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A Stable Finite Horizon Model Predictive Control for Power System Voltage Collapse Prevention

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Abstract— **This paper proposes a finite horizon model predictive control (MPC) method for online voltage collapse prevention. With a hybrid differential-algebraic equations (DAE) model, a bulk electric power system is studied including both continuous dynamics and certain switching behavior critical to voltage instability. Two major contributing factors for long term instability are then discussed to show how such switching mechanisms could lead to unstable dynamic behavior. Based on this model, a safety set concept constructed from a projection of algebraic variables is proposed. Terminal inequality constraints defined by this safety set are adopted to establish convergence properties for the proposed finite horizon MPC algorithm. Constructing this safety set, which is described only by algebraic variables, does not require detailed dynamic state estimation which is not yet available for large-scale power systems. A 10-bus benchmark case for studying voltage collapse is used to illustrate the performance of the control method.**

I. INTRODUCTION

To meet the expectation of being "Smart", electric power grid is developing to maintain a sustainable way of life, while at the same time to face both critical economic and security challenges. Advanced control strategies addressing disturbances through automated prevention, containment and restoration [1] become the key for such a future.

Voltage instability/collapse is a major security concern for power system operation. This phenomenon is often preceded by a slow process of load restoration and generation reactive power saturation, after some initial disturbances. Such a slow dynamic process potentially provides time for implementation of operational decisions aimed at preventing the collapse.

Even with the recent rapid development of communication, metering and data management techniques, the inherent complexity of this interconnected network is constantly posing challenging issues for the adoption of advanced control ideas. Several facts lead to such difficulty: an electric power system is by nature a hybrid nonlinear system with numerous physical (or saturation) limits, constraints and switching devices. Theoretically computing a closed-form system-wide optimal control strategy is impossible. Different types of dynamics with time scales ranging from milliseconds to hours are present in this large-scale network. Some normally well-controlled local dynamic behaviors can significantly contribute to cascading failures propagating system-wide in emergencies. Online metering systems have been well established in electric power systems to measure algebraic states, however dynamic state estimation is still a challenging topic. Lack of dynamic state information prevents some well developed control methods from being practically applied.

Model predictive control (MPC) has attracted the attention of many researchers in recent years. This is partially due to the fact that MPC can handle a constrained nonlinear optimal control problem without requiring a closed-form solution. Since the first introduction of the MPC concept to power system stability control problems in 2002 [2], many publications [3], [4], [5], [6] have appeared to address long-term voltage stability problems, where state estimation and a computation delay are relatively trivial issues. In particular, many researchers [3], [4], [5] have adopted trajectory sensitivities to build the MPC optimization algorithms. This perturbation information can be efficiently extracted from simulation of nonlinear differential-algebraic systems such as power systems.

This paper is an extension to earlier work that adopted trajectory sensitivities in MPC, and discusses several issues in establishing controller stability. Switching mechanisms that lead to unstable voltage behavior are first reviewed based on a hybrid differential-algebraic equation (DAE) model. A safety set concept constructed from a projection of algebraic variables is proposed to represent stability conditions when dynamic state estimation is not available. Terminal inequality constraints defined by this safety set are adopted to establish the proof of the stability for the proposed finite horizon MPC algorithm. A 10-bus benchmark case for voltage collapse study is used to illustrate the performance of the control method.

The paper is organized as follows. Section II discusses a power system dynamic model that captures the hybrid switchings of voltage collapse, and reviews trajectory sensitivity concepts. Section III introduces a finite-horizon MPC algorithm and related controller stability analysis. A example of MPC control for voltage collapse prevention is provided in Section IV and conclusions are presented in Section V.

II. POWER SYSTEM DYNAMIC MODELING

In this section, several issues on power system dynamic modeling and trajectory sensitivities are discussed and reviewed. Based on a hybrid DAE model, two major hybrid switching mechanisms that lead to unstable voltage behavior are described.

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A. Hybrid DAE model

Following disturbances, power systems typically exhibit periods of smooth dynamic behavior, interspersed with discrete events. Accordingly, it is common for dynamic models to consist of nonlinear DAEs coupled with mechanisms for capturing the switching and impulsive effects that are introduced by discrete events, shown as follows:

$$\dot{x} = f(x, y, z; \lambda) \tag{1}$$

$$0 = g^{(0)}(x, y, z; \lambda) \tag{2}$$

$$0 = \begin{cases} g^{(i-)}(x, y, z; \lambda) & y_{d,i} < 0 \\ g^{(i+)}(x, y, z; \lambda) & y_{d,i} > 0 \end{cases} \quad i = 1, \ldots, d \tag{3}$$

$$z^{+} = h_j(x^{-}, y^{-}, z^{-}; \lambda) \quad y_{e,i} = 0, j \in \{1, \ldots, e\} \tag{4}$$

$$\dot{z} = 0 \quad y_{e,i} \neq 0, \quad \forall j \in \{1, \ldots, e\} \tag{5}$$

In this model, x are dynamic state variables with initial values x_0, y are algebraic variables, z are discrete variables and λ are parameters. In the power system context, x would describe quantities such as generator fluxes, and y would include bus voltage magnitudes. Switching of functions from $g^{(i-)}$ to $g^{(i+)}$ can be used to represent saturation or failures and switching of discrete variables z can represent tap changes of transformers and other switching devices. Details of this hybrid DAE models are discussed in [7]. The dynamic behavior described by the model can be represented by the flow functions $x(t) = \phi(x_0, t)$ and $y(t) = \psi(x_0, t)$, obtained by numerically integrating (1)-(5), considering discrete events.

B. Trajectory sensitivity

Trajectory sensitivities are derived by forming the Taylor series expansion of the flow. In terms of the dynamic states $x(t)$, this gives

$$\begin{aligned} \Delta x(t) &= \phi_x(x_0 + \Delta x_0, t) - \phi_x(x_0, t) \\ &= \frac{\partial x(t)}{\partial x_0} \Delta x_0 + \text{ higher order terms} \\ &\approx \Phi(x_0, t) \Delta x_0. \end{aligned} \tag{6}$$

Likewise, the sensitivity of the $y(t)$ trajectory is given by

$$\Delta y(t) \approx \Psi(x_0, t) \Delta x_0. \tag{7}$$

Trajectory sensitivities Φ, Ψ can be mapped through discrete events, and so are well defined for non-smooth systems [8].

C. Switching mechanisms leading to voltage collapse

The major cause of voltage instability is a lack of reactive power supply throughout the network. Long-term voltage instability often involves an initial disturbance and some relatively slow power transfers caused by load restoration, and subsequently by generator reactive power limitation. The load may gradually build up as a consequence of load tap changer (LTC) or thermostat adjustments. Generator reactive power limitation may further exaggerate this load restoration and lead to voltage collapse. Such slow changing processes can last for up to several hours until the point of collapse is reached.

Load restoration and generator reactive power limitation are two major contributing factors to long-term voltage instability. Load restoration, driven by local control devices, is by nature hybrid behavior if the deadband, delay and step change of taps are taken into account. The objective of local control is to restore load bus voltages into deadbands of normal range through discrete tapping or continuous adjustments. Generator reactive power limitation in a voltage collapse process is primarily caused by the thermal limits of the field windings associated with generator exciters. As the major sources of reactive power, generators are locally controlled to quickly ramp up/down their reactive power output within a range determined by thermal limits that prevent field windings from over-heating. The modeling of these two forms of dynamic behavior is briefly discussed as follows.

1) Load restoration: In electricity distribution , a voltage range from 0.95 p.u. to 1.05 p.u. is widely adopted for acceptable steady-state voltages. Voltages tend to vary with changing network conditions. Loads that are dynamically dependent upon discrete tap changing or continuous thermostat adjustments are assumed in steady-state when their bus voltages enter the deadband of the devices. A load model that captures the basic properties of the deadband, saturation, delay and discrete tap change process is described in details can be found in [9], as a hybrid DAE model of the form presented in Section II-A. Under normal conditions, a voltage adjustment is a demand-side response that helps to maintain power quality for users. Under extreme conditions such as voltage collapse, however, load restoration due to the switching of taps could keep stressing and weakening the power network, finally leading to a collapse.

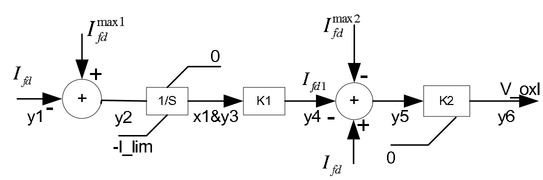

Fig. 1. Over excitation limiter model.

2) Generator reactive power limitation: An IEEE AC4A over-excitation limiter (OXL) model [10] is used in this paper to illustrate the dynamic performance of OXL during the voltage collapse process. Fig. 1 shows the block diagram of this model. The main objective of an OXL control loop is to regulate field current to avoid overheated field windings. This is achieved by changing the field voltage set-point. When a generator is energized, its OXL operates in one of the following three states:

1) If the field current stays below a long-term thermal limit I_{fd}^{max1}, output of the OXL is 0. It has no effect on the generator reactive support.
2) If the field current exceeds the long-term thermal limit I_{fd}^{max1}, but is below a short-term thermal limit I_{fd}^{max2}, OXL does nothing but waits until the heating effects accumulate to a certain amount. Then it will send

a signal to the automatic voltage regulator (AVR) to reduce the field current to a safe range.

3) If the field current exceeds the short-term thermal limit I_{fd}^{max2}, it immediately sends a signal to lower the field current by reducing the field voltage.

During a voltage collapse process, step 3) could significantly reduce the generator reactive power output in a very short time by reducing the excitation voltage, leading to a voltage collapse. To prevent activation of step 3), the field current must stay below the long-term limit I_{fd}^{max1} in steady-state, or at least through long-term dynamic processes. From Fig. 1 it can be observed that when this condition is satisfied, the output of the OXL stays at 0.

The generator reactive power limits discussed here have a saturation form of nonlinearity. However, discrete changes may also occur for brushless exciters, due to their rectifier circuit [10].

D. Safety set for voltage stability

State estimation in electric power systems normally has a different meaning to state estimation in control theory. Using the terms defined in the hybrid DAE model discussed in Section II-A, power system state estimation provides an estimate of the algebraic variables y instead of dynamic states x. Metering devices installed on-site at substations and towers measure algebraic variables such as voltages and phasors in real-time. Energy management systems (EMS) are capable of filtering the data and providing system-wide estimation. This system snapshot is used by operators for real-time analysis. Unfortunately, techniques that are suitable for estimating the dynamic states x are still far from maturity, with industry applications unlikely in the near future. This is largely due to the difficulties imposed by the large-scale nonlinear nature of interconnected power systems.

In this paper, a safety set concept defined by the projection of algebraic variables is used as a pseudo-stability condition for the proposed control strategy. Such a methodology for power system dynamic analysis is not new. For example, a quasi-steady-state simulation method [11], [12] has been widely adopted to establish long-term dynamic behavior for voltage stability investigations. At each simulation step, the fast transient dynamic equations $\dot{x_f} = f(x_f, y, z, \lambda, t)$ are assumed to be in equilibrium $e^* \equiv ((x_f^*(t), y^*(t), z^*(t), t)$ where $f(e^*) = 0$, and the system status is determined by algebraic equations $g(e^*, t) = 0$. Algebraic states y^* can be obtained directly from system measurements and EMS estimation procedures.

A safety set is an approximation of stable equilibria which neglects fast transients, but maintains the nonlinearity and hybrid nature of the system. For example, a bus voltage staying within the deadband of an LTC will prevent future adjustments of tap ratios. From the voltage stability point of view, the safety set can be defined as the region of algebraic state-space bounded by switching limits associated with the slower processes that drive voltage collapse. While the algebraic states remain within this set, no switching events, such as LTC tapping or line tripping, can occur, ensuring long-term voltage collapse is prevented. Even though fast transient dynamics may temporarily perturb system states, it is assumed such transients are quickly damped out, and the system settles back to equilibrium conditions. As an example, a safety set could be defined by,

1) Generator field current I_{fd}. Maintaining field currents I_{fd} below their long-term limits I_{fd}^{max1} prevents overheating of the field windings, and therefore ensures that OXL outputs remain at 0. This is a critical stability condition that can prevent generator tripping.
2) Load bus voltage. A safety set of load-bus voltage deadbands is necessary to prevent tap-changer action and load shedding. Notice that since some loads can be designed to be shed as an emergency remedy for voltage collapse, a subset of load buses (mostly the critical loads or pilot nodes) should be carefully chosen.
3) Transmission line loading. Transmission line loadings are also critical and need to be monitored by the safety set. Consecutive tripping of transmission lines could soon lead to a weak network prone to voltage collapse.

Besides these algebraic variables, angle and frequency may also be monitored. All these algebraic variables with a safety range form a pseudo-stability condition for long-term voltage stability.

Mathematically, the safety set is a time-invariant polyhedral set $\mathbb{Y}_s^* = [y_s^L, y_s^H]$. An operating point y_s staying inside the set

$$y_s^L \leq y_s \leq y_s^H \tag{8}$$

is assumed to be secure from a voltage collapse perspective. In practice, such a set is selected to maintain certain safety margins to increase the robustness of the stability controls.

Let the term $\rho(b, A)$ be a Hausdorff distance [13] between a point b and a set A,

$$\rho(b, A) = \min_{a \in A} \|a - b\|. \tag{9}$$

For $b \in A$, $\rho(b, A) = 0$. In this paper, we only consider polyhedral sets $A = [a_L, a_H]$, where a_L and a_H are vectors describing the set boundary, with 1-norm $\| \cdot \|$ adopted,

$$\rho(b, A) = \frac{1}{2} \left(\|b - a_H\|_1 + \|b - a_L\|_1 - \|a_H - a_L\|_1 \right). \tag{10}$$

Using the Hausdorff distance, this safety set requirement can be translated into a penalty term $C^y \rho(y, \mathbb{Y}_s^*)$ in an optimization objective function for MPC, where C^y is the penalty constant. To solve the problem, the constant term $\|a_H - a_L\|_1$ and scaling $\frac{1}{2}$ can be removed without affecting the final optimization results.

III. MPC for Voltage Stability Enhancement

A. Model predictive control

Model predictive control (MPC) is a discrete-time optimal control strategy. Fig. 2 provides an illustration of the MPC process. Normally, open-loop optimal controls are calculated

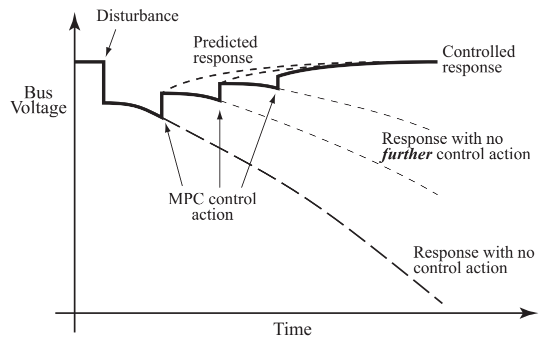

Fig. 2. MPC response.

by solving an optimization problem based on the time-domain prediction of system behavior, after estimating the current system states. The first control in the calculated control sequence will be issued to the system and this process will be repeated when the next system state is available.

B. MPC algorithm

Security requirements are always the first priority for a power system stability control problem. A well-designed MPC strategy should be able to find a control sequence that successfully steers system states from a potential collapse situation to a stable operating point (normally with a certain stability margin). At the same time, disruption to consumers should be minimized. As an example, load control (shedding) is an effective countermeasure for voltage collapse but quite disruptive to consumers, and is therefore less desirable than generation rescheduling. The ultimate goal of the preventive voltage stability control problem is to restore the system to a secure state while minimizing the disruption caused by load shedding.

To take into account the security and optimality requirements, the objective function $\mathcal{V}(y,u,k)$ for MPC includes the penalty terms,

$$\sum_{i=k+1}^{k+N} C_i^y \rho(y_i, \mathbb{Y}_s^*) + C_i^\lambda \lambda_i^k \tag{11}$$

as stage costs, where λ_i^k is the vector of participating load shedding percentage at the i_{th} prediction step. An entry-wise inequality constraint

$$0 \leq \lambda_i(j) \leq \bar{\lambda}_i(j), \quad j = 1 \ldots n \tag{12}$$

is enforced with 0 for no load shedding and $\bar{\lambda}_i(j)$ for all available load being shed. All the other controls such as generation rescheduling are considered to have negligible costs compared to load shedding.

Let the control at time $k-1$ be u^{k-1}, which is a known value at time k. Define the control changes as,

$$\Delta u_i^k = u_i^k - u^{k-1}, \quad i = k, \ldots, k+N-1 \tag{13}$$

where u_i^k refers to the control at time i proposed by MPC at time k.

Let x_k^k be the initial value of state trajectory x at time k. Let the value of x at time i along that trajectory be denoted x_{k+i}^k. With T as the sampling step, we can write,

$$\begin{aligned} x_{k+N}^k = x^k(t_k + NT) &= \phi(x_{k+N-1}^k, u_k^k, T) \\ &= \phi(x_{k+N-2}^k, u_k^k, 2T) \\ &\vdots \\ &= \phi(x_k^k, u_k^k, NT) \end{aligned}$$

with the corresponding y given by $g(x_{k+i}^k, y_{k+i}^k, u_k^k) = 0$. In other words, the nominal discrete-time trajectory (x_{k+1}^k, y_{k+1}^k),. . .,(x_{k+N}^k, y_{k+N}^k) can be obtained by sampling the simulation that begins at the initial value x_k^k, and that runs for time NT. This trajectory is nominal in the sense that the control is held constant at its initial value u_k^k. The aim of MPC is to determine control adjustments $\Delta u_{k+1}^k, \ldots, \Delta u_{k+N-1}^k$ that bring y into the safety set with minimal load shedding.

According to the definition of trajectory sensitivities (6) and (7), the perturbations from the nominal trajectory are given approximately by a linear time-varying discrete-time model,

$$\begin{aligned} \Delta x_{k+i+1}^k = \Phi_x(x_{k+i}^k, u_k^k, T)\Delta x_{k+i}^k + \\ \Phi_u(x_{k+i}^k, u_k^k, T)\Delta u_{k+i}^k \end{aligned} \tag{14}$$

$$\Delta y_{k+i}^k = -g_y^{-1}(g_x \Delta x_{k+i}^k + g_u \Delta u_{k+i-1}^k) \tag{15}$$

where g_x, g_y and g_u are all evaluated at t_{k+i}. (It is assumed that $\Delta u_{k+N}^k \equiv 0$.)

For a multiple step open-loop MPC algorithm, without loss of generality, assuming the initial time $k=0$, by (14) and (15) we have,

$$\Delta x = \Gamma_X^N \Gamma_U^N \Delta u \tag{16}$$

$$\Delta y = -\left(G_Y^N\right)^{-1}\left(G_X^N \Gamma_X^N \Gamma_U^N + G_U^N\right)\Delta u \tag{17}$$

where Δx, Δy and Δu are the N-step change vectors of x, y and u, Γ_U^N is a block diagonal matrix with diagonal blocks $\Phi_u(i)$ (i is the step size, omit other arguments), G_X^N, G_Y^N and G_U^N are block diagonal matrixes with diagonal blocks $g_x(i)$, $g_y(i)$ and $g_u(i)$, respectively, and

$$\Gamma_X^N = \begin{bmatrix} I & 0 & \cdots & \cdots & 0 \\ \vdots & \ddots & \vdots & \vdots & \vdots \\ \prod_{s=2}^k \Phi_x(s) & \cdots & I & 0 & \cdots \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ \prod_{s=2}^N \Phi_x(s) & \cdots & \prod_{s=k+1}^N \Phi_x(s) & \cdots & I \end{bmatrix}. \tag{18}$$

For $N > 1$, let the algebraic variables sampled along the nominal trajectory be y_{k+i}^k and the post-control trajectory be $\bar{y}_{k+i}^k$, then the post-control perturbation can be approximated by,

$$\Delta y_{k+i}^k = \bar{y}_{k+i}^k - y_{k+i}^k = \Psi_u(x_k^k, u_k^k, i)\Delta u^k \tag{19}$$

where $\Psi_u(x_k^k, u_k^k, i)$ is an N-step trajectory sensitivity matrix constructed from the appropriate rows of $G =$

$-\left(G_Y^N\right)^{-1}\left(G_X^N\Gamma_X^N\Gamma_U^N+G_U^N\right)$. The deviations of algebraic states from their nominal trajectory are described (approximately) by a linear function of decision variables Δu^k.

To ensure that the safety set is reached by the closed-loop MPC strategy, a terminal inequality constraint is added to the optimization problem,

$$\min \sum_{i=k}^{k+N-1} C_i^y \rho(y_i, \mathbb{Y}_s^*) + C_i^\lambda \lambda_i^k \tag{20}$$

$$\text{s.t. } y_s^L \le y_{k+N-1}^k + \Psi_u(x_k^k, u_k^k, k+N-1)\Delta u^k \le y_s^H \tag{21}$$

$$\Delta u_i^k \in \Delta\mathbb{U}_k. \tag{22}$$

In this linear programming (LP) problem, reaching the safety set is achieved through penalizing the distance terms in the objective function. Optimal social cost is achieved by minimizing the cost terms of load shedding. The LP form of the MPC algorithm can be solved efficiently for large-scale power system applications.

C. Control Stability

Section II-B showed that trajectory sensitivities $\Phi(x_0,t)$ and $\Psi(x_0,t)$ are first-order approximations of nonlinear sensitivities defined by[1],

$$\bar{\Phi}(x_0, \Delta x_0, t) = \frac{\phi_x(x_0+\Delta x_0, t) - \phi_x(x_0,t)}{\Delta x_0} \tag{23}$$

$$\bar{\Psi}(x_0, \Delta x_0, t) = \frac{\phi_y(x_0+\Delta x_0, t) - \phi_y(x_0,t)}{\Delta x_0}. \tag{24}$$

Even though these nonlinear sensitivities are more accurate, they require iterative time-domain simulation for computation. Trajectory sensitivities, on the other hand, can be computed much more efficiently and generally provide a good approximation of the true nonlinear, non-smooth perturbed trajectory [8].

To establish stability properties of the proposed MPC control strategy, it is assumed that the first priority for MPC is to bring algebraic states y into the safety set. Minimizing the cost related to load shedding λ is of secondary importance. After the safety set has been reached, the penalty term for the λ cost can be added to the objective function. In practice, both objectives can be implemented jointly by using a very small penalty constant C_i^λ. After reaching the safety set, the distance penalty will vanish, leaving only the objective of minimizing the λ cost.

Using the nonlinear sensitivity, controller behavior can be established as follows:

Theorem: Assume $C_i^\lambda = 0$. If the initial algebraic states y_0^0 lie outside the safety set $\mathbb{Y}_s^*$ and the LP problem (20)-(22) using $\bar{\Phi}$ and $\bar{\Psi}$ has feasible solutions, the closed-loop control will drive the algebraic states y into the safety set.

Proof: Let the LP problem at time k have feasible solutions $[u_k^k, \ldots, u_{k+N-1}^k]$, and the objective function evaluated at time k be

$$V^*(k) = \sum_{i=k}^{k+N-1} C_i^y \rho(y_i^*, \mathbb{Y}_s^*). \tag{25}$$

With the terminal inequality constraint and nonlinear sensitivities, the terminal algebraic state will be steered into the safety set with this control sequence with a terminal cost,

$$\rho(y_{k+N-1}^*, \mathbb{Y}_s^*) = 0. \tag{26}$$

Since reaching the safety set means the system is stable, accordingly, we may assume that after applying the control $[u_{k+1}^k, \ldots, u_{k+N-1}^k, u_{k+N-1}^k]$, the final states y_{k+N} remain in the safety set. At the next time step, the objective function becomes,

$$\begin{aligned} V(k+1) &= \sum_{i=k+1}^{k+N-1} C_i^y \rho(y_i^*, \mathbb{Y}_s^*) + C_{k+N}^y \rho(y_{k+N}, \mathbb{Y}_s^*) \\ &= \sum_{i=k+1}^{k+N-1} C_i^y \rho(y_i^*, \mathbb{Y}_s^*) \\ &= V^*(k) - C_k^y \rho(y_k^*, \mathbb{Y}_s^*). \end{aligned} \tag{27}$$

Notice that the above control sequence is only a feasible solution but may not be optimal. For the optimal solution sequence, the cost function $V^*(k+1)$ should be no greater than $V(k+1)$, that is $V^*(k+1) \le V(k+1)$. Therefore $V^*(k+1) \le V^*(k)$. Using the fact that $V^*(k) \ge 0$, the monotonically decreasing objective function values $V^*(k)$ will converge for $k \to \infty$. Therefore, $\forall \delta > 0, \exists M > 0, M \in \mathbb{N}, \forall m > M$,

$$|V^*(m) - V^*(m+1)| \le \delta \tag{28}$$

which means the sequence $\{V^*(m) - V^*(m+1)\}$ will converge to 0.

Notice from (27) that

$$\begin{aligned} 0 \le C_k^y \rho(y_k^*, \mathbb{Y}_s^*) &= V^*(k) - V(k+1) \\ &\le V^*(k) - V^*(k+1). \end{aligned} \tag{29}$$

Therefore, because $C_k^y > 0$, it may be concluded that $\rho(y_k^*, \mathbb{Y}_s^*)$ converges to 0, and hence that the safety set will be reached. ■

IV. Example

The simple 10-bus system shown in Fig. 3 is well established as a benchmark for exploring voltage stability issues [14], [15]. The system includes 3 generators and 2 loads. Load tap changer LTC3 automatically adjusts its tap ratio to regulate the voltage magnitude at load bus 9.

A. Voltage collapse

As illustrated in Fig. 4, an outage of a feeder between buses 5 and 7 will lead to voltage collapse, shown by the steady decline in the voltages at buses 3, 6 and 9. Bus 9 is regulated by the LTC. Initially the voltage at bus 9 starts to recover due to LTC tap ratio adjustments. As a consequence,

[1]Notice that the nonlinear sensitivity $\bar{\Phi}$ and $\bar{\Psi}$ are dependent on Δx_0, while their first-order approximations Φ and Ψ are not.

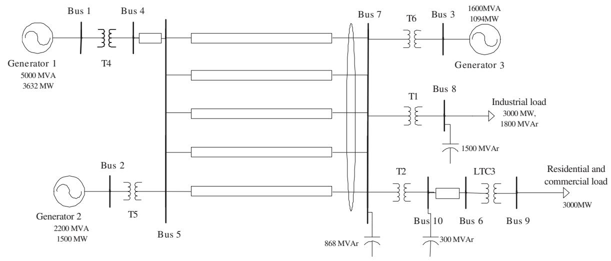

Fig. 3. Benchmark 10-bus voltage collapse test system.

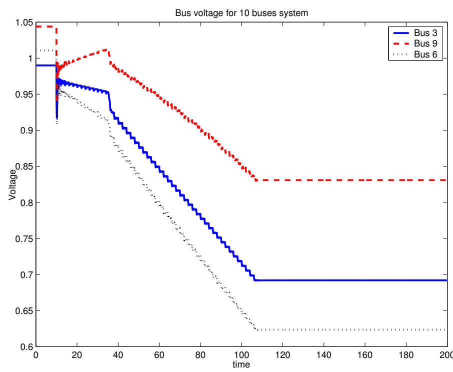

Fig. 4. Bus voltage behavior with no MPC control action.

voltages at the other buses decrease, except generator terminal buses that are regulated by automatic voltage regulators (AVRs). However, at 35 seconds, the increasing field current triggers the over-excitation limiter (OXL) at generator 3, reducing the internal field voltage, which is turn causes a sudden drop in bus voltages. Subsequently, further LTC tap ratio adjustments cause voltages across the system to fall.

B. MPC

Fig. 5 shows the bus voltages that are restored to their safety range after iterative MPC control actions. In addition to load shedding, rescheduling generator active power and terminal voltage set-points help to reduce the required load shedding amount, by providing additional amount of steady state reactive power capability. Table I lists the final control decisions for various combinations of controls. It can be observed that when more controls are available, the required load shedding amount will be reduced.

V. Conclusion

The paper proposes a finite horizon MPC strategy for alleviating voltage collapse. The MPC algorithm is formulated as a linear programming problem that can be efficiently solved for large-scale systems. A terminal safety set is defined to ensure network quantities, including bus voltages and generator field currents,recover to acceptable values. Such a safety set concept is enforced through stage costs and terminal inequality constraints for the finite horizon MPC algorithm. The proposed controller has been tested on a benchmark 10-bus system. Results suggest that the proposed MPC strategy can effectively and efficiently prevent voltage collapse.

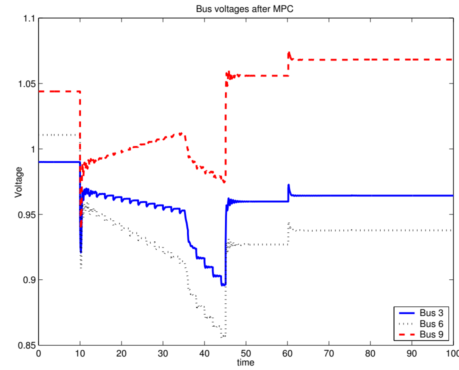

Fig. 5. Bus voltages with MPC control

TABLE I
MPC results for different controls

Case Number	λ_8	T^2_{mech}	T^3_{mech}	V^2_{set}	V^3_{set}
1	10.08%	-	-	-	-
2	7.54%	9	14	-	-
3	8.59%	-	-	0.985	0.983
4	6.17%	9	14	0.985	0.983

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