2011 50th IEEE Conference on Decision and Control and
European Control Conference (CDC-ECC)
Orlando, FL, USA, December 12-15, 2011

Distributed Extremum Seeking and Cooperative Control for Mobile Communication

Chaoyong Li, Zhihua Qu, and Mary Ann Ingram

Abstract— **In this paper, the integrated control and optimization problem of mobile cooperative communication clusters is considered. Each communication channel is modeled by its Shannon capacity outage probability. Hence, connectivity is maintained if the outage probability is less than a certain threshold. The objective of the communication network is to not only maintain communication quality but also extend the coverage. An information theory based performance index is defined to quantify this control objective. Unlike most of the existing results, the proposed cooperative control design does not assume the knowledge of any gradient (of the performance index). Rather, a distributed extremum seeking cooperative control is designed to optimize the connectivity and coverage of each of mobile communication nodes by feeding back only the motion information of its neighboring nodes and by measuring the current performance of its communication channel(s). The proposed approach retains all the advantages of cooperative control (such as requiring only local feedback and tolerating switching topologies) and searches autonomously for the optimal spacings based on typical communication models. Simulation results demonstrate effectiveness of the proposed methodology.**

I. INTRODUCTION

Cooperative control of networked systems is a distributive strategy utilizing contemporary advances in communication of wireless ad hoc network. Cooperative control has received significant amount of interests in the past decades, leading to breakthroughs in many applications, such as formation control [1], [2], attitude synchronization [3], [4], [5], and most recently smart grid [6]. However, most of the existing work on cooperative control generally assumes an ideal communication environment or does not fully consider the communication quality of the resulting configuration. In this paper, we propose a distributed strategy integrating cooperative control with a communication performance metric and extremum-seeking algorithm, such that trade off between control and communication can be achieved. Note that such design leads to an inherent dilemma, communication quality and network connectivity favors moving agents closer together, while formation task (such as area coverage, searching and patrolling) demands agents separated further apart in order to better meet mission statement. In other words, each agent needs to make decision, preferably distributively, to balance communication quality, connectivity, and formation task, such that the overall performance can be optimized, which is of interest in this paper.

Indeed, formation control has been vigorously investigated in the literature [7][8]. The consensus is applying potential field function or its variations to achieve desired formation as well as collision avoidance [9]. In addition, coverage control can also be treated as a special case of formation control, except it is designed to maximize the coverage of sensor network, or to adequately cover a specific area. The most generally adopted treatments to this problem are also based on potential field function [10][11] and voronoi algorithm [12].

Mobile platforms with wireless communication capabilities can often be used as robotic routers to provide and maintain connectivity of the network. To achieve this, modeling of quality of service of communication channel is expected. For instance, metrics of communication quality such as signal-to-noise ratio (SNR) or Shannon capacity [13] are measured online so that the current formation configuration (or relative distances) can be compared to the desired ones. As such, formation control can be accomplished by using communication quality as feedback instead of position information. Moreover, as shown in [14], the quality of a wireless communication link in a vehicular ad hoc network can be estimated by examining the received data packets. In [15], motion control of networked robotic routers is investigated to maintain connectivity of a single user to a base station, which could be either stationary or adversarial. Recent results related to this topic include motion planning and gradient-based control of a robotic sensing network [16] to improve communication quality, optimization of SISO (i.e., single-input-single-output) communication chain under the assumption that the gradient of SNR field is known [17], and an online planing method is introduced in [18] to find a navigation path to meet network connectivity and bandwidth requirements, an opportunistic communication strategy with energy constraint can be found in [19]. However, there are the following key shortcomings in most of the existing studies: absence of an analytical investigation of integrating communication and control issues in mobile communication systems, requirement of online extremum seeking algorithm with the knowledge of gradients.

In this paper, we propose a distributive framework that integrates cooperative control with a communication performance metric. Specifically, the proposed cooperative control scheme is effective and contains no potential field function related terms. Moreover, a uniform performance index capturing the tradeoff between quality of service of communica-

Chaoyong Li and Zhihua Qu are with Dept. Electrical Engineering and Computer Science, University of Central Florida, Orlando, FL 32816, USA. Phone (407) 823-5976. Fax (407) 823-5835 {Qu,Cyli}@mail.ucf.edu

Mary Ann Ingram is with School of Electrical and Computer Engineering, Georgia Institute of Technology, Atlanta, Georgia 30332, USA. Mai@gatech.edu

978-1-61284-799-3/11/$26.00 ©2011 IEEE

tion and network coverage is introduced, although its exact value is not known explicitly at each agent, its maximum condition can be estimated online at each communication link using an adaptive and model-free extremum seeking control scheme, whose effectiveness has been verified in various applications, including anti-lock braking [20], flow control[21], formation flight [22], and communication enhancement [23].

II. Problem Formulation

A. Cooperative formation control

Consider a cluster of n networked mobile agents whose dynamics are described by, for the ith agent

$$\dot{x}_i = u_i, \tag{1}$$

where $x_i \in \Re^m$ is the coordinates of the ith agent, $u_i \in \Re^m$ is the control to be designed.

Connectivity among the group of the agents is described by a piecewise-constant binary matrix $S(t)$. Specifically, there is a time sequence $\{t_k : k \in \aleph\}$ such that $S(t) = S(t_k)$ for all $t \in [t_k, t_{k+1})$, where $\aleph = \{0, 1, ..., \infty\}$,

$$S(t_k) = \begin{bmatrix} 1 & s_{12}(t_k) & \dots & s_{1n}(t_k) \\ s_{21}(t_k) & 1 & \dots & s_{2n}(t_k) \\ \vdots & \vdots & \ddots & \vdots \\ s_{n1}(t_k) & s_{n2}(t_k) & \dots & 1 \end{bmatrix}, \tag{2}$$

$s_{ij}(t) = 1$ if information of $x_j(t)$ is received by the ith agent, and $s_{ij}(t) = 0$ if otherwise. Extensions to high-order linear systems and nonlinear systems can be found in [2], [24] and references therein.

In order to maintain a specified formation among networked agents (1), the linear cooperative control input for agent i can be designed as follows

$$u_i = \mu \sum_{j \in \mathcal{N}_i} \frac{s_{ij}\alpha_{ij}}{\sum_{\ell=1}^{n} s_{i\ell}\alpha_{i\ell}} (x_j - x_i - p_{ij}) \triangleq \mu \sum_{j \in \mathcal{N}_i} d_{ij}(x_j - x_i - p_{ij}) \tag{3}$$

where $\mu \geq 1$ is the control gain, $\mathcal{N}_i$ is the neighboring set for agent i, α_{ij} are piecewise-constant gains as specified in [25], $D = [d_{ij}]$ is nonnegative row-stochastic matrix, and $p_{ij} \triangleq r_{ij}^* e_{ij}$ is the desired separation vector between agents j and i in the inertial frame with r_{ij}^* as the desired distance, e_{ij} is unit directional vector with $||e_{ij}|| = 1$.

In addition, if the networked agents are slow evolving and every agent knows its destination, the overall closed-loop system can be written as

$$\dot{\bar{x}} = \mu \left[(-I_n + D) \otimes I_m \right] \bar{x} \tag{4}$$

where I_ℓ is ℓ-dimension identity matrix, $\otimes$ is Kronecker product, and $\bar{x} = [\bar{x}_1^T \ \bar{x}_2^T \ \dots \ \bar{x}_n^T]^T \in \Re^{mn}$ with $\bar{x}_i = x_i - p_i$ while $p_{ij} = p_i - p_j$.

For notational simplicity, $m = 1$ is set hereafter since $m > 1$ can be handled by analogously proceeding with the technical development in the presence of Kronecker product. It should be pointed out that the description of $\mathcal{N}_i$ varies in different scenarios or mission statement. For instance, $\mathcal{N}_i$ shall consist of the most closest neighbors if a fast convergence is preferred. However, the desired coordination p_i is often time-varying and impossible to be known locally, while the desired separation r_{ij}^* is a better description but needs to be determined online. Hence, rather than using potential field function to achieve desired formation configuration, we consider a design of finding a distributive strategy such that one particular separation r_{ij}^* is achieved over time for all $j \neq i$ and $j \in \mathcal{N}_i$. That is, $e_{ij} = (x_j - x_i)/r_{ij}$ with $r_{ij} \triangleq ||x_i - x_j||$. As such, for agent i, (1) becomes

$$\dot{x}_i = \mu \sum_{j \in \mathcal{N}_i} d_{ij}(1 - r_{ij}^*/r_{ij})(x_j - x_i) \tag{5}$$

It is apparent that input (3) becomes an attractive force if $r_{ij} > r_{ij}^*$, system i moves in favor of eliminating the separation between system j, while (3) becomes repulsive once $r_{ij} < r_{ij}^*$, driving system i away from system j. Hence, the desired separation r_{ij}^* for all $j \in \mathcal{N}_i$ can be ensured asymptotically and distributively provided matrix $S(t)$ or the corresponding graph is cumulatively connected [2]. Indeed, (3) is immune to local minimum problem associated with potential field function. However, local minimum problem may still persist because of communication shadowing, where the short-term average received power is not inversely proportional to distance. Note that communication shadowing will not be addressed further in this paper, for the sake of brevity.

Furthermore, since the closed-loop system (4) is $\infty-$norm preserving and Lyapunov stable [2], the connectedness of network connectivity will be preserved. In other words, if there is no topological changes during time interval $[t_k, \ t_{k+1})$, $S(t)$ remains connected for any $t \in (t_k, \ t_{k+1})$ provided $S(t_k)$ is connected. Therefore, the remaining challenge for cooperative formation control problem is how to find r_{ij}^*, preferably distributively, for any $j \in \mathcal{N}_i$, such that the networked systems (1) with input (3) can be coordinated accordingly and the desired formation mission (i.e., area coverage, surveillance, patrolling, etc.) can be guaranteed.

Remark 1: From implementation point of view, the medium access control (i.e., MAC) layer of the communication network dictates how multiple agents within interference range of each other can access the network. The amount of packet congestion depends on the density of agents, the communication and interference ranges of each agent, the traffic generated by each agent, and the maximum number of retransmissions allowed before dropping a packet. The first two of these are reflected in the connectivity matrix $S(t)$, where any pair of agents with acceptable interference range and reliable link quality are considered as connected (i.e., $s_{ij} = 1$). In addition, these effects can be represented as outage probability or packet loss, latency and jitter. For example, the maximum latency in a two-hop network of 6 nodes is 60 ms with the WirelessHart Standard, with multiple retries [26]. And, latency and jitter of an 802.11 (WiFi) single-hop ad hoc network is up to 10ms and 13 ms [27], respectively. Both of these can be tolerated by the proposed

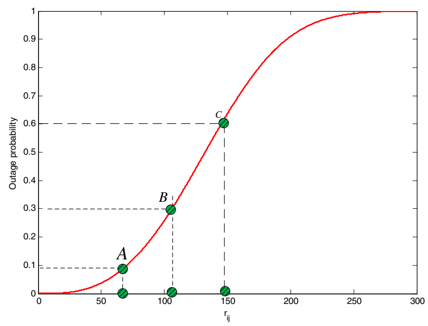


Fig. 1: Outage probability $P[C_{SISO} < \delta]$ with $\delta = 2, \nu = 3, r_0 = 1$, $P_0/\sigma^2 = 10^7$

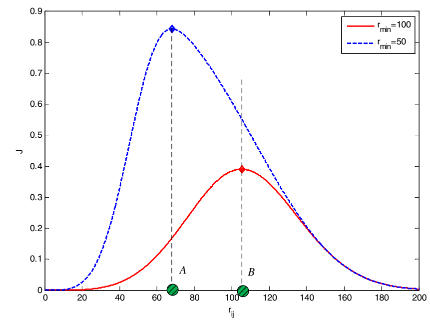


Fig. 2: Performance index J with $r_x = 150, \upsilon = 4$

control.

B. SISO Communication of networked agents

In a mobile ad hoc network, all the systems/agents are moving according to (3), and connectivity is thus changing intermittently. In addition, the communication/broadcasting range of each single agent is uniformly limited, the effective quality of service of communication (measured at the physical layer by packet error rate, or outage rate) depends on many unknown parameters beyond relative position such as multipath fading, shadowing, noise, and interference. As is well known, signal power generally decays with increasing distance, and it is known that the Shannon-Hartley law [13] captures the relationship between distance and communication quality when combined with the empirical radio propagation model. In this paper, we are concerned with preserving the data rate, δ, while extending range r_{ij}, so we consider the capacity outage probability, which, for the SISO link, can be defined as [28]

$$P\left[C_{SISO} < \delta\right] = \exp\left(-(2^{\delta}-1)\frac{\sigma^2}{P_0}\left(\frac{r_{ij}}{r_0}\right)^{\nu}\right) \qquad (6)$$

where C_{SISO} is Shannon capacity, P_0 is the transmitting power, σ is the noise variance, P_0/σ^2 denotes SNR at a reference distance, ν is path loss exponent, r_0 is the reference distance from the transmitter to the receiver, and r_{ij} is effective range.

The outage probability (6) can be used to illustrate the proposed idea of improving communication through local-feedback cooperative control, and it has the intuitive behavior that, as r_{ij} grows, $P\left[C_{SISO} < \delta\right] \to 1$ and, as $r_{ij} \to 0$, $P[C_{SISO} < \delta] \to 0$. Figure 1 shows a typical signal reception outage with respect to r_{ij}. We may wish to impose a design constraint that the outage probability to always be less than $\zeta\%$. This leads to the following inequality constraint:

$$\exp\left(-(2^{\delta}-1)\frac{\sigma^2}{P_0}\left(\frac{r_{ij}}{r_0}\right)^{\nu}\right) \leq \frac{\zeta}{100} \qquad (7)$$

Solving (7) for r_{ij} yields the maximum distance between transmitter and receiver that will give the worst outage probability of $\zeta\%$; call that range r_x. For instance, it is shown in figure 1 outage probability being 9% (i.e., A) or 30% (i.e., B) corresponds to $r_{ij} = 68$ or $r_{ij} = 105$ respectively, and an outage probability of no worse than 60% yields $r_x \leq 150$. However, for communication coverage and vehicle safety, the relative distance should be extended to the maximum possible. To achieve a trade-off between communication quality and network coverage while satisfying (7), we propose the following performance index for any communication channel between agents i and j:

$$J(r_{ij}) = \left[1-\exp\left(-\left(\frac{r_{ij}}{r_{\min}}\right)^{\upsilon}\right)\right]\cdot\exp\left(-\left(\frac{r_{ij}}{r_x}\right)^{\upsilon}\right)$$
$$\cdot\exp\left(-(2^{\delta}-1)\frac{\sigma^2}{P_0}\left(\frac{r_{ij}}{r_0}\right)^{\nu}\right) \quad \forall j \in \mathcal{N}_i \qquad (8)$$

where $r_{\min}$ is the minimum spacing preferred, r_x is the (maximum) spacing that renders the worst tolerable outage probability, and υ is a tuning parameter.

The first multiplicative term in (8) represents a "proximity penalty" that encourages vehicles to separate out to their maximum reliable communication range, while the second multiplicative term ensures (7) is met by making $r_{\min} \leq r_{ij} \leq r_x$, the third multiplicative term penalizes the outage probability.

Performance index $J(r_{ij})$ and its maximum corresponding to $r_x = 150$ and $r_{\min} = 50,\ 100$ are shown in figure 2, their corresponding outage probabilities are marked as A and B in figure 1. It is clear the case with $r_{\min} = 50$ corresponds to a better quality of service (i.e., outage probability 9%), while the case with $r_{\min} = 100$ achieves better network coverage (i.e., $r_{ij}^* = 105$). Accordingly, cooperative control (3) shall be integrated with and assisted by an algorithm of searching for optimal solution r_{ij}^* with respect to $J(r_{ij})$, then consequently arrive at the desired p_{ij}. Once successful, the tradeoff between network coverage and communication quality is ensured.

Remark 2: It should be pointed out that the exact value of $J(r_{ij})$ is not known locally since outage probability can only be measured. In WiFi, for instance, the number of information bits per packet are constantly iterated based on whether the last packet was received correctly or not. Therefore, the probability that the link data rate drops below a certain threshold (that would be an approximation to (6)) could be estimated by observing how often the number of information bits per packet drops below a certain threshold.

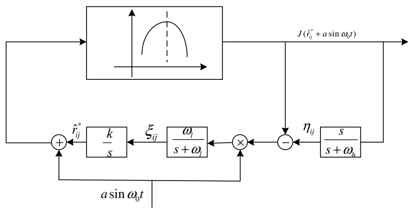


Fig. 3: Diagram of extremum seeking

Specifically, the frequency of samples could be arbitrarily fast with software defined radios for an 802.11-based standard. Then, we could purposefully move the agent around in a small local area, just to sound the channel. In general, the frequency of samples depends on the speed of the vehicle and the carrier frequency, which means the sample period needs to correspond to a distance traveled of at least $1/2$ wavelength. At 2.4 GHz, a half wavelength is 6 cm. With both agents moving at the same speed, then each agent needs to move only $1/4$ wavelength. Since the dynamics model is assumed to be single-order integrator, no limitation imposed on the speed. Hence, the sample period can be made in a scale of ms, outage probability or packet error rate can then be estimated with acceptable accuracy.

III. Extremum Seeking for Cooperative Control

Since neither the exact value of outage probability (6) nor its gradients is known locally, the only available information about communication quality is the measurement of outage probability of each link. Consequently, the maximum condition of performance index $J(r_{ij})$ (8) can not be derived by classical optimization technique, this calls for a model-free scheme to search for r_{ij}^* such that $J(r_{ij}^*) = \max_{j\in\mathscr{N}_i} J(r_{ij})$. Hence, the extremum seeking control fits intuitively into this framework [29]. Its application to the underlying cooperative control problem, however, could be made simpler since (5) is in general a linear system and the performance index $J(r_{ij})$ is independent to the state. Moreover, since the desired outage probability should be ensured at every communication channel, it follows extremum seeking control shall be implemented at each channel, such that the desired separation r_{ij}^* can be determined distributively for all $j \in \mathscr{N}_i$.

As shown in figure 2, there exists $r_{ij} = r_{ij}^*$ such that $J(r_{ij}^*) = \max_{j\in\mathscr{N}_i} J(r_{ij})$, which implies

$$J'(r_{ij}^*) = 0, \quad J''(r_{ij}^*) < 0 \tag{9}$$

Therefore, extremum seeking can be applied to estimate r_{ij}^* [29]. Specifically, without loss of any generality, taking the communication channel between agents i and j for instance, the diagram of extremum seeking is illustrated in figure 3. Hence, the closed-loop system becomes

$$\dot{x}_i = \mu \sum_{j=1}^{n} d_{ij}\left[1 - \frac{\hat{r}_{ij} + a\sin\omega_0 t}{r_{ij}}\right](x_j - x_i) \tag{10}$$

and

$$\begin{cases} \dot{\hat{r}}_{ij} = k\xi_{ij} \\ \dot{\xi}_{ij} = -\omega_l \xi_{ij} + \omega_l\left[J(\hat{r}_{ij} + a\sin\omega_0 t) - \eta_{ij}\right] a\sin\omega_0 t \\ \dot{\eta}_{ij} = -\omega_h \eta_{ij} + \omega_h J(\hat{r}_{ij} + a\sin\omega_0 t) \end{cases} \tag{11}$$

It is known that, convergence of system (5) can be made arbitrarily fast by assigning $\mu \gg 1$ [25][30]. In other words, system (5) can be treated as an inner-loop of extremum seeking, the dynamics of cooperative formation control can then be ignored hereafter in the analysis. The performance of extremum seeking is summarized into the following theorem:

Theorem 1: Consider networked system (1) with input (3) and let $J(r_{ij})$ be performance index for control and communication. It is assumed that the value of $J(r_{ij})$ can be measured (as described in remarks 1 and 2) and reaches its maximum at r_{ij}^*, and the network is initially connected. Then, r_{ij}^* can be estimated distributively by estimation algorithm (11) provided the perturbation frequency $\omega_0 \gg 1$ and a is sufficiently small. Specifically, the estimation error can be ensured to an $O(\frac{1}{\omega_0} + a^3)$ neighborhood of the origin.

Proof: Denoting the estimation error as

$$\tilde{r}_{ij} = \hat{r}_{ij} - r_{ij}^* \qquad \tilde{\eta}_{ij} = \eta_{ij} - J(r_{ij}^*) \tag{12}$$

Then, averaging system (11) around τ with $\tau = \omega_0 t$ yield

$$\frac{d}{d\tau}\begin{bmatrix} \tilde{r}_{ij}^a \\ \xi_{ij}^a \\ \tilde{\eta}_{ij}^a \end{bmatrix} = \frac{1}{\omega_0}\begin{bmatrix} k\xi_{ij}^a \\ -\omega_l \xi_{ij}^a + \frac{\omega_l}{2\pi} a \int_0^{2\pi} \rho(\tilde{r}_{ij}^a + a\sin\vartheta)\sin\vartheta d\vartheta \\ -\omega_h \tilde{\eta}_{ij}^a + \frac{\omega_h}{2\pi}\int_0^{2\pi} \rho(\tilde{r}_{ij}^a + a\sin\vartheta) d\vartheta \end{bmatrix} \tag{13}$$

where $\rho(x) = J(r_{ij}^* + x) - J(r_{ij}^*)$, it follows from (9)

$$\rho(0) = 0, \ \rho'(0) = J(r_{ij}^*) = 0, \ \rho''(0) = J''(r_{ij}^*) < 0$$

Then, the average equilibrium $(\tilde{r}_{ij}^{a,e}, \xi_{ij}^{a,e}, \tilde{\eta}_{ij}^{a,e})$ should satisfy the following relations:

$$\xi_{ij}^{a,e} = 0, \ \int_0^{2\pi} \rho(\tilde{r}_{ij}^{a,e} + a\sin\vartheta)\sin\vartheta d\vartheta = 0$$
$$\tilde{\eta}_{ij}^{a,e} = \frac{1}{2\pi}\int_0^{2\pi} \rho(\tilde{r}_{ij}^{*a,e} + a\sin\vartheta) d\vartheta \tag{14}$$

After several algebraic manipulations, the equilibrium $(\tilde{r}_{ij}^{a,e}, \xi_{ij}^{a,e}, \tilde{\eta}_{ij}^{a,e})$ of system (13) is

$$\begin{bmatrix} \tilde{r}_{ij}^{a,e} \\ \xi_{ij}^{a,e} \\ \tilde{\eta}_{ij}^{a,e} \end{bmatrix} = \begin{bmatrix} -\frac{\rho'''(0)}{8\rho''(0)} a^2 + O(a^3) \\ 0 \\ \frac{\rho''(0)}{4} + O(a^3) \end{bmatrix} \tag{15}$$

This implies that estimation errors $\tilde{r}_{ij}^{a,e}, \tilde{\eta}_{ij}^{a,e} \to 0$ for sufficiently small a. In addition, the Jacobian matrix of (13) at the average equilibrium is

$$\Lambda^a = \frac{1}{\omega_0}\begin{bmatrix} 0 & k & 0 \\ \frac{\omega_l}{2\pi} a \int_0^{2\pi} \rho'(\tilde{r}_{ij}^a + a\sin\vartheta)\sin\vartheta d\vartheta & -\omega_l & 0 \\ \frac{\omega_h}{2\pi}\int_0^{2\pi} \rho'(\tilde{r}_{ij}^a + a\sin\vartheta) d\vartheta & 0 & -\omega_h \end{bmatrix} \tag{16}$$

It follows that Λ^a will be Hurwitz if and only if

$$\int_0^{2\pi} \rho^{'}(\tilde{r}_{ij}^a + a\sin\vartheta)\sin\vartheta d\vartheta = \rho^{''}(0)a\pi + O(a^2) < 0 \quad (17)$$

which indicates the average equilibrium (15) of system (13) is exponentially stable since $\rho^{''}(0) < 0$. Moreover, it is proved that the unique exponentially stable solution $(\tilde{r}_{ij}^{2\pi}, \xi_{ij}^{2\pi}, \tilde{\eta}_{ij}^{2\pi})$ to system (13) satisfies [29]

$$\left| \left| \begin{bmatrix} \tilde{r}_{ij}^{2\pi} - \frac{\rho^{'''}(0)}{8\rho^{''}(0)}a^2 \\ \xi_{ij}^{2\pi} \\ \tilde{\eta}_{ij}^{2\pi} - \frac{\rho^{''}(0)}{4}a^2 \end{bmatrix} \right| \right| \leq O(\frac{1}{\omega_0} + a^3) \quad (18)$$

which implies all solutions $(\tilde{r}_{ij}, \xi_{ij}, \tilde{\eta}_{ij})$ converge to an $O(\frac{1}{\omega_0} + a^3)$ neighborhood of the origin, which implies the estimation error can be made arbitrarily small provided $\omega_0 \gg 1$ and a is sufficiently small.

In addition, average method can also be applied to closed-loop networked control system (10), whose average model is

$$\frac{1}{\omega_0}\frac{dx_i^a}{d\tau} = \mu \sum_{j\in\mathcal{N}_i} \left(1 - \frac{r_{ij}^* + \tilde{r}_{ij}^{a,e}}{r_{ij}}\right) d_{ij}\left(x_j^a - x_i^a\right) - \frac{a\mu}{2\pi} \sum_{j\in\mathcal{N}_i} \int_0^{2\pi} \frac{d_{ij}}{r_{ij}} \left(x_j^a - x_i^a\right)\sin\vartheta d\vartheta \quad (19)$$

Consequently,

$$\frac{dx_i^a}{d\tau} = \omega_0\mu \sum_{j\in\mathcal{N}_i} \left(1 - \frac{r_{ij}^* + \tilde{r}_{ij}^{a,e}}{r_{ij}}\right) d_{ij}\left(x_j^a - x_i^a\right) \quad (20)$$

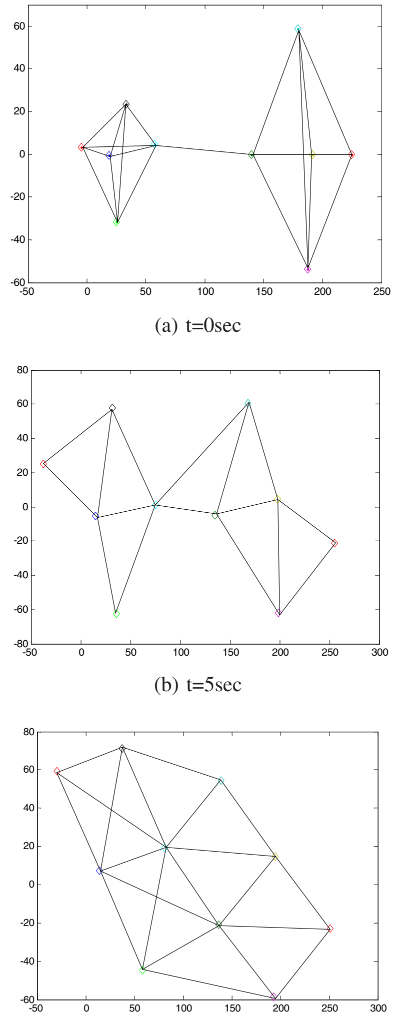
It is apparent that the average system (20) is essentially identical to cooperative control system (5) provided $\tilde{r}_{ij}^{a,e} \to 0$ is satisfied at every pair of connected systems, and as shown in previous analysis, $\tilde{r}_{ij}^{a,e} \to 0$ can be ensured exponentially. Therefore, $r_{ij} \to r_{ij}^*$ can be guaranteed using proposed distributed extremum seeking and cooperative control approach, and the desired separation can be achieved for any $j \in \mathcal{N}_i$ such that both communication quality and network coverage can be ensured. This concludes the proof of theorem 1. □

IV. SIMULATION RESULTS

To illustrate the idea of extremum seeking cooperative control for increased reliability and coverage of communication, consider a group of 10 mobile agents whose initial locations (in meters) are

$$x_1 = [0\ 0]^T,\ x_2 = [25\ 25]^T,\ x_3 = [50\ 0]^T,\ x_4 = [25\ \ -25]^T$$
$$x_5 = [25\ 0]^T,\ x_6 = [150\ 0]^T,\ x_7 = [225\ 0]^T,\ x_8 = [188\ 63]^T$$
$$x_9 = [188\ \ -63]^T,\ x_{10} = [188\ 0]^T.$$

In simulations, cooperative control is implemented to utilize only the motion information received from neighboring vehicles (i.e., mobile agents i and j are considered to be connected, that is, $s_{ij} = 1$) if $r_{ij} \leq r_x = 150$ (its corresponding outage probability is shown at point C as of figure 1). It is straightforward to see that the initial graph is connected



(a) t=0sec

(b) t=5sec

(c) t=30sec

Fig. 4: Evolution of formation movement and connectivity

but the coverage is not spread well and the communication quality among some of the agents are poor.

Specifically, performance index $J(r_{ij})$ is calculated as

$$J(\hat{r}_{ij} + a\sin\omega_0 t) = \left[1 - \exp\left(-\left(\frac{\hat{r}_{ij} + a\sin\omega_0 t}{r_{\min}}\right)^{\upsilon}\right)\right] \cdot \exp\left(-\left(\frac{\hat{r}_{ij} + a\sin\omega_0 t}{r_x}\right)^{\upsilon}\right) \cdot \exp\left(-(2^{\delta} - 1)\frac{\sigma^2}{P_0}\left(\frac{\hat{r}_{ij} + a\sin\omega_0 t}{r_0}\right)^{\nu}\right) \quad (21)$$

with $r_{\min} = 50$.

Parameters characterizing communication quality in simulations are assumed to be the same for all the communication channels. Specifically,

$$\mu = 50,\ \omega_0 = 400,\ \omega_l = 4,\ \omega_h = 25,\ a = 0.5,\ k = 55$$

Evolution of the mobile communication network is shown in figure 4 (in which the presence of a link between any pair of two neighboring agents means their communication channels are considered to be of good quality). It is apparent

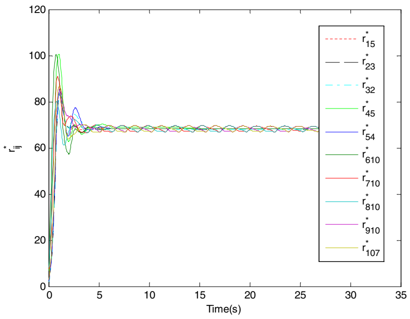


Fig. 5: Estimation of r_{ij}^* at each agent

that the resulting network (at $t = 30$ seconds) provides much improved performance and that separation between the neighboring agents automatically converges to the optimal value of $r_{ij}^* = 68$. Indeed, estimation of r_{ij}^* is performed at each communication channel and, as shown in figure 5, convergence to optimal value r_{ij}^* is achieved within 10 seconds.

V. Conclusion

This paper integrates the cooperative control problem with a communication performance metric by using extremum seeking control, which fits intuitively in such problem since the exact values of communication quality and its gradients are unknown and can only be measured with respect to the distance at each communication channel. Moreover, the proposed formation control scheme is developed based on line-of-sight separation instead of potential field function, such that the desired separation can be rendered over time between any pair of connected agents, and the connectedness of network is preserved. In addition, trade off between communication and control is ensured with the proposed scheme without compromising the minimal communication constraints.

Future work on this topic should focus on extending the proposed results to multiple-input-multiple-output communication problem, applications of vehicles with higher order dynamics are also expected.

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