First, we prove that switching will stop. Without loss of generality, we assume that K_1 is a feasible controller. Thus, we have for each $\tilde{r}_1 \in \mathcal{L}_{\infty e}(\mathbb{N})$ and each $t \geq 0$,

$$\|u_t\|_\infty \leq \beta_1 \|(\tilde{r}_1)_t\|_\infty + \alpha_1 \quad (7)$$
$$\|y_t\|_\infty \leq \beta_2 \|(\tilde{r}_1)_t\|_\infty + \alpha_2. \quad (8)$$

From (8), we have

$$\|(\tilde{r}_1 - y)_t\|_\infty \leq (\beta_2 + 1)\|(\tilde{r}_1)_t\|_\infty + \alpha_2. \quad (9)$$

After squaring (7) and (9), and using the inequality $(a+b)^2 \leq 2a^2 + 2b^2$, we get

$$(\|u_t\|_\infty)^2 \leq 2\beta_1^2(\|(\tilde{r}_1)_t\|_\infty)^2 + 2\alpha_1^2$$
$$(\|(\tilde{r}_1 - y)_t\|_\infty)^2 \leq 2(\beta_2+1)^2(\|(\tilde{r}_1)_t\|_\infty)^2 + 2\alpha_2^2.$$

From (2), we have

$$\begin{aligned} F_\eta(u_t, t) &= \sum_{\tau=0}^{t} u^2(\tau)\eta^{t-\tau} \\ &\leq \sum_{\tau=0}^{t} (\|u_t\|_\infty)^2 \eta^{t-\tau} \\ &\leq \frac{2\beta_1^2(\|(\tilde{r}_1)_t\|_\infty)^2 + 2\alpha_1^2}{1-\eta}, \end{aligned}$$

and

$$F_\eta((\tilde{r}_1 - y)_t, t) \leq \frac{2(\beta_2+1)^2(\|(\tilde{r}_1)_t\|_\infty)^2 + 2\alpha_2^2}{1-\eta}.$$

Let

$$\rho_1 = \frac{2\alpha_1^2 + 2\alpha_2^2}{(1-\eta)c}$$
$$\rho_2 = \frac{2(\beta_2^2 + 2\beta_2 + 1 + \beta_1^2)}{(1-\eta)c}.$$

From (3), we have

$$\begin{aligned} V(K_1, d_t, t) &= \frac{F_\eta((\tilde{r}_1 - y)_t, t) + F_\eta(u_t, t)}{F_\eta((\tilde{r}_1)_t, t) + c} \\ &\leq \frac{F_\eta((\tilde{r}_1 - y)_t, t) + F_\eta(u_t, t)}{c} \\ &\leq \rho_1 + \rho_2(\|(\tilde{r}_1)_t\|_\infty)^2. \end{aligned}$$

If $\|(\tilde{r}_1)_t\|_\infty \leq b$, it is clear that

$$\begin{aligned} V(K_1, d_t, t) &\leq \rho_1 + \rho_2 b^2 \\ &= \left(\frac{\rho_1}{b^2} + \rho_2\right) b^2. \end{aligned}$$

If $\|(\tilde{r}_1)_t\|_\infty > b$, we have

$$\begin{aligned} V(K_1, d_t, t) &\leq \rho_1 + \rho_2(\|(\tilde{r}_1)_t\|_\infty)^2 \\ &\leq \left(\frac{\rho_1}{b^2} + \rho_2\right)(\|(\tilde{r}_1)_t\|_\infty)^2. \end{aligned}$$

Since $B_{K_1}(t) = \max\{\|(\tilde{r}_1)_t\|_\infty, b\}$, we have for each $t \geq 0$

$$V(K_1, d_t, t) \leq \left(\frac{\rho_1}{b^2} + \rho_2\right)(B_{K_1}(t))^2. \quad (10)$$

Define E as

$$E = \min\left\{E \in \mathbb{N} : \theta_0 \lambda^E \geq \frac{\rho_1}{b^2} + \rho_2\right\}.$$

We now prove that after the E^{th} switch, if controller K_1 is switched on, then switching will stop. Suppose K_1 is switched on at time t_g after the g^{th} switch, where $g \geq E$. Then, we have $\tilde{r}_1(t_g+1) = r(t_g+1) \leq b$, so that at time t_g+1,

$$\begin{aligned} B_{K_1}(t_g+1) &= \max\{\|(\tilde{r}_1)_{t_g+1}\|_\infty, b\} \\ &= \max\{\|(\tilde{r}_1)_{t_g}\|_\infty, r(t_g+1), b\} \\ &= \max\{\|(\tilde{r}_1)_{t_g}\|_\infty, b\} \\ &= B_{K_1}(t_g), \end{aligned}$$

while

$$\theta = \theta_0\lambda^g \geq \frac{\rho_1}{b^2} + \rho_2.$$

From (10) and Algorithm I, we have

$$\begin{aligned} V(K_1, d_{t_g+1}, t_g+1) &\leq \left(\frac{\rho_1}{b^2} + \rho_2\right)(B_{K_1}(t_g+1))^2 \\ &\leq \theta_0\lambda^E (B_{K_1}(t_g))^2 \\ &\leq \gamma. \end{aligned}$$

That is, no switching will take place at $t = t_g + 1$. With the same reasoning, no switching will take place at any $t > t_g + 1$.

Now suppose that the active controller switches endlessly. Consider the available controller set Q after the E^{th} switch. If $K_1 \in Q$, it will be switched on after at most $N-1$ switches, because all falsified controllers will eventually be removed from Q, and Q has no more than N controllers. If $K_1 \notin Q$, then at most after $N-1$ switches, Q will be reset, whereupon K_1 will be switched on after at most an additional $N-1$ switches. Hence, switching will eventually stop, and the cost of the final controller will remain bounded.

Now we prove that all $\tilde{r}_i \in \mathcal{L}_\infty(\mathbb{N})$. Suppose that the final controller is $K_f \in \mathbb{K}$ and that the last switch takes place at t_f. With the FRSG given by (1), we have

$$\tilde{r}(K_f, d_t) = r(t), \ \forall t \geq t_f.$$

Thus, with Assumption 2, we have $\tilde{r}_{K_f} \in \mathcal{L}_\infty(\mathbb{N})$. Furthermore, $u, y \in \mathcal{L}_\infty(\mathbb{N})$, because for each $t \geq t_f$, the cost function $V(K_f, d_t, t) < \infty$. Then, because all candidate controllers are SCLI, all $\tilde{r}_i \in \mathcal{L}_\infty(\mathbb{N})$.

Finally, by Theorem 1, we conclude that $(V, \mathbb{K})$ is cost-detectable. By the definition of cost-detectability, the facts that switching stops and that $V(K_f, d_t, t)$ is bounded imply that the stability of $\Gamma(\hat{K}(t), \mathcal{P})$ is unfalsified. Thus, with Algorithm I, $\Gamma(\hat{K}(t), \mathcal{P})$ is stable. □

VII. Conclusion

In this paper, we study unfalsified adaptive control (UAC) for discrete-time systems. We design a controller switching algorithm that uses fading memory data. In the algorithm, the influence of old data is reduced, and current data play a more important role in making decisions regarding whether or not to switch the current controller offline and what controller to select as the next active controller. Our algorithm, combined with fading memory data, can be used in time varying systems in which the plant changes slowly or infrequently. It can detect instability for any currently active destabilizing controller and thus avoid large magnitudes of the output signal. It also helps to attenuate the Dehghani-Anderson-Lanzon (DAL) phenomenon.

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