2011 50th IEEE Conference on Decision and Control and
European Control Conference (CDC-ECC)
Orlando, FL, USA, December 12-15, 2011

Passive Fault-Tolerant Control of a Class of Over-Actuated Nonlinear Systems and Applications to Electric Vehicles

Rongrong Wang and Junmin Wang*, *Member IEEE*

Abstract—**A fault tolerant (FT) controller for a class of over-actuated nonlinear system is presented. As the fault information is unknown before the fault detection and diagnosis (FDD) procedure finishes, the passive FT control is of great necessity in maintaining system stability and achieving acceptable performance. In this study, the actuators' additive fault and the loss-of-effectiveness faults are considered. A FT controller which works for both types of fault is designed. A vehicle control example for a four wheel independently-actuated (FWIA) electric vehicle is given to show the effectiveness of the proposed method.**

I. Introduction

With the increased number of actuators, over-actuated systems, such as ground vehicles and marine vessels, enjoy certain control flexibility and system robustness [10][14][15]. However, the increased number of actuators also increases the chance of a fault, and the fault detection and diagnosis (FDD) is more challenging for over-actuated systems [1]. Some fault-tolerant (FT) control methods have been proposed in the literatures [2][3][4], but most of them were not specifically designed for over-actuated systems. In this study, a passive FT controller for a class of multiple-input-multiple-output (MIMO) over-actuated nonlinear system is presented. The proposed control method has the potential of maintaining system stability and achieving acceptable performance when an actuator fault happens.

Compared to the active FT control, the passive one has the advantage of not requiring the exact actuator fault information which should be given by the FDD function [2]. The passive FT control can also ensure the system stability and desired performance after the fault happens and before the FDD phase ends [3]. Thus, the passive FT controller is important in practical situations. Many passive FT controllers, however, are designed for a certain type of fault [5][6]. As the fault information, such as the fault type, is unknown before the FDD procedure finishes, it may be limited to design a passive FT controller for a specified fault. It is more desirable to design a unified controller which can deal with different kinds of faults. In this study, two types of actuator faults are investigated, the first one is the additive fault and the other one is the loss-of-effectiveness fault. A passive FT controller based on the sliding mode control method which works for both of the fault types is designed.

Due to the redundant actuators, most of the control methods for over-actuated systems use control allocation algorithms to distribute the higher-level control signals to the lower-level actuators [7][10][11]. However, the control allocation algorithms usually require high computational costs, which may discourage the implementations in real-time. In this method, the MIMO over-actuated system is decoupled based on a system transformation method, and the controller design can even be achieved in a single-input single-output (SISO) system fashion. Note that when the fault is estimated, an active FT controller can be adapted or the corresponding weights in the cost function of the original controller can be adjusted to reallocate the control efforts of the actuators. To more clearly show the effectiveness of the proposed FT control method, a vehicle control example of a four wheel independently-actuated (FWIA) electric vehicle is given.

The rest of the paper is organized as follows. The FT control problem of the studied over-actuated nonlinear systems is formulated in section 2. The FT controller is designed and analyzed in section 3. A vehicle control example of a FWIA electric vehicle is introduced in section 4. Simulation results based on a high-fidelity, CarSim®, full-vehicle model are presented in section 5 followed by conclusive remarks.

II. System Modeling and Problem Formulations

A. System Modeling and Transformation

Consider the following over-actuated system

$$\dot{X} = f(X) + BU, \tag{1}$$

where $U = [u_1, u_2, ..., u_n]^T$ is the actual control input vector whose elements correspond to physical actuators, $f(X) = [f_1(X), f_2(X), ..., f_m(X)]^T$, $X = [x_1, x_2, ..., x_m]^T$ is the system state vector, $B \in \mathbb{R}^{m \times n}$ ($m < n$) is the system control effective matrix.

Since $rank(B) = m < n$, there is an invertible matrix $N \in \mathbb{R}^{n \times n}$ such that the following equation holds,

$$B' = BN^{-1}, \tag{2}$$

where $B' = [B_1', B_2', \cdots B_m'] \in \mathbb{R}^{m \times n}$ has a rank m and consists of m column matrices $B_i' \in \mathbb{R}^{m \times m_i}$ $(i = 1, 2, \cdots, m)$ with $rank(B_i')$=1. Based on division of matrix B',

$$\sum_{i=1}^{m} m_i = n. \tag{3}$$

Denote the virtual control as

*Corresponding author. (wang.1862@buckeyemail.osu.edu; wang.1381@osu.edu). This research was partially supported by Office of Naval Research Young Investigator Program (ONR-YIP) Award under Grant N00014-09-1-1018, Honda-OSU Partnership Program, and OSU Transportation Research Endowment Program.

$$V = NU. \tag{4}$$

Based on (2) and (4), we can rewrite (1) as

$$\dot{X} = f(X) + B^{'}V. \tag{5}$$

As $B^{'}$ is divided into *m* column matrices $B_i^{'}$, one can divide $V \in \mathbb{R}^n$ into *m* rows v_i $(i = 1, 2, \cdots, m)$ accordingly. So *V* can be written as

$$V = \begin{bmatrix} v_1 \\ v_2 \\ \vdots \\ v_m \end{bmatrix}, \tag{6}$$

with $v_i = [v_{i\_1}, v_{i\_2}, \cdots, v_{i\_m_i}]^T$ being the *i*th virtual control vector.

Since $rank(B_i^{'})$=1, $B_i^{'}$ can be written as

$$B_i^{'} = [\alpha_{i\_1} b_{i\_1}^{'}, \alpha_{i\_2} b_{i\_1}^{'}, \cdots, \alpha_{i\_m_i} b_{i\_1}^{'}] \tag{7}$$

with $b_{i\_1}^{'}$ being the first column vector of $B_i^{'}$, α_{i_j} being the ratio of $b_{i\_j}^{'}$ to $b_{i\_1}^{'}$. Also denote

$$\beta_{i\_j} = \frac{v_{i\_j}}{v_{i\_1}}. \tag{8}$$

Then the total control effort from the *i*th virtual control v_i is

$$B_i^{'} v_i = \left(\sum_{j=1}^{m_i} \alpha_{i\_j} \beta_{i\_j} \right) b_{i\_1} v_{i\_1}, \tag{9}$$

which means the system (5) can be further written as

$$\dot{X} = f(X) + B^{"} \Phi V^{'}, \tag{10}$$

where $B^{"} = [b_{1\_1}^{'} \quad b_{2\_1}^{'} \quad \cdots \quad b_{m\_1}^{'}]_{m \times m}$, $V^{'} = [v_{1\_1}, v_{2\_1}, \cdots, v_{m\_1}]^T$ with $v_{i\_1} (i = 1, 2, \cdots, m)$ being the first control element of v_i, and

$$\Phi = \begin{bmatrix} \sum_{j=1}^{m_1} \alpha_{1\_j} \beta_{1\_j} & 0 & 0 & 0 \\ 0 & \sum_{j=1}^{m_2} \alpha_{2\_j} \beta_{2\_j} & 0 & 0 \\ 0 & 0 & \ddots & 0 \\ 0 & 0 & 0 & \sum_{j=1}^{m_m} \alpha_{i\_j} \beta_{i\_j} \end{bmatrix}.$$

If β_{i_j} in (8) can be determined, then Φ in (10) will be known, which means that the original over-actuated system (1) can be treated as a system with *m* inputs and *m* outputs Define the cost function for the *i*th control vector v_i as

$$\begin{gathered} J = w_{i\_1} v_{i\_1}^2 + w_{i\_2} v_{i\_2}^2 + \cdots + w_{i\_m_i} v_{i\_m_i}^2 \\ s.t. \left(\sum_{j=1}^{m_i} \alpha_{i\_j} v_{i\_j} \right) b_{i\_1} = C_i \end{gathered}, \tag{11}$$

with w_{i_j} being the weighting factors, C_i being the total control effort from the *i*th virtual control vector. By the Lagrange multiplier method, the above cost function can be minimized if the following holds:

$$v_{i\_j} = \frac{\alpha_{i\_j} \left(\sum_{j=1}^{m_i} \alpha_{i\_j} v_{i\_j} \right)}{w_{i\_j} \sum_{j=1}^{m_i} \frac{\alpha_{i\_j}^2}{w_{i\_j}}}, \tag{12}$$

which means one can write the β_{i_j} in (8) as

$$\beta_{i\_j} = \frac{\alpha_{i\_j} w_{i\_1}}{w_{i\_j}} \cdot \tag{13}$$

In this study, we assume that all the weighting factors are the same, so the Φ in (10) can be written as

$$\Phi = \begin{bmatrix} \sum_{j=1}^{m_1} \alpha_{1\_j}^2 & 0 & 0 & 0 \\ 0 & \sum_{j=1}^{m_2} \alpha_{2\_j}^2 & 0 & 0 \\ 0 & 0 & \ddots & 0 \\ 0 & 0 & 0 & \sum_{j=1}^{m_m} \alpha_{m\_j}^2 \end{bmatrix}.$$

B. *Problem Formulations*

Two types of actuator faults are considered. The first one is the additive fault and the other one is the loss-of-effectiveness fault. Usually, the fault information is unknown before diagnosis. So we aim at designing a unified FT controller which can handle both fault types.

Case 1: Additive fault:

If additive faults happen to the actuators, the control system can be written as

$$\begin{aligned} \dot{X} &= f(X) + B^{'} \left(V + \Delta V(X) \right) \\ &= f(X) + B^{"} \Phi V^{'} + B^{'} \Delta V \\ &= f(X) + B^{"} \Phi \left(V^{'} + \Delta V^{'} \right). \end{aligned} \tag{14}$$

with $\Delta V = N \Delta U(X)$, and

$$\Delta V^{'} = [\Delta v_1^{'}, \Delta v_2^{'}, ..., \Delta v_m^{'}]^T = (B^{"} \Phi)^{-1} B^{'} \Delta V. \tag{15}$$

Note that each element in $\Delta V^{'}$ is bounded as the additive faulty control effort $\Delta U(X)$ is assumed to be bounded.

Case 2: Loss of effectiveness:

If loss-of-effectiveness faults occur to the actuators, the faulty actuators will fail to provide the desired control efforts. Thus, the control system can be written as

$$\begin{aligned} \dot{X} &= f(X) + B^{'} N \Gamma U \\ &= f(X) + B^{'} N \Gamma N^{-1} V \end{aligned} \tag{16}$$

with $\Gamma = diag[\gamma_1, \gamma_2, \cdots, \gamma_n]$. If no such a fault occurs, $\gamma_i = 1$, when a fault happens, we have $0 < \gamma_i < 1$ for the corresponding actuator. As faults happen to the actual actuators, the virtual control signals will be affected. Denote $N \Gamma N^{-1}$ as

$$\Gamma^{'} = N \Gamma N^{-1} = \left[\Gamma_1^{'}, \Gamma_2^{'}, \cdots, \Gamma_m^{'} \right], \tag{17}$$

with $\Gamma_i^{'} = [\Gamma_{i\_1}^{'}, \Gamma_{i\_2}^{'}, \cdots, \Gamma_{i\_m_i}^{'}]$. Thus, we get

$$N\Gamma N^{-1}V=\left[\Gamma_1^{'},\Gamma_2^{'},\cdots,\Gamma_m^{'}\right]\begin{bmatrix}v_1\\v_2\\\vdots\\v_m\end{bmatrix}. \quad (18)$$

From (8), one has $v_i=[\beta_{i\_1}v_{i\_1},\beta_{i\_2}v_{i\_1},\cdots,\beta_{i\_m_i}v_{i\_1}]^T$, which means

$$\Gamma_i^{'}v_i=\left[\Gamma_{i\_1}^{'},\Gamma_{i\_2}^{'},\cdots,\Gamma_{i\_m_i}^{'}\right]\begin{bmatrix}v_{i\_1}\\v_{i\_1}\\\vdots\\v_{i\_m_i}\end{bmatrix} =\left(\sum_{j=1}^{m_i}\beta_{i\_j}\Gamma_{i\_j}^{'}\right)v_{i\_1}. \quad (19)$$

Let $P=\left(\sum_{j=1}^{m_1}\beta_{1\_j}\Gamma_{1\_j}^{'}\ \sum_{j=1}^{m_2}\beta_{2\_j}\Gamma_{2\_j}^{'}\cdots\sum_{j=1}^{m_i}\beta_{m\_j}\Gamma_{m\_j}^{'}\right)$, based on (17) - (19), one has

$$B^{'}PV^{'}=B^{'}N\Gamma N^{-1}V. \quad (20)$$

Denote Ξ as

$$\Xi=\left(B^{''}\Phi\right)^{-1}B^{'}P, \quad (21)$$

Then, based on (20) and (21), one can write the faulty system (16) as

$$\begin{aligned}\dot{X}&=f(X)+B^{'}PV^{'}\\&=f(X)+B^{''}\Phi\Xi V^{'}.\end{aligned} \quad (22)$$

One can see that Ξ defined in (21) represents the effect the actual actuators' faults cause to $V^{'}$.

Rewrite Ξ as

$$\Xi=\Xi^{'}+\left(\Xi-\Xi^{'}\right), \quad (23)$$

with $\Xi^{'}=diag\left[\xi_{11},\xi_{22},\cdots,\xi_{mm}\right]$ being made of the diagonal elements of Ξ. Also denote

$$\Delta V^{'}=(\Xi-\Xi^{'})V^{'}. \quad (24)$$

One can rewrite the faulty system as

$$\dot{X}=f(X)+B^{''}\Phi\Xi V^{'}=f(X)+B^{''}\Phi\left(\Xi^{'}V^{'}+\Delta V^{'}\right). \quad (25)$$

Based on (17) and (21) when there is no fault, $\Xi=I_{m\times m}$ and $\Delta V^{'}=0$. If there is a fault happens, the diagonal element of $\Xi^{'}$ corresponding to the faulty subsystem will change and $\Xi-\Xi^{'}$ will be nonzero as well. Note that as $V^{'}$ is assumed to be finite, the $\Delta V^{'}$ in (24) or (25) can be assumed to be bounded. In the following control design section, a nominal controller is designed for the fault-free system followed by the FT controller design.

III. Controller Designs

A. Nominal Controller Design

There is a matrix $Q\in\mathbb{R}^{m\times m}$ which has the following property,

$$\Lambda=QB^{''}, \quad (26)$$

where $\Lambda=diag[\lambda_1,\lambda_2,\cdots,\lambda_m]$ is a diagonal matrix. Thus, we can redesign the nominal controller in the new states as

$$Z=QX. \quad (27)$$

The system model (14) can be written as

$$\dot{Z}=h(Z)+\Lambda\Phi V^{'}, \quad (28)$$

where $h(Z)=Qf(Q^{-1}Z)$. Denote the reference for the original system as X_r, the reference for the new system is

$$Z_r=QX_r. \quad (29)$$

So the original over-actuated system is decoupled into m SISO systems, and can be written as

$$\begin{cases}\dot{z}_1=h_1(Z)+\lambda_1\phi_1 v_{1\_1}\\\dot{z}_2=h_2(Z)+\lambda_2\phi_2 v_{2\_1}\\\quad\vdots\\\dot{z}_m=h_m(Z)+\lambda_m\phi_m v_{m\_1}\end{cases}, \quad (30)$$

where ϕ_i is the ith diagonal element of Φ. Note that when Z converges to Z_r, X converges to X_r as well. Thus, the control problem can be solved in each of the decoupled channels. That is, design a controller for the following system to make z_i follow the reference z_{ri}.

$$\dot{z}_i=h_i(Z)+\lambda_i\phi_i v_{i\_1}. \quad (31)$$

The following controller

$$v_{oi\_1}=\frac{-h_i(Z)+L_ie_z+\dot{z}_{ri}}{\lambda_i\phi_i}, \quad (32)$$

with L_i being a positive constant can make the tracking error, $e_{zi}=z_{ri}-z_i$, converge to 0 as $t\to\infty$. The actual control can be written as

$$U=N^{-1}V, \quad (33)$$

with each of virtual control signals in V be calculated as

$$v_{oi\_j}=\beta_{i\_j}v_{oi\_1}. \quad (34)$$

B. Passive FT Controller Design

Case 1: Additive fault:

Consider the faulty system as (14) shows, with the help of the matrix Q in (26), we can write the faulty system as

$$\dot{z}_i=h_i(Z)+\lambda_i\phi_i\left(v_{i\_1}+\Delta v_i^{'}\right). \quad (35)$$

Choose a Lyapunov function candidate as

$$V_{zi}=\frac{1}{2}e_{zi}^{\ 2}. \quad (36)$$

The time derivative of V_{zi} can be written as

$$\begin{aligned}\dot{V}_{zi}&=e_{zi}\left(\dot{z}_{ri}-\dot{z}_i\right)\\&=e_{zi}\left(\dot{z}_{ri}-h_i(Z)-\lambda_i\phi_i v_{i\_1}-\lambda_i\phi\Delta v_i^{'}\right)\\&=e_{zi}\left(\dot{z}_{ri}-h_i(Z)-\lambda_i\phi_i v_{oi\_1}+\lambda_i\phi_i\left(v_{oi\_1}-v_{i\_1}+\Delta v_i^{'}\right)\right)\\&=-L_ie_{zi}^2+e_{zi}\lambda_i\phi_i\left(v_{oi\_1}-v_{i\_1}+\Delta v_i^{'}\right).\end{aligned} \quad (37)$$

If we take

$$v_{i\_1}=v_{oi\_1}+K_{a\_i}sign\left(e_{zi}\lambda_i\phi_i\right), \quad (38)$$

where K_{a_i} satisfies

$$K_{a\_i} > |\Delta v_i^{'}|_{\max} \tag{39}$$

with $|\Delta v_i^{'}|_{\max}$ being the upper boundary of $|\Delta v_i^{'}|$, we have $\dot{V}_{zi} \le -L_i e_{zi}^2 - K_{a\_i}\left|\lambda_i\phi_i\right| e_{zi}^2 \le 0$, which means that z_i will converge to z_{ri}. Similarly, FT controllers for other channels can be designed.

Case 2: Loss-of-effectiveness fault:

With the matrix Q, one can write the faulty system (25) as

$$\dot{z}_i = h_i(Z) + \lambda_i\phi_i\xi_{ii}v_{i\_1} + \lambda_i\phi_i\Delta v_i^{'}. \tag{40}$$

Also choose a Lyapunov function candidate as (36) shows, the time derivative of the Lyapunov function can be written as

$$\begin{aligned} \dot{V}_{zi} &= e_{zi}\left(\dot{z}_{ri} - \dot{z}_i\right) \\ &= e_{zi}\left(\dot{z}_{ri} - h_i - \lambda_i\phi_i\xi_{ii}v_{i\_1} - \lambda_i\phi\Delta v_i^{'}\right) \\ &= e_{zi}\left(\dot{z}_{ri} - h_i - \lambda_i\phi_i v_{oi\_1} + \lambda_i\phi_i\left(v_{oi\_1} - \xi_{ii}v_{i\_1} + \Delta v_i^{'}\right)\right) \\ &= -L_i e_{zi}^2 + e_{zi}\lambda_i\phi_i\left(v_{oi\_1} - \xi_{ii}v_{i\_1} + \Delta v_i^{'}\right). \end{aligned} \tag{41}$$

Suppose that the controller is still written as (38) shows, then the above time derivative of the Lyapunov function can be further written as

$$\begin{aligned} \dot{V}_{zi} &\le e_{zi}\lambda_i\phi_i\left(v_{oi\_1} - \xi_{ii}v_{i\_1} + \Delta v_i^{'}\right) \\ &\le e_{zi}\lambda_i\phi_i\left(v_{oi\_1}\left(1-\xi_{ii}\right) - \xi_{ii}K_{l\_i}sign\left(e_{zi}\lambda_i\phi_i\right) + \Delta v_i^{'}\right) \\ &\le \xi_{ii}e_{zi}\lambda_i\phi_i\left(\tfrac{v_{i0\_1}+\Delta v_i^{'}}{\xi_{ii}} - v_{oi\_1} - K_{l\_i}sign\left(e_{zi}\lambda_i\phi_i\right)\right). \end{aligned} \tag{42}$$

If K_{l_i} can be chosen such that

$$K_{l\_i} > \tfrac{|v_{oi\_1}|}{\xi_{ii\_\min}} + \tfrac{|\Delta v_i^{'}|_{\max}}{\xi_{ii\_\min}} + \left|v_{oi\_1}\right| \tag{43}$$

with $\xi_{ii\_\min}$ being the lower boundary of ξ_{ii}, we have

$$sign\left(e_{zi}\lambda_i\phi_i\right) = -sign\left(\tfrac{v_{oi\_1}+\Delta v_i^{'}}{\xi_{ii}} - v_{oi\_1} - K_i sign\left(e_{zi}\lambda_i\phi_i\right)\right), \tag{44}$$

which means $\dot{V}_{zi} \le 0$, and the error will converge to zero.

Remark 1: Modeling error of the system can also be handled with the controller as (38) shows. For the additive fault, if we write the system as

$$\dot{z}_i = h_i(Z) + \Delta h_i(Z) + \lambda_i\phi_i\left(v_{i\_1} + \Delta v_i^{'}\right), \tag{45}$$

where $\Delta h_i(Z)$ is the modeling error. Then the time derivative of the Lyapunov function defined by (36) can be written as

$$\begin{aligned} \dot{V}_{zi} &= e_{zi}\left(\dot{z}_{ri} - h_i(Z) - \Delta h_i(Z) - \lambda_i\phi_i v_{i\_1} - \lambda_i\phi_i\Delta v_i^{'}\right) \\ &= -L_i e_{zi}^2 + e_{zi}\lambda_i\phi_i\left(v_{oi\_1} - v_{i\_1} + \Delta v_i^{'} - \tfrac{\Delta h_i(Z)}{\lambda_i\phi_i}\right). \end{aligned} \tag{46}$$

The control law is still as (38) shows, if K_{a_i} satisfies $K_{a\_i} > |\Delta v_i^{'}|_{\max} + \frac{|\Delta h_i|_{\max}}{|\lambda_i\phi_i|}$ with $|\Delta h_i| \le |\Delta h_i|_{\max}$, $\dot{V}_{zi} \le 0$ still holds. Similarly, for the loss-of-effectiveness fault, the K_{l_i} should satisfy $K_{l\_i} > \frac{|v_{oi\_1}|}{\xi_{ii\_\min}} + \frac{|\Delta v_i^{'}|_{\max}}{\xi_{ii\_\min}} + \left|v_{oi\_1}\right| + \frac{|\Delta h_i|_{\max}}{\xi_{ii\_\min}|\lambda_i\phi_i|}$ to make $\dot{V}_{zi} \le 0$ hold.

Remark 2: As the two FT controllers in the two cases can be written in the same form, the two types of actuator faults can be handled by a uniformed controller which is shown as

$$v_{i\_1} = v_{oi\_1} + K_i sign\left(e_{zi}\lambda_i\phi_i\right), \tag{47}$$

with $K_i = \max\left(K_{a\_i}, K_{l\_i}\right)$.

Remark 3: In order to eliminate the chatting effect caused by the sign function, the sign function can be replaced with a saturation function as

$$sat\left(e_{zi}\lambda_i\phi_i\right) = \begin{cases} \lambda_i\phi_i e_{zi}/\varepsilon_i & if\ |\lambda_i\phi_i e_{zi}| < \varepsilon_i \\ sign\left(e_{zi}\lambda_i\phi_i\right) & if\ |\lambda_i\phi_i e_{zi}| \ge \varepsilon_i \end{cases} \tag{48}$$

where ε_i is the thickness of the saturation function. Note that some hysteresis-based solutions may also be adopted to reduce the chatting effect [8].

IV. A Vehicle Control Example

A. Vehicle Modeling

FWIA electric vehicle is a typical over-actuated system [7]. A FWIA vehicle employs four in-wheel (or hub) motors to drive the four wheels, and the torque and driving/braking mode of each wheel can be controlled independently. A schematic diagram of the vehicle model is shown in Figure 1.

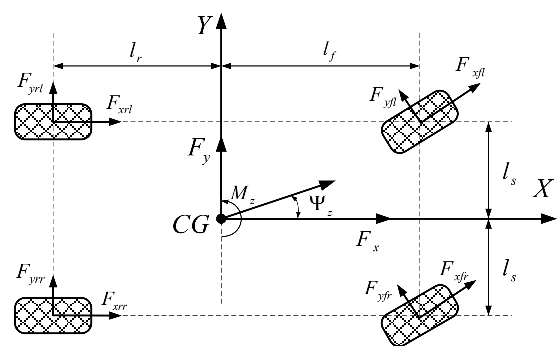

Figure 1. Schematic diagram of a vehicle model.

If the vehicle longitudinal speed and yaw rate are controlled to follow the references, the vehicle model can be written as

$$\begin{bmatrix} \dot{V}_x \\ \dot{\Omega}_z \end{bmatrix} = \begin{bmatrix} f_1(X) \\ f_3(X) \end{bmatrix} + \begin{bmatrix} \frac{1}{M} & \frac{\cos\sigma}{M} & \frac{1}{M} & \frac{\cos\sigma}{M} \\ \frac{-l_s}{I_z} & \frac{l_f\sin\sigma - l_s\cos\sigma}{I_z} & \frac{l_s}{I_z} & \frac{l_f\sin\sigma + l_s\cos\sigma}{I_z} \end{bmatrix} T, \tag{49}$$

where σ is the front wheel steering angle, V_x is the vehicle longitudinal speed, Ω_z is the yaw rate, M and I_z are the vehicle mass and yaw inertia, respectively. $T = \left[T_{rl}\ T_{fl}\ T_{rr}\ T_{fr}\right]^T$ are the motor torques. Please refer to [10] and [12] for the details of $f_1(X)$ and $f_3(X)$. Note that the in-wheel motor torque is directly related to the motor control gain [12]. Thus when a fault happens, the corresponding motor will fail to provide the desired torque.

One can see, from (49), that the two wheels on the same side of the vehicle have same effects on the vehicle longitudinal and yaw motions. So we put T_{fl} and T_{rl}, T_{fr} and T_{rr} into two subspaces, respectively. That is, the (49) can be rewritten as

$$\begin{bmatrix}\dot{V}_x \\ \dot{\Omega}_z\end{bmatrix}=\begin{bmatrix}f_1(X) \\ f_3(X)\end{bmatrix}+\begin{bmatrix}\frac{1}{M} & \frac{\alpha_1}{M} & \frac{1}{M} & \frac{\alpha_2}{M} \\ -\frac{l_s}{I_z} & -\frac{l_s\alpha_1}{I_z} & \frac{l_s}{I_z} & \frac{l_s\alpha_2}{I_z}\end{bmatrix}T_v, \tag{50}$$

where

$$T_v=[T_{v\_rl}\ T_{v\_fl}\ T_{v\_rr}\ T_{v\_fr}]^T=NT, \tag{51}$$

$$\begin{cases}\alpha_1=(r-s)^2+s^2 \\ \alpha_2=(r+s)^2+s^2\end{cases}, \tag{52}$$

$$N=\begin{bmatrix}1 & 0 & 0 & 0 \\ 0 & \frac{r-s}{\alpha_1} & 0 & \frac{-s}{\alpha_2} \\ 0 & 0 & 1 & 0 \\ 0 & \frac{s}{\alpha_1} & 0 & \frac{r+s}{\alpha_2}\end{bmatrix}. \tag{53}$$

with $r=\cos\sigma, s=\frac{l_f\sin\sigma}{2l_s}$. Based on the analysis from (2)-(13), one has

$$\begin{bmatrix}\dot{V}_x \\ \dot{\Omega}_z\end{bmatrix}=\begin{bmatrix}f_1(X) \\ f_3(X)\end{bmatrix}+B''\Phi T_v', \tag{54}$$

with $B''=\begin{bmatrix}\frac{1}{M} & \frac{1}{M} \\ -\frac{l_s}{I_z} & \frac{l_s}{I_z}\end{bmatrix}, \Phi=\begin{bmatrix}1+\alpha_1^2 & 0 \\ 0 & 1+\alpha_2^2\end{bmatrix}$, and $T_v'=[T_{v\_rl}\ T_{v\_rr}]^T$.

B. Controllers Design

If we take B''^{-1} as Q in (54), then $\Lambda=I_{2\times 2}$, and the nominal vehicle model can be written as

$$\dot{Z}=h(Z)+\Phi T_v', \tag{55}$$

with $Z=Q[V_x,\Omega_z]^T$, $h(Z)=Q[f_1(Q^{-1}Z), f_3(Q^{-1}Z)]^T$.

According to (32), the nominal virtual motor torque can be written as

$$T_{ov}'=\Phi^{-1}\left(-h(Z)+Le_z+\dot{Z}_r\right). \tag{56}$$

For the faulty vehicle with additive faults, according to (38) the virtual motor control signal can be written as

$$T_v'=T_{ov}'+K_a sat\left(\Phi e_z\right), \tag{57}$$

where $K_a=[K_{a\_l}\ K_{a\_r}]^T$ with K_{a_i} satisfying ,

$$K_{a\_i}>|\Delta T_i|_{\max}. \tag{58}$$

For the loss-of-effectiveness fault, we have the virtual FT control signal as

$$T_v'=T_{ov}'+K_l sat\left(\Phi e_z\right), \tag{59}$$

where $K_l=[K_{l\_l}\ K_{l\_r}]^T$, with

$$K_{l\_i}>\frac{|T_{oi}|}{\xi_{ii\_\min}}+\frac{|\Delta T_i|_{\max}}{\xi_{ii\_\min}}+\left|T_{oi}'\right|. \tag{60}$$

For the unified FT controller, one needs to choose a sufficiently large K_i to make $K_i=\max(K_{a\_i}, K_{l\_i})$ hold. The actual motor torque is thus written as

$$T=N^{-1}T_v=N^{-1}\left[T_{v\_rl}\quad \alpha_1 T_{v\_rl}\quad T_{v\_rr}\quad \alpha_2 T_{v\_rr}\right]^T. \tag{61}$$

In our previous tests, the in-wheel motor torque was found to be proportional to the motor control signal [13]. Thus the motor control signal can be written as $u_i=T_i/k_i$, with k_i being the motor control gain. Note that each of the fault types can be simulated by manipulating the motor torque control signals.

V. Simulation Studies

Two simulation cases based on a high-fidelity, full-vehicle model constructed in CarSim® were conducted. The vehicle parameters in the simulations were taken from an actual prototyping FWIA electric vehicle with in-wheel motors developed in the authors' group at The Ohio State University [12].

A. Acceleration

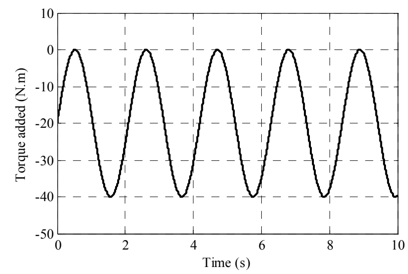

Figure 2. Torque added to the faulty wheel in the straight line acceleration simulation.

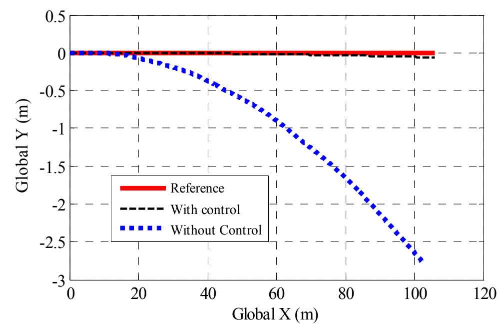

Figure 3. Vehicle trajectories in the straight-line acceleration simulation.

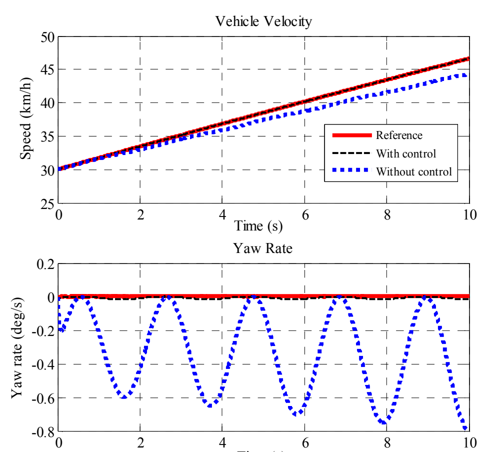

Figure 4. Vehicle speed and yaw rate in the straight-line acceleration simulation.

In this simulation, the desired vehicle speed was accelerated from 30km/h to 47km/h in 10 seconds. An additive fault was introduced to the rear-right in-wheel motor. The added torque is as Figure 2 shows. The vehicle global trajectories are compared in Figure 3. To better show the effectiveness of the proposed controller, the performance of an uncontrolled vehicle which ran on the same road was compared. It can be seen that the proposed control system could control the vehicle well, while the uncontrolled vehicle failed to follow the references as faulty wheel failed to provide the required torque. The vehicle yaw rates and speeds are shown in Figure 4. One can see that the controlled vehicle could follow the reference very well, while the states of the uncontrolled vehicle were significantly affected by the actuator fault.

B. *J-turn simulation*

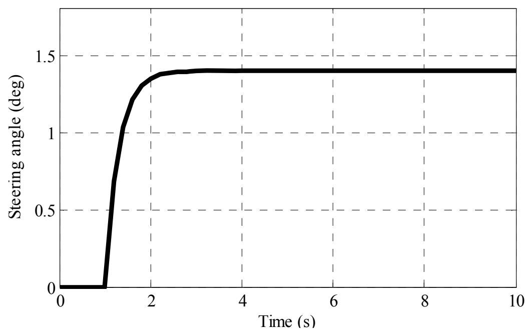

Figure 5. Front wheel steering angle J-turn simulation.

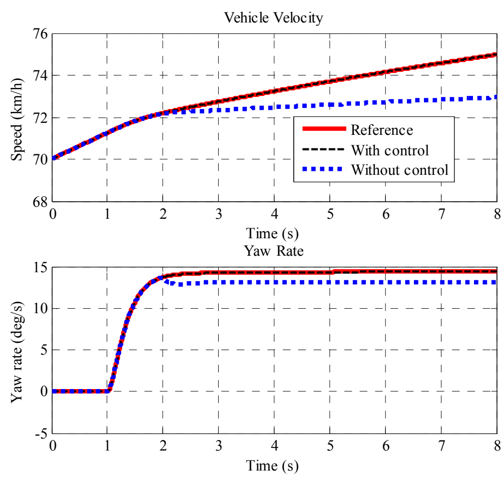

Figure 6. Yaw rates in the J-turn simulation.

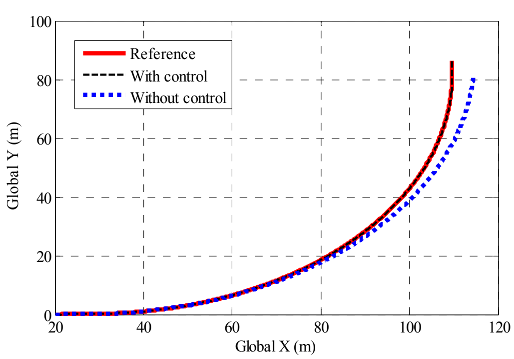

Figure 7. Vehicle trajectories in the J-turn simulation.

In this simulation, a counter-clockwise turn was introduced at 1s. The front wheel steering angle is as Figure 5 shows. A loss-of-effectiveness fault was introduced. The fault decreased the rear-right in-wheel motor control gain to 0.4 times of its nominal value at 2s. The vehicle yaw rates and speeds are compared in Figure 6. One can see that the controlled vehicle followed the references well, while the yaw rate and speed of the uncontrolled vehicle diverted from the references very fast. The vehicle trajectories are shown in Figure 7, where we can see again that the proposed controller works well.

VI. Conclusions

A passive FT controller for a class of over-actuated system is proposed. A passive FT controller which works for both additive and the loss-of-effectiveness actuator fault types is designed. A vehicle control example of a FWIA electric ground vehicle case is given to show the effectiveness of the proposed control method. Simulations using a high-fidelity, CarSim®, full-vehicle model in different scenarios show the effectiveness of the control approach.

References

[1] Nader Meskin and K. Khorasani. “Fault Detection and Isolation of Actuator Faults in Overactuated Systems,” Proceedings of the 2007 American Control Conference, pp. 2527 – 2532, July. 2007.

[2] Youmin Zhang and Jin Jiang, “Bibliographical review on reconfigurable fault-tolerant control systems,” Annual Reviews in Control, Vol.32, pp: 229–252. 2008.

[3] Mouhacine Benosman, “A Survey of Some Recent Results on Nonlinear Fault Tolerant Control,” Mathematical Problems in Engineering, pp: 1-25, 2010.

[4] Xiaodong Zhang, Thomas Parisini, and Marios M. Polycarpou,, “Adaptive fault-tolerant control of nonlinear uncertain systems: an information-based diagnostic approach,” IEEE Transactions on Automatic Control, vol. 49, no. 8, pp. 1259–1274, 2004.

[5] M. Benosman and K.Y. Lum. “Application of passivity and cascade structure to robust control against loss of actuator effectiveness,” International Journal of Robust Nonlinear Control, Vol. 20, pp: 673–693. 2010.

[6] M. Benosman and K.Y. Lum, “Passive Actuators’ Fault-Tolerant Control for Affine Nonlinear Systems,” IEEE Transaction on Control System Technology, Vol. 18, No. 1, pp: 152-163, 2010

[7] J. Wang and R. G. Longoria, “Coordinated and Reconfigurable Vehicle Dynamics Control,” IEEE Transactions on Control Systems Technology, Vol. 17, No. 3, pp. 723 – 732, May 2009.

[8] R. Gupta and A. Ghosh, “Frequency-Domain Characterization of Sliding Mode Control of an Inverter Used in DSTATCOM Application,” IEEE Transaction on Circuits and Systems, VOL. 53, NO. 3, pp: 662-676, March 2006.

[9] J. Tjønnås and T. A. Johansen, “Stabilization of Automotive Vehicles Using Active Steering and Adaptive Brake Control Allocation”, IEEE Transactions on Control Systems Technology, Vol. 18, No.3, pp.545-558, May 2010.

[10] R. Wang and J. Wang, "Fault-Tolerant Control for Electric Ground Vehicles with Independently-Actuated In-Wheel Motors," ASME Transactions Journal of Dynamic Systems, Measurement, and Control (in press), 2011.

[11] H. Alwia and C. Edwards, “Fault tolerant control using sliding modes with on-line control allocation,” Automatica, Vol. 44, No 7, Pages 1859-1866, 2008.

[12] Rongrong Wang and Junmin Wang, "Stability Control of Electric Vehicles with Four Independently Actuated Wheels," Proceedings of the 50th IEEE Conference on Decision and Control, 2011 (accepted).

[13] R. Wang, Y, Chen, D. Feng, X. Huang, and J. Wang, "Development and Performance Characterization of an Electric Ground Vehicle with Independently Actuated In-Wheel Motors," Journal of Power Sources, Vol. 196, No. 8, pp.3962–3971, April 2011.

[14] Yan Chen and Junmin Wang, "Fast and Global Optimal Energy-Efficient Control Allocation with Applications to Over-Actuated Electric Ground Vehicles," IEEE Transactions on Control Systems Technology (in press), 2011 (DOI: 10.1109/TCST.2011.2161989).

[15] Y. Chen and J. Wang, "Adaptive Vehicle Speed Control with Input Injections for Longitudinal Motion Independent Road Frictional Condition Estimation," IEEE Transactions on Vehicular Technology, Vol. 60, No. 3, pp. 839 - 848, 2011.